# System Identification Toolbox ${ }^{\text {Tn }}$ Reference 

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## System Identification Toolbox ${ }^{\mathrm{TM}}$ Reference

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Functions
1

Blocks
2

Functions

## absorbDelay

Replace time delays by poles at $z=0$ or phase shift

## Syntax

sysnd = absorbDelay(sysd)
[sysnd,G] = absorbDelay(sysd)

## Description

sysnd = absorbDelay(sysd) absorbs all time delays of the dynamic system model sysd into the system dynamics or the frequency response data.

For discrete-time models (other than frequency response data models), a delay of k sampling periods is replaced by k poles at $z=0$. For continuous-time models (other than frequency response data models), time delays have no exact representation with a finite number of poles and zeros. Therefore, use pade to compute a rational approximation of the time delay.

For frequency response data models in both continuous and discrete time, absorbDelay absorbs all time delays into the frequency response data as a phase shift.
[sysnd, G] = absorbDelay(sysd) returns the matrix $G$ that maps the initial states of the ss model sysd to the initial states of the sysnd.

## Examples

## Absorb Time Delay into System Dynamics

Create a discrete-time transfer function that has a time delay.

```
z = tf('z',-1);
sysd = (-0.4*z -0.1)/(z^2 + 1.05*z + 0.08);
sysd.InputDelay = 3
sysd =
            -0.4 z - 0.1
    z^(-3)* ------------------
    z^2 + 1.05 z + 0.08
Sample time: unspecified
Discrete-time transfer function.
```

The display of sysd represents the InputDelay as a factor of $z^{\wedge}(-3)$, separate from the system poles that appear in the transfer function denominator.

Absorb the time delay into the system dynamics as poles at $\mathrm{z}=0$.

```
sysnd = absorbDelay(sysd)
```

```
sysnd =
    -0.4 z - 0.1
    z^5 + 1.05 z^4 + 0.08 z^3
Sample time: unspecified
Discrete-time transfer function.
```

The display of sysnd shows that the factor of $z^{\wedge}(-3)$ has been absorbed as additional poles in the denominator.

Verify that sysnd has no input delay.
sysnd.InputDelay
ans $=0$

## Convert Leading Structural Zeros of Polynomial Model to Regular Coefficients

Create a discrete-time polynomial model.
m = idpoly(1,[0 0 0 2 3]);
Convert m to a transfer function model.

```
sys = tf(m)
sys =
    z^(-2) * (2 z^-1 + 3 z^-2)
Sample time: unspecified
Discrete-time transfer function.
```

The numerator of the transfer function, sys, is [0 0 2 3 3 ] and the transport delay, sys.IODelay, is 2. This is because the value of the B polynomial, m. B, has 3 leading zeros. The first fixed zero shows lack of feedthrough in the model. The two zeros after that are treated as input-output delays.

Use absorbDelay to treat the leading zeros as regular B coefficients.

```
m2 = absorbDelay(m);
sys2 = tf(m2)
sys2 =
    2 z^-3 + 3 z^-4
Sample time: unspecified
Discrete-time transfer function.
```

The numerator of sys2 is [ 0000223$]$ and transport delay is 0 . The model m 2 treats the leading zeros as regular coefficients by freeing their values. m2. Structure.B.Free (2:3) is TRUE while m. Structure.B.Free (2:3) is FALSE.

## See Also

hasdelay | pade | totaldelay

Introduced in R2012a

## advice

Analysis and recommendations for data or estimated linear models

## Syntax

```
advice(data)
```

advice(model,data)

## Description

advice(data) displays the following information about the data in the MATLAB ${ }^{\circledR}$ Command Window:

- What are the excitation levels of the signals and how does this affect the model orders? See also pexcit.
- Does it make sense to remove constant offsets and linear trends from the data? See also det rend.
- Is there an indication of output feedback in the data? See also feedback.
- Would a nonlinear ARX model perform better than a linear ARX model?
advice(model, data) displays the following information about the estimated linear model in the MATLAB Command Window:
- Does the model capture essential dynamics of the system and the disturbance characteristics?
- Is the model order higher than necessary?
- Is there potential output feedback in the validation data?


## Input Arguments

## data

Specify data as an iddata object.
model
Specify model as an idtf, idgrey, idpoly, idproc, or idss model object.

## See Also

detrend | feedback |iddata|pexcit
Introduced before R2006a

## addreg

(Not recommended) Add custom regressors to nonlinear ARX model

Note addreg is not recommended. Add linear, polynomial, and custom regressors directly to the idnlarx Regressors property instead. For more information, see "Compatibility Considerations".

## Syntax

$m=$ addreg(model,regressors)
$m=$ addreg(model,regressors,output)

## Description

$m=$ addreg(model,regressors) adds custom regressors to a nonlinear ARX model by appending the CustomRegressors model property. model and $m$ are idnalrx objects. For single-output models, regressors is an object array of regressors you create using customreg or polyreg, or a cell array of character vectors. For multiple-output models, regressors is 1-by-ny cell array of customreg objects or 1-by-ny cell array of cell arrays of character vectors. addreg adds each element of the ny cells to the corresponding model output channel. If regressors is a single regressor, addreg adds this regressor to all output channels.
$m=$ addreg(model,regressors,output) adds regressors regressors to specific output channels output of a multiple-output model. output is a scalar integer or vector of integers, where each integer is the index of a model output channel. Specify several pairs of regressors and output values to add different regressor variables to the corresponding output channels.

## Examples

## Add Regressors to a Nonlinear ARX Model

Create nonlinear ARX model with standard regressors.

```
ml = idnlarx([4 2 1],'wavenet','nlr',[1:3]);
```

Create model with additional custom regressors, specified as a cell array of character vectors.

$$
\text { m2 = addreg(m1,\{'y1(t-2)^2';'u1(t)*y1(t-7)'\}); }
$$

List all standard and custom regressors of m 2 .

```
getreg(m2)
ans = 8xl cell
    {'y1(t-1)' }
    {'y1(t-2)' }
    {'y1(t-3)' }
    {'y1(t-4)' }
    {'ul(t-1)' }
    {'u1(t-2)' }
```

```
{'y1(t-2)^2' }
{'ul(t)*y1(t-7)'}
```

Add Regressors to a Nonlinear ARX Model as customreg Objects
Create nonlinear ARX model with standard regressors.
m1 = idnlarx([4 2 1],'wavenet','nlr',[1:3]);
Create customreg objects.
$r 1=\operatorname{customreg}\left(@(x) x^{\wedge} 2,\left\{' y 1^{\prime}\right\}, 2\right)$
Custom Regressor:
Expression: y1(t-2)^2
Function: @(x) $x^{\wedge} 2$
Arguments: \{'yl'\}
Delays: 2
Vectorized: 0
TimeVariable: 't'
$r 2=$ customreg $\left(@(x, y) x^{*} y,\left\{1^{\prime} u 1^{\prime} y 1 '\right\},[07]\right)$
Custom Regressor:
Expression: u1(t)*y1(t-7)
Function: @(x,y)x*y
Arguments: \{'ul' 'yl'\}
Delays: [0 7]
Vectorized: 0
TimeVariable: 't'
Create a model based on m 1 with custom regressors.
$\mathrm{m} 2=\operatorname{addreg}(\mathrm{m} 1,[r 1 \mathrm{r} 2])$;
List all standard and custom regressors of m 2 .

```
getreg(m2)
ans = 8xl cell
    {'y1(t-1)' }
    {'y1(t-2)' }
    {'y1(t-3)' }
    {'y1(t-4)' }
    {'ul(t-1)' }
    {'ul(t-2)' }
    {'y1(t-2)^2' }
    {'u1(t)*y1(t-7)'}
```


## Compatibility Considerations

addreg is not recommended
Not recommended starting in R2021a

Starting in R2021a, add regressor objects linearRegressor, polynomialRegressor, and customRegressor directly to the idnlarx model Regressor property by using the syntax model.Regressors(end+1) = new_regressor_object. There are no plans to remove addreg at this time.

## See Also

customRegressor|getreg|idnlarx|linearRegressor|nlarx|polynomialRegressor
Topics
"Identifying Nonlinear ARX Models"
Introduced in R2007a

## aic

Akaike's Information Criterion for estimated model

## Syntax

```
value = aic(model)
value = aic(model1,...,modeln)
value = aic( __ ,measure)
```


## Description

value = aic(model) returns the normalized "Akaike's Information Criterion (AIC)" on page 1-12 value for the estimated model.
value $=$ aic(model1, ..., modeln) returns the normalized AIC values for multiple estimated models.
value = aic( $\qquad$ , measure) specifies the type of AIC.

## Examples

## Compute Normalized Akaike's Information Criterion of Estimated Model

Estimate a transfer function model.

```
load iddatal z1;
np = 2;
sys = tfest(z1,np);
```

Compute the normalized Akaike's Information Criterion value.
value = aic(sys)
value $=0.5453$
The value is also computed during model estimation. Alternatively, use the Report property of the model to access this value.
sys.Report.Fit.nAIC
ans $=0.5453$

## Compute Akaike's Information Criterion Metrics of Estimated Model

Estimate a transfer function model.

```
load iddatal z1;
np = 2;
sys = tfest(z1,np);
```

Compute the normalized Akaike's Information Criterion (AIC) value. This syntax is equivalent to aic_raw = aic(sys).
aic_raw = aic(sys,'nAIC')
aic raw $=0.5453$
Compute the raw AIC value.

```
aic_raw = aic(sys,'aic')
aic raw = 1.0150e+03
```

Compute the sample-size corrected AIC value.

```
aic_c = aic(sys,'AICc')
aic c = 1.0153e+03
```

Compute the Bayesian Information Criteria (BIC) value.

```
bic = aic(sys,'BIC')
bic = 1.0372e+03
```

These values are also computed during model estimation. Alternatively, use the Report. Fit property of the model to access these values.

```
sys.Report.Fit
ans = struct with fields:
    FitPercent: 70.7720
        LossFcn: 1.6575
            MSE: 1.6575
            FPE: 1.7252
            AIC: 1.0150e+03
            AICc: 1.0153e+03
            nAIC: 0.5453
            BIC: 1.0372e+03
```

Pick Model with Optimal Tradeoff Between Accuracy and Complexity Using AICc Criterion
Estimate multiple Output-Error (OE) models and use the small sample-size corrected Akaike's Information Criterion (AICc) value to pick the one with optimal tradeoff between accuracy and complexity.

Load the estimation data.
load iddata2
Specify model orders varying in 1:4 range.

```
nf = 1:4;
nb = 1:4;
nk = 0:4;
```

Estimate OE models with all possible combinations of chosen order ranges.

```
NN = struc(nf,nb,nk);
models = cell(size(NN,1),1);
for ct = 1:size(NN,1)
    models{ct} = oe(z2, NN(ct,:));
end
```

Compute the small sample-size corrected AIC values for the models, and return the smallest value.

```
V = aic(models{:},'AICc');
[Vmin,I] = min(V);
```

Return the optimal model that has the smallest AICc value.

```
models{I}
ans =
Discrete-time OE model: y(t) = [B(z)/F(z)]u(t) + e(t)
    B(z) = 1.067 z^-2
    F(z) = 1 - 1.824 z^-1 + 1.195 z^-2 - 0.2307 z^-3
Sample time: 0.1 seconds
Parameterization:
    Polynomial orders: nb=1 nf=3 nk=2
    Number of free coefficients: 4
    Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.
Status:
Estimated using OE on time domain data "z2".
Fit to estimation data: 86.53%
FPE: 0.9809, MSE: 0.9615
```


## Input Arguments

## model - Identified model

idtf|idgrey|idpoly|idproc|idss|idnlarx, |idnlhw|idnlgrey
Identified model, specified as one of the following model objects:

- idtf
- idgrey
- idpoly
- idproc
- idss
- idnlarx, except nonlinear ARX model that includes a binary-tree or neural network nonlinearity estimator
- idnlhw
- idnlgrey
measure - Type of AIC
'nAIC' (default) | 'aic'| 'AICc' | 'BIC'

Type of AIC, specified as one of the following values:

- 'nAIC' - Normalized AIC
- 'aic' - Raw AIC
- 'AICc' - Small sample-size corrected AIC
- 'BIC' - Bayesian Information Criteria

See "Akaike's Information Criterion (AIC)" on page 1-12 for more information.

## Output Arguments

## value - Value of quality metric

scalar | vector
Value of the quality measure, returned as a scalar or vector. For multiple models, value is a row vector where value( $k$ ) corresponds to the kth estimated model modelk.

## More About

## Akaike's Information Criterion (AIC)

Akaike's Information Criterion (AIC) provides a measure of model quality obtained by simulating the situation where the model is tested on a different data set. After computing several different models, you can compare them using this criterion. According to Akaike's theory, the most accurate model has the smallest AIC. If you use the same data set for both model estimation and validation, the fit always improves as you increase the model order and, therefore, the flexibility of the model structure.

Akaike's Information Criterion (AIC) includes the following quality metrics:

- Raw AIC, defined as:

$$
A I C=N^{*} \log \left(\operatorname{det}\left(\frac{1}{N} \sum_{1}^{N} \varepsilon\left(t, \widehat{\theta}_{N}\right)\left(\varepsilon\left(t, \widehat{\theta}_{N}\right)\right)^{T}\right)\right)+2 n_{p}+N^{*}\left(n_{y} *(\log (2 \Pi)+1)\right)
$$

where:

- $N$ is the number of values in the estimation data set
- $\varepsilon(t)$ is a $n_{y}$-by- 1 vector of prediction errors
- $\theta_{N}$ represents the estimated parameters
- $n_{p}$ is the number of estimated parameters
- $n_{y}$ is the number of model outputs
- Small sample-size corrected AIC, defined as:

$$
A I C c=A I C+2 n_{p} * \frac{n_{p}+1}{N-n_{p}-1}
$$

- Normalized AIC, defined as:

$$
n A I C=\log \left(\operatorname{det}\left(\frac{1}{N} \sum_{1}^{N} \varepsilon\left(t, \widehat{\theta}_{N}\right)\left(\varepsilon\left(t, \widehat{\theta}_{N}\right)\right)^{T}\right)\right)+\frac{2 n_{p}}{N}
$$

- Bayesian Information Criteria, defined as:

$$
B I C=N^{*} \log \left(\operatorname{det}\left(\frac{1}{N} \sum_{1}^{N} \varepsilon\left(t, \widehat{\theta}_{N}\right)\left(\varepsilon\left(t, \widehat{\theta}_{N}\right)\right)^{T}\right)\right)+N^{*}\left(n_{y} * \log (2 \Pi)+1\right)+n_{p} * \log (N)
$$

## Tips

- The software computes and stores all types of Akaike's Information Criterion metrics during model estimation. If you want to access these values, see the Report. Fit property of the model.


## References

[1] Ljung, L. System Identification: Theory for the User, Upper Saddle River, NJ, Prentice-Hall PTR, 1999. See sections about the statistical framework for parameter estimation and maximum likelihood method and comparing model structures.

## See Also

fpe |goodnessOfFit
Topics
"Loss Function and Model Quality Metrics"
Introduced before R2006a

## append

Group models by appending their inputs and outputs

## Syntax

sys $=$ append(sys1,sys2,...,sysN)

## Description

sys = append(sys1,sys2,...,sysN) appends the inputs and outputs of the models sys1,...,sysN to form the augmented model sys depicted below.


For systems with transfer functions $H_{1}(s), \ldots, H_{\mathrm{N}}(s)$, the resulting system sys has the block-diagonal transfer function

$$
\left[\begin{array}{cccc}
H_{1}(s) & 0 & \ldots & 0 \\
0 & H_{2}(s) & \cdots & \vdots \\
\vdots & \vdots & \ddots & 0 \\
0 & \cdots & 0 & H_{N}(s)
\end{array}\right]
$$

For state-space models sys1 and sys2 with data $\left(A_{1}, B_{1}, C_{1}, D_{1}\right)$ and $\left(A_{2}, B_{2}, C_{2}, D_{2}\right)$, append(sys1,sys2) produces the following state-space model:

$$
\begin{aligned}
& {\left[\begin{array}{l}
\dot{x}_{1} \\
\dot{x}_{2}
\end{array}\right]=\left[\begin{array}{cc}
A_{1} & 0 \\
0 & A_{2}
\end{array}\right]\left[\begin{array}{l}
x_{1} \\
x_{2}
\end{array}\right]+\left[\begin{array}{cc}
B_{1} & 0 \\
0 & B_{2}
\end{array}\right]\left[\begin{array}{l}
u_{1} \\
u_{2}
\end{array}\right]} \\
& {\left[\begin{array}{l}
y_{1} \\
y_{2}
\end{array}\right]=\left[\begin{array}{cc}
C_{1} & 0 \\
0 & C_{2}
\end{array}\right]\left[\begin{array}{l}
x_{1} \\
x_{2}
\end{array}\right]+\left[\begin{array}{cc}
D_{1} & 0 \\
0 & D_{2}
\end{array}\right]\left[\begin{array}{l}
u_{1} \\
u_{2}
\end{array}\right]}
\end{aligned}
$$

## Arguments

The input arguments sys1,..., sysN can be model objects s of any type. Regular matrices are also accepted as a representation of static gains, but there should be at least one model in the input list. The models should be either all continuous, or all discrete with the same sample time. When appending models of different types, the resulting type is determined by the precedence rules (see "Rules That Determine Model Type" (Control System Toolbox) for details).

There is no limitation on the number of inputs.

## Examples

## Append Inputs and Outputs of Models

Create a SISO transfer function.
sys1 = tf(1,[10]);
size(sys1)
Transfer function with 1 outputs and 1 inputs.
Create a SISO continuous-time state-space model.
sys2 $=\mathrm{ss}(1,2,3,4)$;
size(sys2)
State-space model with 1 outputs, 1 inputs, and 1 states.
Append the inputs and outputs of sys1, a SISO static gain system, and sys2. The resulting model should be a 3 -input, 3-output state-space model.

```
sys = append(sys1,10,sys2)
sys =
    A =
        }\begin{array}{l}{\mathrm{ x1 }}\\{\times1}
    B =
            u1 u2 u3
        x1 1 0 0
        x2 0 0 2
    C =
        x1 x2
    y1 1 0
    y2 0 0
    y3 0 3
    D =
            u1 u2 u3
    y1 0 0 0
    y2 0 10 0
    y3 0 0 4
```

Continuous-time state-space model.
size(sys)
State-space model with 3 outputs, 3 inputs, and 2 states.

## See Also

connect | feedback | parallel | series

Introduced in R2012a

## ar

Estimate parameters of AR model or ARI model for scalar time series

## Syntax

```
sys = ar(y,n)
sys = ar(y,n,approach,window)
sys = ar(y,n,
```

$\qquad$

``` ,Name, Value)
sys \(=\operatorname{ar}(\mathrm{y}, \mathrm{n}\),
``` \(\qquad\)
``` ,opt)
[sys,refl] = ar(y,n,approach,
``` \(\qquad\)
``` )
```


## Description

sys $=\operatorname{ar}(\mathrm{y}, \mathrm{n})$ estimates the parameters of an AR on page 1-25 idpoly model sys of order n using a least-squares method. The model properties include covariances (parameter uncertainties) and estimation goodness of fit.
sys $=\operatorname{ar}(y, n$, approach, window) uses the algorithm specified by approach and the prewindowing and postwindowing specification in window. To specify window while accepting the default value for approach, use [] in the third position of the syntax.
sys $=a r(y, n$, $\qquad$ ,Name, Value) specifies additional options using one or more name-value pair arguments. For instance, using the name-value pair argument 'IntegrateNoise' , 1 estimates an ARI on page 1-25 model, which is useful for systems with nonstationary disturbances. Specify Name, Value after any of the input argument combinations in the previous syntaxes.
sys $=\operatorname{ar}(\mathrm{y}, \mathrm{n}$, $\qquad$ , opt) specifies estimation options using the option set opt.
[sys,refl] = ar(y,n,approach,___) returns an AR model along with the reflection coefficients refl when approach is the lattice-based method 'burg' or 'gl'.

## Examples

## AR Model

Estimate an AR model and compare its response with the measured output.
Load the data, which contains the time series z 9 with noise.

```
load iddata9 z9
```

Estimate a fourth-order AR model.

```
sys \(=\operatorname{ar}(z 9,4)\)
sys =
Discrete-time AR model: \(A(z) y(t)=e(t)\)
    \(A(z)=1-0.8369 z^{\wedge}-1-0.4744 z^{\wedge}-2-0.06621 z^{\wedge}-3+0.4857 z^{\wedge}-4\)
Sample time: 0.0039063 seconds
```

```
Parameterization:
    Polynomial orders: na=4
    Number of free coefficients: 4
    Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.
Status:
Estimated using AR ('fb/now') on time domain data "z9".
Fit to estimation data: 79.38%
FPE: 0.5189, MSE: 0.5108
```

The output displays the polynomial containing the estimated parameters alongside other estimation details. Under Status, Fit to estimation data shows that the estimated model has 1-stepahead prediction accuracy above $75 \%$.

You can find additional information about the estimation results by exploring the estimation report, sys. Report. For instance, you can retrieve the parameter covariance.

```
covar = sys.Report.Parameters.FreeParCovariance
covar = 4×4
\begin{tabular}{rrrr}
0.0015 & -0.0015 & -0.0005 & 0.0007 \\
-0.0015 & 0.0027 & -0.0008 & -0.0004 \\
-0.0005 & -0.0008 & 0.0028 & -0.0015 \\
0.0007 & -0.0004 & -0.0015 & 0.0014
\end{tabular}
```

For more information on viewing the estimation report, see "Estimation Report".

## Compare Burg's Method with Forward-Backward Approach

Given a sinusoidal signal with noise, compare the spectral estimates of Burg's method with those found using the forward-backward approach.

Generate an output signal and convert it into an iddata object.

```
y = sin([1:300]') + 0.5*randn(300,1);
y = iddata(y);
```

Estimate fourth-order AR models using Burg's method and using the default forward-backward approach. Plot the model spectra together.

```
sys_b = ar(y,4,'burg');
sys_fb = ar(y,4);
spectrum(sys_b,sys_fb)
legend('Burg',''Forward-Backward')
```


## Power Spectrum



The two responses match closely throughout most of the frequency range.

## ARI Model

Estimate an ARI model, which includes an integrator in the noise source.
Load the data, which contains the time series z 9 with noise.
load iddata9 z9
Integrate the output signal.
y = cumsum(z9.y);
Estimate an AR model with 'IntegrateNoise' set to true. Use the least-squares method ' ls '. Because $y$ is a vector and not an iddata object, specify Ts.

Ts = z9.Ts;
sys = ar(y,4,'ls','Ts',Ts,'IntegrateNoise',true);
Predict the model output using 5-step prediction and compare the result with the integrated output signal y.

```
compare(y,sys,5)
```



## Modify Default Options

Modify the default options for the AR function.
Load the data, which contains a time series z 9 with noise.
load iddata9 z9
Modify the default options so that the function uses the ' 1 s ' approach and does not estimate covariance.

```
opt = arOptions('Approach','ls','EstimateCovariance',false)
opt =
Option set for the ar command:
    Approach: 'ls'
            Window: 'now'
        DataOffset: 0
    EstimateCovariance: 0
    MaxSize: 250000
Description of options
```

Estimate a fourth-order AR model using the updated options.

```
sys = ar(z9,4,opt);
```


## Retrieve Reflection Coefficients for Burg's Method

Retrieve reflection coefficients and loss functions when using Burg's method.
Lattice-based approaches such, as Burg's method 'burg' and geometric lattice 'gl', compute reflection coefficients and corresponding loss function values as part of the estimation process. Use a second output argument to retrieve these values.

Generate an output signal and convert it into an iddata object.

```
y = sin([1:300]') + 0.5*randn(300,1);
y = iddata(y);
```

Estimate a fourth-order AR model using Burg's method and include an output argument for the reflection coefficients.

```
[sys,refl] = ar(y,4,'burg');
refl
refl = 2×5
\begin{tabular}{rrrrr}
0 & -0.3562 & 0.4430 & 0.5528 & 0.2385 \\
0.8494 & 0.7416 & 0.5960 & 0.4139 & 0.3904
\end{tabular}
```


## Input Arguments

## y - Time-series data

iddata object | double vector
Time-series data, specified as one of the following:

- An iddata object that contains a single output channel and an empty input channel.
- A double column vector containing output-channel data. When you specify y as a vector, you must also specify the sample time Ts.
n - Model order
positive integer
Model order, specified as a positive integer. The value of n determines the number of $A$ parameters in the AR model.

Example: ar(idy, 2) computes a second-order AR model from the single-channel iddata object idy

## approach - Algorithm for computing AR model <br> 'fb' (default)|'burg'|'gl'|'ls'|'yw'

Algorithm for computing the AR model, specified as one of the following values:

- 'burg ': Burg's lattice-based method. Solves the lattice filter equations using the harmonic mean of forward and backward squared prediction errors.
- ' fb ': (Default) Forward-backward approach. Minimizes the sum of a least-squares criterion for a forward model, and the analogous criterion for a time-reversed model.
- 'gl ': Geometric lattice approach. Similar to Burg's method, but uses the geometric mean instead of the harmonic mean during minimization.
- ' ls ': Least-squares approach. Minimizes the standard sum of squared forward-prediction errors.
- 'yw': Yule-Walker approach. Solves the Yule-Walker equations, formed from sample covariances.

All of these algorithms are variants of the least-squares method. For more information, see "Algorithms" on page 1-26.

Example: ar(idy, 2,'ls') computes an AR model using the least-squares approach

## window - Prewindowing and postwindowing

'now'|'pow'|'ppw'|'prw
Prewindowing and postwindowing outside the measured time interval (past and future values), specified as one of the following values:

- 'now' : No windowing. This value is the default except when you set approach to ' yw '. Only measured data is used to form regression vectors. The summation in the criteria starts at the sample index equal to $\mathrm{n}+1$.
- 'pow' : Postwindowing. Missing end values are replaced with zeros and the summation is extended to time $\mathrm{N}+\mathrm{n}$ ( N is the number of observations).
- 'ppw ': Prewindowing and postwindowing. The software uses this value whenever you select the Yule-Walker approach 'yw', regardless of your window specification.
- 'prw': Prewindowing. Missing past values are replaced with zeros so that the summation in the criteria can start at time equal to zero.

Example: $\operatorname{ar(idy,2,'yw','ppw')~computes~an~AR~model~using~the~Yule-Walker~approach~with~}$ prewindowing and postwindowing.

## opt - Estimation options

arOptions option set
Estimation options for AR model identification, specified as an arOptions option set. opt specifies the following options:

- Estimation approach
- Data windowing technique
- Data offset
- Maximum number of elements in a segment of data

For more information, see arOptions. For an example, see "Modify Default Options" on page 1-20.

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.
Example: 'IntegrateNoise', true adds an integrator in the noise source

## Ts - Sample time

1 (default) | positive scalar
Sample time, specified as the comma-separated pair consisting of 'Ts' and the sample time in seconds. If $y$ is a double vector, then you must specify 'Ts '.
Example: ar (y_signal, $2, ' T s ', 0.08$ ) computes a second-order AR model with sample time of 0.08 seconds

## IntegrateNoise - Add integrator to noise channel

false (default) | logical vector
Noise-channel integration option for estimating ARI on page 1-25 models, specified as the commaseparated pair consisting of 'IntegrateNoise' and a logical. Noise integration is useful in cases where the disturbance is nonstationary.

When using 'IntegrateNoise', you must also integrate the output-channel data. For an example, see "ARI Model" on page 1-19.

## Output Arguments

## sys - AR or ARI model

idpoly model object
AR on page 1-25 or ARI on page 1-25 model that fits the given estimation data, returned as a discrete-time idpoly model object. This model is created using the specified model orders, delays, and estimation options.

Information about the estimation results and options used is stored in the Report property of the model. Report has the following fields.

| Report <br> Field | Description |
| :--- | :--- |
| Status | Summary of the model status, which indicates whether the model was created by <br> construction or obtained by estimation. |
| Method | Estimation command used. |


| Report <br> Field | Description |  |
| :--- | :--- | :--- |
| Fit | Quantitative assessment of the estimation, returned as a structure. See "Loss Function <br> and Model Quality Metrics" for more information on these quality metrics. The <br> structure has the following fields: |  |
|  | Field | Description |
|  | FitPerce <br> nt | Normalized root mean squared error (NRMSE) measure of how well the <br> response of the model fits the estimation data, expressed as the <br> percentage fit = 100(1-NRMSE). |
|  | LossFcn | Value of the loss function when the estimation completes. |
|  | MSE <br> Mean squared error (MSE) measure of how well the response of the <br> model fits the estimation data. |  |
|  | FPE | Final prediction error for the model. |
|  | AIC | Raw Akaike Information Criteria (AIC) measure of model quality. |
|  | AICc | Small sample-size corrected AIC. |
| nAIC | Normalized AIC. |  |
|  | BIC | Bayesian Information Criteria (BIC). |
| Parameter <br> s | Estimated values of model parameters. <br> OptionsUs <br> edOption set used for estimation. If no custom options were configured, this is a set of <br> default options. See arOptions for more information. |  |
| RandState | State of the random number stream at the start of estimation. Empty, [ ], if <br> randomization was not used during estimation. For more information, see rng. |  |


| Report Field | Description |  |
| :---: | :---: | :---: |
| DataUsed | Attributes of the data used for estimation, returned as a structure with the following fields: |  |
|  | Field | Description |
|  | Name | Name of the data set. |
|  | Type | Data type. |
|  | Length | Number of data samples. |
|  | Ts | Sample time. |
|  | InterSam ple | Input intersample behavior, returned as one of the following values: <br> - 'zoh ' - Zero-order hold maintains a piecewise-constant input signal between samples. <br> - ' foh ' - First-order hold maintains a piecewise-linear input signal between samples. <br> - 'bl' - Band-limited behavior specifies that the continuous-time input signal has zero power above the Nyquist frequency. |
|  | Input0ff set | Offset removed from time-domain input data during estimation. For nonlinear models, it is []. |
|  | Output0f fset | Offset removed from time-domain output data during estimation. For nonlinear models, it is [ ]. |

For more information on using Report, see "Estimation Report".

## refl - Reflection coefficients and loss functions

array
Reflection coefficients and loss functions, returned as a 2-by-2 array. For the two lattice-based approaches 'burg' and ' gl ', refl stores the reflection coefficients in the first row and the corresponding loss function values in the second row. The first column of refl is the zeroth-order model, and the $(2,1)$ element of refl is the norm of the time series itself. For an example, see "Retrieve Reflection Coefficients for Burg's Method" on page 1-21.

## More About

## AR (Autoregressive) Model

The AR model structure has no input, and is given by the following equation:

$$
A(q) y(t)=e(t)
$$

This model structure accommodates estimation for scalar time-series data, which have no input channel. The structure is a special case of the ARX structure.

## ARI (Autoregressive Integrated) Model

The ARI model is an AR model with an integrator in the noise channel. The ARI model structure is given by the following equation:

$$
A(q) y(t)=\frac{1}{1-q^{-1}} e(t)
$$

## Algorithms

AR and ARI model parameters are estimated using variants of the least-squares method. The following table summarizes the common names for methods with a specific combination of approach and window argument values.

| Method | Approach and Windowing |
| :--- | :--- |
| Modified covariance method | (Default) Forward-backward approach with no <br> windowing |
| Correlation method | Yule-Walker approach with prewindowing and <br> postwindowing |
| Covariance method | Least squares approach with no windowing. arx <br> uses this routine |

## References

[1] Marple, S. L., Jr. Chapter 8. Digital Spectral Analysis with Applications. Englewood Cliffs, NJ: Prentice Hall, 1987.

## See Also

arOptions|arx|etfe|forecast|iddata|idpoly|ivar|pem|spa|spectrum

## Topics

"What Are Time Series Models?"
"Representing Data in MATLAB Workspace"
"Identify Time Series Models at the Command Line"

## Introduced in R2006a

## armax

Estimate parameters of ARMAX, ARIMAX, ARMA, or ARIMA model using time-domain data

## Syntax

```
sys = armax(data,[na nb nc nk])
sys = armax(data,[na nb nc nk],Name,Value)
sys = armax(data,init_sys)
sys = armax(data,
```

$\qquad$

``` ,opt)
[sys,ic] = armax(
``` \(\qquad\)

\section*{Description}

\section*{Estimate an ARMAX Model}
sys = armax(data,[na nb nc nk]) estimates the parameters of an ARMAX on page 1-40 or an ARMA on page 1-41 idpoly model sys using the prediction-error method and the polynomial orders specified in [na nb nc nk]. The model properties include estimation covariances (parameter uncertainties) and goodness of fit between the estimated and the measured data.
sys = armax(data,[na nb nc nk],Name, Value) specifies additional options using one or more name-value pair arguments. For instance, using the name-value pair argument
'IntegrateNoise' , 1 estimates an ARIMAX on page 1-41 or ARIMA on page 1-41 model, which is useful for systems with nonstationary disturbances.

\section*{Configure Initial Parameters}
sys = armax(data,init_sys) uses the discrete-time linear model init_sys to configure the initial parameterization.

\section*{Specify Additional Options}
sys = armax (data, \(\qquad\) , opt) incorporates an option set opt that specifies options such as estimation objective, handling of initial conditions, regularization, and numerical search method to use for estimation. Specify opt after any of the previous input-argument combinations.

\section*{Return Estimated Initial Conditions}
[sys,ic] = armax( \(\qquad\) ) returns the estimated initial conditions as an initialCondition object. Use this syntax if you plan to simulate or predict the model response using the same estimation input data and then compare the response with the same estimation output data. Incorporating the initial conditions yields a better match during the first part of the simulation.

\section*{Examples}

\section*{Estimate ARMAX Model}

Estimate an ARMAX model and view the fit of the model output to the estimation data.

Load the measurement data in iddata object z2.
load iddata2 z2
Estimate an ARMAX model with second-order \(A, B\), and \(C\) polynomials and a transport delay of one sample period.
```

na = 2;
nb = 2;
nc = 2;
nk = 1;
sys = armax(z2,[na nb nc nk])
sys =
Discrete-time ARMAX model: A(z)y(t) = B(z)u(t) + C(z)e(t)
A(z) = 1 - 1.512 z^-1 + 0.7006 z^-2
B(z) = -0.2606 z^-1 + 1.664 z^-2
C(z) = 1 - 1.604 z^-1 + 0.7504 z^-2
Sample time: 0.1 seconds
Parameterization:
Polynomial orders: na=2 nb=2 nc=2 nk=1
Number of free coefficients: 6
Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.
Status:
Estimated using ARMAX on time domain data "z2".
Fit to estimation data: 85.89% (prediction focus)
FPE: 1.086, MSE: 1.054

```

The output displays the polynomial containing the estimated parameters alongside the estimation details. Under Status, Fit to estimation data shows that the estimated model has 1 -stepahead prediction accuracy above \(80 \%\).

Compare the model simulated output to the measured data.
```

compare(z2,sys)

```


The fit of the simulated model to the measured data is nearly the same as the estimation fit.

\section*{ARMA Model}

Estimate an ARMA model and compare its response with both the measured output and an AR model.
Load the data, which contains the time series z 9 with noise.
load iddata9 z9
Estimate a fourth-order ARMA model with a first-order \(C\) polynomial.
```

na = 4;
nc = 1;
sys = armax(z9,[na nc]);

```

Estimate a fourth-order AR model.
sys_ar = ar(z9,na);
Compare the model outputs with the measured data.
compare(z9,sys,sys_ar)

Simulated Response Comparison


The ARMA model has the better fit to the data.

\section*{Specify Estimation Options}

Estimate an ARMAX model from measured data and specify estimation options.
Load the data and create an iddata object. Initialize option set opt, and set options for Focus, SearchMethod, MaxIterations, and Display. Then estimate the ARMAX model using the updated option set.
```

load twotankdata;
z = iddata(y,u,0.2);
opt = armaxOptions;
opt.Focus = 'simulation';
opt.SearchMethod = 'lm';
opt.SearchOptions.MaxIterations = 10;
opt.Display = 'on';
sys = armax(z,[2 2 2 1],opt);

```

The termination conditions for measured component of the model shown in the progress viewer is that the maximum number of iterations were reached.

To improve results, re-estimate the model using a greater value for MaxIterations, or continue iterations on the previously estimated model as follows:
sys2 \(=\operatorname{armax}(z, s y s) ;\)
compare(z,sys,sys2)

where sys 2 refines the parameters of sys to improve the fit to data.

\section*{ARMAX Model with Regularization}

Estimate a regularized ARMAX model by converting a regularized ARX model.
Load data.
load regularizationExampleData.mat m0simdata;
Estimate an unregularized ARMAX model of order 30.
```

m1 = armax(m0simdata(1:150),[30 30 30 1]);

```

Estimate a regularized ARMAX model by determining the Lambda value by trial and error.
opt = armaxOptions;
opt.Regularization.Lambda = 1;
m2 = armax(m0simdata(1:150),[30 3030 1],opt);
Obtain a lower order ARMAX model by converting a regularized ARX model and then performing order reduction.
```

opt1 = arxOptions;
[L,R] = arxRegul(m0simdata(1:150),[30 30 1]);
opt1.Regularization.Lambda = L;
opt1.Regularization.R = R;
m0 = arx(m0simdata(1:150),[30 30 1],opt1);
mr = idpoly(balred(idss(m0),7));
Compare the model outputs against the data.
opt2 = compareOptions('InitialCondition','z');
compare(m0simdata(150:end),m1,m2,mr,opt2);

```

Simulated Response Comparison


\section*{ARIMA Model}

Estimate a fourth-order ARIMA model for univariate time-series data.
Load data that contains a time series with noise.
load iddata9 z9;
Integrate the output signal and use the result to replace the original output signal in z9.
z9.y = cumsum(z9.y);
Estimate a fourth-order ARIMA model with a first-orderCpolynomial by setting 'IntegrateNoise' to true.
```

model = armax(z9,[4 1],'IntegrateNoise',true);

```

Predict the model output using 10-step ahead prediction, and compare the predicted output with the estimation data.
```

compare(z9,model,10)

```


\section*{Estimate ARMAX Models Iteratively}

Estimate ARMAX models of varying orders iteratively from measured data.
Load dryer2 data and perform estimation for combinations of polynomial orders na, nb, nc, and input delay nk.
```

load dryer2;
z = iddata(y2,u2,0.08,'Tstart',0);
na = 2:4;
nc = 1:2;
nk = 0:2;
models = cell(1,18);
ct = 1;
for i = 1:3
na_ = na(i);
nb = na ;
for j = \overline{1}:2

```
```

        nc_ = nc(j);
        for k = 1:3
            nk = nk(k);
            models{ct} = armax(z,[na_ nb_ nc_ nk_]);
            ct = ct+1;
        end
    end
    end

```

Stack the estimated models and compare their simulated responses to the estimation data z .
```

models = stack(1,models{:});

```
compare(z,models)

Simulated Response Comparison


\section*{Initialize ARMAX Model Parameters Using State-Space Model}

Load the estimation data.
load iddata2 z2
Estimate a state-space model of order 3 from the estimation data.
sys0 = n4sid(z2,3);
Estimate an ARMAX model using the previously estimated state-space model to initialize the parameters.
```

sys = armax(z2,sys0);

```

\section*{Obtain Initial Conditions}

Load the data.
```

load iddatalic z1i

```

Estimate a second-order ARMAX model sys and return the initial conditions in ic.
```

na = 2;
nb = 2;
nc = 2;
nk = 1;
[sys,ic] = armax(zli,[na nb nc nk]);
iC
ic =
initialCondition with properties:
A: [2x2 double]
X0: [2x1 double]
C: [0 1]
Ts: 0.1000

```
ic is an initialCondition object that encapsulates the free response of sys, in state-space form, to the initial state vector in X0. You can incorporate ic when you simulate sys with the z1i input signal and compare the response with the z1i output signal.

\section*{Input Arguments}

\section*{data - Time-domain estimation data}
iddata object
Time-domain estimation data, specified as an iddata object. For ARMA and ARIMA time-series models, the input channel in data must be empty. For examples, see "ARMA Model" on page 1-29 and "ARIMA Model" on page 1-32.

\section*{[na nb nc nk] - Polynomial orders}
integer row vector | row vector of integer matrices | scalar
Polynomial orders and delays for the model, specified as a 1-by-4 vector or vector of matrices [na nb nc nk]. The polynomial order is equal to the number of coefficients to estimate in that polynomial.

For an ARMA or ARIMA time-series model, which has no input, set [na nb nc nk] to [na nc]. For an example, see "ARMA Model" on page 1-29.

For a model with \(N y\) outputs and \(N u\) inputs:
- na is the order of the polynomial \(A(q)\), specified as an \(N y\)-by- \(N y\) matrix of nonnegative integers.
- nb is the order of the polynomial \(B(q)+1\), specified as an \(N y\)-by- \(N u\) matrix of nonnegative integers.
- nc is the order of the polynomial \(C(q)\), specified as a column vector of nonnegative integers of length \(N y\).
- nk is the input-output delay, also known at the transport delay, specified as an \(N y\)-by- \(N u\) matrix of nonnegative integers. nk is represented in ARMAX models by fixed leading zeros of the \(B\) polynomial.

For an example, see "Estimate ARMAX Model" on page 1-27.

\section*{init_sys - System for configuring initial parameterization}
discrete-time linear model
System for configuring the initial parameterization of sys, specified as a discrete-time linear model. You obtain init_sys by either performing an estimation using measured data or by direct construction using commands such as idpoly and idss.

If init_sys is an ARMAX model, armax uses the parameter values of init_sys as the initial guess for estimation. To configure initial guesses and constraints for \(A(q), B(q)\), and \(C(q)\), use the Structure property of init_sys. For example:
- To specify an initial guess for the \(A(q)\) term of init_sys, set init_sys.Structure.A.Value as the initial guess.
- To specify constraints for the \(B(q)\) term of init_sys:
- Set init_sys.Structure.B.Minimum to the minimum \(B(q)\) coefficient values.
- Set init_sys.Structure.B.Maximum to the maximum \(B(q)\) coefficient values.
- Set init_sys.Structure.B.Free to indicate which \(B(q)\) coefficients are free for estimation.

If init_sys is not a polynomial model with the ARMAX structure, the software first converts init_sys to an ARMAX model. armax uses the parameters of the resulting model as the initial guess for estimating sys.

If opt is not specified and init_sys was obtained by estimation, then the estimation options from init_sys.Report.OptionsUsed are used.

For an example, see "Initialize ARMAX Model Parameters Using State-Space Model" on page 1-34.

\section*{opt - Estimation options}
armax0ptions option set
Estimation options for ARMAX model identification, specified as an armaxOptions option set. Options specified by opt include the following:
- Initial condition handling - Use this option to determine how the initial conditions are set or estimated.
- Input and output data offsets - Use these options to remove offsets from data during estimation.
- Regularization - Use this option to control the tradeoff between bias and variance errors during the estimation process.

\section*{Name-Value Pair Arguments}

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

\section*{Example: 'InputDelay' , 2 applies an input delay of two sample periods to all input channels}

\section*{InputDelay - Input delays}

0 (default) | integer scalar | positive integer vector
Input delays expressed as integer multiples of the sample time, specified as the comma-separated pair consisting of 'InputDelay' and one of the following:
- \(\quad N_{u}\)-by-1 vector, where \(N_{u}\) is the number of inputs - Each entry is a numerical value representing the input delay for the corresponding input channel.
- Scalar value - Apply the same delay to all input channels.
- 0 - No input delays.

Example: armax (data, [2 1 1 0],'InputDelay', 1) estimates a second-order ARX model with first-order \(B\) and \(C\) polynomials that has an input delay of two samples.

\section*{IODelay - Transport delays}

0 (default) | scalar | matrix
Transport delays for each input-output pair, expressed as integer multiples of the sample time, and specified as the comma-separated pair consisting of 'IODelay' and one of the following:
- \(N_{y}\)-by- \(N_{u}\) matrix, where \(N_{y}\) is the number of outputs and \(N_{u}\) is the number of inputs - Each entry is an integer value representing the transport delay for the corresponding input-output pair.
- Scalar value - Apply the same delay to all input-output pairs.
'IODelay' is useful as a replacement for the nk order. You can factor out max (nk-1,0) lags as the 'IODelay ' value. For nk > 1, armax (na, nb, nk) is equivalent to armax (na,nb,1,'IODelay',nk-1).

\section*{IntegrateNoise - Addition of integrators in noise channel}
false (default) | logical vector
Addition of integrators in the noise channel, specified as the comma-separated pair consisting of ' IntegrateNoise' and a logical vector of length \(N_{y}\), where \(N_{y}\) is the number of outputs.

Setting 'IntegrateNoise' to true for a particular output results in the model
\[
A(q) y(t)=B(q) u(t-n k)+\frac{C(q)}{1-q^{-1}} e(t)
\]
where \(\frac{1}{1-q^{-1}}\) is the integrator in the noise channel, \(e(t)\).
Use 'IntegrateNoise' to create ARIMA or ARIMAX models.
For an example, see "ARIMA Model" on page 1-32.

\section*{Output Arguments}
sys - ARMAX model
idpoly object

ARMAX model that fits the given estimation data, returned as a discrete-time idpoly object. This model is created using the specified model orders, delays, and estimation options.

Information about the estimation results and options used is stored in the Report property of the model. Report has the following fields.
\begin{tabular}{|c|c|c|}
\hline Report Field & \multicolumn{2}{|l|}{Description} \\
\hline Status & \multicolumn{2}{|l|}{Summary of the model status, which indicates whether the model was created by construction or obtained by estimation.} \\
\hline Method & \multicolumn{2}{|l|}{Estimation command used.} \\
\hline InitialCo ndition & \multicolumn{2}{|l|}{\begin{tabular}{l}
Handling of initial conditions during model estimation, returned as one of the following values: \\
- 'zero' - The initial conditions were set to zero. \\
- 'estimate ' - The initial conditions were treated as independent estimation parameters. \\
- 'backcast' - The initial conditions were estimated using the best least squares fit. \\
This field is especially useful to view how the initial conditions were handled when the InitialCondition option in the estimation option set is 'auto'.
\end{tabular}} \\
\hline Fit & \multicolumn{2}{|l|}{Quantitative assessment of the estimation, returned as a structure. See "Loss Function and Model Quality Metrics" for more information on these quality metrics. The structure has the following fields:} \\
\hline & Field & Description \\
\hline & FitPerce nt & Normalized root mean squared error (NRMSE) measure of how well the response of the model fits the estimation data, expressed as the percentage fit \(=100(1-\) NRMSE \()\). \\
\hline & LossFen & Value of the loss function when the estimation completes. \\
\hline & MSE & Mean squared error (MSE) measure of how well the response of the model fits the estimation data. \\
\hline & FPE & Final prediction error for the model. \\
\hline & AIC & Raw Akaike Information Criteria (AIC) measure of model quality. \\
\hline & AICc & Small sample-size corrected AIC. \\
\hline & nAIC & Normalized AIC. \\
\hline & BIC & Bayesian Information Criteria (BIC). \\
\hline \begin{tabular}{l}
Parameter \\
s
\end{tabular} & \multicolumn{2}{|l|}{Estimated values of model parameters.} \\
\hline OptionsUs
ed & \multicolumn{2}{|l|}{Option set used for estimation. If no custom options were configured, this is a set of default options. See armax0ptions for more information.} \\
\hline RandState & \multicolumn{2}{|l|}{State of the random number stream at the start of estimation. Empty, [ ], if randomization was not used during estimation. For more information, see rng.} \\
\hline
\end{tabular}
\begin{tabular}{|c|c|c|}
\hline Report Field & \multicolumn{2}{|l|}{Description} \\
\hline \multirow[t]{9}{*}{DataUsed} & \multicolumn{2}{|l|}{Attributes of the data used for estimation, returned as a structure with the following fields:} \\
\hline & Field & Description \\
\hline & Name & Name of the data set. \\
\hline & Type & Data type. \\
\hline & Length & Number of data samples. \\
\hline & Ts & Sample time. \\
\hline & InterSam ple & \begin{tabular}{l}
Input intersample behavior, returned as one of the following values: \\
- 'zoh ' - Zero-order hold maintains a piecewise-constant input signal between samples. \\
- ' foh ' - First-order hold maintains a piecewise-linear input signal between samples. \\
- 'bl' - Band-limited behavior specifies that the continuous-time input signal has zero power above the Nyquist frequency.
\end{tabular} \\
\hline & Input0ff set & Offset removed from time-domain input data during estimation. For nonlinear models, it is []. \\
\hline & Output0f fset & Offset removed from time-domain output data during estimation. For nonlinear models, it is [ ]. \\
\hline \multirow[t]{10}{*}{Terminati on} & \multicolumn{2}{|l|}{Termination conditions for the iterative search used for prediction error minimization, returned as a structure with the following fields:} \\
\hline & Field & Description \\
\hline & WhyStop & Reason for terminating the numerical search. \\
\hline & Iteratio ns & Number of search iterations performed by the estimation algorithm. \\
\hline & First0rd erOptima lity & \(\infty\)-norm of the gradient search vector when the search algorithm terminates. \\
\hline & FcnCount & Number of times the objective function was called. \\
\hline & UpdateNo rm & Norm of the gradient search vector in the last iteration. Omitted when the search method is 'lsqnonlin' or 'fmincon'. \\
\hline & LastImpr ovement & Criterion improvement in the last iteration, expressed as a percentage. Omitted when the search method is 'lsqnonlin' or 'fmincon'. \\
\hline & \[
\begin{array}{|l|l|}
\hline \text { Algorith } \\
\hline
\end{array}
\] & Algorithm used by ' lsqnonlin' or 'fmincon' search method. Omitted when other search methods are used. \\
\hline & \multicolumn{2}{|l|}{For estimation methods that do not require numerical search optimization, the Termination field is omitted.} \\
\hline
\end{tabular}

For more information on using Report, see "Estimation Report".

\section*{ic - Initial conditions}
initialCondition object| object array of initialCondition values
Estimated initial conditions, returned as an initialCondition object or an object array of initialCondition values.
- For a single-experiment data set, ic represents, in state-space form, the free response of the transfer function model ( \(A\) and \(C\) matrices) to the estimated initial states ( \(x_{0}\) ).
- For a multiple-experiment data set with \(N_{e}\) experiments, ic is an object array of length \(N_{e}\) that contains one set of initialCondition values for each experiment.

If armax returns ic values of 0 and the you know that you have non-zero initial conditions, set the 'InitialCondition ' option in armax0ptions to 'estimate' and pass the updated option set to armax. For example:
```

opt = armaxOptions('InitialCondition,'estimate')
[sys,ic] = armax(data,np,nz,opt)

```

The default 'auto' setting of 'InitialCondition' uses the 'zero' method when the initial conditions have a negligible effect on the overall estimation-error minimization process. Specifying 'estimate' ensures that the software estimates values for ic.

For more information, see initialCondition. For an example of using this argument, see "Obtain Initial Conditions" on page 1-35.

\section*{More About}

\section*{ARMAX Model}

The ARMAX (Autoregressive Moving Average with Extra Input) model structure is:
\[
\begin{aligned}
& y(t)+a_{1} y(t-1)+\ldots+a_{n_{a}} y\left(t-n_{a}\right)= \\
& b_{1} u\left(t-n_{k}\right)+\ldots+b_{n_{b}} u\left(t-n_{k}-n_{b}+1\right)+ \\
& \quad c_{1} e(t-1)+\ldots+c_{n_{C}} e\left(t-n_{c}\right)+e(t)
\end{aligned}
\]

A more compact way to write the difference equation is
\[
A(q) y(t)=B(q) u\left(t-n_{k}\right)+C(q) e(t)
\]
where
- \(y(t)\) - Output at time \(t\)
- \(n_{a}\) - Number of poles
- \(n_{b}\) - Number of zeroes plus 1
- \(n_{C}\) - Number of \(C\) coefficients
- \(n_{k}\) - Number of input samples that occur before the input affects the output, also called the dead time in the system
- \(y(t-1) \ldots y\left(t-n_{a}\right)\) - Previous outputs on which the current output depends
- \(u\left(t-n_{k}\right) \ldots u\left(t-n_{k}-n_{b}+1\right)\) - Previous and delayed inputs on which the current output depends
- \(e(t-1) . . . e\left(t-n_{c}\right)-\) White-noise disturbance value

The parameters na, nb, and nc are the orders of the ARMAX model, and nk is the delay. \(q\) is the delay operator. Specifically,
\[
\begin{aligned}
& A(q)=1+a_{1} q^{-1}+\ldots+a_{n_{a}} q^{-n_{a}} \\
& B(q)=b_{1}+b_{2} q^{-1}+\ldots+b_{n_{b}} q^{-n_{b}+1} \\
& C(q)=1+c_{1} q^{-1}+\ldots+c_{n_{c}} q^{-n_{C}}
\end{aligned}
\]

\section*{ARMA Time-Series Model}

The ARMA (Autoregressive Moving Average) model is a special case of an "ARMAX Model" on page 140 with no input channels. The ARMA single-output model structure is given by the following equation:
\[
A(q) y(t)=C(q) e(t)
\]

\section*{ARIMAX Model}

The ARIMAX (Autoregressive Integrated Moving Average with Extra Input) model structure is similar to the ARMAX model, except that it contains an integrator in the noise source \(e(t)\) :
\[
A(q) y(t)=B(q) u(t-n k)+\frac{C(q)}{\left(1-q^{-1}\right)} e(t)
\]

\section*{ARIMA Model}

The ARIMA (Autoregressive Integrated Moving Average) model structure is a reduction of the ARIMAX model with no inputs:
\[
A(q) y(t)=\frac{C(q)}{\left(1-q^{-1}\right)} e(t)
\]

\section*{Algorithms}

An iterative search algorithm minimizes a robustified quadratic prediction error criterion. The iterations are terminated when any of the following is true:
- Maximum number of iterations is reached.
- Expected improvement is less than the specified tolerance.
- Lower value of the criterion cannot be found.

You can get information about the stopping criteria using sys.Report.Termination.
Use the armax0ptions option set to create and configure options affecting the estimation results. In particular, set the search algorithm attributes, such as MaxIterations and Tolerance, using the 'SearchOptions' property.

When you do not specify initial parameter values for the iterative search as an initial model, they are constructed in a special four-stage LS-IV algorithm.

The cutoff value for the robustification is based on the Advanced. ErrorThreshold estimation option and on the estimated standard deviation of the residuals from the initial parameter estimate. The cutoff value is not recalculated during the minimization. By default, no robustification is performed; the default value of ErrorThreshold option is 0 .

To ensure that only models corresponding to stable predictors are tested, the algorithm performs a stability test of the predictor. Generally, both \(C(q)\) and \(F(q)\) (if applicable) must have all zeros inside the unit circle.

Minimization information is displayed on the screen when the estimation option 'Display ' is ' \(0 n\) ' or 'Full'. When 'Display' is 'Full', both the current and the previous parameter estimates are displayed in column-vector form, and the parameters are listed in alphabetical order. Also, the values of the criterion function (cost) are given and the Gauss-Newton vector and its norm are displayed. When 'Display' is ' \(0 n\) ', only the criterion values are displayed.

\section*{Alternatives}
armax does not support continuous-time model estimation. Use tfest to estimate a continuous-time transfer function model, or ssest to estimate a continuous-time state-space model.
armax supports only time-domain data. For frequency-domain data, use oe to estimate an OutputError (OE) model.

\section*{References}
[1] Ljung, L. System Identification: Theory for the User, Second Edition. Upper Saddle River, NJ: Prentice-Hall PTR, 1999. See chapter about computing the estimate.

\section*{Extended Capabilities}

\section*{Automatic Parallel Support}

Accelerate code by automatically running computation in parallel using Parallel Computing Toolbox \({ }^{\mathrm{TM}}\).
Parallel computing support is available for estimation using the lsqnonlin search method (requires Optimization Toolbox \({ }^{\mathrm{TM}}\) ). To enable parallel computing, use armax0ptions, set SearchMethod to 'lsqnonlin', and set SearchOptions.Advanced. UseParallel to true.

For example:
```

opt = armaxOptions;
opt.SearchMethod = 'lsqnonlin';
opt.SearchOptions.Advanced.UseParallel = true;

```

\section*{See Also}
aic|armax0ptions |arx|bj|compare|fpe|iddata|idpoly | oe|polyest|ssest|tfest

\section*{Topics}
"What Are Polynomial Models?"
"What Are Time Series Models?"
"Estimate Models Using armax"
"Estimation Report"
"Loss Function and Model Quality Metrics"
"Regularized Estimates of Model Parameters"
"Apply Initial Conditions when Simulating Identified Linear Models"
Introduced in R2006a

\section*{armaxOptions}

Option set for armax

\section*{Syntax}
opt = armax0ptions
opt \(=\) armaxOptions(Name, Value)

\section*{Description}
opt \(=\) armaxOptions creates the default options set for armax.
opt = armax0ptions(Name, Value) creates an option set with the options specified by one or more Name, Value pair arguments.

\section*{Input Arguments}

\section*{Name-Value Pair Arguments}

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, ... , NameN, ValueN.

\section*{InitialCondition - Handling of initial conditions \\ 'auto' (default)|'zero'|'estimate'|'backcast'}

Handling of initial conditions during estimation, specified as one of the following values:
- 'zero' - The initial conditions are set to zero.
- 'estimate' - The initial conditions are treated as independent estimation parameters.
- 'backcast' - The initial conditions are estimated using the best least squares fit.
- 'auto ' - The software chooses the method to handle initial conditions based on the estimation data.

\section*{Focus - Error to be minimized}
'prediction' (default)|'simulation'
Error to be minimized in the loss function during estimation, specified as the comma-separated pair consisting of 'Focus' and one of the following values:
- 'prediction' - The one-step ahead prediction error between measured and predicted outputs is minimized during estimation. As a result, the estimation focuses on producing a good predictor model.
- 'simulation' - The simulation error between measured and simulated outputs is minimized during estimation. As a result, the estimation focuses on making a good fit for simulation of model response with the current inputs.

The Focus option can be interpreted as a weighting filter in the loss function. For more information, see "Loss Function and Model Quality Metrics".

\section*{WeightingFilter - Weighting prefilter}
[ ] (default) | vector | matrix | cell array | linear system
Weighting prefilter applied to the loss function to be minimized during estimation. To understand the effect of WeightingFilter on the loss function, see "Loss Function and Model Quality Metrics".

Specify WeightingFilter as one of the following values:
- [ ] - No weighting prefilter is used.
- Passbands - Specify a row vector or matrix containing frequency values that define desired passbands. You select a frequency band where the fit between estimated model and estimation data is optimized. For example, [ \(w l, w h\) ], where \(w l\) and wh represent lower and upper limits of a passband. For a matrix with several rows defining frequency passbands, [w1l,w1h;w2l,w2h;w3l,w3h; ...], the estimation algorithm uses the union of the frequency ranges to define the estimation passband.

Passbands are expressed in rad/TimeUnit for time-domain data and in FrequencyUnit for frequency-domain data, where TimeUnit and FrequencyUnit are the time and frequency units of the estimation data.
- SISO filter - Specify a single-input-single-output (SISO) linear filter in one of the following ways:
- A SISO LTI model
- \(\{A, B, C, D\}\) format, which specifies the state-space matrices of a filter with the same sample time as estimation data.
- \{numerator, denominator\} format, which specifies the numerator and denominator of the filter as a transfer function with same sample time as estimation data.

This option calculates the weighting function as a product of the filter and the input spectrum to estimate the transfer function.

\section*{EnforceStability - Control whether to enforce stability of model}
false (default) | true
Control whether to enforce stability of estimated model, specified as the comma-separated pair consisting of 'EnforceStability' and either true or false.

This option is not available for multi-output models with a non-diagonal \(A\) polynomial array.
Data Types: logical

\section*{EstimateCovariance - Control whether to generate parameter covariance data true (default) | false}

Controls whether parameter covariance data is generated, specified as true or false.
If EstimateCovariance is true, then use getcov to fetch the covariance matrix from the estimated model.

\section*{Display - Specify whether to display the estimation progress \\ 'off' (default)|'on'}

Specify whether to display the estimation progress, specified as one of the following values:
- 'on' - Information on model structure and estimation results are displayed in a progress-viewer window.
- 'off' - No progress or results information is displayed.

\section*{InputOffset - Removal of offset from time-domain input data during estimation}
[ ] (default) | vector of positive integers | matrix
Removal of offset from time-domain input data during estimation, specified as the comma-separated pair consisting of 'InputOffset' and one of the following:
- A column vector of positive integers of length \(N u\), where \(N u\) is the number of inputs.
- [ ] - Indicates no offset.
- \(N u\)-by-Ne matrix - For multi-experiment data, specify InputOffset as an Nu -by-Ne matrix. Nu is the number of inputs, and \(N e\) is the number of experiments.

Each entry specified by InputOffset is subtracted from the corresponding input data.

\section*{OutputOffset - Removal of offset from time-domain output data during estimation}
[] (default) | vector | matrix
Removal of offset from time-domain output data during estimation, specified as the comma-separated pair consisting of 'OutputOffset' and one of the following:
- A column vector of length \(N y\), where \(N y\) is the number of outputs.
- [ ] - Indicates no offset.
- \(N y\)-by-Ne matrix - For multi-experiment data, specify OutputOffset as a \(N y\)-by-Ne matrix. \(N y\) is the number of outputs, and \(N e\) is the number of experiments.

Each entry specified by OutputOffset is subtracted from the corresponding output data.

\section*{Regularization - Options for regularized estimation of model parameters \\ structure}

Options for regularized estimation of model parameters. For more information on regularization, see "Regularized Estimates of Model Parameters".

Regularization is a structure with the following fields:
- Lambda - Constant that determines the bias versus variance tradeoff.

Specify a positive scalar to add the regularization term to the estimation cost.
The default value of zero implies no regularization.
Default: 0
- R - Weighting matrix.

Specify a vector of nonnegative numbers or a square positive semi-definite matrix. The length must be equal to the number of free parameters of the model.

For black-box models, using the default value is recommended. For structured and grey-box models, you can also specify a vector of np positive numbers such that each entry denotes the confidence in the value of the associated parameter.

The default value of 1 implies a value of eye(npfree), where npfree is the number of free parameters.

\section*{Default: 1}
- Nominal - The nominal value towards which the free parameters are pulled during estimation.

The default value of zero implies that the parameter values are pulled towards zero. If you are refining a model, you can set the value to 'model' to pull the parameters towards the parameter values of the initial model. The initial parameter values must be finite for this setting to work.

Default: 0

\section*{SearchMethod - Numerical search method used for iterative parameter estimation}
'auto' (default)|'gn'|'gna'|'lm'|'grad'|'lsqnonlin'|'fmincon'
Numerical search method used for iterative parameter estimation, specified as the comma-separated pair consisting of 'SearchMethod' and one of the following:
- 'auto' - A combination of the line search algorithms, 'gn', 'lm', 'gna', and 'grad' methods is tried in sequence at each iteration. The first descent direction leading to a reduction in estimation cost is used.
- 'gn' - Subspace Gauss-Newton least squares search. Singular values of the Jacobian matrix less than GnPinvConstant*eps*max(size(J))*norm(J) are discarded when computing the search direction. \(J\) is the Jacobian matrix. The Hessian matrix is approximated as \(J^{T} J\). If there is no improvement in this direction, the function tries the gradient direction.
- 'gna' - Adaptive subspace Gauss-Newton search. Eigenvalues less than gamma*max(sv) of the Hessian are ignored, where \(s v\) contains the singular values of the Hessian. The Gauss-Newton direction is computed in the remaining subspace. gamma has the initial value InitialGnaTolerance (see Advanced in 'Search0ptions' for more information). This value is increased by the factor LMStep each time the search fails to find a lower value of the criterion in fewer than five bisections. This value is decreased by the factor \(2 *\) LMStep each time a search is successful without any bisections.
- 'lm' - Levenberg-Marquardt least squares search, where the next parameter value is - pinv (H \(+\mathrm{d} * \mathrm{I}) * \mathrm{grad}\) from the previous one. \(H\) is the Hessian, \(I\) is the identity matrix, and grad is the gradient. \(d\) is a number that is increased until a lower value of the criterion is found.
- 'grad' - Steepest descent least squares search.
- 'lsqnonlin' - Trust-region-reflective algorithm of lsqnonlin. Requires Optimization Toolbox software.
- 'fmincon' - Constrained nonlinear solvers. You can use the sequential quadratic programming (SQP) and trust-region-reflective algorithms of the fmincon solver. If you have Optimization Toolbox software, you can also use the interior-point and active-set algorithms of the fmincon solver. Specify the algorithm in the SearchOptions.Algorithm option. The fmincon algorithms may result in improved estimation results in the following scenarios:
- Constrained minimization problems when there are bounds imposed on the model parameters.
- Model structures where the loss function is a nonlinear or non smooth function of the parameters.
- Multi-output model estimation. A determinant loss function is minimized by default for multioutput model estimation. fmincon algorithms are able to minimize such loss functions directly. The other search methods such as ' lm ' and 'gn' minimize the determinant loss function by alternately estimating the noise variance and reducing the loss value for a given noise variance value. Hence, the fmincon algorithms can offer better efficiency and accuracy for multi-output model estimations.

\section*{SearchOptions - Option set for the search algorithm}
search option set
Option set for the search algorithm, specified as the comma-separated pair consisting of 'Search0ptions ' and a search option set with fields that depend on the value of SearchMethod.

\section*{SearchOptions Structure When SearchMethod is Specified as 'gn', 'gna', 'lm', 'grad', or 'auto'}
\begin{tabular}{|l|l|l|}
\hline \begin{tabular}{l} 
Field \\
Name
\end{tabular} & Description & Default \\
\hline \begin{tabular}{l} 
Toleran \\
ce
\end{tabular} & \begin{tabular}{l} 
Minimum percentage difference between the current value of the loss \\
function and its expected improvement after the next iteration, specified as \\
a positive scalar. When the percentage of expected improvement is less \\
than Tolerance, the iterations stop. The estimate of the expected loss- \\
function improvement at the next iteration is based on the Gauss-Newton \\
vector computed for the current parameter value.
\end{tabular} & 0.01 \\
\hline \begin{tabular}{l} 
MaxIter \\
ations
\end{tabular} & \begin{tabular}{l} 
Maximum number of iterations during loss-function minimization, specified \\
as a positive integer. The iterations stop when MaxIterations is reached \\
or another stopping criterion is satisfied, such as Tolerance.
\end{tabular} & 20 \\
\begin{tabular}{l} 
Setting MaxIterations = 0 returns the result of the start-up procedure. \\
Use sys. Report. Termination. Iterations to get the actual number \\
of iterations during an estimation, where sys is an idtf model.
\end{tabular} & \\
\hline
\end{tabular}
\begin{tabular}{|c|c|c|c|}
\hline Field Name & \multicolumn{2}{|l|}{Description} & Default \\
\hline \multirow[t]{11}{*}{\begin{tabular}{l}
Advance \\
d
\end{tabular}} & \multicolumn{3}{|l|}{Advanced search settings, specified as a structure with the following fields:} \\
\hline & Field Name & Description & Default \\
\hline & GnPinvCons tant & Jacobian matrix singular value threshold, specified as a positive scalar. Singular values of the Jacobian matrix that are smaller than GnPinvConstant*max(size(J)*norm(J)*eps) are discarded when computing the search direction. Applicable when SearchMethod is 'gn'. & 10000 \\
\hline & InitialGna Tolerance & Initial value of gamma, specified as a positive scalar. Applicable when SearchMethod is 'gna'. & 0.0001 \\
\hline & LMStartVal ue & Starting value of search-direction length \(d\) in the Levenberg-Marquardt method, specified as a positive scalar. Applicable when SearchMethod is ' 1 m '. & 0.001 \\
\hline & LMStep & Size of the Levenberg-Marquardt step, specified as a positive integer. The next value of the search-direction length \(d\) in the Levenberg-Marquardt method is LMStep times the previous one. Applicable when SearchMethod is ' lm '. & 2 \\
\hline & MaxBisecti ons & Maximum number of bisections used for line search along the search direction, specified as a positive integer. & 25 \\
\hline & MaxFunctio nEvaluatio ns & Maximum number of calls to the model file, specified as a positive integer. Iterations stop if the number of calls to the model file exceeds this value. & Inf \\
\hline & MinParamet erChange & Smallest parameter update allowed per iteration, specified as a nonnegative scalar. & 0 \\
\hline & RelativeIm provement & Relative improvement threshold, specified as a nonnegative scalar. Iterations stop if the relative improvement of the criterion function is less than this value. & 0 \\
\hline & StepReduct ion & \begin{tabular}{l}
Step reduction factor, specified as a positive scalar that is greater than 1 . The suggested parameter update is reduced by the factor StepReduction after each try. This reduction continues until MaxBisections tries are completed or a lower value of the criterion function is obtained. \\
StepReduction is not applicable for SearchMethod ' lm' (Levenberg-Marquardt method).
\end{tabular} & 2 \\
\hline
\end{tabular}

\section*{SearchOptions Structure When SearchMethod is Specified as ' Isqnonlin'}
\begin{tabular}{|l|l|l|}
\hline \begin{tabular}{l} 
Field \\
Name
\end{tabular} & Description & Default \\
\hline \begin{tabular}{l} 
Function \\
Toleranc \\
e
\end{tabular} & \begin{tabular}{l} 
Termination tolerance on the loss function that the software \\
minimizes to determine the estimated parameter values, \\
specified as a positive scalar.
\end{tabular} & \(1 \mathrm{e}-5\) \\
\hline \begin{tabular}{l} 
The value of FunctionTolerance is the same as that of \\
opt.Search0ptions. Advanced. TolFun.
\end{tabular} & (e-6 \\
\hline \begin{tabular}{l} 
StepTole \\
rance
\end{tabular} & \begin{tabular}{l} 
Termination tolerance on the estimated parameter values, \\
specified as a positive scalar.
\end{tabular} & \begin{tabular}{l} 
The value of StepTolerance is the same as that of \\
opt. SearchOptions . Advanced. TolX.
\end{tabular}
\end{tabular}

SearchOptions Structure When SearchMethod is Specified as 'fmincon'
\begin{tabular}{|c|c|c|}
\hline Field Name & Description & Default \\
\hline Algorithm & \begin{tabular}{l}
fmincon optimization algorithm, specified as one of the following: \\
- 'sqp' - Sequential quadratic programming algorithm. The algorithm satisfies bounds at all iterations, and it can recover from NaN or Inf results. It is not a large-scale algorithm. For more information, see "Large-Scale vs. MediumScale Algorithms" (Optimization Toolbox). \\
- 'trust-regionreflective' - Subspace trust-region method based on the interior-reflective Newton method. It is a largescale algorithm. \\
- 'interior-point' -Large-scale algorithm that requires Optimization Toolbox software. The algorithm satisfies bounds at all iterations, and it can recover from NaN or Inf results. \\
- 'active-set' - Requires Optimization Toolbox software. The algorithm can take large steps, which adds speed. It is not a large-scale algorithm. \\
For more information about the algorithms, see "Constrained Nonlinear Optimization Algorithms" (Optimization Toolbox) and "Choosing the Algorithm" (Optimization Toolbox).
\end{tabular} & 'sqp' \\
\hline
\end{tabular}
\begin{tabular}{|l|l|l|}
\hline Field Name & Description & Default \\
\hline FunctionTolerance & \begin{tabular}{l} 
Termination tolerance on the \\
loss function that the software \\
minimizes to determine the \\
estimated parameter values, \\
specified as a positive scalar.
\end{tabular} & \(1 \mathrm{e}-6\) \\
\hline StepTolerance & \begin{tabular}{l} 
Termination tolerance on the \\
estimated parameter values, \\
specified as a positive scalar.
\end{tabular} & \(1 \mathrm{e}-6\) \\
\hline MaxIterations & \begin{tabular}{l} 
Maximum number of iterations \\
during loss function \\
minimization, specified as a \\
positive integer. The iterations \\
stop when MaxIterations is \\
reached or another stopping \\
criterion is satisfied, such as \\
FunctionTolerance.
\end{tabular} & 100 \\
\hline
\end{tabular}

\section*{Advanced - Additional advanced options}
structure
Additional advanced options, specified as a structure with the following fields:
- ErrorThreshold - Specifies when to adjust the weight of large errors from quadratic to linear.

Errors larger than ErrorThreshold times the estimated standard deviation have a linear weight in the loss function. The standard deviation is estimated robustly as the median of the absolute deviations from the median of the prediction errors and divided by 0.7. For more information on robust norm choices, see section 15.2 of [2].

ErrorThreshold \(=0\) disables robustification and leads to a purely quadratic loss function. When estimating with frequency-domain data, the software sets ErrorThreshold to zero. For time-domain data that contains outliers, try setting ErrorThreshold to 1.6.

Default: 0
- MaxSize - Specifies the maximum number of elements in a segment when input-output data is split into segments.

MaxSize must be a positive integer.
Default: 250000
- StabilityThreshold - Specifies thresholds for stability tests.

StabilityThreshold is a structure with the following fields:
- s-Specifies the location of the right-most pole to test the stability of continuous-time models. A model is considered stable when its right-most pole is to the left of \(s\).

\section*{Default: 0}
- \(z\) - Specifies the maximum distance of all poles from the origin to test stability of discretetime models. A model is considered stable if all poles are within the distance \(z\) from the origin.

Default: \(1+\) sqrt(eps)
- AutoInitThreshold - Specifies when to automatically estimate the initial condition.

The initial condition is estimated when
\[
\frac{\left\|y_{p, z}-y_{\text {meas }}\right\|}{\left\|y_{p, e}-y_{\text {meas }}\right\|}>\text { AutoInitThreshold }
\]
- \(y_{\text {meas }}\) is the measured output.
- \(y_{p, z}\) is the predicted output of a model estimated using zero initial conditions.
- \(y_{p, e}\) is the predicted output of a model estimated using estimated initial conditions.

Applicable when InitialCondition is 'auto'.
Default: 1.05

\section*{Output Arguments}

\section*{opt - Options set for armax}
armax0ptions option set
Option set for armax, returned as an armaxOptions option set.

\section*{Examples}

\section*{Create Default Options Set for ARMAX Estimation}
```

opt = armaxOptions;

```

\section*{Specify Options for ARMAX Estimation}

Create an option set for armax to use the 'simulation' Focus and to set the Display to 'on'.
```

opt = armaxOptions('Focus','simulation','Display','on');

```

Alternatively, use dot notation to set the values of opt.
```

opt = armaxOptions;
opt.Focus = 'simulation';
opt.Display = 'on';

```

\section*{Compatibility Considerations}

\section*{Renaming of Estimation and Analysis Options}

The names of some estimation and analysis options were changed in R2018a. Prior names still work. For details, see the R2018a release note "Renaming of Estimation and Analysis Options".

\section*{References}
[1] Wills, Adrian, B. Ninness, and S. Gibson. "On Gradient-Based Search for Multivariable System Estimates". Proceedings of the 16th IFAC World Congress, Prague, Czech Republic, July 3-8, 2005. Oxford, UK: Elsevier Ltd., 2005.
[2] Ljung, L. System Identification: Theory for the User. Upper Saddle River, NJ: Prentice-Hall PTR, 1999.

\section*{See Also}
armax|idfilt
Topics
"Loss Function and Model Quality Metrics"

Introduced in R2012a

\section*{arOptions}

Option set for ar

\section*{Syntax}
opt = arOptions
opt \(=\) arOptions(Name, Value)

\section*{Description}
opt \(=\) arOptions creates the default options set for ar.
opt \(=\) arOptions(Name, Value) creates an option set with the options specified by one or more Name, Value pair arguments.

\section*{Input Arguments}

\section*{Name-Value Pair Arguments}

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, ... , NameN, ValueN.

\section*{Approach}

Technique used for AR model estimation.
Approach is specified as one of the following values:
- 'fb' - Forward-backward approach.
- 'ls' - Least-squares method.
- 'yw' - Yule-Walker approach.
- 'burg' - Burg's method.
- 'gl' - Geometric lattice method.

Default: 'fb'
Window
Data windowing technique.
Window determines how the data outside the measured time interval (past and future values) is handled.

Window is specified as one of the following values:
- 'now' - No windowing.
- 'prw' - Pre-windowing.
- 'pow' - Post-windowing.
- 'ppw' - Pre- and post-windowing.

This option is ignored when you use the Yule-Walker approach.
Default: 'now'

\section*{DataOffset}

Data offset level that is removed before estimation of parameters.
Specify Data0ffset as a double scalar. For multiexperiment data, specify Data0ffset as a vector of length \(N e\), where \(N e\) is the number of experiments. Each entry of the vector is subtracted from the corresponding data.

Default: [ ] (no offsets)

\section*{MaxSize}

Specifies the maximum number of elements in a segment when input/output data is split into segments.

If larger matrices are needed, the software uses loops for calculations. Use this option to manage the trade-off between memory management and program execution speed. The original data matrix must be smaller than the matrix specified by MaxSize.

MaxSize must be a positive integer.
Default: 250000

\section*{Output Arguments}

\section*{opt}

Option set containing the specified options for ar.

\section*{Examples}

\section*{Create Default Options Set for AR Estimation \\ opt = arOptions;}

\section*{Specify Options for AR Estimation}

Create an options set for ar using the least squares algorithm for estimation. Set Window to 'ppw'.
opt = arOptions('Approach','ls','Window','ppw');
Alternatively, use dot notation to set the values of opt.
opt = arOptions;
opt.Approach = 'ls';
opt.Window = 'ppw';

\section*{See Also}
ar

Introduced in R2012a

\section*{arx}

Estimate parameters of ARX, ARIX, AR, or ARI model

\section*{Syntax}
```

sys = arx(data,[na nb nk])
sys = arx(data,[na nb nk],Name,Value)
sys = arx(data,[na nb nk],___,opt)
[sys,ic] = arx(

```
\(\qquad\)
``` )
```


## Description

sys = arx(data, [na nb nk]) estimates the parameters of an ARX on page 1-67 or an AR on page 1-68 idpoly model sys using a least-squares method and the polynomial orders specified in [na nb nk]. The model properties include covariances (parameter uncertainties) and goodness of fit between the estimated and measured data.
sys = arx(data,[na nb nk],Name, Value) specifies additional options using one or more namevalue pair arguments. For instance, using the name-value pair argument 'IntegrateNoise' , 1 estimates an ARIX on page 1-68 or ARI structure model, which is useful for systems with nonstationary disturbances.
sys = arx(data,[na nb nk],__, opt) specifies estimation options using the option set opt. Specify opt after all other input arguments.
[sys,ic] = arx( __ ) returns the estimated initial conditions as an initialCondition object. Use this syntax if you plan to simulate or predict the model response using the same estimation input data and then compare the response with the same estimation output data. Incorporating the initial conditions yields a better match during the first part of the simulation.

## Examples

## ARX Model

Generate output data based on a specified ARX model and use the output data to estimate the model.
Specify a polynomial model sys0 with the ARX structure. The model includes an input delay of one sample, expressed as a leading zero in the B polynomial.

```
A = [1 -1.5 0.7];
B = [0 1 0.5];
sys0 = idpoly(A,B);
```

Generate a measured input signal u that contains random binary noise and an error signal e that contains normally distributed noise. With these signals, simulate the measured output signal y of sys0.
u = iddata([],idinput(300,'rbs'));
e = iddata([],randn(300,1));
$y=\operatorname{sim}(s y s 0,[u e]) ;$

Combine $y$ and $u$ into a single iddata object $z$. Estimate a new ARX model using $z$ and the same polynomial orders and input delay as the original model.

```
z = [y,u];
sys = arx(z,[2 2 1])
sys =
Discrete-time ARX model: A(z)y(t) = B(z)u(t) + e(t)
    A(z) = 1 - 1.524 z^-1 + 0.7134 z^-2
    B(z) = z^-1 + 0.4748 z^-2
Sample time: 1 seconds
Parameterization:
    Polynomial orders: na=2 nb=2 nk=1
    Number of free coefficients: 4
    Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.
Status:
Estimated using ARX on time domain data "z".
Fit to estimation data: 81.36% (prediction focus)
FPE: 1.025, MSE: 0.9846
```

The output displays the polynomial containing the estimated parameters alongside other estimation details. Under Status, Fit to estimation data shows that the estimated model has 1 -stepahead prediction accuracy above $80 \%$.

## AR Model

Estimate a time-series AR model using the arx function. An AR model has no measured input.
Load the data, which contains the time series z 9 with noise.
load iddata9 z9
Estimate a fourth-order AR model by specifying only the na order in [na nb nk].
sys = $\operatorname{arx}(z 9,4)$;
Examine the estimated A polynomial parameters and the fit of the estimate to the data.

```
param = sys.Report.Parameters.ParVector
param = 4×I
    -0.7923
    -0.4780
    -0.0921
        0.4698
fit = sys.Report.Fit.FitPercent
fit = 79.4835
```


## ARIX Model

Estimate the parameters of an ARIX model. An ARIX model is an ARX model with integrated noise.
Specify a polynomial model sys0 with an ARX structure. The model includes an input delay of one sample, expressed as a leading zero in $B$.

```
A = [lllll
B = [0 1 0.5];
sys0 = idpoly(A,B);
```

Simulate the output signal of sys0 using the random binary input signal $u$ and the normally distributed error signal e.
u = iddata([],idinput(300,'rbs'));
e = iddata([],randn(300,1));
$y=\operatorname{sim}(s y s 0,[u$ e]);
Integrate the output signal and store the result yi in the iddata object zi.

```
yi = iddata(cumsum(y.y),[]);
zi = [yi,u];
```

Estimate an ARIX model from zi. Set the name-value pair argument 'IntegrateNoise' to true.

```
sys = arx(zi,[2 2 1],'IntegrateNoise',true);
```

Predict the model output using 5-step prediction and compare the result with yi.

```
compare(zi,sys,5)
```

5-Step Predicted Response Comparison


## ARX Model with Regularization

Use arxRegul to determine regularization constants automatically and use the values for estimating an FIR model with an order of 50 .

Obtain the lambda and $R$ values.

```
load regularizationExampleData eData;
orders = [0 50 0];
[lambda,R] = arxRegul(eData,orders);
```

Use the returned lambda and $R$ values for regularized ARX model estimation.

```
opt = arxOptions;
opt.Regularization.Lambda = lambda;
opt.Regularization.R = R;
sys = arx(eData,orders,opt);
```


## Obtain Initial Conditions

Load the data.

```
load iddatalic zli
```

Estimate a second-order ARX model sys and return the initial conditions in ic.

```
na = 2;
nb = 2;
nk = 1;
[sys,ic] = arx(zli,[na nb nk]);
iC
ic =
    initialCondition with properties:
    A: [2x2 double]
    X0: [2x1 double]
    C: [0 2]
    Ts: 0.1000
```

ic is an initialCondition object that encapsulates the free response of sys, in state-space form, to the initial state vector in X0. You can incorporate ic when you simulate sys with the zli input signal and compare the response with the zli output signal.

## Input Arguments

## data - Estimation data

iddata object | frd object | idfrd object
Estimation data, specified as an iddata object, an frd object, or an idfrd frequency-response object. For AR and ARI time-series models, the input channel in data must be empty.

## [na nb nk] - Polynomial orders and delays

integer row vector | row vector of integer matrices | scalar
Polynomial orders and delays for the model, specified as a 1-by-3 vector or vector of matrices [ na nb nk ]. The polynomial order is equal to the number of coefficients to estimate in that polynomial.

For an AR or ARI time-series model, which has no input, set [na nb nk] to the scalar na. For an example, see "AR Model" on page 1-60.

For a model with $N_{y}$ outputs and $N_{u}$ inputs:

- na is the order of polynomial $A(q)$, specified as an $N_{y}$-by- $N_{y}$ matrix of nonnegative integers.
- nb is the order of polynomial $B(q)+1$, specified as an $N_{y}$-by- $N_{u}$ matrix of nonnegative integers.
- nk is the input-output delay, also known as the transport delay, specified as an $N_{y}$-by- $N_{u}$ matrix of nonnegative integers. nk is represented in ARX models by fixed leading zeros in the $B$ polynomial.

For instance, suppose that without transport delays, sys.b is [5 6].

- Because sys. $\mathrm{b}+1$ is a second-order polynomial, $\mathrm{nb}=2$.
- Specify a transport delay of $n k=3$. Specifying this delay adds three leading zeros to sys.b so that sys.b is now [0 0056 6], while nb remains equal to 2 .
- These coefficients represent the polynomial $B(q)=5 q^{-3}+6 q^{-4}$.

You can also implement transport delays using the name-value pair argument 'IODelay '.

Example: arx(data, [2 1 1]) computes, from an iddata object, a second-order ARX model with one input channel that has an input delay of one sample.

## opt - Estimation options

arx0ptions option set
Estimation options for ARX model identification, specified as an arOptions option set. Options specified by opt include the following:

- Initial condition handling - Use this option only for frequency-domain data. For time-domain data, the signals are shifted such that unmeasured signals are never required in the predictors.
- Input and output data offsets - Use these options to remove offsets from time-domain data during estimation.
- Regularization - Use this option to control the tradeoff between bias and variance errors during the estimation process.

For more information, see arxOptions. For an example, see "ARX Model with Regularization" on page 1-62.

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

## Example: 'IntegrateNoise', true adds an integrator in the noise source s

## InputDelay - Input delays

0 (default) | integer scalar | positive integer vector
Input delays expressed as integer multiples of the sample time, specified as the comma-separated pair consisting of 'InputDelay' and one of the following:

- $N_{u}$-by-1 vector, where $N_{u}$ is the number of inputs - Each entry is a numerical value representing the input delay for the corresponding input channel.
- Scalar value - Apply the same delay to all input channels.
 input channel that has an input delay of three samples.


## IODelay - Transport delays

0 (default) | integer scalar | integer array
Transport delays for each input-output pair, expressed as integer multiples of the sample time, and specified as the comma-separated pair consisting of 'IODelay' and one of the following:

- $N_{y}$-by- $N_{u}$ matrix, where $N_{y}$ is the number of outputs and $N_{u}$ is the number of inputs - Each entry is an integer value representing the transport delay for the corresponding input-output pair.
- Scalar value - Apply the same delay is applied to all input-output pairs. This approach is useful when the input-output delay parameter nk results in a large number of fixed leading zeros in the $B$
polynomial. You can factor out max (nk-1,0) lags by moving those lags from nk into the 'IODelay' value.

For instance, suppose that you have a system with two inputs, where the first input has a delay of three samples and the second input has a delay of six samples. Also suppose that the $B$ polynomials for these inputs are order $n$. You can express these delays using the following:

- nk $=[36]-$ This results in B polynomials of [0 00 b11 ... bln] and [0 00000 b21 ... b2n].
- nk = [3 6] and 'IODelay',3 - This results in B polynomials of [b11 ... b1n] and [0 0 0 b21 ... b2n].


## IntegrateNoise - Addition of integrators in noise channel

false (default) | logical vector
Addition of integrators in the noise channel, specified as the comma-separated pair consisting of 'IntegrateNoise ' and a logical vector of length $N y$, where $N y$ is the number of outputs.

Setting ' IntegrateNoise' to true for a particular output creates an ARIX on page 1-68 or ARI model for that channel. Noise integration is useful in cases where the disturbance is nonstationary.

When using 'IntegrateNoise', you must also integrate the output channel data. For an example, see "ARIX Model" on page 1-61.

## Output Arguments

sys - ARX model
idpoly object
ARX model that fits the estimation data, returned as a discrete-time idpoly object. This model is created using the specified model orders, delays, and estimation options.

Information about the estimation results and options used is stored in the Report property of the model. Report has the following fields.

| Report <br> Field | Description |
| :--- | :--- |
| Status | Summary of the model status, which indicates whether the model was created by <br> construction or obtained by estimation. |
| Method | Estimation command used. |
| InitialCo <br> ndition | Handling of initial conditions during model estimation, returned as one of the following <br> values: |
| - 'zero' - The initial conditions were set to zero. |  |
| - 'estimate ' - The initial conditions were treated as independent estimation |  |
| parameters. |  |
| This field is especially useful to view how the initial conditions were handled when the |  |
| InitialCondition option in the estimation option set is 'auto'. |  |


| Report <br> Field | Description |  |
| :--- | :--- | :--- |
| Fit | Quantitative assessment of the estimation, returned as a structure. See "Loss Function <br> and Model Quality Metrics" for more information on these quality metrics. The <br> structure has the following fields: |  |
|  | Field | Description |
|  | FitPerce <br> nt | Normalized root mean squared error (NRMSE) measure of how well the <br> response of the model fits the estimation data, expressed as the <br> percentage fit = 100(1-NRMSE). |
|  | LossFcn | Value of the loss function when the estimation completes. |
|  | MSE <br> Mean squared error (MSE) measure of how well the response of the <br> model fits the estimation data. |  |
|  | FPE | Final prediction error for the model. |
|  | AIC | Raw Akaike Information Criteria (AIC) measure of model quality. |
|  | AICc | Small sample-size corrected AIC. |
| nAIC | Normalized AIC. |  |
|  | BIC | Bayesian Information Criteria (BIC). |
| Parameter <br> s | Estimated values of model parameters. <br> OptionsUs <br> edOption set used for estimation. If no custom options were configured, this is a set of <br> default options. See arx0ptions for more information. |  |
| RandState | State of the random number stream at the start of estimation. Empty, [ ], if <br> randomization was not used during estimation. For more information, see rng. |  |


| Report Field | Description |  |
| :---: | :---: | :---: |
| DataUsed | Attributes of the data used for estimation, returned as a structure with the following fields: |  |
|  | Field | Description |
|  | Name | Name of the data set. |
|  | Type | Data type. |
|  | Length | Number of data samples. |
|  | Ts | Sample time. |
|  | InterSam ple | Input intersample behavior, returned as one of the following values: <br> - 'zoh ' - Zero-order hold maintains a piecewise-constant input signal between samples. <br> - ' foh ' - First-order hold maintains a piecewise-linear input signal between samples. <br> - 'bl' - Band-limited behavior specifies that the continuous-time input signal has zero power above the Nyquist frequency. |
|  | Input0ff set | Offset removed from time-domain input data during estimation. For nonlinear models, it is [ ]. |
|  | Output0f fset | Offset removed from time-domain output data during estimation. For nonlinear models, it is []. |

For more information on using Report, see "Estimation Report".

## ic - Initial conditions

initialCondition object | object array of initialCondition values
Estimated initial conditions, returned as an initialCondition object or an object array of initialCondition values.

- For a single-experiment data set, ic represents, in state-space form, the free response of the transfer function model ( $A$ and $C$ matrices) to the estimated initial states ( $x_{0}$ ).
- For a multiple-experiment data set with $N_{e}$ experiments, ic is an object array of length $N_{e}$ that contains one set of initialCondition values for each experiment.

For more information, see initialCondition. For an example of using this argument, see "Obtain Initial Conditions" on page 1-62.

## More About

## ARX Structure

The ARX model name stands for Autoregressive with Extra Input, because, unlike the AR model, the ARX model includes an input term. ARX is also known as Autoregressive with Exogenous Variables, where the exogenous variable is the input term. The ARX model structure is given by the following equation:

$$
\begin{aligned}
& y(t)+a_{1} y(t-1)+\ldots+a_{n a} y(t-n a)= \\
& b_{1} u(t-n k)+\ldots+b_{n b} u(t-n b-n k+1)+e(t)
\end{aligned}
$$

The parameters na and $n b$ are the orders of the ARX model, and $n k$ is the delay.

- $y(t)$ - Output at time $t$
- $n_{a}$ - Number of poles
- $n_{b}$ - Number of zeros
- $n_{k}$ - Number of input samples that occur before the input affects the output, also called the dead time in the system
- $y(t-1) \ldots y\left(t-n_{a}\right)-$ Previous outputs on which the current output depends
- $u\left(t-n_{k}\right) \ldots u\left(t-n_{k}-n_{b}+1\right)-$ Previous and delayed inputs on which the current output depends
- $e(t)$ - White-noise disturbance value

A more compact way to write the difference equation is

$$
A(q) y(t)=B(q) u\left(t-n_{k}\right)+e(t)
$$

q is the delay operator. Specifically,

$$
\begin{aligned}
& A(q)=1+a_{1} q^{-1}+\ldots+a_{n_{a}} q^{-n_{a}} \\
& B(q)=b_{1}+b_{2} q^{-1}+\ldots+b_{n_{b}} q^{-n_{b}+1}
\end{aligned}
$$

## ARIX Model

The ARIX (Autoregressive Integrated with Extra Input) model is an ARX model with an integrator in the noise channel. The ARIX model structure is given by the following equation:

$$
A(q) y(t)=B(q) u(t-n k)+\frac{1}{1-q^{-1}} e(t)
$$

where $\frac{1}{1-q^{-1}}$ is the integrator in the noise channel, $e(t)$.

## AR Time-Series Models

For time-series data that contains no inputs, one output, and the $A$ polynomial order na, the model has an AR structure of order na.

The AR (Autoregressive) model structure is given by the following equation:

$$
A(q) y(t)=e(t)
$$

## ARI Model

The ARI (Autoregressive Integrated) model is an AR model with an integrator in the noise channel.
The ARI model structure is given by the following equation:

$$
A(q) y(t)=\frac{1}{1-q^{-1}} e(t)
$$

## Multiple-Input, Single-Output Models

For multiple-input, single-output systems (MISO) with $n u$ inputs, $n b$ and $n k$ are row vectors where the $i$ th element corresponds to the order and delay associated with the ith input in column vector $u(t)$. Similarly, the coefficients of the $B$ polynomial are row vectors. The ARX MISO structure is then given by the following equation:

$$
A(q) y(t)=B_{1}(q) u_{1}\left(t-n k_{1}\right)+B_{2}(q) u_{2}\left(t-n k_{2}\right)+\cdots+B_{n u}(q) u_{n u}\left(t-n k_{n u}\right)
$$

## Multiple-Input, Multiple-Output Models

For multiple-input, multiple-output systems, na, nb, and nk contain one row for each output signal.
In the multiple-output case, arx minimizes the trace of the prediction error covariance matrix, or the norm

$$
\sum_{t=1}^{N} e^{T}(t) e(t)
$$

To transform this norm to an arbitrary quadratic norm using a weighting matrix Lambda

$$
\sum_{t=1}^{N} e^{T}(t) \Lambda^{-1} e(t)
$$

use the following syntax:

```
opt = arxOptions('OutputWeight',inv(lambda))
m = arx(data,orders,opt)
```


## Initial Conditions

For time-domain data, the signals are shifted such that unmeasured signals are never required in the predictors. Therefore, there is no need to estimate initial conditions.

For frequency-domain data, it might be necessary to adjust the data by initial conditions that support circular convolution.

Set the ' InitialCondition' estimation option (see arxOptions) to one of the following values:

- 'zero' - No adjustment
- 'estimate' - Perform adjustment to the data by initial conditions that support circular convolution
- 'auto' - Automatically choose 'zero' or 'estimate' based on the data


## Algorithms

QR factorization solves the overdetermined set of linear equations that constitutes the least-squares estimation problem.

Without regularization, the ARX model parameters vector $\theta$ is estimated by solving the normal equation

$$
\left(J^{T} J\right) \theta=J^{T} y
$$

where $J$ is the regressor matrix and $y$ is the measured output. Therefore,
$\theta=\left(J^{T} J\right)^{-1} J^{T} y$
Using regularization adds the regularization term
$\theta=\left(J^{T} J+\lambda R\right)^{-1} J^{T} y$
where $\lambda$ and $R$ are the regularization constants. For more information on the regularization constants, see arx0ptions.

When the regression matrix is larger than the MaxSize specified in arxOptions, the data is segmented and QR factorization is performed iteratively on the data segments.

## See Also

ar|armax|arx0ptions|arxRegul|arxstruc|iddata|idfrd|idinput|iv4

## Topics

"What Are Polynomial Models?"
"What Are Time Series Models?"
"Estimate Polynomial Models at the Command Line"
"Regularized Estimates of Model Parameters"
"Estimating Models Using Frequency-Domain Data"
"Apply Initial Conditions when Simulating Identified Linear Models"
Introduced before R2006a

## arxOptions

Option set for arx

## Syntax

```
opt = arxOptions
```

opt = arxOptions(Name, Value)

## Description

opt $=$ arxOptions creates the default options set for arx.
opt $=$ arxOptions(Name, Value) creates an option set with the options specified by one or more Name, Value pair arguments.

## Input Arguments

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

## InitialCondition - Handling of initial conditions

'auto' (default)|'zero'|'estimate'
Handling of initial conditions during estimation using frequency-domain data, specified as the comma-separated pair consisting of 'InitialCondition' and one of the following values:

- 'zero' - The initial conditions are set to zero.
- 'estimate' - The initial conditions are treated as independent estimation parameters.
- 'auto ' - The software chooses the method to handle initial conditions based on the estimation data.


## Focus - Error to be minimized

'prediction' (default)|'simulation'
Error to be minimized in the loss function during estimation, specified as the comma-separated pair consisting of 'Focus ' and one of the following values:

- 'prediction' - The one-step ahead prediction error between measured and predicted outputs is minimized during estimation. As a result, the estimation focuses on producing a good predictor model.
- 'simulation' - The simulation error between measured and simulated outputs is minimized during estimation. As a result, the estimation focuses on making a good fit for simulation of model response with the current inputs.

The Focus option can be interpreted as a weighting filter in the loss function. For more information, see "Loss Function and Model Quality Metrics".

## WeightingFilter - Weighting prefilter

[ ] (default) | vector | matrix | cell array | linear system
Weighting prefilter applied to the loss function to be minimized during estimation. To understand the effect of WeightingFilter on the loss function, see "Loss Function and Model Quality Metrics".

Specify WeightingFilter as one of the following values:

- [ ] - No weighting prefilter is used.
- Passbands - Specify a row vector or matrix containing frequency values that define desired passbands. You select a frequency band where the fit between estimated model and estimation data is optimized. For example, [ $\mathrm{wl} \mathrm{l}, \mathrm{wh}$ ] where wl and $w h$ represent lower and upper limits of a passband. For a matrix with several rows defining frequency passbands, [w1l,w1h;w2l,w2h;w3l,w3h; ...], the estimation algorithm uses the union of the frequency ranges to define the estimation passband.

Passbands are expressed in rad/TimeUnit for time-domain data and in FrequencyUnit for frequency-domain data, where TimeUnit and FrequencyUnit are the time and frequency units of the estimation data.

- SISO filter - Specify a single-input-single-output (SISO) linear filter in one of the following ways:
- A SISO LTI model
- $\{A, B, C, D\}$ format, which specifies the state-space matrices of a filter with the same sample time as estimation data.
- \{numerator, denominator\} format, which specifies the numerator and denominator of the filter as a transfer function with same sample time as estimation data.

This option calculates the weighting function as a product of the filter and the input spectrum to estimate the transfer function.

- Weighting vector - Applicable for frequency-domain data only. Specify a column vector of weights. This vector must have the same length as the frequency vector of the data set, Data.Frequency. Each input and output response in the data is multiplied by the corresponding weight at that frequency.


## EnforceStability - Control whether to enforce stability of model <br> false (default) | true

Control whether to enforce stability of estimated model, specified as the comma-separated pair consisting of 'EnforceStability' and either true or false.

This option is not available for multi-output models with a non-diagonal $A$ polynomial array.

## Data Types: logical

## EstimateCovariance - Control whether to generate parameter covariance data true (default) | false

Controls whether parameter covariance data is generated, specified as true or false.
If EstimateCovariance is true, then use getcov to fetch the covariance matrix from the estimated model.

## Display - Specify whether to display the estimation progress <br> 'off' (default)|'on'

Specify whether to display the estimation progress, specified as one of the following values:

- 'on' - Information on model structure and estimation results are displayed in a progress-viewer window.
- 'off' - No progress or results information is displayed.


## InputOffset - Removal of offset from time-domain input data during estimation

[ ] (default) | vector of positive integers $\mid$ matrix
Removal of offset from time-domain input data during estimation, specified as the comma-separated pair consisting of 'InputOffset' and one of the following:

- A column vector of positive integers of length $N u$, where $N u$ is the number of inputs.
- [] - Indicates no offset.
- $N u$-by-Ne matrix - For multi-experiment data, specify Input0ffset as an $N u$-by- $N e$ matrix. $N u$ is the number of inputs, and $N e$ is the number of experiments.

Each entry specified by Input0ffset is subtracted from the corresponding input data.

## OutputOffset - Removal of offset from time-domain output data during estimation

[ ] (default) | vector | matrix
Removal of offset from time-domain output data during estimation, specified as the comma-separated pair consisting of 'OutputOffset' and one of the following:

- A column vector of length $N y$, where $N y$ is the number of outputs.
- [] - Indicates no offset.
- Ny-by-Ne matrix - For multi-experiment data, specify OutputOffset as a $N y$-by-Ne matrix. Ny is the number of outputs, and $N e$ is the number of experiments.

Each entry specified by Output0ffset is subtracted from the corresponding output data.

## OutputWeight - Weight of prediction errors in multi-output estimation

[ ] (default) | positive semidefinite, symmetric matrix
Weight of prediction errors in multi-output estimation, specified as one of the following values:

- Positive semidefinite, symmetric matrix (W). The software minimizes the trace of the weighted prediction error matrix trace (E'*E*W/N) where:
- $E$ is the matrix of prediction errors, with one column for each output, and $W$ is the positive semidefinite, symmetric matrix of size equal to the number of outputs. Use $W$ to specify the relative importance of outputs in multiple-output models, or the reliability of corresponding data.
- $\quad N$ is the number of data samples.
- [ ] - No weighting is used. Specifying as [] is the same as eye ( Ny ) , where Ny is the number of outputs.

This option is relevant only for multi-output models.

## Regularization - Options for regularized estimation of model parameters

[ ] (default) | positive semidefinite, symmetric matrix

Options for regularized estimation of model parameters, specified as a structure with the following fields:

- Lambda - Constant that determines the bias versus variance tradeoff.

Specify a positive scalar to add the regularization term to the estimation cost.
The default value of zero implies no regularization.
Default: 0

- R - Weighting matrix.

Specify a positive scalar or a positive definite matrix. The length of the matrix must be equal to the number of free parameters ( $n p$ ) of the model. For ARX model, $n p=\operatorname{sum}(\operatorname{sum}([n a n b])$.

Default: 1

- Nominal - This option is not used for ARX models.

Default: 0
Use arxRegul to automatically determine Lambda and $R$ values.
For more information on regularization, see "Regularized Estimates of Model Parameters".

## Advanced - Additional advanced options

structure
Additional advanced options, specified as a structure with the following fields:

- MaxSize - Specifies the maximum number of elements in a segment when input-output data is split into segments.

MaxSize must be a positive integer.
Default: 250000

- StabilityThreshold - Specifies thresholds for stability tests.

StabilityThreshold is a structure with the following fields:

- $s$ - Specifies the location of the right-most pole to test the stability of continuous-time models. A model is considered stable when its right-most pole is to the left of $s$.

Default: 0

- z - Specifies the maximum distance of all poles from the origin to test stability of discretetime models. A model is considered stable if all poles are within the distance $z$ from the origin.

Default: 1+sqrt(eps)

## Output Arguments

opt - Options set for arx
arx0ptions option set
Option set for arx, returned as an arx0ptions option set.

## Examples

## Create Default Options Set for ARX Estimation <br> opt = arxOptions;

## Specify Options for ARX Estimation

Create an options set for arx using zero initial conditions for estimation. Set Display to ' on '.
opt = arxOptions('InitialCondition','zero','Display','on');
Alternatively, use dot notation to set the values of opt.

```
opt = arxOptions;
opt.InitialCondition = 'zero';
opt.Display = 'on';
```


## Compatibility Considerations

## Renaming of Estimation and Analysis Options

The names of some estimation and analysis options were changed in R2018a. Prior names still work. For details, see the R2018a release note "Renaming of Estimation and Analysis Options".

See Also<br>arx|arxRegul|idfilt<br>Topics<br>"Loss Function and Model Quality Metrics"<br>Introduced in R2012a

## arxRegul

Determine regularization constants for ARX model estimation

## Syntax

```
[lambda,R] = arxRegul(data,orders)
[lambda,R] = arxRegul(data,orders,options)
[lambda,R] = arxRegul(data,orders,Name,Value)
[lambda,R] = arxRegul(data,orders,options,Name,Value)
```


## Description

[lambda,R] = arxRegul(data,orders) returns the regularization constants used for ARX model estimation. Use the regularization constants in arxOptions to configure the regularization options for ARX model estimation.
[lambda,R] = arxRegul(data,orders,options) specifies regularization options such as regularization kernel and I/O offsets.
[lambda,R] = arxRegul(data,orders,Name, Value) specifies model structure attributes, such as noise integrator and input delay, using one or more Name, Value pair arguments.
[lambda,R] = arxRegul(data,orders,options,Name, Value) specifies both regularization options and model structure attributes.

## Examples

## Determine Regularization Constants for ARX Model Estimation Using Default Kernel

```
load iddatal z1;
orders = [10 10 1];
[Lambda,R] = arxRegul(z1,orders);
```

The ARX model is estimated using the default regularization kernel TC.
Use the Lambda and $R$ values for ARX model estimation.

```
opt = arxOptions;
opt.Regularization.Lambda = Lambda;
opt.Regularization.R = R;
model = arx(zl,orders,opt);
```


## Specify a Regularization Kernel

Specify ' DC' as the regularization kernel and obtain a regularized ARX model of order [|10 10 1|].

```
load iddatal z1;
orders = [10 10 1];
```

```
option = arxRegulOptions('RegularizationKernel','DC');
[Lambda,R] = arxRegul(z1,orders,option);
```

Use the Lambda and R values for ARX model estimation.

```
arx0pt = arx0ptions;
arxOpt.Regularization.Lambda = Lambda;
arx0pt.Regularization.R = R;
model = arx(z1,orders,arxOpt);
```


## Specify Noise Source Integrator

Specify to include a noise source integrator in the noise component of the model.

```
load iddatal z1;
orders = [10 10 1];
[Lambda,R] = arxRegul(z1,orders,'IntegrateNoise',true);
```


## Specify Regularization Kernel And Noise Integrator

Specify the regularization kernel and include a noise source integrator in the noise component of the model.

```
load iddatal z1;
orders = [10 10 1];
opt = arxRegulOptions('RegularizationKernel','DC');
[Lambda,R] = arxRegul(z1,orders,opt,'IntegrateNoise',true);
```


## Input Arguments

## data - Estimation data

iddata object
Estimation data, specified as an iddata object.

## orders - ARX model orders

matrix of nonnegative integers
ARX model orders [na nb nc], specified as a matrix of nonnegative integers. See the arx reference page for more information on model orders.

## options - Regularization options

arxRegulOptions options set
Regularization options, specified as an options set you create using arxRegulOptions.

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Example: [Lambda, R] = arxRegul(z1,orders,option,'InputDelay',10);

## InputDelay - Input delay

0 (default)| positive integer
Input delay, specified as a positive, nonzero numeric value representing the number of samples.
Example: [Lambda, R] = arxRegul(z1,orders,'InputDelay',10) ;
Data Types: double

## IntegrateNoise - Noise source integrator

false (default) | true
Noise source integrator, specified as a logical. Specifies whether the noise source e(t) should contain an integrator. The default is false, indicating the noise integrator is off. To turn it on, change the value to true.

Example: [Lambda, R] = arxRegul(z1,orders,'IntegrateNoise',true);
Data Types: logical

## Output Arguments

## lambda - Constant that determines bias versus variance trade-off positive scalar

Constant that determines the bias versus variance trade-off, returned as a positive scalar.

## R - Weighting matrix

vector of nonnegative numbers | square positive semi-definite matrix
Weighting matrix, returned as a vector of nonnegative numbers or a positive definite matrix.

## Algorithms

Without regularization, the ARX model parameters vector $\theta$ is estimated by solving the normal equation
$\left(J^{T} J\right) \theta=J^{T} y$
where $J$ is the regressor matrix and $y$ is the measured output. Therefore,
$\theta=\left(J^{T} J\right)^{-1} J^{T} y$
Using regularization adds the regularization term
$\theta=\left(J^{T} J+\lambda R\right)^{-1} J^{T} y$
where $\lambda$ and $R$ are the regularization constants. For more information on the regularization constants, see arx0ptions.

## References

[1] T. Chen, H. Ohlsson, and L. Ljung. "On the Estimation of Transfer Functions, Regularizations and Gaussian Processes - Revisited", Automatica, Volume 48, August 2012.

## See Also

arx|arx0ptions|arxRegulOptions

## Topics

"Estimate Regularized ARX Model Using System Identification App"
"Regularized Estimates of Model Parameters"

Introduced in R2013b

## arxRegulOptions

Option set for arxRegul

## Syntax

opt = arxRegulOptions
opt = arxRegulOptions(Name,Value)

## Description

opt $=$ arxRegulOptions creates a default option set for arxRegul.
opt = arxRegulOptions(Name, Value) creates an options set with the options specified by one or more name-value pair arguments.

## Examples

## Create Default Options Set for Determining Regularization Constants opt = arxRegulOptions;

## Specify Regularizing Kernel for ARX Model Estimation

```
opt = arxRegulOptions('RegularizationKernel','DC');
```


## Input Arguments

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Valuel, . . . NameN, ValueN.
Example: option = arxRegulOptions('RegularizationKernel', 'DC') specifies 'DC' as the regularization kernel.

## RegularizationKernel - Regularization kernel <br> 'TC' (default)|'SE'|'SS'|'HF'|'DI'|'DC'

Regularization kernel, specified as one of the following values:

- 'TC' - Tuned and correlated kernel
- 'SE' - Squared exponential kernel
- 'SS' - Stable spline kernel
- 'HF ' - High frequency stable spline kernel
- 'DI' - Diagonal kernel
- 'DC' - Diagonal and correlated kernel

The specified kernel is used for regularized estimation of impulse response for all input-output channels. Regularization reduces variance of estimated model coefficients and produces a smoother response by trading variance for bias.

For more information about these choices, see [1].
Data Types: char

## Input0ffset - Offset levels present in the input signals of estimation data

[ ] (default) | vector | matrix
Offset levels present in the input signals of time-domain estimation data, specified as one of the following:

- An Nu-element column vector, where Nu is the number of inputs. For multi-experiment data, specify a Nu-by-Ne matrix, where Ne is the number of experiments. The offset value Input0ffset ( $i, j$ ) is subtracted from the $i^{\text {th }}$ input signal of the $j^{\text {th }}$ experiment.
- [] - No offsets.

Data Types: double

## OutputOffset - Output signal offset levels

[ ] (default) | vector | matrix
Output signal offset level of time-domain estimation data, specified as one of the following:

- An Ny-element column vector, where Ny is the number of outputs. For multi-experiment data, specify a Ny-by-Ne matrix, where Ne is the number of experiments. The offset value OputOffset ( $i, j$ ) is subtracted from the $i^{\text {th }}$ output signal of the $j^{\text {th }}$ experiment.
- [] - No offsets.

The specified values are subtracted from the output signals before using them for estimation.
Data Types: double

## Advanced - Advanced estimation options

## structure

Advanced options for regularized estimation, specified as a structure with the following fields:

- MaxSize - Maximum allowable size of Jacobian matrices formed during estimation, specified as a large positive number.

Default: 250e3

- SearchMethod - Search method for estimating regularization parameters, specified as one of the following values:
- 'gn': Quasi-Newton line search.
- 'fmincon ': Trust-region-reflective constrained minimizer. In general, 'fmincon' is better than 'gn' for handling bounds on regularization parameters that are imposed automatically during estimation.

Default: 'fmincon'

## Output Arguments

opt - Regularization options
arxRegulOptions options set
Regularization options, returned as an arxRegulOptions options set.

## Compatibility Considerations

## Renaming of Estimation and Analysis Options

The names of some estimation and analysis options were changed in R2018a. Prior names still work. For details, see the R2018a release note "Renaming of Estimation and Analysis Options".

## References

[1] T. Chen, H. Ohlsson, and L. Ljung. "On the Estimation of Transfer Functions, Regularizations and Gaussian Processes - Revisited", Automatica, Volume 48, August 2012.

See Also<br>arxRegul<br>Topics<br>"Regularized Estimates of Model Parameters"<br>Introduced in R2014a

## arxstruc

Compute loss functions for single-output ARX models

## Syntax

$\mathrm{V}=\operatorname{arxstruc}(z e, z v, N N)$

## Arguments

## ze

Estimation data set can be iddata or idfrd object.
zv
Validation data set can be iddata or idfrd object.
NN
Matrix defines the number of different ARX-model structures. Each row of NN is of the form:
nn = [na nb nk]

## Description

Note Use arxstruc for single-output systems only. arxstruc supports both single-input and multiple-input systems.
$\mathrm{V}=$ arxstruc(ze,zv,NN) returns V , which contains the loss functions in its first row. The remaining rows of V contain the transpose of NN , so that the orders and delays are given just below the corresponding loss functions. The last column of V contains the number of data points in ze.

The output argument V is best analyzed using selstruc. The selection of a suitable model structure based on the information in $v$ is normally done using selstruc.

## Examples

## Generate Model-Order Combinations and Estimate Single-Input ARX Model

Create an ARX model for generating data.
$\mathrm{A}=\left[\begin{array}{lll}1 & -1.5 & 0.7\end{array}\right] ;$
$B=[0$ 1 0.5];
m 0 = idpoly(A,B);
Generate random input and additive noise signals.
u = iddata([],idinput(400,'rbs'));
e = iddata([],0.1*randn(400,1));
Simulate the model output using the defined input and error signals.
$y=\operatorname{sim}(m 0,[u \quad e]) ;$
z = [y,u];
Generate model-order combinations for estimation. Specify a delay of 1 for all models, and a model order range between 1 and 5 for na and nb.
$N N=\operatorname{struc}(1: 5,1: 5,1) ;$
Estimate ARX models and compute the loss function for each model order combination. The input data is split into estimation and validation data sets.

V = arxstruc(z(1:200),z(201:400),NN);
Select the model order with the best fit to the validation data.
order $=$ selstruc (V,0);
Estimate an ARX model of selected order.
M = arx(z,order);

## Generate Model-Order Combinations and Estimate Multi-Input ARX Model

Create estimation and validation data sets.

```
load co2data;
```

Ts $=0.5$; Sample time is 0.5 min
ze = iddata(Output_exp1,Input_exp1,Ts);
zv = iddata(Output_exp2,Input_exp2,Ts);

Generate model-order combinations for:

- $\mathrm{na}=2: 4$
- $\mathrm{nb}=2: 5$ for the first input, and 1 or 4 for the second input.
- $n k=1: 4$ for the first input, and 0 for the second input.

NN $=\operatorname{struc}(2: 4,2: 5,[14], 1: 4,0)$;
Estimate an ARX model for each model order combination.
V = arxstruc (ze,zv,NN);
Select the model order with the best fit to the validation data.

```
order = selstruc(V,0);
```

Estimate an ARX model of selected order.

```
M = arx(ze,order);
```


## Tips

Each of ze and $z v$ is an iddata object containing output-input data. Frequency-domain data and idfrd objects are also supported. Models for each of the model structures defined by NN are
estimated using the data set ze. The loss functions (normalized sum of squared prediction errors) are then computed for these models when applied to the validation data set $z v$. The data sets ze and $z v$ need not be of equal size. They could, however, be the same sets, in which case the computation is faster.

## See Also

arx|idpoly|ivstruc| selstruc|struc
Introduced before R2006a

## balred

Model order reduction

## Syntax

[rsys,info] = balred(sys,order)
[~,info] = balred(sys)
[___] = balred(__, opts)
balred(sys)

## Description

[rsys,info] = balred(sys,order) computes a reduced-order approximation rsys of the LTI model sys. The desired order (number of states) is specified by order. You can try multiple orders at once by setting order to a vector of integers, in which case rsys is an array of reduced models. bal red also returns a structure info with additional information like the Hankel singular values (HSV), error bound, regularization level and the Cholesky factors of the gramians.
[~,info] = balred(sys) returns the structure info without computing the reduced-order model. You can use this information to select the reduced order order based on your desired fidelity.

Note When performance is a concern, avoid computing the Hankel singular values twice by using the information obtained from the above syntax to select the desired model order and then use rsys = balred(sys,order,info) to compute the reduced-order model.
[ ___ ] = balred(__, opts) computes the reduced model using the options set opts that you specify using bal redOptions. You can specify additional options for eliminating states, using absolute vs. relative error control, emphasizing certain time or frequency bands, and separating the stable and unstable modes. See bal redOptions to create and configure the option set opts.
bal red(sys) displays the Hankel singular values and approximation error on a plot. Use hsvplot to customize this plot.

## Examples

## Reduced-Order Model using Hankel Singular Values

For this example, use the Hankel singular value plot to select suitable order and compute the reduced-order model.

For this instance, generate a random discrete-time state-space model with 40 states.
rng(0)
sys = drss(40);
Plot the Hankel singular values using bal red.
balred(sys)


For this example, select order of 16 since it is the first order with an absolute error less than 1e-4. In general, you select the order based on the desired absolute or relative fidelity. Then, compute the reduced-order model.

```
rsys = balred(sys,16);
```

Verify the absolute error by plotting the singular value response using sigma.

```
sigma(sys,sys-rsys)
```



Observe from the plot that the error, represented by the red curve, is below - $80 \mathrm{~dB}(1 \mathrm{e}-4)$.

## Array of Reduced-Order Models

For this example, consider a random continuous-time state-space model with 65 states.

```
rng(0)
```

sys = rss(65);
size(sys)

State-space model with 1 outputs, 1 inputs, and 65 states.
Visualize the Hankel singular values on a plot.
balred(sys)


For this instance, compute reduced-order models with 25, 30 and 35 states.
order = [25,30,35];
rsys = balred(sys,order);
size(rsys)
$3 \times 1$ array of state-space models.
Each model has 1 outputs, 1 inputs, and between 25 and 35 states.

## Reduced-Order Approximation with Offset Option

Compute a reduced-order approximation of the system given by:

$$
G(s)=\frac{(s+0.5)(s+1.1)(s+2.9)}{\left(s+10^{-6}\right)(s+1)(s+2)(s+3)} .
$$

Create the model.
sys $=\operatorname{zpk}([-0.5-1.1-2.9],[-1 e-6-2-1-3], 1) ;$
Exclude the pole at $s=10^{-6}$ from the stable term of the stable/unstable decomposition. To do so, set the Offset option of bal redOptions to a value larger than the pole you want to exclude.

```
opts = balredOptions('Offset',0.001,'StateProjection','Truncate');
```

Visualize the Hankel singular values (HSV) and the approximation error. balred(sys,opts)


Observe that the first HSV is red which indicates that it is associated with an unstable mode.
Now, compute a second-order approximation with the specified options.

```
[rsys,info] = balred(sys,2,opts);
rsys
rsys =
    0.99113 (s+0.5235)
    (s+1e-06) (s+1.952)
```

Continuous-time zero/pole/gain model.
Notice that the pole at - le-6 appears unchanged in the reduced model rsys.
Compare the responses of the original and reduced-order models.
bodeplot(sys,rsys,'r--')


Observe that the bode response of the original model and the reduced-order model nearly match.

## Model Reduction in a Particular Frequency Band

Reduce a high-order model with a focus on the dynamics in a particular frequency range.
Load a model and examine its frequency response.
load('high0rderModel.mat','G') bodeplot(G)


G is a 48th-order model with several large peak regions around $5.2 \mathrm{rad} / \mathrm{s}, 13.5 \mathrm{rad} / \mathrm{s}$, and $24.5 \mathrm{rad} / \mathrm{s}$, and smaller peaks scattered across many frequencies. Suppose that for your application you are only interested in the dynamics near the second large peak, between $10 \mathrm{rad} / \mathrm{s}$ and $22 \mathrm{rad} / \mathrm{s}$. Focus the model reduction on the region of interest to obtain a good match with a low-order approximation. Use bal redOptions (Control System Toolbox) to specify the frequency interval for bal red.
bopt = balredOptions('StateProjection','Truncate','FreqIntervals', [10,22]);
GLim10 = balred(G,10,bopt);
GLim18 = balred(G,18,bopt);
Examine the frequency responses of the reduced-order models. Also, examine the difference between those responses and the original response (the absolute error).

```
subplot(2,1,1);
bodemag(G,GLim10,GLim18,logspace(0.5,1.5,100));
title('Bode Magnitude Plot')
legend('Original','Order 10','Order 18');
subplot(2,1,2);
bodemag(G-GLim10,G-GLim18,logspace(0.5,1.5,100));
title('Absolute Error Plot')
legend('Order 10','Order 18');
```



With the frequency-limited energy computation, even the 10th-order approximation is quite good in the region of interest.

## Model-Order Reduction with Relative Error Approximation

For this example, consider the SISO state-space model cdrom with 120 states. You can use absolute or relative error control when approximating models with bal red. This example compares the two approaches when applied to a 120 -state model of a portable CD player device crdom [1,2] on page 10 .

Load the CD player model cdrom.

```
load cdromData.mat cdrom
size(cdrom)
State-space model with 1 outputs, 1 inputs, and 120 states.
```

To compare results with absolute vs. relative error control, create one option set for each approach.

```
opt_abs = balredOptions('ErrorBound','absolute','StateProjection','truncate');
opt_rel = balredOptions('ErrorBound','relative','StateProjection','truncate');
```

Compute reduced-order models of order 15 with both approaches.

```
rsys_abs = balred(cdrom,15,opt_abs);
rsys_rel = balred(cdrom,15,opt_rel);
size(rsys_abs)
State-space model with 1 outputs, 1 inputs, and 15 states.
size(rsys_rel)
State-space model with 1 outputs, 1 inputs, and 15 states.
```

Plot the Bode response of the original model along with the absolute-error and relative-error reduced models.

```
bo = bodeoptions;
bo.PhaseMatching = 'on';
bodeplot(cdrom,'b.',rsys_abs,'r',rsys_rel,'g',bo)
legend('Original (120 stātes)','Absolute Error (15 states)','Relative Error (15 states)')
```



Observe that the Bode response of:

- The relative-error reduced model rsys_rel nearly matches the response of the original model sys across the complete frequency range.
- The absolute-error reduced model rsys_abs matches the response of the original model sys only in areas with the most gain.


## References

1 Benchmark Examples for Model Reduction, Subroutine Library in Systems and Control Theory (SLICOT). The CDROM data set is reproduced with permission, see BSD3-license for details.
2 A.Varga, "On stochastic balancing related model reduction", Proceedings of the 39th IEEE Conference on Decision and Control (Cat. No.00CH37187), Sydney, NSW, 2000, pp. 2385-2390 vol.3, doi: 10.1109/CDC.2000.914156.

## Input Arguments

sys - Dynamic system
dynamic system model
Dynamic system, specified as a SISO or MIMO dynamic system model (Control System Toolbox). Dynamic systems that you can use can be continuous-time or discrete-time numeric LTI models, such as $\mathrm{tf}, \mathrm{zpk}$, or ssmodels.

When sys has unstable poles, balred decomposes sys to its stable and unstable parts and only the stable part is approximated. Use bal redOptions to specify additional options for the stable/unstable decomposition.
bal red does not support frequency response data models, uncertain and generalized state-space models, PID models or sparse model objects.

## order - Desired number of states

integer | vector of integers
Desired number of states, specified as an integer or a vector of integers. You can try multiple orders at once by setting order to a vector of integers, in which case rys is returned as an array of reduced models.

You can also use the Hankel singular values and error bound information to select the reduced-model order based on the desired model fidelity.

## opts - Additional options for model reduction

options set
Additional options for model reduction, specified as an options set. You can specify additional options for eliminating states, using absolute vs. relative error control, emphasizing certain time or frequency bands, and separating the stable and unstable modes.

See bal redOptions to create and configure the option set opts.

## Output Arguments

## rsys - Reduced-order model

dynamic system model | array of dynamic system models
Reduced-order model, returned as a dynamic system model or an array of dynamic system models.

## info - Additional information about the LTI model

structure
Additional information about the LTI model, returned as a structure with the following fields:

- HSV - Hankel singular values (state contributions to the input/output behavior). In state coordinates that equalize the input-to-state and state-to-output energy transfers, the Hankel singular values measure the contribution of each state to the input/output behavior. Hankel singular values are to model order what singular values are to matrix rank. In particular, small Hankel singular values signal states that can be discarded to simplify the model.
- ErrorBound - Bound on absolute or relative approximation error. info. ErrorBound (J+1) bounds the error for order J.
- Regularization - Regularization level $\square$ (for relative error only). Here, sys is replaced by [sys , $\rho^{*} I$ ] or [sys; $\rho^{*} I$ ] that ensures a well-defined relative error at all frequencies.
- Rr, Ro - Cholesky factors of gramians.


## Algorithms

1 bal red first decomposes $G$ into its stable and unstable parts:

$$
G=G_{s}+G_{u}
$$

2 When you specify ErrorBound as absolute, balred uses the balanced truncation method of [1] to reduce $G_{s}$. This computes the Hankel singular values (HSV) $\sigma_{j}$ based on the controllability and observability gramians. For order $r$, the absolute error $\left\|G_{s}-G_{r}\right\|_{\infty}$ is bounded by $2 \sum_{j=r+1}^{n} \sigma_{j}$. Here, $n$ is the number of states in $G_{s}$.
3 When you specify ErrorBound as relative, bal red uses the balanced stochastic truncation method of [2] to reduce $G_{s}$. For square $G_{s}$, this computes the HSV $\sigma_{j}$ of the phase matrix $F=\left(W^{\prime}\right)^{-1} G$ where $W(s)$ is a stable, minimum-phase spectral factor of $G G^{\prime}$ :
$W^{\prime}(s) W(s)=G(s) G^{\prime}(s)$
For order $r$, the relative error $\left\|G_{s}-1\left(G_{S}-G_{r}\right)\right\|_{\infty}$ is bounded by:
$\prod_{j=r+1}^{H}\left(\frac{1+\sigma_{j}}{1-\sigma_{j}}\right)-1 \approx 2 \sum_{j=r+1}^{n} \sigma_{j}$
when, $2 \sum_{j=r+1}^{n} \sigma_{j} \ll 1$.

## Alternative Functionality

## App

## Model Reducer

## Live Editor Task

Reduce Model Order (Control System Toolbox)

## Compatibility Considerations

MatchDC option honored when specified frequency or time intervals exclude DC
Behavior changed in R2017b

When you use bal red for model reduction, you can use bal redOptions to restrict the computation to specified frequency or time intervals. If the StateProjection option of balredOptions is set to 'MatchDC' (the default value), then bal red attempts to match the DC gain of the original and reduced models, even if the specified intervals exclude DC (frequency $=0$ or time $=\operatorname{Inf}$ ).

Prior to R2017b, if you specified time or frequency intervals that excluded DC, bal red did not attempt to match the DC gain of the original and reduced models, even if StateProjection = 'MatchDC'.

## References

[1] Varga, A., "Balancing-Free Square-Root Algorithm for Computing Singular Perturbation Approximations," Proc. of 30th IEEE CDC, Brighton, UK (1991), pp. 1062-1065.
[2] Green, M., "A Relative Error Bound for Balanced Stochastic Truncation", IEEE Transactions on Automatic Control, Vol. 33, No. 10, 1988

## See Also

## Functions

balredOptions

## Apps

Model Reducer

## Live Editor Tasks <br> Reduce Model Order

## Topics

"Model Reduction Basics" (Control System Toolbox)
"Balanced Truncation Model Reduction" (Control System Toolbox)

## Introduced before R2006a

## bandwidth

Frequency response bandwidth

## Syntax

$\mathrm{fb}=$ bandwidth(sys)
$\mathrm{fb}=$ bandwidth(sys,dbdrop)

## Description

$\mathrm{fb}=$ bandwidth(sys) returns the bandwidth of the SISO dynamic system model sys. The bandwidth is the first frequency where the gain drops below $70.79 \%(-3 \mathrm{~dB})$ of its DC value. The bandwidth is expressed in rad/TimeUnit, where TimeUnit is the TimeUnit property of sys.

This command requires a Control System Toolbox ${ }^{\mathrm{TM}}$ license.
$\mathrm{fb}=$ bandwidth(sys,dbdrop) returns the bandwidth for a specified gain drop.

## Examples

## Compute System Bandwidth

Compute the bandwidth of the transfer function sys $=1 /(s+1)$.

```
sys = tf(1,[1 1]);
fb = bandwidth(sys)
fb = 0.9976
```

This result shows that the gain of sys drops to 3 dB below its DC value at around $1 \mathrm{rad} / \mathrm{s}$.

## Find Bandwidth of System with Custom Gain Drop

Compute the frequency at which the gain of a system drops to 3.5 dB below its DC value. Create a state-space model.

```
A = [-2,-1;1,0];
B = [1;0];
C = [1,2];
D = 1;
sys = ss(A,B,C,D);
```

Find the 3.5 dB bandwidth of sys.

```
dbdrop = -3.5;
```

$\mathrm{fb}=$ bandwidth(sys,dbdrop)
$\mathrm{fb}=0.8348$

## Find Bandwidth of Model Array

Find the bandwidth of each entry in a 5-by-1 array of transfer function models. Use a for loop to create the array, and confirm its dimensions.

```
sys = tf(zeros(1,1,5));
s = tf('s');
for m = 1:5
    sys(:,:,m) = m/(s^2+s+m);
end
size(sys)
5x1 array of transfer functions.
Each model has 1 outputs and 1 inputs.
```

Find the bandwidths.

```
fb = bandwidth(sys)
```

$\mathrm{fb}=5 \times 1$
1.2712
1.9991
2.5298
2.9678
3.3493
bandwidth returns an array in which each entry is the bandwidth of the corresponding entry in sys. For instance, the bandwidth of sys (:,:,2) is $\mathrm{fb}(2)$.

## Input Arguments

## sys - Dynamic system

dynamic system model \| model array
Dynamic system, specified as a SISO dynamic system model or an array of SISO dynamic system models. Dynamic systems that you can use include:

- Continuous-time or discrete-time numeric LTI models such as tf, zpk, or ss models.
- Frequency-response data models such as frd models. For such models, bandwidth uses the first frequency point to approximate the DC gain.

If sys is an array of models, bandwidth returns an array of the same size, where each entry is the bandwidth of the corresponding model in sys. For more information on model arrays, see "Model Arrays" (Control System Toolbox).

## dbdrop - Gain drop

-3 (default) | negative scalar
Gain drop in dB , specified as a real negative scalar.

## Output Arguments

## fb - Frequency response bandwidth

scalar | array
Frequency response bandwidth, returned as a scalar or an array. If sys is:

- A single model, then fb is the bandwidth of sys.
- A model array, then fb is an array of the same size as the model array sys. Each entry is the bandwidth of the corresponding entry in sys.
fb is expressed in rad/TimeUnit, where TimeUnit is the TimeUnit property of sys.


## See Also <br> bodeplot|dcgain|issiso

Introduced before R2006a

## bj

Estimate Box-Jenkins polynomial model using time domain data

## Syntax

```
sys = bj(data, [nb nc nd nf nk])
sys = bj(data,[nb nc nd nf nk], Name,Value)
sys = bj(data, init_sys)
sys = bj(data, ___, opt)
[sys,ic] = bj(___)
```


## Description

sys = bj(data, [nb nc nd nf nk]) estimates a Box-Jenkins polynomial model, sys, using the time-domain data, data. [nb nc nd nf nk] define the orders of the polynomials used for estimation.
sys = bj(data,[nb nc nd nf nk], Name,Value) estimates a polynomial model with additional options specified by one or more Name, Value pair arguments.
sys = bj(data, init_sys) estimates a Box-Jenkins polynomial using the polynomial model init_sys to configure the initial parameterization of sys.
sys = bj(data, __ , opt) estimates a Box-Jenkins polynomial using the option set, opt, to specify estimation behavior.
[sys,ic] = bj( $\qquad$ ) returns the estimated initial conditions as an initialCondition object. Use this syntax if you plan to simulate or predict the model response using the same estimation input data and then compare the response with the same estimation output data. Incorporating the initial conditions yields a better match during the first part of the simulation.

## Input Arguments

## data

Estimation data.
data is an iddata object that contains time-domain input and output signal values.
You cannot use frequency-domain data for estimating Box-Jenkins models.

## Default:

## [nb nc nd nf nk]

A vector of matrices containing the orders and delays of the Box-Jenkins model. Matrices must contain nonnegative integers.

- nb is the order of the B polynomial plus 1 (Ny-by-Nu matrix)
- nc is the order of the C polynomial plus 1 (Ny-by-1 matrix)
- nd is the order of the D polynomial plus 1 (Ny-by-1 matrix)
- nf is the order of the F polynomial plus 1 (Ny-by-Nu matrix)
- nk is the input delay (in number of samples, Ny-by-Nu matrix) where Nu is the number of inputs and Ny is the number of outputs.


## opt

Estimation options.
opt is an options set that configures, among others, the following:

- estimation objective
- initial conditions
- numerical search method to be used in estimation

Use bjOptions to create the options set.

## init_sys

Polynomial model that configures the initial parameterization of sys.
init sys must be an idpoly model with the Box-Jenkins structure that has only $B, C, D$ and $F$ polynomials active. bj uses the parameters and constraints defined in init_sys as the initial guess for estimating sys.

Use the Structure property of init_sys to configure initial guesses and constraints for $B(q), F(q)$, $C(q)$ and $D(q)$.

To specify an initial guess for, say, the $C(q)$ term of init_sys, set init_sys.Structure.C.Value as the initial guess.

To specify constraints for, say, the $B(q)$ term of init_sys:

- set init_sys.Structure.B.Minimum to the minimum $B(q)$ coefficient values
- set init_sys.Structure.B.Maximum to the maximum $B(q)$ coefficient values
- set init_sys.Structure.B.Free to indicate which $B(q)$ coefficients are free for estimation

You can similarly specify the initial guess and constraints for the other polynomials.

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

## InputDelay

Input delays. InputDelay is a numeric vector specifying a time delay for each input channel. Specify input delays in integer multiples of the sample time Ts. For example, InputDelay $=3$ means a delay of three sampling periods.

For a system with Nu inputs, set InputDelay to an Nu-by-1 vector, where each entry is a numerical value representing the input delay for the corresponding input channel. You can also set InputDelay to a scalar value to apply the same delay to all channels.

Default: 0 for all input channels

## IODelay

Transport delays. IODelay is a numeric array specifying a separate transport delay for each input/ output pair.

Specify transport delays as integers denoting delay of a multiple of the sample time Ts.
For a MIMO system with Ny outputs and Nu inputs, set IODelay to a Ny-by-Nu array, where each entry is a numerical value representing the transport delay for the corresponding input/output pair. You can also set IODelay to a scalar value to apply the same delay to all input/output pairs.

Default: 0 for all input/output pairs

## IntegrateNoise

Logical specifying integrators in the noise channel.
IntegrateNoise is a logical vector of length $N y$, where $N y$ is the number of outputs.
Setting IntegrateNoise to true for a particular output results in the model:

$$
y(t)=\frac{B(q)}{F(q)} u(t-n k)+\frac{C(q)}{D(q)} \frac{e(t)}{1-q^{-1}}
$$

Where, $\frac{1}{1-q^{-1}}$ is the integrator in the noise channel, $e(t)$.
Default: false $(N y, 1)(N y$ is the number of outputs)

## Output Arguments

## sys

BJ model that fits the estimation data, returned as a discrete-time idpoly object. This model is created using the specified model orders, delays, and estimation options.

Information about the estimation results and options used is stored in the Report property of the model. Report has the following fields:

| Report <br> Field | Description |
| :--- | :--- |
| Status | Summary of the model status, which indicates whether the model was created by <br> construction or obtained by estimation. |
| Method | Estimation command used. |


| Report Field | Description |  |
| :---: | :---: | :---: |
| InitialCo ndition | Handling of initial conditions during model estimation, returned as one of the following values: <br> - 'zero' - The initial conditions were set to zero. <br> - 'estimate' - The initial conditions were treated as independent estimation parameters. <br> - 'backcast' - The initial conditions were estimated using the best least squares fit. <br> This field is especially useful to view how the initial conditions were handled when the InitialCondition option in the estimation option set is 'auto'. |  |
| Fit | Quantitative assessment of the estimation, returned as a structure. See "Loss Function and Model Quality Metrics" for more information on these quality metrics. The structure has the following fields: |  |
|  | Field | Description |
|  | FitPerce nt | Normalized root mean squared error (NRMSE) measure of how well the response of the model fits the estimation data, expressed as the percentage fit = 100(1-NRMSE). |
|  | LossFcn | Value of the loss function when the estimation completes. |
|  | MSE | Mean squared error (MSE) measure of how well the response of the model fits the estimation data. |
|  | FPE | Final prediction error for the model. |
|  | AIC | Raw Akaike Information Criteria (AIC) measure of model quality. |
|  | AICc | Small sample-size corrected AIC. |
|  | nAIC | Normalized AIC. |
|  | BIC | Bayesian Information Criteria (BIC). |
| Parameter <br> s | Estimated values of model parameters. |  |
| OptionsUs ed | Option set used for estimation. If no custom options were configured, this is a set of default options. See bjOptions for more information. |  |
| RandState | State of the random number stream at the start of estimation. Empty, [ ], if randomization was not used during estimation. For more information, see rng. |  |


| Report Field | Description |  |
| :---: | :---: | :---: |
| DataUsed | Attributes of the data used for estimation, returned as a structure with the following fields: |  |
|  | Field | Description |
|  | Name | Name of the data set. |
|  | Type | Data type. |
|  | Length | Number of data samples. |
|  | Ts | Sample time. |
|  | InterSam ple | Input intersample behavior, returned as one of the following values: <br> - 'zoh ' - Zero-order hold maintains a piecewise-constant input signal between samples. <br> - ' foh ' - First-order hold maintains a piecewise-linear input signal between samples. <br> - 'bl' - Band-limited behavior specifies that the continuous-time input signal has zero power above the Nyquist frequency. |
|  | Input0ff set | Offset removed from time-domain input data during estimation. For nonlinear models, it is []. |
|  | Output0f fset | Offset removed from time-domain output data during estimation. For nonlinear models, it is [ ]. |
| Terminati on | Termination conditions for the iterative search used for prediction error minimization, returned as a structure with the following fields: |  |
|  | Field | Description |
|  | WhyStop | Reason for terminating the numerical search. |
|  | Iteratio ns | Number of search iterations performed by the estimation algorithm. |
|  | First0rd erOptima lity | $\infty$-norm of the gradient search vector when the search algorithm terminates. |
|  | FcnCount | Number of times the objective function was called. |
|  | UpdateNo rm | Norm of the gradient search vector in the last iteration. Omitted when the search method is 'lsqnonlin' or 'fmincon'. |
|  | LastImpr ovement | Criterion improvement in the last iteration, expressed as a percentage. Omitted when the search method is 'lsqnonlin' or 'fmincon'. |
|  | $\begin{array}{\|l\|l\|} \hline \text { Algorith } \\ \hline \end{array}$ | Algorithm used by ' lsqnonlin' or 'fmincon' search method. Omitted when other search methods are used. |
|  | For estimation methods that do not require numerical search optimization, the Termination field is omitted. |  |

For more information on using Report, see "Estimation Report".

## [nb nc nd nf nk]

A vector of matrices containing the orders and delays of the Box-Jenkins model. Matrices must contain nonnegative integers.

- nb is the order of the B polynomial plus 1 (Ny-by-Nu matrix)
- nc is the order of the C polynomial plus 1 (Ny-by-1 matrix)
- nd is the order of the D polynomial plus 1 (Ny-by-1 matrix)
- $n f$ is the order of the F polynomial plus 1 (Ny-by-Nu matrix)
- nk is the input delay (in number of samples, Ny-by-Nu matrix) where Nu is the number of inputs and Ny is the number of outputs.


## ic

Estimated initial conditions, returned as an initialCondition object or an object array of initialCondition values.

- For a single-experiment data set, ic represents, in state-space form, the free response of the transfer function model ( $A$ and $C$ matrices) to the estimated initial states ( $x_{0}$ ).
- For a multiple-experiment data set with $N_{e}$ experiments, ic is an object array of length $N_{e}$ that contains one set of initialCondition values for each experiment.

If bj returns ic values of 0 and the you know that you have non-zero initial conditions, set the 'InitialCondition' option in bjOptions to 'estimate' and pass the updated option set to bj. For example:

```
opt = bjOptions('InitialCondition,'estimate')
[sys,ic] = bj(data,[nb nc nd nf nk],opt)
```

The default 'auto' setting of 'InitialCondition' uses the 'zero' method when the initial conditions have a negligible effect on the overall estimation-error minimization process. Specifying 'estimate' ensures that the software estimates values for ic.

For more information, see initialCondition. For an example of using this argument, see "Obtain Initial Conditions" on page 1-109.

## Examples

## Identify SISO Box-Jenkins Model

Estimate the parameters of a single-input, single-output Box-Jenkins model from measured data.

```
load iddatal z1;
nb = 2;
nc = 2;
nd = 2;
nf = 2;
nk = 1;
sys = bj(z1,[nb nc nd nf nk]);
```

sys is a discrete-time idpoly model with estimated coefficients. The order of sys is as described by $n b, n c, n d, n f$, and $n k$.

Use getpvec to obtain the estimated parameters and getcov to obtain the covariance associated with the estimated parameters.

## Estimate a Multi-Input, Single-Output Box-Jenkins Model

Estimate the parameters of a multi-input, single-output Box-Jenkins model from measured data.

```
load iddata8
nb = [2 1 1];
nc = 1;
nd = 1;
nf = [\begin{array}{lll}{2 1 2];}\end{array}]
nk = [5 10 15];
sys = bj(z8,[nb nc nd nf nk]);
```

sys estimates the parameters of a model with three inputs and one output. Each of the inputs has a delay associated with it.

## Estimate Box-Jenkins Model Using Regularization

Estimate a regularized BJ model by converting a regularized ARX model.
Load data.
load regularizationExampleData.mat m0simdata;
Estimate an unregularized BJ model of order 30.
m1 = bj(m0simdata(1:150),[15 151515 1]);
Estimate a regularized BJ model by determining Lambda value by trial and error.

```
opt = bjOptions;
opt.Regularization.Lambda = 1;
m2 = bj(m0simdata(1:150),[15 15 15 15 1],opt);
```

Obtain a lower-order BJ model by converting a regularized ARX model followed by order reduction.
opt1 = arx0ptions;
[L,R] = arxRegul(m0simdata(1:150),[30 30 1]);
opt1.Regularization.Lambda $=\mathrm{L}$;
opt1.Regularization. $\mathrm{R}=\mathrm{R}$;
m0 = arx(m0simdata(1:150),[30 30 1],opt1);
$\mathrm{mr}=$ idpoly(balred(idss(m0),7));
Compare the model outputs against data.

```
opt2 = compareOptions('InitialCondition','z');
```

compare(m0simdata(150:end),m1,m2,mr,opt2);

## Simulated Response Comparison



## Configure Estimation Options

Estimate the parameters of a single-input, single-output Box-Jenkins model while configuring some estimation options.

Generate estimation data.
$\mathrm{B}=\left[\begin{array}{lll}0 & 1 & 0.5\end{array}\right] ;$
$\mathrm{C}=\left[\begin{array}{lll}1 & -1 & 0.2\end{array}\right] ;$

$\mathrm{F}=\left[\begin{array}{lll}1 & -1.5 & 0.7\end{array}\right] ;$
sys0 $=$ idpoly(1, B, C, D, F, 0.1);
e = iddata([],randn(200,1));
u = iddata([],idinput(200));
y = sim(sys0,[u e]);
data $=$ [y u];
data is a single-input, single-output data set created by simulating a known model.
Estimate initial Box-Jenkins model.
nb $=2$;
nc = 2;
nd $=2$;
nf = 2;

```
nk = 1;
init_sys = bj(data,[2 2 2 2 1]);
```

Create an estimation option set to refine the parameters of the estimated model.

```
opt = bjOptions;
```

opt.Display = 'on';
opt.SearchOptions.MaxIterations = 50;
opt is an estimation option set that configures the estimation to iterate 50 times at most and display the estimation progress.

Reestimate the model parameters using the estimation option set.
sys = bj(data,init_sys,opt);
sys is estimated using init_sys for the initial parameterization for the polynomial coefficients.
To view the estimation result, enter sys.Report.

## Estimate MIMO Box-Jenkins Model

Estimate a multi-input, multi-output Box-Jenkins model from estimated data.
Load measured data.

```
load iddata1 z1
load iddata2 z2
data = [z1 z2(1:300)];
```

data contains the measured data for two inputs and two outputs.
Estimate the model.

```
nb = [2 2; 3 4];
nc = [2;2];
nd = [2;2];
nf = [1 0; 2 2];
nk = [1 1; 0 0];
sys = bj(data,[nb nc nd nf nk]);
```

The polynomial order coefficients contain one row for each output.
sys is a discrete-time idpoly model with two inputs and two outputs.

## Obtain Initial Conditions

Load the data.
load iddatalic zli
Estimate a second-order Box-Jenkins model sys and return the initial conditions in ic.

```
nb = 2;
nc = 2;
nd = 2;
nf = 2;
nk = 1;
[sys,ic] = bj(zli,[nb nc nd nf nk]);
ic
ic =
    initialCondition with properties:
        A: [4\times4 double]
        X0: [4x1 double]
        C: [0.8744 0.5426 0.4647 -0.5285]
        Ts: 0.1000
```

ic is an initialCondition object that encapsulates the free response of sys, in state-space form, to the initial state vector in X0. You can incorporate ic when you simulate sys with the z1i input signal and compare the response with the zli output signal.

## More About

## Box-Jenkins Model Structure

The general Box-Jenkins model structure is:

$$
y(t)=\sum_{i=1}^{n u} \frac{B_{i}(q)}{F_{i}(q)} u_{i}\left(t-n k_{i}\right)+\frac{C(q)}{D(q)} e(t)
$$

where $n u$ is the number of input channels.
The orders of Box-Jenkins model are defined as follows:

$$
n b: \quad B(q)=b_{1}+b_{2} q^{-1}+\ldots+b_{n b} q^{-n b+1}
$$

$$
n c: \quad C(q)=1+c_{1} q^{-1}+\ldots+c_{n c} q^{-n c}
$$

$$
n d: \quad D(q)=1+d_{1} q^{-1}+\ldots+d_{n d} q^{-n d}
$$

$$
n f: \quad F(q)=1+f_{1} q^{-1}+\ldots+f_{n f} q^{-n f}
$$

## Alternatives

To estimate a continuous-time model, use:

- tfest - returns a transfer function model
- ssest - returns a state-space model
- bj to first estimate a discrete-time model and convert it a continuous-time model using d2c.


## References

[1] Ljung, L. System Identification: Theory for the User, Upper Saddle River, NJ, Prentice-Hall PTR, 1999.

## Extended Capabilities

## Automatic Parallel Support

Accelerate code by automatically running computation in parallel using Parallel Computing Toolbox ${ }^{\mathrm{TM}}$.
Parallel computing support is available for estimation using the lsqnonlin search method (requires Optimization Toolbox). To enable parallel computing, use bjOptions, set SearchMethod to 'lsqnonlin', and set SearchOptions.Advanced.UseParallel to true.

For example:
opt $=$ bjOptions;
opt.SearchMethod = 'lsqnonlin';
opt.SearchOptions.Advanced.UseParallel = true;

## See Also

armax|arx|bj0ptions|compare|d2c|forecast|iddata|idpoly|iv4|oe|polyest| sim|ssest|tfest

## Topics

"Regularized Estimates of Model Parameters"
"Apply Initial Conditions when Simulating Identified Linear Models"
Introduced before R2006a

## bjOptions

Option set for bj

## Syntax

opt = bjOptions
opt $=$ bjOptions(Name,Value)

## Description

opt $=$ bjOptions creates the default options set for bj.
opt $=$ bjOptions(Name, Value) creates an option set with the options specified by one or more Name, Value pair arguments.

## Input Arguments

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, ... , NameN, ValueN.

## InitialCondition - Handling of initial conditions <br> 'auto' (default)|'zero'|'estimate' |'backcast'

Handling of initial conditions during estimation, specified as one of the following values:

- 'zero' - The initial conditions are set to zero.
- 'estimate' - The initial conditions are treated as independent estimation parameters.
- 'backcast' - The initial conditions are estimated using the best least squares fit.
- 'auto ' - The software chooses the method to handle initial conditions based on the estimation data.


## Focus - Error to be minimized

'prediction' (default)|'simulation'
Error to be minimized in the loss function during estimation, specified as the comma-separated pair consisting of 'Focus' and one of the following values:

- 'prediction' - The one-step ahead prediction error between measured and predicted outputs is minimized during estimation. As a result, the estimation focuses on producing a good predictor model.
- 'simulation' - The simulation error between measured and simulated outputs is minimized during estimation. As a result, the estimation focuses on making a good fit for simulation of model response with the current inputs.

The Focus option can be interpreted as a weighting filter in the loss function. For more information, see "Loss Function and Model Quality Metrics".

## WeightingFilter - Weighting prefilter

## [] (default) | vector | matrix | cell array | linear system

Weighting prefilter applied to the loss function to be minimized during estimation. To understand the effect of WeightingFilter on the loss function, see "Loss Function and Model Quality Metrics".

Specify WeightingFilter as one of the following values:

- [ ] - No weighting prefilter is used.
- Passbands - Specify a row vector or matrix containing frequency values that define desired passbands. You select a frequency band where the fit between estimated model and estimation data is optimized. For example, [ $w l, w h$ ], where $w l$ and wh represent lower and upper limits of a passband. For a matrix with several rows defining frequency passbands, [w1l,w1h;w2l,w2h;w3l,w3h;...], the estimation algorithm uses the union of the frequency ranges to define the estimation passband.

Passbands are expressed in rad/TimeUnit for time-domain data and in FrequencyUnit for frequency-domain data, where TimeUnit and FrequencyUnit are the time and frequency units of the estimation data.

- SISO filter - Specify a single-input-single-output (SISO) linear filter in one of the following ways:
- A SISO LTI model
- $\{A, B, C, D\}$ format, which specifies the state-space matrices of a filter with the same sample time as estimation data.
- \{numerator, denominator\} format, which specifies the numerator and denominator of the filter as a transfer function with same sample time as estimation data.

This option calculates the weighting function as a product of the filter and the input spectrum to estimate the transfer function.

## EnforceStability - Control whether to enforce stability of model

false (default)|true
Control whether to enforce stability of estimated model, specified as the comma-separated pair consisting of 'EnforceStability' and either true or false.

Data Types: logical

## EstimateCovariance - Control whether to generate parameter covariance data true (default) | false

Controls whether parameter covariance data is generated, specified as true or false.
If EstimateCovariance is true, then use getcov to fetch the covariance matrix from the estimated model.

## Display - Specify whether to display the estimation progress

'off' (default)|'on'
Specify whether to display the estimation progress, specified as one of the following values:

- ' on ' - Information on model structure and estimation results are displayed in a progress-viewer window.
- 'off' - No progress or results information is displayed.


## InputOffset - Removal of offset from time-domain input data during estimation

[ ] (default) | vector of positive integers | matrix
Removal of offset from time-domain input data during estimation, specified as the comma-separated pair consisting of 'InputOffset' and one of the following:

- A column vector of positive integers of length $N u$, where $N u$ is the number of inputs.
- [] - Indicates no offset.
- $N u$-by-Ne matrix - For multi-experiment data, specify InputOffset as an Nu -by-Ne matrix. Nu is the number of inputs, and $N e$ is the number of experiments.

Each entry specified by InputOffset is subtracted from the corresponding input data.

## OutputOffset - Removal of offset from time-domain output data during estimation

[] (default) | vector | matrix
Removal of offset from time-domain output data during estimation, specified as the comma-separated pair consisting of 'OutputOffset' and one of the following:

- A column vector of length $N y$, where $N y$ is the number of outputs.
- [ ] - Indicates no offset.
- $N y$-by-Ne matrix - For multi-experiment data, specify OutputOffset as a $N y$-by-Ne matrix. $N y$ is the number of outputs, and $N e$ is the number of experiments.

Each entry specified by OutputOffset is subtracted from the corresponding output data.

## Regularization - Options for regularized estimation of model parameters <br> structure

Options for regularized estimation of model parameters. For more information on regularization, see "Regularized Estimates of Model Parameters".

Regularization is a structure with the following fields:

- Lambda - Constant that determines the bias versus variance tradeoff.

Specify a positive scalar to add the regularization term to the estimation cost.
The default value of zero implies no regularization.
Default: 0

- R - Weighting matrix.

Specify a vector of nonnegative numbers or a square positive semi-definite matrix. The length must be equal to the number of free parameters of the model.

For black-box models, using the default value is recommended. For structured and grey-box models, you can also specify a vector of np positive numbers such that each entry denotes the confidence in the value of the associated parameter.

The default value of 1 implies a value of eye(npfree), where npfree is the number of free parameters.

## Default: 1

- Nominal - The nominal value towards which the free parameters are pulled during estimation.

The default value of zero implies that the parameter values are pulled towards zero. If you are refining a model, you can set the value to 'model' to pull the parameters towards the parameter values of the initial model. The initial parameter values must be finite for this setting to work.

Default: 0

## SearchMethod - Numerical search method used for iterative parameter estimation

'auto' (default)|'gn'|'gna'|'lm'|'grad'|'lsqnonlin'|'fmincon'
Numerical search method used for iterative parameter estimation, specified as the comma-separated pair consisting of 'SearchMethod' and one of the following:

- 'auto' - A combination of the line search algorithms, 'gn', 'lm', 'gna', and 'grad' methods is tried in sequence at each iteration. The first descent direction leading to a reduction in estimation cost is used.
- 'gn' - Subspace Gauss-Newton least squares search. Singular values of the Jacobian matrix less than GnPinvConstant*eps*max(size(J))*norm(J) are discarded when computing the search direction. $J$ is the Jacobian matrix. The Hessian matrix is approximated as $J^{T} J$. If there is no improvement in this direction, the function tries the gradient direction.
- 'gna' - Adaptive subspace Gauss-Newton search. Eigenvalues less than gamma*max(sv) of the Hessian are ignored, where $s v$ contains the singular values of the Hessian. The Gauss-Newton direction is computed in the remaining subspace. gamma has the initial value InitialGnaTolerance (see Advanced in 'Search0ptions' for more information). This value is increased by the factor LMStep each time the search fails to find a lower value of the criterion in fewer than five bisections. This value is decreased by the factor $2 *$ LMStep each time a search is successful without any bisections.
- 'lm' - Levenberg-Marquardt least squares search, where the next parameter value is - pinv ( H $+\mathrm{d} * \mathrm{I}) * \mathrm{grad}$ from the previous one. $H$ is the Hessian, $I$ is the identity matrix, and grad is the gradient. $d$ is a number that is increased until a lower value of the criterion is found.
- 'grad' - Steepest descent least squares search.
- 'lsqnonlin' - Trust-region-reflective algorithm of lsqnonlin. Requires Optimization Toolbox software.
- 'fmincon' - Constrained nonlinear solvers. You can use the sequential quadratic programming (SQP) and trust-region-reflective algorithms of the fmincon solver. If you have Optimization Toolbox software, you can also use the interior-point and active-set algorithms of the fmincon solver. Specify the algorithm in the SearchOptions.Algorithm option. The fmincon algorithms may result in improved estimation results in the following scenarios:
- Constrained minimization problems when there are bounds imposed on the model parameters.
- Model structures where the loss function is a nonlinear or non smooth function of the parameters.
- Multi-output model estimation. A determinant loss function is minimized by default for multioutput model estimation. fmincon algorithms are able to minimize such loss functions directly. The other search methods such as ' lm ' and 'gn' minimize the determinant loss function by alternately estimating the noise variance and reducing the loss value for a given noise variance value. Hence, the fmincon algorithms can offer better efficiency and accuracy for multi-output model estimations.


## SearchOptions - Option set for the search algorithm

search option set
Option set for the search algorithm, specified as the comma-separated pair consisting of 'Search0ptions ' and a search option set with fields that depend on the value of SearchMethod.

## Search0ptions Structure When SearchMethod is Specified as 'gn', 'gna', 'lm', 'grad', or 'auto'

| Field <br> Name | Description | Default |
| :--- | :--- | :--- |
| Toleran <br> ce | Minimum percentage difference between the current value of the loss <br> function and its expected improvement after the next iteration, specified as <br> a positive scalar. When the percentage of expected improvement is less <br> than Tolerance, the iterations stop. The estimate of the expected loss- <br> function improvement at the next iteration is based on the Gauss-Newton <br> vector computed for the current parameter value. | 0.01 |
| MaxIter <br> ations | Maximum number of iterations during loss-function minimization, specified <br> as a positive integer. The iterations stop when MaxIterations is reached <br> or another stopping criterion is satisfied, such as Tolerance. | 20 |
| Setting MaxIterations = 0 returns the result of the start-up procedure. <br> Use sys. Report. Termination. Iterations to get the actual number <br> of iterations during an estimation, where sys is an idtf model. |  |  |


| Field Name | Description |  | Default |
| :---: | :---: | :---: | :---: |
|  | Advanced search settings, specified as a structure with the following fields: |  |  |
|  | Field Name | Description | Default |
|  | GnPinvCons tant | Jacobian matrix singular value threshold, specified as a positive scalar. Singular values of the Jacobian matrix that are smaller than GnPinvConstant*max(size(J)*norm(J)*eps) are discarded when computing the search direction. Applicable when SearchMethod is 'gn'. | 10000 |
|  | InitialGna Tolerance | Initial value of gamma, specified as a positive scalar. Applicable when SearchMethod is 'gna'. | 0.0001 |
|  | LMStartVal ue | Starting value of search-direction length $d$ in the Levenberg-Marquardt method, specified as a positive scalar. Applicable when SearchMethod is ' lm'. | 0.001 |
|  | LMStep | Size of the Levenberg-Marquardt step, specified as a positive integer. The next value of the search-direction length $d$ in the Levenberg-Marquardt method is LMStep times the previous one. Applicable when SearchMethod is ' 1 m '. | 2 |
|  | MaxBisecti ons | Maximum number of bisections used for line search along the search direction, specified as a positive integer. | 25 |
|  | MaxFunctio nEvaluatio ns | Maximum number of calls to the model file, specified as a positive integer. Iterations stop if the number of calls to the model file exceeds this value. | Inf |
|  | MinParamet erChange | Smallest parameter update allowed per iteration, specified as a nonnegative scalar. | 0 |
|  | RelativeIm provement | Relative improvement threshold, specified as a nonnegative scalar. Iterations stop if the relative improvement of the criterion function is less than this value. | 0 |
|  | StepReduct ion | Step reduction factor, specified as a positive scalar that is greater than 1 . The suggested parameter update is reduced by the factor StepReduction after each try. This reduction continues until MaxBisections tries are completed or a lower value of the criterion function is obtained. <br> StepReduction is not applicable for SearchMethod ' lm' (Levenberg-Marquardt method). | 2 |

## SearchOptions Structure When SearchMethod is Specified as ' Isqnonlin'

| Field Name | Description | Default |
| :---: | :---: | :---: |
| Function Toleranc e | Termination tolerance on the loss function that the software minimizes to determine the estimated parameter values, specified as a positive scalar. <br> The value of FunctionTolerance is the same as that of opt.SearchOptions.Advanced.TolFun. | 1e-5 |
| StepTole rance | Termination tolerance on the estimated parameter values, specified as a positive scalar. <br> The value of StepTolerance is the same as that of opt. SearchOptions.Advanced.TolX. | 1e-6 |
| MaxItera tions | Maximum number of iterations during loss-function minimization, specified as a positive integer. The iterations stop when MaxIterations is reached or another stopping criterion is satisfied, such as FunctionTolerance. <br> The value of MaxIterations is the same as that of opt. SearchOptions.Advanced.MaxIter. | 20 |
| Advanced | Advanced search settings, specified as an option set for lsqnonlin. <br> For more information, see the Optimization Options table in "Optimization Options" (Optimization Toolbox). | Use optimset('lsqnonl in') to create a default option set. |

SearchOptions Structure When SearchMethod is Specified as 'fmincon'

| Field Name | Description | Default |
| :---: | :---: | :---: |
| Algorithm | fmincon optimization algorithm, specified as one of the following: <br> - 'sqp' - Sequential quadratic programming algorithm. The algorithm satisfies bounds at all iterations, and it can recover from NaN or Inf results. It is not a large-scale algorithm. For more information, see "Large-Scale vs. MediumScale Algorithms" (Optimization Toolbox). <br> - 'trust-regionreflective' - Subspace trust-region method based on the interior-reflective Newton method. It is a largescale algorithm. <br> - 'interior-point' -Large-scale algorithm that requires Optimization Toolbox software. The algorithm satisfies bounds at all iterations, and it can recover from NaN or Inf results. <br> - 'active-set' - Requires Optimization Toolbox software. The algorithm can take large steps, which adds speed. It is not a large-scale algorithm. <br> For more information about the algorithms, see "Constrained Nonlinear Optimization Algorithms" (Optimization Toolbox) and "Choosing the Algorithm" (Optimization Toolbox). | 'sqp' |


| Field Name | Description | Default |
| :--- | :--- | :--- |
| FunctionTolerance | Termination tolerance on the <br> loss function that the software <br> minimizes to determine the <br> estimated parameter values, <br> specified as a positive scalar. | $1 \mathrm{e}-6$ |
| StepTolerance | Termination tolerance on the <br> estimated parameter values, <br> specified as a positive scalar. | $1 \mathrm{e}-6$ |
| MaxIterations | Maximum number of iterations <br> during loss function <br> minimization, specified as a <br> positive integer. The iterations <br> stop when MaxIterations is <br> reached or another stopping <br> criterion is satisfied, such as <br> FunctionTolerance. | 100 |

## Advanced - Additional advanced options

structure
Additional advanced options, specified as a structure with the following fields:

- ErrorThreshold - Specifies when to adjust the weight of large errors from quadratic to linear.

Errors larger than ErrorThreshold times the estimated standard deviation have a linear weight in the loss function. The standard deviation is estimated robustly as the median of the absolute deviations from the median of the prediction errors, divided by 0.7 . For more information on robust norm choices, see section 15.2 of [2].

ErrorThreshold $=0$ disables robustification and leads to a purely quadratic loss function. When estimating with frequency-domain data, the software sets ErrorThreshold to zero. For time-domain data that contains outliers, try setting ErrorThreshold to 1.6.

Default: 0

- MaxSize - Specifies the maximum number of elements in a segment when input-output data is split into segments.

MaxSize must be a positive integer.
Default: 250000

- StabilityThreshold - Specifies thresholds for stability tests.

StabilityThreshold is a structure with the following fields:

- s-Specifies the location of the right-most pole to test the stability of continuous-time models. A model is considered stable when its right-most pole is to the left of $s$.

Default: 0

- $z$ - Specifies the maximum distance of all poles from the origin to test stability of discretetime models. A model is considered stable if all poles are within the distance $z$ from the origin.

Default: 1+sqrt(eps)

- AutoInitThreshold - Specifies when to automatically estimate the initial condition.

The initial condition is estimated when

$$
\frac{\left\|y_{p, z}-y_{\text {meas }}\right\|}{\left\|y_{p, e}-y_{\text {meas }}\right\|}>\text { AutoInitThreshold }
$$

- $y_{\text {meas }}$ is the measured output.
- $y_{p, z}$ is the predicted output of a model estimated using zero initial states.
- $y_{p, e}$ is the predicted output of a model estimated using estimated initial states.

Applicable when InitialCondition is 'auto'.
Default: 1.05

## Output Arguments

## opt - Options set for $\mathbf{b j}$

bj0ptions option set
Option set for bj , returned as an bjOptions option set.

## Examples

## Create Default Options Set for Box-Jenkins Estimation

```
opt = bjOptions;
```


## Specify Options for Box-Jenkins Estimation

Create an options set for bj using zero initial conditions for estimation. Set Display to ' on '.
opt = bjOptions('InitialCondition','zero','Display','on');
Alternatively, use dot notation to set the values of opt.

```
opt = bjOptions;
opt.InitialCondition = 'zero';
opt.Display = 'on';
```


## Compatibility Considerations

## Renaming of Estimation and Analysis Options

The names of some estimation and analysis options were changed in R2018a. Prior names still work. For details, see the R2018a release note "Renaming of Estimation and Analysis Options".

## References

[1] Wills, Adrian, B. Ninness, and S. Gibson. "On Gradient-Based Search for Multivariable System Estimates". Proceedings of the 16th IFAC World Congress, Prague, Czech Republic, July 3-8, 2005. Oxford, UK: Elsevier Ltd., 2005.
[2] Ljung, L. System Identification: Theory for the User. Upper Saddle River, NJ: Prentice-Hall PTR, 1999.

## See Also

bj|idfilt
Topics
"Loss Function and Model Quality Metrics"

Introduced in R2012a

## blkdiag

Block-diagonal concatenation of models

## Syntax

sys = blkdiag(sys1,sys2,...,sysN)

## Description

sys = blkdiag(sys1,sys2,...,sysN) produces the aggregate system
$\left[\begin{array}{cccc}\text { sys1 } & 0 & . & 0 \\ 0 & \text { sys2 } & . & : \\ : & . & . & 0 \\ 0 & . . & 0 & \text { sysN }\end{array}\right]$
blkdiag is equivalent to append.

## Examples

## Perform Block-Diagonal Concatenation

Perform block-diagonal concatenation of a transfer function model and a state-space model.
Create the SISO continuous-time transfer function model, $1 / \mathrm{s}$.
sys1 = tf(1,[10]);
Create a SISO continuous-time state-space model with state-space matrices 1,2,3, and 4 .
sys2 = ss(1,2,3,4);
Concatenate sys1, a SISO static gain system, and sys2. The resulting model is a 3-input, 3-output state-space model.

```
sys = blkdiag(sys1,10,sys2)
sys =
    A =
        < }\begin{array}{r}{\textrm{x}1}\\{\times1}\\{\times1}
        x2 0 1
    B =
            u1 u2 u3
        x1 1 00 0
        x2 0 0 2
    C =
```

```
l}\begin{array}{rrr}{x1}&{x2}\\{y1 1}&{0}
    y1 100
    y3 0 3
    D =
        u1 u2 u3
    y1 0 0 0
    y2 rrrre
```

Continuous-time state-space model.

Alternatively, use the append command.
sys = append(sys1,10,sys2);

## See Also

append|feedback|parallel|series

## Introduced in R2009a

## bode

Bode plot of frequency response, or magnitude and phase data

## Syntax

```
bode(sys)
bode(sys1,sys2,...,sysN)
bode(sys1,LineSpec1,...,sysN,LineSpecN)
bode(
```

$\qquad$

``` , w)
[mag,phase,wout] = bode(sys)
[mag,phase,wout] = bode(sys,w)
[mag,phase,wout,sdmag,sdphase] = bode(sys,w)
```


## Description

bode(sys) creates a Bode plot of the frequency response of a dynamic system model sys. The plot displays the magnitude (in dB ) and phase (in degrees) of the system response as a function of frequency. bode automatically determines frequencies to plot based on system dynamics.

If sys is a multi-input, multi-output (MIMO) model, then bode produces an array of Bode plots, each plot showing the frequency response of one I/O pair.
bode(sys1, sys2, ...,sysN) plots the frequency response of multiple dynamic systems on the same plot. All systems must have the same number of inputs and outputs.
bode(sys1,LineSpec1, . . . sysN, LineSpecN) specifies a color, line style, and marker for each system in the plot.
bode( $\qquad$ ,w) plots system responses for frequencies specified by w.

- If $w$ is a cell array of the form $\{w m i n, w m a x\}$, then bode plots the response at frequencies ranging between wmin and wmax.
- If $w$ is a vector of frequencies, then bode plots the response at each specified frequency.

You can use w with any of the input-argument combinations in previous syntaxes.
[mag, phase, wout] = bode(sys) returns the magnitude and phase of the response at each frequency in the vector wout. The function automatically determines frequencies in wout based on system dynamics. This syntax does not draw a plot.
[mag, phase, wout] = bode(sys,w) returns the response data at the frequencies specified by w .

- If $w$ is a cell array of the form $\{w m i n, w m a x\}$, then wout contains frequencies ranging between wmin and wmax.
- If $w$ is a vector of frequencies, then wout = $w$.
[mag, phase, wout,sdmag,sdphase] = bode(sys,w) also returns the estimated standard deviation of the magnitude and phase values for the identified model sys. If you omit $w$, then the function automatically determines frequencies in wout based on system dynamics.


## Examples

## Bode Plot of Dynamic System

Create a Bode plot of the following continuous-time SISO dynamic system.

$$
H(s)=\frac{s^{2}+0.1 s+7.5}{s^{4}+0.12 s^{3}+9 s^{2}} .
$$

H $=\mathrm{tf}\left(\left[\begin{array}{lll}1 & 0.1 & 7.5\end{array}\right],\left[\begin{array}{lll}1 & 0.12 & 9 \\ 0 & 0\end{array}\right]\right)$;
bode(H)

bode automatically selects the plot range based on the system dynamics.

## Bode Plot at Specified Frequencies

Create a Bode plot over a specified frequency range. Use this approach when you want to focus on the dynamics in a particular range of frequencies.

```
H = tf([-0.1,-2.4,-181,-1950],[1,3.3,990,2600]);
bode(H,{1,100})
grid on
```



The cell array $\{1,100\}$ specifies the minimum and maximum frequency values in the Bode plot. When you provide frequency bounds in this way, the function selects intermediate points for frequency response data.

Alternatively, specify a vector of frequency points to use for evaluating and plotting the frequency response.

```
w = [1 5 10 15 20 23 31 40 44 50 85 100];
bode(H,w,'--')
grid on
```


bode plots the frequency response at the specified frequencies only.

## Compare Bode Plots of Several Dynamic Systems

Compare the frequency response of a continuous-time system to an equivalent discretized system on the same Bode plot.

Create continuous-time and discrete-time dynamic systems.
$H=t f([10.17 .5],[10.12900]) ;$
Hd = c2d(H, 0.5,'zoh');
Create a Bode plot that displays both systems.
bode(H,Hd)


The Bode plot of a discrete-time system includes a vertical line marking the Nyquist frequency of the system.

## Bode Plot with Specified Line Attributes

Specify the line style, color, or marker for each system in a Bode plot using the LineSpec input argument.

```
H = tf([1 0.1 7.5],[1 0.12 9 0 0]);
```

Hd = c2d(H, 0.5,'zoh');
bode(H,'r',Hd,'b--')


The first LineSpec, ' $r$ ', specifies a solid red line for the response of $H$. The second LineSpec, 'b--' , specifies a dashed blue line for the response of Hd .

## Obtain Magnitude and Phase Data

Compute the magnitude and phase of the frequency response of a SISO system.
If you do not specify frequencies, bode chooses frequencies based on the system dynamics and returns them in the third output argument.

```
H = tf([1 0.1 7.5],[1 0.12 9 0 0]);
[mag,phase,wout] = bode(H);
```

Because H is a SISO model, the first two dimensions of mag and phase are both 1. The third dimension is the number of frequencies in wout.

```
size(mag)
ans = 1\times3
    1 1
        4 1
length(wout)
```

```
ans = 41
```

Thus, each entry along the third dimension of mag gives the magnitude of the response at the corresponding frequency in wout.

## Magnitude and Phase of MIMO System

For this example, create a 2-output, 3-input system.

```
rng(0,'twister'); % For reproducibility
H = rss(4,2,3);
```

For this system, bode plots the frequency responses of each I/O channel in a separate plot in a single figure.
bode(H)


Compute the magnitude and phase of these responses at 20 frequencies between 1 and 10 radians.

```
w = logspace(0,1,20);
[mag,phase] = bode(H,w);
```

mag and phase are three-dimensional arrays, in which the first two dimensions correspond to the output and input dimensions of H , and the third dimension is the number of frequencies. For instance, examine the dimensions of mag.

```
size(mag)
ans = 1\times3
    2 3 20
```

Thus, for example, $\operatorname{mag}(1,3,10)$ is the magnitude of the response from the third input to the first output, computed at the 10 th frequency in $w$. Similarly, phase $(1,3,10)$ contains the phase of the same response.

## Bode Plot of Identified Model

Compare the frequency response of a parametric model, identified from input/output data, to a nonparametric model identified using the same data.

Identify parametric and nonparametric models based on data.

```
load iddata2 z2;
w = linspace(0,10*pi,128);
sys_np = spa(z2,[],w);
sys_p = tfest(z2,2);
```

Using the spa and tfest commands requires System Identification Toolbox ${ }^{\mathrm{TM}}$ software.
sys_np is a nonparametric identified model. sys_p is a parametric identified model.
Create a Bode plot that includes both systems.

```
bode(sys_np,sys_p,w);
```

legend('sys-np','sys-p')

## Bode Diagram



You can display the confidence region on the Bode plot by right-clicking the plot and selecting Characteristics > Confidence Region.

## Obtain Magnitude and Phase Standard Deviation Data of Identified Model

Compute the standard deviation of the magnitude and phase of an identified model. Use this data to create a $3 \sigma$ plot of the response uncertainty.

Identify a transfer function model based on data. Obtain the standard deviation data for the magnitude and phase of the frequency response.

```
load iddata2 z2;
sys_p = tfest(z2,2);
w = linspace(0,10*pi,128);
[mag,ph,w,sdmag,sdphase] = bode(sys_p,w);
```

Using the tfest command requires System Identification Toolbox ${ }^{\mathrm{TM}}$ software.
sys_p is an identified transfer function model. sdmag and sdphase contain the standard deviation data for the magnitude and phase of the frequency response, respectively.

Use the standard deviation data to create a $3 \sigma$ plot corresponding to the confidence region.

```
mag = squeeze(mag);
sdmag = squeeze(sdmag);
semilogx(w,mag,'b',w,mag+3*sdmag,'k:',w,mag-3*sdmag,'k:');
```



## Input Arguments

## sys - Dynamic system

dynamic system model | model array
Dynamic system, specified as a SISO or MIMO dynamic system model or array of dynamic system models. Dynamic systems that you can use include:

- Continuous-time or discrete-time numeric LTI models, such as tf , zpk, or ss models.
- Generalized or uncertain LTI models such as genss or uss models. (Using uncertain models requires Robust Control Toolbox ${ }^{\mathrm{TM}}$ software.)
- For tunable control design blocks, the function evaluates the model at its current value for both plotting and returning frequency response data.
- For uncertain control design blocks, the function plots the nominal value and random samples of the model. When you use output arguments, the function returns frequency response data for the nominal model only.
- Frequency-response data models such as frd models. For such models, the function plots the response at frequencies defined in the model.
- Identified LTI models, such as idtf, idss, or idproc models. For such models, the function can also plot confidence intervals and return standard deviations of the frequency response. See "Bode Plot of Identified Model" on page 1-133.

If sys is an array of models, the function plots the frequency responses of all models in the array on the same axes.

## LineSpec - Line style, marker, and color

character vector | string
Line style, marker, and color, specified as a string or vector of one, two, or three characters. The characters can appear in any order. You do not need to specify all three characteristics (line style, marker, and color). For example, if you omit the line style and specify the marker, then the plot shows only the marker and no line. For more information about configuring this argument, see the LineSpec input argument of the plot function.

Example: 'r--' specifies a red dashed line
Example: '*b' specifies blue asterisk markers
Example: 'y' specifies a yellow line
w- Frequencies
\{wmin,wmax\} | vector
Frequencies at which to compute and plot frequency response, specified as the cell array \{wmin, wmax\} or as a vector of frequency values.

- If $w$ is a cell array of the form $\{w m i n, w m a x\}$, then the function computes the response at frequencies ranging between wmin and wmax.
- If $w$ is a vector of frequencies, then the function computes the response at each specified frequency. For example, use logspace to generate a row vector with logarithmically spaced frequency values.

Specify frequencies in units of rad/TimeUnit, where TimeUnit is the TimeUnit property of the model.

## Output Arguments

## mag - Magnitude of system response

3-D array
Magnitude of the system response in absolute units, returned as a 3-D array. The dimensions of this array are (number of system outputs) $\times$ (number of system inputs) $\times$ (number of frequency points).

- For SISO systems, mag $(1,1, k)$ gives the magnitude of the response at the kth frequency in $w$ or wout. For an example, see "Obtain Magnitude and Phase Data" on page 1-131.
- For MIMO systems, mag ( $i, j, k$ ) gives the magnitude of the response at the kth frequency from the jth input to the ith output. For an example, see "Magnitude and Phase of MIMO System" on page 1-132.

To convert the magnitude from absolute units to decibels, use:

```
magdb = 20*log10(mag)
```


## phase - Phase of system response

3-D array
Phase of the system response in degrees, returned as a 3-D array. The dimensions of this array are (number of outputs) $\times$ (number of inputs) $\times$ (number of frequency points).

- For SISO systems, mag ( $1,1, k$ ) gives the phase of the response at the kth frequency in w or wout. For an example, see "Obtain Magnitude and Phase Data" on page 1-131.
- For MIMO systems, mag ( $i, j, k$ ) gives the phase of the response at the kth frequency from the $j$ th input to the ith output. For an example, see "Magnitude and Phase of MIMO System" on page 1-132.
wout - Frequencies
vector
Frequencies at which the function returns the system response, returned as a column vector. The function chooses the frequency values based on the model dynamics, unless you specify frequencies using the input argument $w$.

Frequency values are in radians per TimeUnit, where TimeUnit is the value of the TimeUnit property of sys.

## sdmag - Standard deviation of magnitude

3-D array | [ ]
Estimated standard deviation of the magnitude of the response at each frequency point, returned as a 3-D array. sdmag has the same dimensions as mag.

If sys is not an identified LTI model, sdmag is [].

## sdphase - Standard deviation of phase <br> 3-D array | [ ]

Estimated standard deviation of the phase of the response at each frequency point, returned as a 3-D array. sdphase has the same dimensions as phase.

If sys is not an identified LTI model, sdphase is [ ].

## Tips

- When you need additional plot customization options, use bodeplot instead.


## Algorithms

bode computes the frequency response as follows:
1 Compute the zero-pole-gain ( zpk ) representation of the dynamic system.
2 Evaluate the gain and phase of the frequency response based on the zero, pole, and gain data for each input/output channel of the system.

- For continuous-time systems, bode evaluates the frequency response on the imaginary axis $s$ $=j \omega$ and considers only positive frequencies.
- For discrete-time systems, bode evaluates the frequency response on the unit circle. To facilitate interpretation, the command parameterizes the upper half of the unit circle as:

$$
z=e^{j \omega T_{s}}, \quad 0 \leq \omega \leq \omega_{N}=\frac{\Pi}{T_{s}},
$$

where $T_{s}$ is the sample time and $\omega_{N}$ is the Nyquist frequency. The equivalent continuous-time frequency $\omega$ is then used as the $x$-axis variable. Because $H\left(e^{j \omega T_{S}}\right)$ is periodic with period $2 \omega_{N}$, bode plots the response only up to the Nyquist frequency $\omega_{N}$. If sys is a discrete-time model with unspecified sample time, bode uses $T_{s}=1$.

## See Also

bodeplot | freqresp| nyquist | spectrum | step

## Topics

"Plot Bode and Nyquist Plots at the Command Line"
"Dynamic System Models"

## Introduced before R2006a

## bodemag

Magnitude-only Bode plot of frequency response

## Syntax

bodemag(sys)
bodemag(sys1,sys2,...,sysN)
bodemag(sys1,LineSpec1, ...,sysN,LineSpecN)
bodemag ( $\qquad$ , w)

## Description

bodemag enables you to generate magnitude-only plots to visualize the magnitude frequency response of a dynamic system.

For a more comprehensive function, see bode. bode provides magnitude and phase information. If you have System Identification toolbox, bode also returns the computed values, including statistical estimates.

For more customizable plotting options, see bodeplot.
bodemag (sys) creates a Bode magnitude plot of the frequency response of the dynamic system model sys. The plot displays the magnitude (in dB ) of the system response as a function of frequency. bodemag automatically determines frequencies to plot based on system dynamics.

If sys is a multi-input, multi-output (MIMO) model, then bodemag produces an array of Bode magnitude plots in which each plot shows the frequency response of one I/O pair.
bodemag (sys1, sys2, ...,sysN) plots the frequency response of multiple dynamic systems on the same plot. All systems must have the same number of inputs and outputs.
bodemag (sys1,LineSpec1, . . . sysN, LineSpecN) specifies a color, line style, and marker for each system in the plot.
bodemag ( $\qquad$ ,w) plots system responses for frequencies specified by w.

- If $w$ is a cell array of the form $\{w m i n, w m a x\}$, then bodemag plots the response at frequencies ranging between wmin and wmax.
- If $w$ is a vector of frequencies, then bodemag plots the response at each specified frequency.

You can use this syntax with any of the input-argument combinations in previous syntaxes.

## Examples

## Bode Magnitude Plot of Dynamic System

Create a Bode magnitude plot of the following continuous-time SISO dynamic system.

$$
H(s)=\frac{s^{2}+0.1 s+7.5}{s^{4}+0.12 s^{3}+9 s^{2}}
$$

H = tf([1 0.1 7.5],[1 0.12 9000$])$;
bodemag (H)

bodemag automatically selects the plot range based on the system dynamics.

## Bode Magnitude Plot at Specified Frequencies

Create a Bode magnitude plot over a specified frequency range. Use this approach when you want to focus on the dynamics in a particular range of frequencies.


The cell array $\{1,100\}$ specifies the minimum and maximum frequency values in the Bode magnitude plot. When you provide frequency bounds in this way, the function selects intermediate points for frequency response data.

Alternatively, specify a vector of frequency points to use for evaluating and plotting the frequency response.

```
w = [11 5 10 15 20 23 31 40 44 50 85 100];
bodemag(H,w,'.-')
grid on
```


bodemag plots the frequency response at the specified frequencies only.

## Compare Bode Magnitude Plots of Several Dynamic Systems

Compare the magnitude of the frequency response of a continuous-time system to an equivalent discretized system on the same Bode plot.

Create continuous-time and discrete-time dynamic systems.
H = tf([1 0.1 7.5],[1 0.12 9000$])$;
Hd = c2d(H, 0.5,'zoh');
Create a Bode magnitude plot that displays the responses of both systems.
bodemag (H,Hd)


The Bode magnitude plot of a discrete-time system includes a vertical line marking the Nyquist frequency of the system.

## Bode Magnitude Plot with Specified Line and Marker Attributes

Specify the color, linestyle, or marker for each system in a Bode magnitude plot using the LineSpec input arguments.

```
H = tf([1 0.1 7.5],[1 0.12 9 0 0]);
```

Hd = c2d(H, 0.5,'zoh'); bodemag(H,'r',Hd,'b--')


The first LineSpec argument ' $r$ ' specifies a solid red line for the response of H . The second LineSpec argument 'b-- ' specifies a dashed blue line for the response of Hd .

## Magnitude of MIMO System

For this example, create a 2 -output, 3 -input system.

```
rng(0,'twister'); % For reproducibility
```

H = rss (4, 2, 3) ;

For this system, bodemag plots the magnitude-only frequency responses of each I/O channel in a separate plot in a single figure.
bodemag (H)


## Input Arguments

## sys - Dynamic system

dynamic system model | model array
Dynamic system, specified as a SISO or MIMO dynamic system model or array of dynamic system models. Dynamic systems that you can use include:

- Continuous-time or discrete-time numeric LTI models, such as tf, zpk, or ss models.
- Generalized or uncertain LTI models such as genss or uss models. (Using uncertain models requires Robust Control Toolbox software.)
- For tunable control design blocks, the function evaluates the model at its current value for both plotting and returning frequency response data.
- For uncertain control design blocks, the function plots the nominal value and random samples of the model. When you use output arguments, the function returns frequency response data for the nominal model only.
- Frequency-response data models such as frd models. For such models, the function plots the response at frequencies defined in the model.
- Identified LTI models, such as idtf, idss, or idproc models.

If sys is an array of models, the function plots the frequency responses of all models in the array on the same axes.

## LineSpec - Line style, marker, and color

character vector | string
Line style, marker, and color, specified as a string or vector of one, two, or three characters. The characters can appear in any order. You do not need to specify all three characteristics (line style, marker, and color). For example, if you omit the line style and specify the marker, then the plot shows only the marker and no line. For more information about configuring this argument, see the LineSpec input argument of the plot function.

## Example: 'r--' specifies a red dashed line

Example: '*b' specifies blue asterisk markers
Example: 'y' specifies a yellow line

## w- Frequencies

\{wmin, wmax\} | vector
Frequencies at which to compute and plot frequency response, specified as the cell array \{wmin,wmax $\}$ or as a vector of frequency values.

- If $w$ is a cell array of the form $\{w m i n, w m a x\}$, then the function computes the index at frequencies ranging between wmin and wmax.
- If $w$ is a vector of frequencies, then the function computes the index at each specified frequency. For example, use logspace to generate a row vector with logarithmically spaced frequency values.

Specify frequencies in units of rad/TimeUnit, where TimeUnit is the TimeUnit property of the model.

## Algorithms

bodemag computes the frequency response as follows:
1 Compute the zero-pole-gain ( zpk ) representation of the dynamic system.
2 Evaluate the gain and phase of the frequency response based on the zero, pole, and gain data for each input/output channel of the system.

- For continuous-time systems, bodemag evaluates the frequency response on the imaginary axis $s=j \omega$ and considers only positive frequencies.
- For discrete-time systems, bodemag evaluates the frequency response on the unit circle. To facilitate interpretation, the command parameterizes the upper half of the unit circle as:

$$
z=e^{j \omega T_{s}}, \quad 0 \leq \omega \leq \omega_{N}=\frac{\Pi}{T_{s}},
$$

where $T_{s}$ is the sample time and $\omega_{N}$ is the Nyquist frequency. The equivalent continuous-time frequency $\omega$ is then used as the $x$-axis variable. Because $H\left(e^{j \omega T_{S}}\right)$ is periodic with period $2 \omega_{N}$, bodemag plots the response only up to the Nyquist frequency $\omega_{N}$. If sys is a discrete-time model with unspecified sample time, bodemag uses $T_{s}=1$.

## See Also

bode | bodeplot | freqresp|nyquist | spectrum | step

## Topics

"Plot Bode and Nyquist Plots at the Command Line"
"Dynamic System Models"
Introduced in R2012a

## bodeoptions

Create list of Bode plot options

## Syntax

P = bodeoptions
P = bodeoptions('cstprefs')

## Description

P = bodeoptions returns a default set of plot options for use with the bodeplot. You can use these options to customize the Bode plot appearance using the command line. This syntax is useful when you want to write a script to generate plots that look the same regardless of the preference settings of the MATLAB session in which you run the script.

P = bodeoptions('cstprefs') initializes the plot options with the options you selected in the Control System and System Identification Toolbox Preferences Editor. For more information about the editor, see "Toolbox Preferences Editor". This syntax is useful when you want to change a few plot options but otherwise use your default preferences. A script that uses this syntax may generate results that look different when run in a session with different preferences.

The following table summarizes the Bode plot options.

| Option | Description |
| :---: | :---: |
| Title, XLabel, YLabel | Label text and style, specified as a structure with the following fields: <br> - String - Label text, specified as a character vector, for example 'Amplitude'. <br> - FontSize - Default: 8 <br> - FontWeight - Default: 'Normal' <br> - Font Angle - Default: 'Normal' <br> - Color - Vector of RGB values ranging from 0 to 1. Default: [0, 0, 0] <br> - Interpreter - Default: 'tex' |
| TickLabel | Tick label style, specified as a structure with the following fields: <br> - FontSize Default: 8 <br> - FontWeight - Default: 'Normal' <br> - Font Angle - Default: 'Normal' <br> - Color - Vector of RGB values ranging from 0 to 1 . Default: [0, 0, 0] |
| Grid | Show or hide the grid <br> Specified as one of the following values: 'off' \| 'on' Default: 'off' |


| Option | Description |
| :---: | :---: |
| GridColor | Color of the grid lines <br> Specified as one of the following: Vector of RGB values in the range [0, 1] \| <br> character vector of color name $\mid$ ' none ' . For example, for yellow color, specify as one of the following: $\left.\begin{array}{lll}1 & 1 & 0\end{array}\right]$, 'yellow', or ' $y$ '. <br> Default: [0.15, 0.15, 0.15] |
| XlimMode, YlimMode | Axis limit modes. Default: 'auto' |
| Xlim, Ylim | Axes limits, specified as an array of the form [min, max] |
| IOGrouping | Grouping of input-output pairs <br> Specified as one of the following values: 'none'\|'inputs'|'outputs'|'all' <br> Default: ' none' |
| InputLabels, OutputLabels | Input and output label styles |
| InputVisible, OutputVisible | Visibility of input and output channels |
| ConfidenceRegionNumbe rSD | Number of standard deviations to use to plotting the response confidence region (identified models only). <br> Default: 1. |


| Option | Description |
| :---: | :---: |
| FreqUnits | Frequency units, specified as one of the following values: <br> - 'Hz' <br> - 'rad/second' <br> - 'rpm' <br> - 'kHz' <br> - 'MHz' <br> - 'GHz' <br> - 'rad/nanosecond' <br> - 'rad/microsecond' <br> - 'rad/millisecond' <br> - 'rad/minute' <br> - 'rad/hour' <br> - 'rad/day' <br> - 'rad/week' <br> - 'rad/month' <br> - 'rad/year' <br> - 'cycles/nanosecond' <br> - 'cycles/microsecond' <br> - 'cycles/millisecond' <br> - 'cycles/hour' <br> - 'cycles/day' <br> - 'cycles/week' <br> - 'cycles/month' <br> - 'cycles/year' |
| FreqScale | Frequency scale <br> Specified as one of the following values: 'linear' \| 'log' <br> Default: ' $\log$ ' |
| MagUnits | Magnitude units Specified as one of the following values: ' dB ' \| 'abs ' Default: ' dB ' |
| MagScale | Magnitude scale <br> Specified as one of the following values: 'linear' \| 'log' <br> Default: ' linear' |
| MagVisible | Magnitude plot visibility <br> Specified as one of the following values: 'on ' \| 'off' <br> Default: ' on ' |
| MagLowerLimMode | Enables a lower magnitude limit <br> Specified as one of the following values: 'auto' \| 'manual' <br> Default: 'auto' |
| MagLowerLim | Specifies the lower magnitude limit |


| Option | Description |
| :---: | :---: |
| PhaseUnits | Phase units <br> Specified as one of the following values: 'deg ' \| ' rad ' Default: 'deg' |
| PhaseVisible | Phase plot visibility <br> Specified as one of the following values: 'on ' \| 'off' Default: ' on ' |
| PhaseWrapping | Enables phase wrapping <br> Specified as one of the following values: 'on ' \| 'off' <br> When you set PhaseWrapping to 'on' , the plot wraps accumulated phase at the value specified by the PhaseWrappingBranch property. <br> Default: 'off' |
| PhaseWrappingBranch | Phase value at which the plot wraps accumulated phase when PhaseWrapping is set to 'on'. <br> Default: - 180 (phase wraps into the interval $\left[-180^{\circ}, 180^{\circ}\right.$ ) ) |
| PhaseMatching | Enables phase matching <br> Specified as one of the following values: 'on ' \| 'off' <br> Default: ' off' |
| PhaseMatchingFreq | Frequency for matching phase |
| PhaseMatchingValue | The value to which phase responses are matched closely |

## Examples

## Create Bode Plot with Custom Settings

Create a Bode plot that suppresses the phase plot and uses frequency units Hz instead of the default radians/second. Otherwise, the plot uses the settings that are saved in the toolbox preferences.

First, create an options set based on the toolbox preferences.
opts = bodeoptions('cstprefs');
Change properties of the options set.

```
opts.PhaseVisible = 'off';
```

opts.FreqUnits = 'Hz';

Create a plot using the options.
h = bodeplot(tf(1,[1,1]),opts);


Depending on your own toolbox preferences, the plot you obtain might look different from this plot. Only the properties that you set explicitly, in this example PhaseVisible and FreqUnits, override the toolbox preferences.

## Custom Plot Settings Independent of Preferences

Create a Bode plot that uses 14 -point red text for the title. This plot should look the same, regardless of the preferences of the MATLAB session in which it is generated.

First, create a default options set.
opts = bodeoptions;
Change properties of the options set.
opts.Title.FontSize = 14;
opts.Title.Color = [1 0 0];
opts.FreqUnits = 'Hz';
Create a plot using the options.
h = bodeplot(tf(1,[1,1]),opts);


Because opts begins with a fixed set of options, the plot result is independent of the toolbox preferences of the MATLAB session.

```
See Also
bode | bodeplot | getoptions| setoptions| showConfidence
Introduced in R2012a
```


## bodeplot

Plot Bode frequency response with additional plot customization options

## Syntax

```
h = bodeplot(sys)
bodeplot(sys)
bodeplot(sys1,sys2,...)
bodeplot(AX,...)
bodeplot(..., plotoptions)
bodeplot(sys,w)
```


## Description

$\mathrm{h}=$ bodeplot(sys) plot the Bode magnitude and phase of the dynamic system model sys and returns the plot handle $h$ to the plot. You can use this handle to customize the plot with the getoptions and setoptions commands.
bodeplot (sys) draws the Bode plot of the model sys. The frequency range and number of points are chosen automatically.
bodeplot (sys1, sys $2, \ldots$ ) graphs the Bode response of multiple models sys 1, sys $2, \ldots$ on a single plot. You can specify a color, line style, and marker for each model, as in
bodeplot(sys1,'r',sys2,'y--',sys3,'gx')
bodeplot (AX, ...) plots into the axes with handle AX.
bodeplot(..., plotoptions) plots the Bode response with the options specified in plotoptions. Type
help bodeoptions
for a list of available plot options. See "Match Phase at Specified Frequency" on page 1-156 for an example of phase matching using the PhaseMatchingFreq and PhaseMatchingValue options.
bodeplot (sys,w) draws the Bode plot for frequencies specified by w. When w = \{wmin, wmax \}, the Bode plot is drawn for frequencies between wmin and wmax (in rad/TimeUnit, where TimeUnit is the time units of the input dynamic system, specified in the TimeUnit property of sys.). When w is a user-supplied vector $w$ of frequencies, in rad/TimeUnit, the Bode response is drawn for the specified frequencies.

See logspace to generate logarithmically spaced frequency vectors.

## Examples

## Change Bode Plot Options with Plot Handle

Generate a Bode plot.
sys = rss(5);
h = bodeplot(sys);


Change the units to Hz and suppress the phase plot. To do so, edit properties of the plot handle, h . setoptions(h,'FreqUnits','Hz','PhaseVisible','off');


The plot automatically updates when you call setoptions.

## Match Phase at Specified Frequency

Create a Bode plot of a dynamic system.
sys = tf(1,[1 1]);
h = bodeplot(sys);


Fix the phase at $1 \mathrm{rad} / \mathrm{s}$ to 750 degrees. To do so, get the plot properties. Then alter the properties PhaseMatchingFreq and PhaseMatchingValue to match a phase to a specified frequency.
$\mathrm{p}=$ getoptions(h);
p.PhaseMatching = 'on';
p. PhaseMatchingFreq = 1;
p.PhaseMatchingValue $=750$;

Update the plot.
setoptions(h,p);


The first bode plot has a phase of -45 degrees at a frequency of $1 \mathrm{rad} / \mathrm{s}$. Setting the phase matching options so that at $1 \mathrm{rad} / \mathrm{s}$ the phase is near 750 degrees yields the second Bode plot. Note that, however, the phase can only be $-45+N * 360$, where $N$ is an integer, and so the plot is set to the nearest allowable phase, namely 675 degrees (or $2 * 360-45=675$ ).

## Display Confidence Regions of Identified Models

Compare the frequency responses of identified state-space models of order 2 and 6 along with their 2 $\sigma$ confidence regions.

```
load iddata1
sys1 = n4sid(z1, 2);
sys2 = n4sid(z1, 6);
```

Both models produce about 70\% fit to data. However, sys2 shows higher uncertainty in its frequency response, especially close to Nyquist frequency as shown by the plot:

```
w = linspace(8,10*pi,256);
h = bodeplot(sys1,sys2,w);
setoptions(h,'PhaseMatching','on','ConfidenceRegionNumberSD',2);
```



Right-click the plot and select Characteristics > Confidence Region to turn on the confidence region characteristic. Alternatively, type showConfidence(h) to plot the confidence region.

## Frequency Response of Identified Parametric and Nonparametric Models

Compare the frequency response of a parametric model, identified from input/output data, to a nonparametric model identified using the same data. Identify parametric and non-parametric models based on data.

```
load iddata2 z2;
w = linspace(0,10*pi,128);
sys_np = spa(z2,[],w);
sys_p = tfest(z2,2);
```

spa and tfest require System Identification Toolbox ${ }^{\mathrm{TM}}$ software. sys_np is a nonparametric identified model. sys_p is a parametric identified model.

Create a Bode plot that includes both systems.
opt = bodeoptions;
opt.PhaseMatching = 'on';
bodeplot(sys_np,sys_p,w,opt);


## Tips

You can change the properties of your plot, for example the units. For information on the ways to change properties of your plots, see "Ways to Customize Plots" (Control System Toolbox).

## See Also

bode | bodeoptions | getoptions | setoptions | showConfidence
Introduced before R2006a

## c2d

Convert model from continuous to discrete time

## Syntax

sysd = c2d(sysc,Ts)
sysd $=$ c2d(sysc,Ts,method)
sysd $=$ c2d(sysc,Ts,opts)
[sysd,G] = c2d( $\qquad$ )

## Description

sysd $=c 2 d(s y s c, T s)$ discretizes the continuous-time dynamic system model sysc using zeroorder hold on the inputs and a sample time of Ts.
sysd $=$ c2d(sysc,Ts,method) specifies the discretization method.
sysd $=$ c2d(sysc,Ts,opts) specifies additional options for the discretization.
[sysd,G] = c2d( $\qquad$ ), where sysc is a state-space model, returns a matrix, G that maps the continuous initial conditions $x_{0}$ and $u_{0}$ of the state-space model to the discrete-time initial state vector $x[0]$.

## Examples

## Discretize a Transfer Function

Discretize the following continuous-time transfer function:

$$
H(s)=e^{-0.3 s} \frac{s-1}{s^{2}+4 s+5} .
$$

This system has an input delay of 0.3 s . Discretize the system using the triangle (first-order-hold) approximation with sample time $\mathrm{Ts}=0.1 \mathrm{~s}$.
$H=t f\left(\left[\begin{array}{ll}1 & -1\end{array}\right],\left[\begin{array}{lll}1 & 4 & 5\end{array}\right]\right.$, InputDelay', 0.3$) ;$
Hd = c2d(H,0.1,'foh');
Compare the step responses of the continuous-time and discretized systems.
step(H,'-',Hd,'--')


## Discretize Model with Fractional Delay Absorbed into Coefficients

Discretize the following delayed transfer function using zero-order hold on the input, and a $10-\mathrm{Hz}$ sampling rate.

$$
H(s)=e^{-0.25 s} \frac{10}{s^{2}+3 s+10} .
$$

h = tf(10,[1 3 10],'IODelay',0.25);
hd $=c 2 d(h, 0.1)$
hd $=$

$$
0.01187 z^{\wedge} 2+0.06408 z+0.009721
$$

$z^{\wedge}(-3) *------------------------------$
$z^{\wedge} 2-1.655 z+0.7408$
Sample time: 0.1 seconds
Discrete-time transfer function.
In this example, the discretized model hd has a delay of three sampling periods. The discretization algorithm absorbs the residual half-period delay into the coefficients of hd.

Compare the step responses of the continuous-time and discretized models.
step(h,'--',hd,'-')


## Discretize Model With Approximated Fractional Delay

Create a continuous-time state-space model with two states and an input delay.
sys $=\operatorname{ss}(t f([1,2],[1,4,2]))$;
sys.InputDelay $=2.7$
sys =
$A=$

|  | x1 | x2 |
| ---: | ---: | ---: |
| x1 | -4 | -2 |
| $\times 2$ | 1 | 0 |

$B=$

$$
\begin{array}{rr} 
& \mathrm{ul} \\
\times 1 & 2 \\
\times 2 & 0
\end{array}
$$

C =
$\begin{array}{rr}x 1 & x 2 \\ 0.5 & 1\end{array}$

```
    D =
        ul
    y1 0
    Input delays (seconds): 2.7
Continuous-time state-space model.
```

Discretize the model using the Tustin discretization method and a Thiran filter to model fractional delays. The sample time Ts $=1$ second.

```
opt = c2dOptions('Method','tustin','FractDelayApproxOrder',3);
sysd1 = c2d(sys,1,opt)
sysd1 =
    A =
\begin{tabular}{rrrrrr} 
& & \(\times 1\) & \(\times 2\) & \(x 3\) & \(\times 4\) \\
\(\times 1\) & -0.4286 & -0.5714 & -0.00265 & 0.06954 & 2.286 \\
\(\times 2\) & 0.2857 & 0.7143 & -0.001325 & 0.03477 & 1.143 \\
\(\times 3\) & 0 & 0 & -0.2432 & 0.1449 & -0.1153 \\
\(\times 4\) & 0 & 0 & 0.25 & 0 & 0 \\
x5 & 0 & 0 & 0 & 0.125 & 0
\end{tabular}
    B =
        u1
        0.002058
        x2 0.001029
        x3 8
        x4 0
        x5 0
    C =
        rrrrrrer
    D =
        u1
        y1 0.001029
Sample time: 1 seconds
Discrete-time state-space model.
```

The discretized model now contains three additional states $\times 3, \times 4$, and $\times 5$ corresponding to a thirdorder Thiran filter. Since the time delay divided by the sample time is 2.7 , the third-order Thiran filter ('FractDelayApproxOrder' = 3) can approximate the entire time delay.

## Discretize Identified Model

Estimate a continuous-time transfer function, and discretize it.

```
load iddata1
syslc = tfest(z1,2);
sysld = c2d(sys1c,0.1,'zoh');
```

Estimate a second order discrete-time transfer function.
sys2d = tfest(z1,2,'Ts',0.1);
Compare the response of the discretized continuous-time transfer function model, sys1d, and the directly estimated discrete-time model, sys2d.
compare(z1,sys1d,sys2d)


The two systems are almost identical.

## Build Predictor Model

Discretize an identified state-space model to build a one-step ahead predictor of its response.
Create a continuous-time identified state-space model using estimation data.
load iddata2
sysc = ssest(z2,4);
Predict the 1 -step ahead predicted response of sysc.
predict(sysc,z2)


Discretize the model.
sysd = c2d(sysc,0.1,'zoh');
Build a predictor model from the discretized model, sysd.
[A,B,C,D,K] = idssdata(sysd);
Predictor $=$ ss(A-K*C,[K B-K*D],C,[0 D],0.1);
Predictor is a two-input model which uses the measured output and input signals ([z1.y z1.u]) to compute the 1 -step predicted response of sysc.

Simulate the predictor model to get the same response as the predict command.
lsim(Predictor,[z2.y,z2.u])


The simulation of the predictor model gives the same response as predict(sysc,z2).

## Input Arguments

## sysc - Continuous-time dynamic system

dynamic system model
Continuous-time model, specified as a dynamic system model such as idtf, idss, or idpoly. sysc cannot be a frequency response data model. sysc can be a SISO or MIMO system, except that the 'matched ' discretization method supports SISO systems only.
sysc can have input/output or internal time delays; however, the 'matched', 'impulse', and ' least-squares ' methods do not support state-space models with internal time delays.

The following identified linear systems cannot be discretized directly:

- idgrey models whose FunctionType is 'c '. Convert to idss model first.
- idproc models. Convert to idtf or idpoly model first.


## Ts - Sample time

positive scalar
Sample time, specified as a positive scalar that represents the sampling period of the resulting discrete-time system. Ts is in TimeUnit, which is the sysc.TimeUnit property.

## method - Discretization method

'zoh' (default)|'foh'|'impulse'|'tustin'|'matched'|'least-squares'
Discretization method, specified as one of the following values:

- 'zoh' - Zero-order hold (default). Assumes the control inputs are piecewise constant over the sample time Ts.
- 'foh ' - Triangle approximation (modified first-order hold). Assumes the control inputs are piecewise linear over the sample time Ts.
- 'impulse ' - Impulse invariant discretization
- 'tustin' - Bilinear (Tustin) method. To specify this method with frequency prewarping (formerly known as the 'prewarp' method), use the PrewarpFrequency option of c2dOptions.
- 'matched ' - Zero-pole matching method
- 'least-squares' - Least-squares method
- 'damped ' - Damped Tustin approximation based on the TRBDF2 formula for sparse models only.

For information about the algorithms for each conversion method, see "Continuous-Discrete Conversion Methods".

## opts - Discretization options

c2dOptions object
Discretization options, specified as a c2d0ptions object. For example, specify the prewarp frequency, order of the Thiran filter or discretization method as an option.

## Output Arguments

## sysd - Discrete-time model

dynamic system model
Discrete-time model, returned as a dynamic system model of the same type as the input system sysc.
When sysc is an identified (IDLTI) model, sysd:

- Includes both measured and noise components of sysc. The innovations variance $\lambda$ of the continuous-time identified model sysc, stored in its NoiseVarianceproperty, is interpreted as the intensity of the spectral density of the noise spectrum. The noise variance in sysd is thus $\lambda / T s$.
- Does not include the estimated parameter covariance of sysc. If you want to translate the covariance while discretizing the model, use translatecov.


## G - Mapping of continuous initial conditions of state-space model to discrete-time initial state vector

matrix
Mapping of continuous-time initial conditions $x_{0}$ and $u_{0}$ of the state-space model sysc to the discretetime initial state vector $x[0]$, returned as a matrix. The mapping of initial conditions to the initial state vector is as follows:

$$
x[0]=G \cdot\left[\begin{array}{l}
x_{0} \\
u_{0}
\end{array}\right]
$$

For state-space models with time delays, c2d pads the matrix $G$ with zeroes to account for additional states introduced by discretizing those delays. See "Continuous-Discrete Conversion Methods" for a discussion of modeling time delays in discretized systems.

## See Also

c2d0ptions |d2c|d2d|thiran|translatecov

## Topics

"Dynamic System Models"
"Transforming Between Discrete-Time and Continuous-Time Representations"
"Continuous-Discrete Conversion Methods"
Introduced before R2006a

## c2dOptions

Create option set for continuous- to discrete-time conversions

## Syntax

opts = c2dOptions
opts = c2dOptions('OptionName', OptionValue)

## Description

opts = c2d0ptions returns the default options for c2d.
opts = c2dOptions('OptionName', OptionValue) accepts one or more comma-separated name/value pairs that specify options for the c2d command. Specify OptionName inside single quotes.

## Input Arguments

## Name-Value Pair Arguments

## Method

Discretization method, specified as one of the following values:

| 'zoh ' | Zero-order hold, where c2d assumes the control inputs are piecewise constant <br> over the sample time Ts. <br> Triangle approximation (modified first-order hold), where c2d assumes the <br> control inputs are piecewise linear over the sample time Ts. (See [1] on page <br> 1-171, p. 228.) |
| :--- | :--- |
| ' foh ' | Impulse-invariant discretization. <br> Bilinear (Tustin) approximation. By default, c2d discretizes with no prewarp <br> and rounds any fractional time delays to the nearest multiple of the sample <br> time. To include prewarp, use the PrewarpFrequency option. To approximate <br> fractional time delays, use theFractDelayApproxOrder option. |
| 'tustin ' | Zero-pole matching method. (See [1] on page 1-171, p. 224.) By default, c2d <br> rounds any fractional time delays to the nearest multiple of the sample time. <br> To approximate fractional time delays, use the FractDelayApprox0rder <br> option. |
| 'matched ' least-squares 'Least-squares method. Minimize the error between the frequency responses <br> of the continuous-time and discrete-time systems up to the Nyquist frequency. |  |

For information about the algorithms for each conversion method, see "Continuous-Discrete Conversion Methods".

Default: 'zoh'

## PrewarpFrequency

Prewarp frequency for 'tustin' method, specified in rad/TimeUnit, where TimeUnit is the time units, specified in the TimeUnit property, of the discretized system. Takes positive scalar values. A value of 0 corresponds to the standard 'tustin' method without prewarp.

## Default: 0

## FitOrder

Fit order for ' least-squares ' method, specified as an integer. Specifies the order of the discretetime model to be fitted to the continuous-time frequency response. Leave the default option 'auto' to use the order of the continuous-time model, and change it to an integer N to use an $\mathrm{N}^{\text {th }}$-order fit. Reducing the order helps with unstable poles or pole/zero cancellations at $z=-1$.

Default: 'auto'

## FractDelayApproxOrder

Maximum order of the Thiran filter used to approximate fractional delays in the 'tustin' and 'matched ' methods. Takes integer values. A value of 0 means that c2d rounds fractional delays to the nearest integer multiple of the sample time.

## Default: 0

## Examples

## Discretize Two Models Using Tustin Discretization Method

Generate two random continuous-time state-space models.

```
sys1 = rss(3,2,2);
sys2 = rss(4,4,1);
```

Create an option set for c2d to use the Tustin discretization method and $3.4 \mathrm{rad} / \mathrm{s}$ prewarp frequency.

```
opt = c2dOptions('Method','tustin','PrewarpFrequency',3.4);
```

Discretize the models, sys1 and sys2, using the same option set, but different sample times.

```
dsys1 = c2d(sys1,0.1,opt);
dsys2 = c2d(sys2,0.2,opt);
```


## References

[1] Franklin, G.F., Powell, D.J., and Workman, M.L., Digital Control of Dynamic Systems (3rd Edition), Prentice Hall, 1997.

## See Also

c2d

Introduced in R2012a

## canon

Canonical state-space realization

## Syntax

```
csys = canon(sys,type)
csys = canon(sys,'modal',condt)
[csys,T]= canon(
```

$\qquad$

``` )
```


## Description

csys = canon(sys, type) transforms the linear model sys into a canonical state-space model csys. type specifies whether csys is in modal or companion form.

For information on controllable and observable canonical forms, see "Canonical State-Space Realizations".
csys = canon(sys,'modal', condt) specifies an upper bound condt on the condition number of the block-diagonalizing transformation. Use condt if you have close lying eigenvalues in csys.
[csys,T]= canon( $\qquad$ ) also returns the state-coordinate transformation matrix T that relates the states of the state-space model sys to the states of csys.

## Examples

## Convert State-Space Model to Companion Canonical Form

aircraftPitchSSModel.mat contains the state-space matrices of an aircraft where the input is elevator deflection angle $\delta$ and the output is the aircraft pitch angle $\theta$.

$$
\begin{aligned}
{\left[\begin{array}{c}
\dot{\alpha} \\
\dot{q} \\
\dot{\theta}
\end{array}\right] } & =\left[\begin{array}{ccc}
-0.313 & 56.7 & 0 \\
-0.0139 & -0.426 & 0 \\
0 & 56.7 & 0
\end{array}\right]\left[\begin{array}{l}
\alpha \\
q \\
\theta
\end{array}\right]+\left[\begin{array}{c}
0.232 \\
0.0203 \\
0
\end{array}\right][\delta] \\
y & =\left[\begin{array}{lll}
0 & 0 & 1
\end{array}\right]\left[\begin{array}{l}
\alpha \\
q \\
\theta
\end{array}\right]+[0][\delta]
\end{aligned}
$$

Load the model data to the workspace and create the state-space model sys.

```
load('aircraftPitchSSModel.mat');
sys = ss(A,B,C,D)
sys =
    A =
        rrrer
```

```
    x2 -0.0139 -0.426 0
    x3 0
    B =
        \times1 0.232
        x2 0.0203
        x3 0
    C =
        x1 r2 re3
    D =
        u1
    y1 0
Continuous-time state-space model.
```

Convert the resultant state-space model sys to companion canonical form.

```
csys = canon(sys,'companion')
csys =
    A =
        rrrr
    B =
        u1
        x1 1
        x2 0
        x3 0
    C =
        l
    D =
        ul
    y1 0
Continuous-time state-space model.
```

csys is the companion canonical form of sys.

## Convert State-Space Model to Modal Canonical Form

pendulumCartSSModel.mat contains the state-space model of an inverted pendulum on a cart where the outputs are the cart displacement x and the pendulum angle $\theta$. The control input u is the horizontal force on the cart.

$$
\begin{gathered}
{\left[\begin{array}{c}
\dot{x} \\
\ddot{x} \\
\dot{\theta} \\
\ddot{\theta}
\end{array}\right]=\left[\begin{array}{cccc}
0 & 1 & 0 & 0 \\
0 & -0.1 & 3 & 0 \\
0 & 0 & 0 & 1 \\
0 & -0.5 & 30 & 0
\end{array}\right]\left[\begin{array}{l}
x \\
\dot{x} \\
\theta \\
\dot{\theta}
\end{array}\right]+\left[\begin{array}{l}
0 \\
2 \\
0 \\
5
\end{array}\right] u} \\
y=\left[\begin{array}{llll}
1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0
\end{array}\right]\left[\begin{array}{c}
x \\
\dot{x} \\
\theta \\
\dot{\theta}
\end{array}\right]+\left[\begin{array}{l}
0 \\
0
\end{array}\right] u
\end{gathered}
$$

First, load the state-space model sys to the workspace.
load('pendulumCartSSModel.mat','sys');
Convert sys to modal canonical form and extract the transformation matrix.

```
[csys,T] = canon(sys,'modal')
csys =
    A =
        rrrrr
    B =
        M 41
        x2 6.298
        x3 12.8
        x4 12.05
    C=
        lrrrr
    D =
        ul
        y1 0
        y2 0
Continuous-time state-space model.
T = 4×4
\begin{tabular}{rrrr}
0.0625 & 1.2500 & -0.0000 & -0.1250 \\
0 & 4.1986 & 0.0210 & -0.4199 \\
0 & 0.2285 & -13.5873 & 2.4693 \\
0 & -0.2251 & 13.6287 & 2.4995
\end{tabular}
```

csys is the modal canonical form of sys, while T represents the transformation between the state vectors of sys and csys.

## Convert Zero-Pole-Gain Model to Modal Canonical Form

For this example, consider the following system with doubled poles and clusters of close poles:

$$
\operatorname{sys}(s)=100 \frac{(s-1)(s+1)}{s(s+10)(s+10.0001)(s-(1+i))^{2}(s-(1-i))^{2}}
$$

Create a zpk model of this system and convert it to modal canonical form using the string 'modal'.
sys $=$ zpk([1-1],[0 -10 -10.0001 1+1i 1-1i 1+1i 1-1i],100);
csys1 = canon(sys,'modal');
csys1.A

```
ans = 7\times7
```

| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 0 | 1.0000 | 1.0000 | 0 | 0 | 0 | 0 |
| 0 | -1.0000 | 1.0000 | 3.2459 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1.0000 | 1.0000 | 0 | 0 |
| 0 | 0 | 0 | -1.0000 | 1.0000 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | -10.0000 | 4.0571 |
| 0 | 0 | 0 | 0 | 0 | 0 | -10.0001 |

Csys1.B
ans $=7 \times 1$
0.1600
-0.0052
0.0427
-0.0975
0.5319
4.0095
sys has a pair of poles at $s=-10$ and $s=-10.0001$, and two complex poles of multiplicity 2 at $s=$ $1+i$ and $s=1-i$. As a result, the modal form csys 1 is a state-space model with a block of size 2 for the two poles near $s=-10$, and a block of size 4 for the complex eigenvalues.

Now, separate the two poles near $s=-10$ by increasing the value of the condition number of the block-diagonalizing transformation. Use a value of 1 e 10 for this example.

```
csys2 = canon(sys,'modal',1e10);
csys2.A
ans = 7×7
```

| 0 |  | 0 | 0 | 0 | 0 | 0 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 0 | 1.0000 | 1.0000 | 0 | 0 | 0 | 0 |
| 0 | -1.0000 | 1.0000 | 3.2459 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1.0000 | 1.0000 | 0 | 0 |
| 0 | 0 | 0 | -1.0000 | 1.0000 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | -10.0000 | 0 |

0
0
0
0
0
$0-10.0001$

```
format shortE
csys2.B
ans = 7\times1
    1.6000e-01
    -5.1885e-03
    4.2687e-02
    -9.7508e-02
    5.3187e-01
    1.6267e+05
    1.6267e+05
```

The A matrix of csys2 includes separate diagonal elements for the poles near $s=-10$. Increasing the condition number results in some very large values in the B matrix.

## Convert System to Companion Canonical Form

The file icEngine. mat contains one data set with 1500 input-output samples collected at the a sampling rate of 0.04 seconds. The input $u(t)$ is the voltage $(V)$ controlling the By-Pass Idle Air Valve (BPAV), and the output $\mathrm{y}(\mathrm{t})$ is the engine speed (RPM/100).

Use the data in icEngine. mat to create a state-space model with identifiable parameters.

```
load icEngine.mat
z = iddata(y,u,0.04);
sys = n4sid(z,4,'InputDelay',2);
```

Convert the identified state-space model sys to companion canonical form.

```
csys = canon(sys,'companion');
```

Obtain the covariance of the resulting form by running a zero-iteration update to model parameters.

```
opt = ssestOptions;
opt.SearchOptions.MaxIterations = 0;
csys = ssest(z,csys,opt);
```

Compare frequency response confidence bounds of sys to csys.

```
h = bodeplot(sys,csys,'r.');
showConfidence(h)
```


## Bode Diagram



The frequency response confidence bounds are identical.

## Input Arguments

## sys - Dynamic system

dynamic system model
Dynamic system, specified as a SISO, or MIMO dynamic system model. Dynamic systems that you can use include:

- Continuous-time or discrete-time numeric LTI models, such as tf, zpk, ss, or pid models.
- Generalized or uncertain LTI models such as genss or uss models. (Using uncertain models requires Robust Control Toolbox software.)

The resulting canonical state-space model assumes

- current values of the tunable components for tunable control design blocks.
- nominal model values for uncertain control design blocks.
- Identified LTI models, such as idtf, idss, idproc, idpoly, and idgrey models.

You cannot use frequency-response data models such as frd models.

## type - Transformation type

'modal' (default)|'companion'

Transformation type, specified as either 'modal' or ' companion '. If type is unspecified, then canon converts the specified dynamic system model to modal canonical form by default.

The companion canonical form is the same as the observable canonical form. For information on controllable and observable canonical forms, see "Canonical State-Space Realizations".

- Modal Form

In modal form, $A$ is a block-diagonal matrix. The block size is typically 1-by-1 for real eigenvalues and 2-by-2 for complex eigenvalues. However, if there are repeated eigenvalues or clusters of nearby eigenvalues, the block size can be larger.

For example, for a system with eigenvalues $\left(\lambda_{1}, \sigma \pm j \omega, \lambda_{2}\right)$, the modal $A$ matrix is of the form

$$
\left[\begin{array}{cccc}
\lambda_{1} & 0 & 0 & 0 \\
0 & \sigma & \omega & 0 \\
0 & -\omega & \sigma & 0 \\
0 & 0 & 0 & \lambda_{2}
\end{array}\right]
$$

- Companion Form

In the companion realization, the characteristic polynomial of the system appears explicitly in the rightmost column of the $A$ matrix. For a system with characteristic polynomial

$$
P(s)=s^{n}+\alpha_{1} s^{n-1}+\ldots+\alpha_{n-1} s+\alpha_{n}
$$

the corresponding companion $A$ matrix is

$$
A=\left[\begin{array}{cccccc}
0 & 0 & 0 & \ldots & 0 & -\alpha_{n} \\
1 & 0 & 0 & \ldots & 0 & -\alpha_{n-1} \\
0 & 1 & 0 & \ldots & 0 & -\alpha_{n-2} \\
0 & 0 & 1 & \ldots & 0 & -\alpha_{n-3} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & \ldots & 1 & -\alpha_{1}
\end{array}\right]
$$

The companion transformation requires that the system is controllable from the first input. The transformation to companion form is based on the controllability matrix which is almost always numerically singular for mid-range orders. Hence, avoid using it when possible.

The companion canonical form is the same as the observable canonical form. For more information on observable and controllable canonical forms, see "Canonical State-Space Realizations".

## condt - Upper bound on the condition number of the block-diagonalizing transformation

1e8 (default) | positive scalar
Upper bound on the condition number of the block-diagonalizing transformation, specified as a positive scalar. This argument is available only when type is set to 'modal '.

Increase condt to reduce the size of the eigenvalue clusters in the $A$ matrix of csys. Setting condt $=$ Inf diagonalizes matrix $A$.

## Output Arguments

## csys - Canonical state-space form of the dynamic model

ss model object
Canonical state-space form of the dynamic model, returned as an ss model object. csys is a statespace realization of sys in the canonical form specified by type.

## T - Transformation matrix

## matrix

Transformation matrix, returned as an $n$-by-n matrix, where $n$ is the number of states. $T$ is the transformation between the state vector $x$ of the state-space model sys and the state vector $x_{c}$ of csys:
$x_{c}=T x$

This argument is available only when sys is an ss model object.

## Limitations

- You cannot use frequency-response data models to convert to canonical state-space form.
- The companion form is poorly conditioned for most state-space computations, that is, the transformation to companion form is based on the controllability matrix which is almost always numerically singular for mid-range orders. Hence, avoid using it when possible.


## Algorithms

The canon command uses the bdschur command to convert sys into modal form and to compute the transformation T. If sys is not a state-space model, canon first converts it to state space using ss.

The reduction to companion form uses a state similarity transformation based on the controllability matrix [1].

## References

[1] Kailath, T. Linear Systems, Prentice-Hall, 1980.

```
See Also
ctrb|ctrbf|genss|idgrey|idpoly|idproc|idss|idtf|pid|ss|ss2ss|tf|uss|zpk
Topics
"Canonical State-Space Realizations"
```

Introduced before R2006a

## chgFreqUnit

Change frequency units of frequency-response data model

## Syntax

sys_new = chgFreqUnit(sys,newfrequnits)

## Description

sys_new = chgFreqUnit(sys, newfrequnits) changes units of the frequency points in sys to newfrequits. Both Frequency and FrequencyUnit properties of sys adjust so that the frequency responses of sys and sys_new match.

## Input Arguments

## sys

Frequency-response data ( frd , idfrd, or genfrd) model

## newfrequnits

New units of frequency points, specified as one of the following values:

- 'rad/TimeUnit'
- 'cycles/TimeUnit'
- 'rad/s'
- 'Hz'
- 'kHz'
- 'MHz'
- 'GHz'
- 'rpm'
rad/TimeUnit and cycles/TimeUnit express frequency units relative to the system time units specified in the TimeUnit property.

Default: 'rad/TimeUnit'

## Output Arguments

sys_new
Frequency-response data model of the same type as sys with new units of frequency points. The frequency response of sys_new is same as sys.

## Examples

## Change Frequency Units of Frequency-Response Data Model

Create a frequency-response data model.
load('AnalyzerData');
sys = frd(resp,freq);
The data file AnalyzerData has column vectors freq and resp. These vectors contain 256 test frequencies and corresponding complex-valued frequency response points, respectively. The default frequency units of sys is rad/TimeUnit, where TimeUnit is the system time units.

Change the frequency units.
sys1 = chgFreqUnit(sys,'rpm');
The FrequencyUnit property of sys1 is rpm.
Compare the Bode responses of sys and sys1.
bodeplot(sys,'r',sys1,'y--');
legend('sys','sysl')


The magnitude and phase of sys and sys 1 match because chgFreqUnit command changes the units of frequency points in sys without modifying system behavior.

Change the FrequencyUnit property of sys to compare the Bode response with the original system.
sys2 = sys;
sys2.FrequencyUnit = 'rpm';

```
bodeplot(sys,'r',sys2,'gx');
```

legend('sys','sys2');


Changing the FrequencyUnit property changes the system behavior. Therefore, the Bode responses of sys and sys2 do not match. For example, the original corner frequency at about $2 \mathrm{rad} / \mathrm{s}$ changes to approximately 2 rpm (or $0.2 \mathrm{rad} / \mathrm{s}$ ).

## Tips

- Use chgFreqUnit to change the units of frequency points without modifying system behavior.


## See Also

chgTimeUnit|frd|idfrd

## Topics

"Specify Frequency Units of Frequency-Response Data Model" (Control System Toolbox)

## Introduced in R2012a

## chgTimeUnit

Change time units of dynamic system

## Syntax

sys_new = chgTimeUnit(sys,newtimeunits)

## Description

sys_new = chgTimeUnit(sys,newtimeunits) changes the time units of sys to newtimeunits. The time- and frequency-domain characteristics of sys and sys_new match.

## Input Arguments

## sys

Dynamic system model
newtimeunits
New time units, specified as one of the following values:

- 'nanoseconds'
- 'microseconds'
- 'milliseconds'
- 'seconds'
- 'minutes'
- 'hours'
- 'days'
- 'weeks'
- 'months'
- 'years'

Default: 'seconds'

## Output Arguments

sys_new
Dynamic system model of the same type as sys with new time units. The time response of sys_new is same as sys.

If sys is an identified linear model, both the model parameters as and their minimum and maximum bounds are scaled to the new time units.

## Examples

## Change Time Units of Dynamic System Model

Create a transfer function model.

```
num = [4 2];
den = [ll 3 10];
sys = tf(num,den);
```

By default, the time unit of sys is 'seconds '. Create a new model with the time units changed to minutes.
sys1 = chgTimeUnit(sys,'minutes');
This command sets the TimeUnit property of sys1 to 'minutes', without changing the dynamics. To confirm that the dynamics are unchanged, compare the step responses of sys and sys1.

```
stepplot(sys,'r',sys1,'y--');
legend('sys','sysl');
```


## Step Response



The step responses are the same.
If you change the TimeUnit property of the system instead of using chgTimeUnit, the dynamics of the system do change. To see this, change the TimeUnit property of a copy of sys and compare the step response with the original system.

```
sys2 = sys;
sys2.TimeUnit = 'minutes';
stepplot(sys,'r',sys2,'gx');
legend('sys','sys2');
```



The step responses of sys and sys2 do not match. For example, the original rise time of 0.04 seconds changes to 0.04 minutes.

## Tips

- Use chgTimeUnit to change the time units without modifying system behavior.


## See Also

chgFreqUnit|frd|idpoly|idproc|idss|idtf|pid|ss|tf|zpk

## Topics

"Specify Model Time Units" (Control System Toolbox)

## Introduced in R2012a

## clone

Copy online parameter estimation System object

## Syntax

obj_clone = clone(obj)

## Description

obj_clone = clone(obj) creates a copy of the online parameter estimation System object ${ }^{\text {TM }}$, obj, with the same property values. If the object you clone is locked, the new object is also locked.
clone is not supported for code generation using MATLAB Coder ${ }^{\mathrm{TM}}$.

Note If you want to copy an existing System object and then modify properties of the copied object, use the clone command. Do not create additional objects using syntax obj2 $=\mathrm{obj}$. Any changes made to the properties of the new System object created this way (obj2) also change the properties of the original System object (obj).

## Examples

## Clone an Online Estimation System Object

Create a System object ${ }^{\mathrm{TM}}$ for online estimation of an ARX model with default properties.

```
obj = recursiveARX
obj =
    recursiveARX with properties:
                        A: []
                        B: []
                            InitialA: [1 2.2204e-16]
                            InitialB: [0 2.2204e-16]
            ParameterCovariance: []
        InitialParameterCovariance: [2x2 double]
            EstimationMethod: 'ForgettingFactor'
            ForgettingFactor:
            EnableAdaptation: true
                History: 'Infinite'
            InputProcessing: 'Sample-based'
                    DataType: 'double'
```

Use clone to generate an object with the same properties as the original object.

```
obj2 = clone(obj)
obj2 =
    recursiveARX with properties:
```

```
                        A: []
            B: []
    InitialA: [1 2.2204e-16]
    InitialB: [0 2.2204e-16]
    ParameterCovariance: []
InitialParameterCovariance: [2x2 double]
    EstimationMethod: 'ForgettingFactor'
    ForgettingFactor: 1
    EnableAdaptation: true
            History: 'Infinite'
        InputProcessing: 'Sample-based'
                            DataType: 'double'
```


## Input Arguments

## obj - System object for online parameter estimation

recursiveAR object | recursiveARMA object | recursiveARX object| recursiveARMAX object | recursive0E object | recursiveBJ object| recursiveLS object

System object for online parameter estimation, created using one of the following commands:

- recursiveAR
- recursiveARMA
- recursiveARX
- recursiveARMAX
- recursiveOE
- recursiveBJ
- recursiveLS


## Output Arguments

## obj_clone - Copy of online estimation System object

System object
Copy of online estimation System object, obj, returned as a System object with the same properties as obj.

## See Also

isLocked | recursiveAR|recursiveARMA | recursiveARMAX | recursiveARX| recursiveBJ | recursiveLS | recursive0E | release | reset \| step

Topics
"What Is Online Estimation?"

Introduced in R2015b

## clone

Copy online state estimation object

## Syntax

obj_clone = clone(obj)

## Description

obj_clone = clone(obj) creates a copy of the online state estimation object obj with the same property values.

If you want to copy an existing object and then modify properties of the copied object, use the clone command. Do not create additional objects using syntax obj2 $=\mathrm{obj}$. Any changes made to the properties of the new object created in this way (obj2) also change the properties of the original object (obj).

## Examples

## Clone an Online State Estimation Object

Create an extended Kalman filter object for a van der Pol oscillator with two states and one output. To create the object, use the previously written and saved state transition and measurement functions, vdpStateFcn.m and vdpMeasurementFcn.m. Specify the initial state values for the two states as [2;0].

```
obj = extendedKalmanFilter(@vdpStateFcn,@vdpMeasurementFcn,[2;0])
obj =
    extendedKalmanFilter with properties:
        HasAdditiveProcessNoise: 1
            StateTransitionFcn: @vdpStateFcn
        HasAdditiveMeasurementNoise: 1
            MeasurementFcn: @vdpMeasurementFcn
        StateTransitionJacobianFcn: []
            MeasurementJacobianFcn: []
                            State: [2x1 double]
                            StateCovariance: [2\times2 double]
                ProcessNoise: [2x2 double]
            MeasurementNoise: 1
```

Use clone to generate an object with the same properties as the original object.

```
obj2 = clone(obj)
obj2 =
    extendedKalmanFilter with properties:
HasAdditiveProcessNoise: 1
```

StateTransitionFcn: @vdpStateFcn
HasAdditiveMeasurementNoise: 1
MeasurementFcn: @vdpMeasurementFcn
StateTransitionJacobianFcn: []
MeasurementJacobianFcn: []
State: [2x1 double]
StateCovariance: [2x2 double] ProcessNoise: [2x2 double]
MeasurementNoise: 1

Modify the MeasurementNoise property of obj2.
obj2. MeasurementNoise = 2;
Verify that the MeasurementNoise property of original object obj remains unchanged and equals 1 .
obj.MeasurementNoise
ans $=1$

## Input Arguments

```
obj - Object for online state estimation
extendedKalmanFilter object|unscentedKalmanFilter object| particleFilter object
```

Object for online state estimation of a nonlinear system, created using one of the following commands:

- extendedKalmanFilter
- unscentedKalmanFilter
- particleFilter


## Output Arguments

## obj_clone - Clone of online state estimation object

extendedKalmanFilter object|unscentedKalmanFilter object| particleFilter object
Clone of online state estimation object obj, returned as an extendedKalmanFilter, unscentedKalmanFilter or particleFilter object with the same properties as obj.

## See Also

correct | extendedKalmanFilter|initialize| particleFilter|predict|
unscentedKalmanFilter

## Topics

"What Is Online Estimation?"

## Introduced in R2016b

## compare

Compare identified model output and measured output

## Syntax

```
compare(data,sys)
compare(data,sys,kstep)
compare(data,sys,LineSpec,kstep)
compare(data,sysl,...,sysN,kstep)
compare(data,sys1,LineSpec1,...,sysN,LineSpecN,kstep)
compare(
```

$\qquad$

``` ,opt)
[y,fit,ic] = compare(
``` \(\qquad\)
``` )
```


## Description

## Plot Results

compare (data, sys) simulates the response of a dynamic system model, and superimposes that response over plotted measurement data. The plot also displays the normalized root mean square (NRMSE) measure of the goodness of the fit between simulated response and measurement data. Use this function when you want to evaluate a set of candidate models identified from the same measurement data, or when you want to validate a model you have selected. You can use compare with time-domain or frequency-domain models and data.
compare(data, sys, kstep) also predicts the response of sys, using a prediction horizon specified by kstep. Prediction uses output measurements as well as input measurements to project a future response. kstep represents the number of time samples between the timepoint of each output measurement and the timepoint of the resulting predicted response. For more information on prediction, see "Simulate and Predict Identified Model Output".
compare(data, sys,LineSpec, kstep) also specifies the line type, marker symbol, and color for the model response.
compare(data, sys1, ...,sysN, kstep) compares the responses of multiple dynamic systems on the same axes. compare automatically chooses the line specifications.
compare(data, sys1,LineSpec1, ...,sysN,LineSpecN, kstep) also compares the responses of multiple systems on the same axes using the line type, marker symbol, and color specified for each system.
compare (__ , opt) configures the comparison using an option set. Options include initial condition handling, data offsets, and data selection. You can use this syntax with any of the previous inputargument combinations.

## Return Results

[y,fit,ic] = compare (__ ) returns the model response $y$, goodness of fit value fit, and the initial conditions ic. This syntax does not generate a plot, so any line specifications are ignored.

## Examples

## Compare Response of Estimated Model to Measured Data

Identify a linear model and visualize the simulated model response with the data from which it was generated.

Load input/output measurements z1, and identify a third-order state-space model sys.
load iddatal z1;
sys = ssest(z1,3);
sys is a continuous-time identified state-space (idss) model.
Use compare to simulate the sys response and plot it alongside the data z1.
figure
compare(z1,sys)


The plot illustrates the differences between the model response and the original data. The percentage shown in the legend is the NRMSE fitness value. It represents how close the predicted model output is to the data.

To change display options in the plot, right-click the plot to access the context menu. For example:

- To plot the error between the predicted output and measured output, select Error Plot.
- To view the confidence region for the simulated response, select Characteristics -> ConfidenceRegion.
- To specify number of standard deviations to plot, double-click the plot and open the Property Editor dialog box. In the Options tab, specify the number of standard deviations in Confidence Region for Identified Models. The default value is 1 standard deviation.


## Compare Predicted Response of Identified Time-Domain Model to Measured Data

Identify a linear model and visualize the predicted model response with the data from which it was computed.

Identify a third-order state-space model using the input/output measurements in z 1 .

```
load iddatal z1;
sys = ssest(z1,3);
```

sys is a continuous-time identified state-space (idss) model.
Now use compare to plot the predicted response. Prediction differs from simulation in that it uses both measured input and measured output when computing the system response. The prediction horizon defines how far in the future to predict, relative to your current measured output point. For this example, set the prediction horizon kstep to 10 steps, and use compare to plot the predicted response against the original measurement data.

```
kstep = 10;
compare(z1,sys,kstep)
```



In this plot, each sys data point represents the predicted output associated with output measurement data that was taken at least 10 steps earlier. For instance, the point at $t=15 \mathrm{~s}$ is based on output measurements taken at or prior to $t=5 \mathrm{~s}$. The calculation of this $t=15 \mathrm{~s}$ sys data point also uses input measurements up to $t=15 \mathrm{~s}$, just as a simulation would.

The plot illustrates the differences between the model response and the original data. The percentage shown in the legend is the NRMSE fitness value. It represents how closely the predicted model output matches the data.

To change display and simulation options in the plot, right-click the plot to access the context menu. For example, to plot the error between the predicted output and measured output, select Error Plot from the context menu. To change the prediction horizon value, or to toggle between simulation and prediction, select Prediction Horizon from the context menu.

## Compare Multiple Identified Models to Measured Time-Domain Data

Identify several model types for the same data, and compare the results to see which best fits the data.

Load the data, which contains iddata object z1 with single input and output.
load iddatal;
From z1, identify a model for each of the following linear forms:

- ARMAX (idpoly) of orders 2,3 , and 1 , with dead time of 0
- State space (idss) with three states
- Transfer function (idtf) with three poles
sys_armax $=\operatorname{armax}\left(z 1,\left[\begin{array}{lll}2 & 3 & 1\end{array}\right]\right)$;
sys_ss = ssest(z1,3);
sys_tf = tfest(z1,3);
Using compare, plot the simulated responses for the three models with $z 1$.
compare(z1,sys_armax,sys_ss,sys_tf)

Simulated Response Comparison


For this set of data, along with the default settings for all the models, the transfer-function form has the best NRMSE fit. However, the fits for all models are within about $1 \%$ of each other.

You can interactively control which model responses are displayed in the plot by right-clicking on the plot and hovering over Systems.

## Compare Multiple Estimated Models to Measured Frequency-Domain Data

Compare the outputs of multiple estimated models of differing types to measured frequency-domain data.

For this example, estimate a process model and an output-error polynomial from frequency response data.

```
load demofr % frequency response data
zfr = AMP.*exp(1i*PHA*pi/180);
Ts = 0.1;
data = idfrd(zfr,W,Ts);
sys1 = procest(data,'P2UDZ');
sys2 = oe(data,[2 2 1]);
```

sys1, an idproc model, is a continuous-time process model. sys2, an idpoly model, is a discretetime output-error model.

Compare the frequency response of the estimated models to data.

```
compare(data,sys1,'g',sys2,'r');
```

Frequency Response Comparison


The two models have NRMSE fit values that are nearly equal with respect to the data from which they were calculated.

## Compare Estimated Model to Data and Specify Comparison Options

Modify default behavior when you compare an estimated model to measured data.
Estimate a transfer function for measured data.
load iddatal z1;
sys $=$ tfest(z1,3);
sys is a continuous-time identified transfer function (idtf) model.
Suppose you want your initial conditions to be zero. The default for compare is to estimate initial conditions from the data.

Create an option set to specify the initial condition handling. To use zero for initial conditions, specify 'z' for the 'InitialCondition' option.
opt = compareOptions('InitialCondition','z');
Compare the estimated transfer function model output to the measured data using the comparison option set.

```
compare(z1,sys,opt)
```



## Obtain Initial Conditions

Load the data.
load iddata2 $z 2$
Split the data into estimation and validation sets.

```
z2e = z2(1:200);
z2v = z2(201:400);
plot(z2e,z2v)
```



Estimate a state-space model and a transfer function model using the estimation data.

```
sys_ss = ssest(z2e,2);
sys_tf = tfest(z2e,2,1);
```

Use compare to obtain initial conditions for sys_ss.

```
[y_ss,fit_ss,ic_ss] = compare(z2e,sys_ss);
ic_ss
ic_ss = 2\times1
    -0.0018
    0.0016
```

ic_ss is a numeric vector of initial states.

```
[y_tf,fit_tf,ic_tf] = compare(z2e,sys_tf);
```

ic_tf
$i c \_t f=$
initialCondition with properties:

```
    A: [2x2 double]
X0: [2x1 double]
    C: [-1.6093 5.1442]
Ts: 0
```

ic_tf is an initialCondition object that contains, in state-space form, a model of the free response of sys_tf to the initial conditions. A and C contain the free-response information and X0 contains the initial states.

Now obtain initial conditions for both models at once using the validation data.

```
[y_sstf,fit_sstf,ic_sstf] = compare(z2v,sys_ss,sys_tf);
ic_sstf
ic_sstf=2\times1 cell array
    {2x1 double }
    {1x1 initialCondition}
```

ic_sstf is a cell array that contains an initial state vector for sys_ss and an initialCondition object for sys_tf.
compare can provide initial conditions for an existing model with any measurement data set.

## Input Arguments

## data - Validation data

iddata object|idfrd object|frd object
Validation data, specified as an iddata, idfrd, or frd object.
If sys is:

- An iddata object, then data must be an iddata object with matching domain, number of experiments and time or frequency vectors
- A frequency-response data (FRD) model (defined as either idfrd or frd), then data must also be FRD
- A parametric model (such as idss), then data can be iddata or FRD
data can represent either time-domain or frequency-domain data when comparing with linear models. data must be time-domain data when comparing with a nonlinear model.

For examples, see:

- "Compare Predicted Response of Identified Time-Domain Model to Measured Data" on page 1-193
- "Compare Multiple Estimated Models to Measured Frequency-Domain Data" on page 1-195


## sys - Identified model

dynamic system model | iddata object | model array
Identified model, specified as a dynamic system model, an iddata object, or a model array.

When the time or frequency units of data do not match the units of sys, compare rescales sys to match the units of data.

## kstep - Prediction Horizon - steps ahead to predict

## Inf (default) | integer

Prediction horizon, specified as one of the following:

- Inf - Compare simulated response of sys to data.
- Positive finite integer - Compare predicted response of sys to data, where each predicted response point is based not only on measured input data up to that timepoint, but also on measured output data up to kstep timepoints earlier.
compare ignores kstep when sys is an iddata object, an FRD model, or a dynamic system with no noise component. compare also ignores kstep when using frequency response validation data.

If you specify kstep that is greater than the number of data samples, compare sets kstep to Inf and provides a warning message.

For more information on simulation and prediction, see "Simulate and Predict Identified Model Output".

For an example, see "Compare Predicted Response of Identified Time-Domain Model to Measured Data" on page 1-193.

## LineSpec - Line style, marker, and color

character vector
Line style, marker, and color of both the line and marker, specified as a character vector, such as ' b ' or 'b+:'.

For more information about configuring LineSpec, see the Linespec input argument of plot. For an example, see "Compare Multiple Estimated Models to Measured Frequency-Domain Data" on page 1-195.

## opt - Comparison options

compareOptions object
Comparison options, specified as an option set you create using compareOptions.
Available options include:

- Handling of initial conditions
- Sample range for computing fit numbers
- Data offsets
- Output weighting

For examples, see:

- "Compare Estimated Model to Data and Specify Comparison Options" on page 1-196
- "Resolve Fit Value Differences Between Model Identification and compare Command"


## Output Arguments

## y - Model response

iddata object | idfrd object | cell array | array
Model response, returned as an iddata object, an idfrd object, a cell array, or an array. The output depends on the models and data you provide, as follows:

- For a single model and single-experiment data set, $y$ is an iddata object or idfrd object
- For multimodel comparisons, $y$ is a cell array with one iddata or idfrd object entry for each input model
- For multiexperiment data, y is a cell array with one entry for each experiment
- For multimodel comparisons using multiexperiment data, y is an $N_{s y s}$-by- $N_{\text {exp }}$ cell array, where $N_{s y s}$ is the number of models, and $N_{\text {exp }}$ is the number of experiments
- If sys is a model array, $y$ is an array with an element corresponding to each model in sys and experiment in data. For more information on model arrays, see stack

If kstep is not specified or is Inf, then compare returns the simulated response in $y$.
Otherwise, compare returns the predicted response. Measured output values in data up to time $t_{n-}$ kstep are used to predict the output of sys at time $t_{n}$. For more information on simulation and prediction, see "Simulate and Predict Identified Model Output".

The compare response computation requires specification of initial condition handling. By default, compare estimates the initial conditions to maximize the fit to data. See compare0ptions for more information on how compare determines the initial conditions to use.

## fit - NRMSE fitness value

vector | matrix | cell array
NRMSE fitness value indicator of how well the simulated or predicted model response matches the measurement data, returned as a vector, a matrix, or a cell array. The output depends on the models and data you provide, as follows:

- If data is an iddata object, fit is a vector of length $N_{y}$, where $N_{y}$ is the number of outputs
- If data is an FRD model, fit is an $N_{y}$-by- $N_{u}$ matrix, where $N_{u}$ is the number of inputs in data
- For a single model and single-experiment data set, fit is a vector or matrix
- For multimodel comparisons, fit is a cell array with one entry for each input model
- For multiexperiment data, fit is a cell array with one entry for each experiment
- For multimodel comparisons using multiexperiment data, fit is an $N_{\text {sys }}$-by- $N_{\text {exp }}$ cell array, where $N_{\text {sys }}$ is the number of models, and $N_{\text {exp }}$ is the number of experiments
- If sys is a model array, fit is an array with an element corresponding to each model in sys and experiment in data
compare calculates fit (in percentage) using:

$$
\text { fit }=100\left(1-\frac{\|y-\widehat{y}\|}{\|y-\operatorname{mean}(y)\|}\right)
$$

where $y$ is the validation data output and $\widehat{y}$ is the output of sys.

For FRD models - compare calculates fit by comparing the complex frequency response. The fits of the magnitude and phase curves shown in the compare plot are not computed by compare separately.

## ic - Initial conditions used to compute system response <br> [ ] | vector | initialCondition object | cell array

Initial conditions used to compute system response, returned as an empty array, a vector, an initialCondition object, or a cell array.

For a single model and single-experiment data, the form of the output depends on the model type.

- For state-space models, ic is a numeric vector that contains the initial states.
- For transfer function and polynomial models, ic is an initialCondition object. An initialCondition represents, in state-space form, the free response of the model ( $A$ and $C$ matrices) to the estimated initial states ( $x_{0}$ ).
- When sys is an frd or iddata object, ic is the empty array [ ], because initial conditions cannot be used with these objects.

For multiple model and/or experiments, the form of the output is as follows:

- For multimodel comparisons, ic is a cell or object array, with one vector, matrix, or initialCondition entry for each input model.
- For multiexperiment data, ic is a cell or object array, with one entry for each experiment.
- For multimodel comparisons using multiexperiment data, ic is an $N_{s y s}$-by- $N_{\text {exp }}$ cell or object array, where $N_{s y s}$ is the number of models and $N_{\text {exp }}$ is the number of experiments.
- If sys is a model array, ic is an array with an element corresponding to each model in sys and experiment in data.

By default, compare uses findstates to estimate the initial states in ic. To change this behavior, set the 'InitialCondition' option in opt (see compareOptions). If you have input/output history that immediately precedes your start point, you can set 'InitialCondition' to that history data. compare then uses data2state to compute the end state of the history data, and thus the start state for the simulation. Other choices include setting initial conditions to zero, or to specific values that you determine previously. For more information about finding initial conditions, see "Estimate Initial Conditions for Simulating Identified Models".

If you are using an estimation model that does not explicitly use states, compare first converts the model to its state-space representation and then maps the data to the initial states. compare then packages the initial state vector and the A and C state-space matrices into an initialCondition object. For an example of using compare with such a model, see "Obtain Initial Conditions" on page 1-197.

## Tips

- The NRMSE fit result you obtain with compare may not precisely match the fit value reported in model identification. These differences typically arise from mismatches in initial conditions, and in the differences in the prediction horizon defaults for identification and for validation. The differences are generally small, and should not impact your model selection and validation workflow. For more information, see "Resolve Fit Value Differences Between Model Identification and compare Command".
- compare matches the input/output channels in data and sys based on the channel names. Thus, it is possible to evaluate models that do not use all the input channels that are available in data. This flexibility allows you to compare multiple models which were each identified independently from different sets of input/output channels.
- The compare plot allows you to vary key parameters. For example, you can interactively control:
- Whether you generate a simulated or predicted response
- Prediction horizon value
- Initial condition handling
- Which experiment data you view
- Which system models you view

To access the controls, right-click the plot to bring up the options menu.

## See Also

chgFreqUnit | chgTimeUnit | compareOptions | forecast | goodness0fFit | plot | predict | sim

## Topics

"Compare Simulated Output with Measured Validation Data"
"Validating Models After Estimation"
"Model Validation"
"Estimate Initial Conditions for Simulating Identified Models"
"Apply Initial Conditions when Simulating Identified Linear Models"

## Introduced in R2006a

## compareOptions

Option set for compare

## Syntax

opt = compareOptions
opt = compareOptions(Name,Value)

## Description

opt = compareOptions creates the default options set for compare.
opt = compareOptions(Name, Value) creates an option set with the options specified by one or more Name, Value pair arguments.

## Input Arguments

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

## Samples

Data for which compare calculates fit values.
Specify Samples as a vector containing the data sample indices. For multiexperiment data, use a cell array of $N e$ vectors, where $N e$ is the number of experiments.

## InitialCondition

Handling of initial conditions.
Specify InitialCondition as one of the following:

- 'z' - Zero initial conditions.
- 'e' - Estimate initial conditions such that the prediction error for observed output is minimized.

For nonlinear grey-box models, only those initial states i that are designated as free in the model (sys.InitialStates(i).Fixed = false) are estimated. To estimate all the states of the model, first specify all the Nx states of the idnlgrey model sys as free.
for $i=1: N x$
sys.InitialStates(i).Fixed = false;
end
Similarly, to fix all the initial states to values specified in sys. InitialStates, first specify all the states as fixed in the sys. InitialStates property of the nonlinear grey-box model.

- 'd' - Similar to 'e', but absorbs nonzero delays into the model coefficients. The delays are first converted to explicit model states, and the initial values of those states are also estimated and returned.

Use this option for linear models only.

- Vector or Matrix - Initial guess for state values, specified as a numerical column vector of length equal to the number of states. For multiexperiment data, specify a matrix with Ne columns, where $N e$ is the number of experiments. Otherwise, use a column vector to specify the same initial conditions for all experiments. Use this option for state-space (idss and idgrey) and nonlinear models (idnlarx, idnlhw, and idnlgrey) only.
- initialCondition object - initialCondition object that represents a model of the free response of the system to initial conditions. For multiexperiment data, specify a 1 -by- $N_{e}$ array of objects, where $N_{e}$ is the number of experiments.

Use this option for individual linear models only. If you are analyzing more than one model and want to specify an initialCondition object for each model, you must specify the object and use compare for each model separately.

- Structure with the following fields, which contain the historical input and output values for a time interval immediately before the start time of the data used by compare:

| Field | Description |
| :--- | :--- |
| Input | Input history, specified as a matrix with $N u$ columns, where $N u$ is the number of <br> input channels. For time series models, use [ ]. The number of rows must be <br> greater than or equal to the model order. |
| Output | Output history, specified as a matrix with Ny columns, where Ny is the number of <br> output channels. The number of rows must be greater than or equal to the model <br> order. |

For multiexperiment data, configure the initial conditions separately for each experiment by specifying InitialCondition as a structure array with $N e$ elements. To specify the same initial conditions for all experiments, use a single structure.

The software uses data2state to map the historical data to states. If your model is not idss, idgrey, idnlgrey, or idnlarx, the software first converts the model to its state-space representation and then maps the data to states. If conversion of your model to idss is not possible, the estimated states are returned empty.

- x0obj - Specification object created using idpar. Use this object for discrete-time state-space (idss and idgrey) and nonlinear grey-box (idnlgrey) models only. Use x0obj to impose constraints on the initial states by fixing their value or specifying minimum or maximum bounds.

Default: 'e'

## InputOffset

Removes offset from time domain input data for model response computation.
Specify as a column vector of length $N u$, where $N u$ is the number of inputs.
Use [ ] to indicate no offset.
For multiexperiment data, specify InputOffset as a $N u$-by- $N e$ matrix. $N u$ is the number of inputs and $N e$ is the number of experiments.

Each entry specified by InputOffset is subtracted from the corresponding input data.
Default: []
OutputOffset
Removes offset from time-domain output data for model response prediction.
Specify as a column vector of length $N y$, where $N y$ is the number of outputs.
Use [ ] to indicate no offset.
For multiexperiment data, specify OutputOffset as a $N y$-by-Ne matrix. $N y$ is the number of outputs and $N e$ is the number of experiments.

Each entry specified by Output0ffset is subtracted from the corresponding output data before computing the model response. After computing the model response, the software adds the offset to the response to give the final model response.

## Default: []

## OutputWeight

Weight of output for initial condition estimation.
OutputWeight requires one of the following values:

- [ ] - No weighting is used. This option is the same as using eye(Ny) for the output weight. Ny is the number of outputs.
- 'noise' - Inverse of the noise variance stored with the model.
- Matrix of doubles - A positive semi-definite matrix of dimension $N y$-by- $N y . N y$ is the number of outputs.

Default: []

## Output Arguments

opt
Option set containing the specified options for compare.

## Examples

## Create Default Options Set for Model Comparison

Create a default options set for compare.
opt = compareOptions;

## Specify Options for Model Comparison

Create an options set for compare using zero initial conditions. Set the input offset to 5 .
opt = compareOptions('InitialCondition','z','InputOffset',5);
Alternatively, use dot notation to set the values of opt.
opt = compareOptions;
opt.InitialCondition = 'z';
opt.InputOffset = 5;

## See Also

compare

Introduced in R2012a

## correct

Correct state and state estimation error covariance using extended or unscented Kalman filter, or particle filter and measurements

## Syntax

[CorrectedState,CorrectedStateCovariance] = correct(obj,y)
[CorrectedState,CorrectedStateCovariance] = correct(obj,y,Um1,..., Umn)

## Description

The correct command updates the state and state estimation error covariance of an extendedKalmanFilter, unscentedKalmanFilter or particleFilter object using measured system outputs. To implement extended or unscented Kalman filter, or particle filter, use the correct and predict commands together. If the current output measurement exists, you can use correct and predict. If the measurement is missing, you can only use predict. For information about the order in which to use the commands, see "Using predict and correct Commands" on page 1-215.
[CorrectedState, CorrectedStateCovariance] = correct(obj,y) corrects the state estimate and state estimation error covariance of an extended or unscented Kalman filter, or particle filter object obj using the measured output y.

You create obj using the extendedKalmanFilter, unscentedKalmanFilter or particleFilter commands. You specify the state transition function and measurement function of your nonlinear system in obj. You also specify whether the process and measurement noise terms are additive or nonadditive in these functions. The State property of the object stores the latest estimated state value. Assume that at time step $k$, obj. State is $\widehat{x}[k \mid k-1]$. This value is the state estimate for time $k$, estimated using measured outputs until time $k-1$. When you use the correct command with measured system output $y[k]$, the software returns the corrected state estimate $\widehat{x}[k \mid k]$ in the CorrectedState output. Where $\widehat{x}[k \mid k]$ is the state estimate at time k , estimated using measured outputs until time $k$. The command returns the state estimation error covariance of $\widehat{x}[k \mid k]$ in the CorrectedStateCovariance output. The software also updates the State and StateCovariance properties of obj with these corrected values.

Use this syntax if the measurement function $h$ that you specified in obj. MeasurementFcn has one of the following forms:

- $y(k)=h(x(k))-$ for additive measurement noise.
- $y(k)=h(x(k), v(k))-$ for nonadditive measurement noise.

Where $y(k), x(k)$, and $v(k)$ are the measured output, states, and measurement noise of the system at time step $k$. The only inputs to $h$ are the states and measurement noise.
[CorrectedState,CorrectedStateCovariance] = correct(obj,y,Um1,...,Umn) specifies additional input arguments, if the measurement function of the system requires these inputs. You can specify multiple arguments.

Use this syntax if the measurement function $h$ has one of the following forms:

- $y(k)=h(x(k), U m 1, \ldots, U m n)-$ for additive measurement noise.
- $y(k)=h(x(k), v(k), U m 1, \ldots, U m n)-$ for nonadditive measurement noise.
correct command passes these inputs to the measurement function to calculate the estimated outputs.


## Examples

## Estimate States Online Using Extended Kalman Filter

Estimate the states of a van der Pol oscillator using an extended Kalman filter algorithm and measured output data. The oscillator has two states and one output.

Create an extended Kalman filter object for the oscillator. Use previously written and saved state transition and measurement functions, vdpStateFcn.m and vdpMeasurementFcn.m. These functions describe a discrete-approximation to a van der Pol oscillator with the nonlinearity parameter mu equal to 1 . The functions assume additive process and measurement noise in the system. Specify the initial state values for the two states as [1;0]. This is the guess for the state value at initial time k , based on knowledge of system outputs until time $\mathrm{k}-1, \hat{x}[k \mid k-1]$.

```
obj = extendedKalmanFilter(@vdpStateFcn,@vdpMeasurementFcn,[1;0]);
```

Load the measured output data y from the oscillator. In this example, use simulated static data for illustration. The data is stored in the vdp_data. mat file.

```
load vdp_data.mat y
```

Specify the process noise and measurement noise covariances of the oscillator.

```
obj.ProcessNoise = 0.01;
obj.MeasurementNoise = 0.16;
```

Initialize arrays to capture results of the estimation.

```
residBuf = [];
xcorBuf = [];
xpredBuf = [];
```

Implement the extended Kalman filter algorithm to estimate the states of the oscillator by using the correct and predict commands. You first correct $\hat{x}[k \mid k-1]$ using measurements at time $k$ to get $\hat{x}[k \mid k]$. Then, you predict the state value at the next time step $\hat{x}[k+1 \mid k]$ using $\hat{x}[k \mid k]$, the state estimate at time step $k$ that is estimated using measurements until time $k$.

To simulate real-time data measurements, use the measured data one time step at a time. Compute the residual between the predicted and actual measurement to assess how well the filter is performing and converging. Computing the residual is an optional step. When you use residual, place the command immediately before the correct command. If the prediction matches the measurement, the residual is zero.

After you perform the real-time commands for the time step, buffer the results so that you can plot them after the run is complete.

```
for k = 1:size(y)
    [Residual,ResidualCovariance] = residual(obj,y(k));
```

```
[CorrectedState,CorrectedStateCovariance] = correct(obj,y(k));
[PredictedState,PredictedStateCovariance] = predict(obj);
    residBuf(k,:) = Residual;
    xcorBuf(k,:) = CorrectedState';
    xpredBuf(k,:) = PredictedState';
```

end

When you use the correct command, obj. State and obj. StateCovariance are updated with the corrected state and state estimation error covariance values for time step k, CorrectedState and CorrectedStateCovariance. When you use the predict command, obj. State and obj. StateCovariance are updated with the predicted values for time step $\mathrm{k}+1$, PredictedState and PredictedStateCovariance. When you use the residual command, you do not modify any obj properties.

In this example, you used correct before predict because the initial state value was $\hat{\chi}[k \mid k-1]$, a guess for the state value at initial time $k$ based on system outputs until time $k$ - 1 . If your initial state value is $\hat{x}[k-1 \mid k-1]$, the value at previous time $k-1$ based on measurements until $k-1$, then use the predict command first. For more information about the order of using predict and correct, see "Using predict and correct Commands" on page 1-215.

Plot the estimated states, using the postcorrection values.
plot (xcorBuf(:,1), xcorBuf(:,2))
title('Estimated States')

Estimated States


Plot the actual measurement, the corrected estimated measurement, and the residual. For the measurement function in vdpMeasurementFcn, the measurement is the first state.
$M=[y, x \operatorname{corBuf}(:, 1), r e s i d B u f] ;$
plot(M)
grid on
title('Actual and Estimated Measurements, Residual')
legend('Measured','Estimated','Residual')


The estimate tracks the measurement closely. After the initial transient, the residual remains relatively small throughout the run.

## Estimate States Online using Particle Filter

Load the van der Pol ODE data, and specify the sample time.
vdpODEdata.mat contains a simulation of the van der Pol ODE with nonlinearity parameter mu=1, using ode45, with initial conditions [2;0]. The true state was extracted with sample time $\mathrm{dt}=$ 0.05 .
load ('vdpODEdata.mat','xTrue','dt')
tSpan = 0:dt:5;
Get the measurements. For this example, a sensor measures the first state with a Gaussian noise with standard deviation 0.04.

```
sqrtR = 0.04;
yMeas = xTrue(:,1) + sqrtR*randn(numel(tSpan),1);
```

Create a particle filter, and set the state transition and measurement likelihood functions.
myPF = particleFilter(@vdpParticleFilterStateFcn,@vdpMeasurementLikelihoodFcn);
Initialize the particle filter at state [2; 0] with unit covariance, and use 1000 particles.

```
initialize(myPF,1000,[2;0],eye(2));
```

Pick the mean state estimation and systematic resampling methods.

```
myPF.StateEstimationMethod = 'mean';
```

myPF.ResamplingMethod = 'systematic';

Estimate the states using the correct and predict commands, and store the estimated states.

```
xEst = zeros(size(xTrue));
for k=1:size(xTrue,1)
    xEst(k,:) = correct(myPF,yMeas(k));
    predict(myPF);
end
```

Plot the results, and compare the estimated and true states.

```
figure(1)
plot(xTrue(:,1),xTrue(:,2),'x',xEst(:,1),xEst(:,2),'ro')
legend('True','Estimated')
```



## Specify State Transition and Measurement Functions with Additional Inputs

Consider a nonlinear system with input $u$ whose state $x$ and measurement $y$ evolve according to the following state transition and measurement equations:

$$
\begin{aligned}
& x[k]=\sqrt{x[k-1]+u[k-1]}+w[k-1] \\
& y[k]=x[k]+2 * u[k]+v[k]^{2}
\end{aligned}
$$

The process noise $w$ of the system is additive while the measurement noise $v$ is nonadditive.
Create the state transition function and measurement function for the system. Specify the functions with an additional input $u$.
$\mathrm{f}=@(\mathrm{x}, \mathrm{u})(\operatorname{sqrt}(\mathrm{x}+\mathrm{u}))$;
$h=@(x, v, u)\left(x+2 * u+v^{\wedge} 2\right)$;
f and h are function handles to the anonymous functions that store the state transition and measurement functions, respectively. In the measurement function, because the measurement noise is nonadditive, $v$ is also specified as an input. Note that $v$ is specified as an input before the additional input $u$.

Create an extended Kalman filter object for estimating the state of the nonlinear system using the specified functions. Specify the initial value of the state as 1 and the measurement noise as nonadditive.
obj = extendedKalmanFilter(f,h,1,'HasAdditiveMeasurementNoise',false);
Specify the measurement noise covariance.
obj. MeasurementNoise $=0.01$;
You can now estimate the state of the system using the predict and correct commands. You pass the values of $u$ to predict and correct, which in turn pass them to the state transition and measurement functions, respectively.

Correct the state estimate with measurement $\mathrm{y}[\mathrm{k}]=0.8$ and input $\mathrm{u}[\mathrm{k}]=0.2$ at time step k .

```
correct(obj,0.8,0.2)
```

Predict the state at the next time step, given $u[k]=0.2$.
predict(obj,0.2)
Retrieve the error, or residual, between the prediction and the measurement.
[Residual, ResidualCovariance] = residual(obj,0.8,0.2);

## Input Arguments

obj - Extended or unscented Kalman filter, or particle filter object
extendedKalmanFilter object | unscentedKalmanFilter object | particleFilter object

Extended or unscented Kalman filter, or particle filter object for online state estimation, created using one of the following commands:

- extendedKalmanFilter - Uses the extended Kalman filter algorithm.
- unscentedKalmanFilter - Uses the unscented Kalman filter algorithm.
- particleFilter - Uses the particle filter algorithm.


## y - Measured system output

vector
Measured system output at the current time step, specified as an $N$-element vector, where $N$ is the number of measurements.

## Um1, . . . , Umn - Additional input arguments to measurement function <br> input arguments of any type

Additional input arguments to the measurement function of the system, specified as input arguments of any type. The measurement function, $h$, is specified in the MeasurementFcn or MeasurementLikelihoodFcn property of obj. If the function requires input arguments in addition to the state and measurement noise values, you specify these inputs in the correct command syntax. correct command passes these inputs to the measurement or the measurement likelihood function to calculate estimated outputs. You can specify multiple arguments.

For example, suppose that your measurement or measurement likelihood function calculates the estimated system output $y$ using system inputs $u$ and current time $k$, in addition to the state $x$ :
$y(k)=h(x(k), u(k), k)$
Then when you perform online state estimation at time step $k$, specify these additional inputs in the correct command syntax:
[CorrectedState, CorrectedStateCovariance] $=\operatorname{correct}(\mathrm{obj}, \mathrm{y}, \mathrm{u}(\mathrm{k}), \mathrm{k})$;

## Output Arguments

## CorrectedState - Corrected state estimate

vector
Corrected state estimate, returned as a vector of size $M$, where $M$ is the number of states of the system. If you specify the initial states of obj as a column vector then $M$ is returned as a column vector, otherwise $M$ is returned as a row vector.

For information about how to specify the initial states of the object, see the extendedKalmanFilter, unscentedKalmanFilter and particleFilter reference pages.

## CorrectedStateCovariance - Corrected state estimation error covariance

 matrixCorrected state estimation error covariance, returned as an $M$-by- $M$ matrix, where $M$ is the number of states of the system.

## More About

## Using predict and correct Commands

After you have created an extended or unscented Kalman filter, or particle filter object, obj, to implement the estimation algorithms, use the correct and predict commands together.

At time step k, correct command returns the corrected value of states and state estimation error covariance using measured system outputs $\mathrm{y}[\mathrm{k}]$ at the same time step. If your measurement function has additional input arguments $\mathrm{U}_{\mathrm{m}}$, you specify these as inputs to the correct command. The command passes these values to the measurement function.
[CorrectedState,CorrectedCovariance] = correct(obj,y,Um)
The correct command updates the State and StateCovariance properties of the object with the estimated values, CorrectedState and CorrectedCovariance.

The predict command returns the prediction of state and state estimation error covariance at the next time step. If your state transition function has additional input arguments $U_{s}$, you specify these as inputs to the predict command. The command passes these values to the state transition function.
[PredictedState, PredictedCovariance] = predict(obj,Us)
The predict command updates the State and StateCovariance properties of the object with the predicted values, PredictedState and PredictedCovariance.

If the current output measurement exists at a given time step, you can use correct and predict. If the measurement is missing, you can only use predict. For details about how these commands implement the algorithms, see "Extended and Unscented Kalman Filter Algorithms for Online State Estimation".

The order in which you implement the commands depends on the availability of measured data $\mathrm{y}, \mathrm{U}_{\mathrm{s}}$, and $\mathrm{U}_{\mathrm{m}}$ for your system:

- correct then predict - Assume that at time step $k$, the value of obj. State is $\widehat{x}[k \mid k-1]$. This value is the state of the system at time $k$, estimated using measured outputs until time $k-1$. You also have the measured output $y[k]$ and inputs $U_{5}[k]$ and $U_{m}[k]$ at the same time step.

Then you first execute the correct command with measured system data y [k] and additional inputs $U_{m}[k]$. The command updates the value of obj. State to be $\widehat{x}[k \mid k]$, the state estimate for time $k$, estimated using measured outputs up to time $k$. When you then execute the predict command with input $U_{s}[k]$, obj. State now stores $\widehat{x}[k+1 \mid k]$. The algorithm uses this state value as an input to the correct command in the next time step.

- predict then correct - Assume that at time step k, the value of obj. State is $\widehat{x}[k-1 \mid k-1]$. You also have the measured output $y[k]$ and input $U_{m}[k]$ at the same time step but you have $\mathrm{U}_{\mathrm{s}}[\mathrm{k}-1]$ from the previous time step.

Then you first execute the predict command with input $\mathrm{U}_{\mathrm{s}}[\mathrm{k}-1]$. The command updates the value of obj. State to $\widehat{x}[k \mid k-1]$. When you then execute the correct command with input arguments $y[k]$ and $U_{m}[k]$, obj. State is updated with $\widehat{x}[k \mid k]$. The algorithm uses this state value as an input to the predict command in the next time step.

Thus, while in both cases the state estimate for time $k, \widehat{x}[k \mid k]$ is the same, if at time $k$ you do not have access to the current state transition function inputs $U_{s}[k]$, and instead have $U_{s}[k-1]$, then use predict first and then correct.

For an example of estimating states using the predict and correct commands, see "Estimate States Online Using Extended Kalman Filter" on page 1-209 or "Estimate States Online using Particle Filter" on page 1-1118.

## See Also

clone|extendedKalmanFilter|initialize|particleFilter|predict|residual| unscentedKalmanFilter

## Topics

"Nonlinear State Estimation Using Unscented Kalman Filter and Particle Filter"
"Generate Code for Online State Estimation in MATLAB"
"What Is Online Estimation?"
"Extended and Unscented Kalman Filter Algorithms for Online State Estimation"

## Introduced in R2016b

## cra

Estimate impulse response using prewhitened-based correlation analysis

## Syntax

ir=cra(data)
[ir,R,cl] = cra(data, M, na,plot)

## Description

ir=cra(data) estimates the impulse response for the time-domain data, data.
[ir,R,cl] = cra(data, M, na, plot) estimates correlation/covariance information, R, and the $99 \%$ confidence level for the impulse response, cl .

The cra command first computes an autoregressive model for the input $u$ as $A(q) u(t)=e(t)$, where $e$ is uncorrelated (white) noise, $q$ is the time-shift operator, and $A(q)$ is a polynomial of order na. The command then filters $u$ and output data $y$ with $A(q)$ to obtain the prewhitened data. The command then computes and plots the covariance functions of the prewhitened $y$ and $u$ and the crosscorrelation function between them. Positive values of the lag variable then correspond to an influence from $u$ to later values of $y$. In other words, significant correlation for negative lags is an indication of feedback from $y$ to $u$ in the data. A properly scaled version of this correlation function is also an estimate of the system impulse response. This is also plotted along with $99 \%$ confidence levels. The output argument ir is this impulse response estimate, so that its first entry corresponds to lag zero. (Negative lags are excluded in ir.) In the plot, the impulse response is scaled so that it corresponds to an impulse of height $1 / T$ and duration $T$, where $T$ is the sample time of the data.

## Input Arguments

## data

Input-output data.
Specify data as an iddata object containing time-domain data only.
data should contain data for a single-input, single-output experiment. For the multivariate case, apply cra to two signals at a time, or use impulse.

## M

Number of lags for which the covariance/correlation functions are computed.
M specifies the number of lags for which the covariance/correlation functions are computed. These are from -M to M , so that the length of R is $2 \mathrm{M}+1$. The impulse response is computed from 0 to M .

Default: 20
na
Order of the AR model to which the input is fitted.

For the prewhitening, the input is fitted to an AR model of order na.
Use na $=0$ to obtain the covariance and correlation functions of the original data sequences.
Default: 10
plot
Plot display control.
Specify plot as one of the following integers:

- 0 - No plots are displayed.
- 1 - Plots the estimated impulse response with a $99 \%$ confidence region.
- 2 - Plots all the covariance functions.

Default: 1

## Output Arguments

## ir

Estimated impulse response.
The first entry of ir corresponds to lag zero. (Negative lags are excluded in ir.)
R
Covariance/correlation information.

- The first column of R contains the lag indices.
- The second column contains the covariance function of the (possibly filtered) output.
- The third column contains the covariance function of the (possibly prewhitened) input.
- The fourth column contains the correlation function. The plots can be redisplayed by cra(R).


## cl

$99 \%$ significance level for the impulse response.

## Examples

## Estimate the Impulse Response of an ARX Model

Compare a second-order ARX model's impulse response with the one obtained by correlation analysis.

```
load iddatal
z = zl;
ir = cra(z);
m = arx(z,[2 2 1]);
imp = [1;zeros(20,1)];
irth = sim(m,imp);
subplot(211)
```

```
plot([ir irth])
title('Impulse Responses')
subplot(212)
plot([cumsum(ir),cumsum(irth)])
title('Step Responses')
```



## Alternatives

An often better alternative to cra is impulseest, which use a high-order FIR model to estimate the impulse response.

## See Also

impulse|impulseest|spa|step
Introduced before R2006a

## customnet

Custom network function for nonlinear ARX and Hammerstein-Wiener models

## Description

A customnet object implements a custom network function, and is a nonlinear mapping function for estimating nonlinear ARX and Nonlinear Hammerstein-Weiner models. The mapping function, which is also referred to as a nonlinearity, uses a combination of linear weights, an offset and a nonlinear function to compute its output. The nonlinear function contains custom unit functions that operate on a ridge combination (weighted linear sum) of inputs.


Mathematically, a customnet network is a function that maps $m$ inputs $X(t)=\left[x\left(t_{1}\right), x_{2}(t), \ldots, \chi_{m}(t)\right]^{\mathrm{T}}$ to a scalar output $y(t)$ using the following relationship:

$$
y(t)=y_{0}+(X(t)-\bar{X})^{T} P L+S(X(t))
$$

Here:

- $X(t)$ is an $m$-by- 1 vector of inputs, or regressors, with mean $\bar{X}$.
- $y_{0}$ is the output offset, a scalar.
- $\quad P$ is an $m$-by- $p$ projection matrix, where $m$ is the number of regressors and is $p$ is the number of linear weights. $m$ must be greater than or equal to $p$.
- $L$ is a $p$-by- 1 vector of weights.
- $S(X)$ is a sum of dilated and translated custom unit functions. The total number of unit functions is referred to as the number of units $n$ of the network.

For the definition of the unit function term $S(X)$, see "More About" on page 1-226.
Use customnet as the value of the OutputFen property of an idnlarx model or the InputNonlinearity and OutputLinearity properties of an idnlhw object. For example, specify customnet when you estimate an idnlarx model with the following command.
sys = nlarx(data, regressors,customnet)
When $n l a r x$ estimates the model, it essentially estimates the parameters of the customnet function.
You can configure the customnet function to disable components and fix parameters. To omit the linear component, set LinearFcn. Use to false. To omit the offset, set Offset. Use to false. To specify known values for the linear function and the offset, set their Value attributes directly and set the corresponding Free attributes to False. Use evaluate to compute the output of the function for a given vector of inputs.

## Creation

## Syntax

```
C = customnet(H)
S = customnet(H,numUnits)
S = customnet(H,numUnits,UseLinearFcn)
S = customnet(H,numUnits,UseLinearFcn,UseOffset)
```


## Description

$\mathrm{C}=$ customnet $(\mathrm{H})$ creates a nonlinear mapping object with a user-defined unit function using the function handle $H$. H must point to a function of the form [ $f, g, a]=$ function_name ( $x$ ), where $f$ is the value of the function, $g=d f / d x$, and a indicates the unit function active range [-a a] where $g$ is significantly nonzero. Hammerstein-Wiener models require that your custom function have only one input and one output.
$S=$ customnet ( $H$, numUnits) specifies the number of unit functions numUnits.
$S=$ customnet ( $H$, numUnits, UseLinearFcn) specifies whether the function uses a linear function as a subcomponent.
$S=$ customnet (H, numUnits, UseLinearFcn, UseOffset) specifies whether the function uses an offset term $y_{0}$ parameter.

## Input Arguments

## H - Function handle

function handle
Function handle that points to a custom function of the form $[f, g, a]=$ function_name $(x)$, specified as a function handle. The function that H points to must be vectorized. That is, for a vector or matrix $x$, the output arguments $f$ and $g$ must have the same size as $x$ when computed element by element.

This argument sets the C.NonlinearFcn.UnitFcn property.

## numUnits - Number of units

10 (default) | positive integer
Number of units, specified as a positive integer. numUnits determines the number of custom unit functions.

This argument sets the C.NonlinearFcn.NumberOfUnits property.

## UseLinearFcn - Option to use linear function <br> true (default) | false

Option to use the linear function subcomponent, specified as true or false. This argument sets the value of the C.LinearFcn.Use property.

## UseOffset - Option to use offset term

true (default) | false
Option to use an offset term, specified as true or false. This argument sets the value of the C.Offset.Use property.

## Properties

## Input - Input signal information

signal property values
Input signal information for signals used for estimation, specified as vectors of $m$ property-specific values, where $m$ is the number of input signals. The Input properties for each input signal are as follows:

- Name - Names of the input signals, specified as a 1-by-m string or character array, where $m$ is the number of inputs
- Mean - Mean of the input signals, specified as a numeric scalar
- Range - Ranges of the input signals, specified as a 2-by-m numeric array that contains the minimum and maximum values


## Output - Output signal information

signal property values
Output signal information, specified as property-specific values. The Output properties are as follows:

- Name - Name of the output signal, specified as a string or a character array
- Mean - Mean of the output signal, specified as a numeric scalar
- Range - Range of the output signal, specified as 2-by-1 numeric array that contains the minimum and maximum values.


## LinearFcn - Parameters of linear function

linear function property values (default)
Parameters of the linear function, specified as follows:

- Use - Option to use the linear function in the custom network, specified as a scalar logical. The default value is true.
- Value - Linear weights that compose $L^{\prime}$, specified as a 1-by- $p$ vector.
- InputProjection - Input projection matrix $P$, specified as an $m$-by- $p$ matrix, that transforms the detrended input vector of length $m$ into a vector of length $p$. For Hammerstein-Wiener models, InputProjection is equal to 1.
- Free - Option to update entries of Value during estimation, specified as a 1-by-p logical vector. The software honors the Free specification only if the starting value of Value is finite. The default value is true.
- Minimum - Minimum bound on Value, specified as a 1-by-p vector. If Minimum is specified with a finite value and the starting value of Value is finite, then the software enforces that minimum bound during model estimation.
- Maximum - Maximum bound on Value, specified as a 1-by-p vector. If Maximum is specified with a finite value and the starting value of Value is finite, then the software enforces that maximum bound during model estimation.
- SelectedInputIndex - Indices of customnet inputs (see Input. Name) that are used as inputs to the linear function, specified as an 1-by- $n_{r}$ integer vector, where $n_{r}$ is the number of inputs. For nonlinear ARX models, the RegressorUsage property determines these indices. For Hammerstein-Wiener models, SelectedInputIndex is always 1.

The software computes the output of LinearFcn as $\left.F_{L}(t)=(X(t)-\bar{X})^{T} P L\right)$.

## Offset - Parameters of offset term

offset property values
Parameters of the offset term, specified as follows:

- Use - Option to use the offset in the custom network, specified as a scalar logical. The default value is true.
- Value - Offset value, specified as a scalar.
- Free - Option to update Value during estimation, specified as a scalar logical. The software honors the Free specification of false only if the value of Value is finite. The default value is true.
- Minimum - Minimum bound on Value, specified as a numeric scalar or -Inf. If Minimum is specified with a finite value and the value of Value is finite, then the software enforces that minimum bound during model estimation. The default value is - Inf.
- Maximum - Maximum bound on Value, specified as a numeric scalar or Inf. If Maximum is specified with a finite value and the starting value of Value is finite, then the software enforces that maximum bound during model estimation. The default value is Inf.


## NonlinearFcn - Parameters of nonlinear function

nonlinear function property values
Parameters of the nonlinear function, specified as follows:

- UnitFcn - Function handle that points to a custom function of the form [f,g,a] = function_name (x). The function that UnitFcn points to must be vectorized. That is, for a vector or matrix $x$, the output arguments $f$ and $g$ must have the same size as $x$ when computed element by element.
- NumberOfUnits - Number of units, specified as a positive integer. NumberOfUnits determines the number $n$ of custom functions.
- Parameters - Parameters of customnet, specified as in the following table:

| Field Name | Description | Default |
| :--- | :--- | :--- |
|  |  |  |
| InputProject <br> ion | Projection matrix $Q$, specified as an $m$-by- $q$ matrix. $Q$ <br> transforms the detrended input vector $(X-\bar{X})$ of length $m$ <br> into a vector of length $q$. Typically, $Q$ has the same <br> dimensions as the linear projection matrix $P$. In this case, $q$ is <br> equal to $p$, which is the number of linear weights. <br> For Hammerstein-Wiener models, InputProjection is <br> equal to 1. | [ ] |
| OutputCoeffi <br> cient | custom function output coefficients $s_{i}$, specified as an $n$-by-1 <br> vector. | [ ] |
| Translation | Translation matrix, specified as an $n$-by- $q$ matrix of <br> translation row vectors $c_{i}$. | [] |
| Dilation | Dilation coefficients $b_{i}$, specified as an $n$-by- 1 vector. | [ ] |

- Free - Option to estimate parameters, specified as a logical scalar. If all the parameters have finite values, such as when the customnet object corresponds to a previously estimated model, then setting Free to false causes the parameters of the nonlinear function $S(X)$ to remain unchanged during estimation. The default value is true.
- SelectedInputIndex - Indices of customnet inputs (see Input. Name) that are used as inputs to the nonlinear function, specified as an 1-by- $n_{r}$ integer vector, where $n_{r}$ is the number of inputs. For nonlinear ARX models, the RegressorUsage property determines these indices. For Hammerstein-Wiener models, SelectedInputIndex is always 1.


## Examples

## Estimate Nonlinear ARX Model with customnet as Output Function

Load the data
load iddata1 z1
Create a customnet object that uses gaussunit as the unit function.
H = @gaussunit;
C = customnet(@gaussunit);

Set properties of C using dot notation. Fix the value of the offset to 0.2 and set the number of unit functions to 15 .

```
C.Offset.Value = 0.2;
C.Offset.Free = false;
C.NonlinearFcn.NumberOfUnits = 15
C =
    customnet with properties:
        UnitFcn: @gaussunit
        NonlinearFcn: [1\times1 nlident.internal.RidgenetFcn]
            Input: [1\times1 idpack.Channel]
            Output: [1×1 idpack.Channel]
            LinearFcn: [1×1 nlident.internal.UseProjectedLinearFcn]
            Offset: [1\times1 nlident.internal.ChooseableOffset]
```

Create model regressors.

```
Reg = linearRegressor([z1.OutputName z1.InputName],{1:4 0:4});
```

Estimate the nonlinear ARX model.

```
sys = nlarx(z1,Reg,C)
sys =
Nonlinear ARX model with 1 output and 1 input
    Inputs: u1
    Outputs: y1
    Standard regressors corresponding to the orders:
        na = 4, nb = 5, nk = 0
Sample time: 0.1 seconds
Status:
Estimated using NLARX on time domain data "z1".
Fit to estimation data: 0.06041% (prediction focus)
FPE: 75.67, MSE: 19.38
```


## Estimate Hammerstein-Wiener Model with customnet as Output Nonlinearity

Load the data.

```
load throttledata
```

Create a customnet object C that uses gaussunit as the unit function. Include input arguments that specify 10 units and exclude the linear function and the offset.

```
H = @gaussunit;
C = customnet(@gaussunit,10,false,false)
C =
Custom Network
Nonlinear Function: Custom Network with 10 units and "gaussunit" unit function
```

```
Linear Function: not in use
Output Offset: not in use
            Input: [1×1 idpack.Channel]
            Output: [1×1 idpack.Channel]
        LinearFcn: [1\times1 nlident.internal.UseProjectedLinearFcn]
    NonlinearFcn: [1\times1 nlident.internal.CustomNLFcn]
            Offset: [1×1 nlident.internal.ChooseableOffset]
```

Estimate the Hammerstein-Wiener Model using orders [ $\begin{gathered}4 \\ 4\end{gathered} \quad 1$ 1], no input nonlinearity, and C as the output nonlinearity.

```
sys = nlhw(ThrottleData,[4 4 1],[],C)
sys =
Hammerstein-Wiener model with 1 output and 1 input
    Linear transfer function corresponding to the orders nb = 4, nf = 4, nk = 1
    Input nonlinearity: absent
    Output nonlinearity: Custom Network with 10 units
Sample time: 0.01 seconds
Status:
Estimated using NLHW on time domain data "ThrottleData".
Fit to estimation data: 72.35%
FPE: 108.1, MSE: 84.36
```


## More About

## Custom Nonlinear Function $S(X)$

The custom nonlinear function is a sum of the dilated and translated unit functions, and is described by the following equation:

$$
S(X)=\sum_{i=1}^{n} s_{i} f\left((X-\bar{X})^{T} Q b_{i}+c_{i}\right)
$$

Here:

- $Q$ is an $m$-by- $q$ projection matrix, where $m \geq q$.
- $s_{1}, s_{2}, \ldots, s_{n}$ are scalar weights called output coefficients.
- $b_{1}, b_{2}, \ldots, b_{n}$ are $q$-by-1 vectors called dilation coefficients .
- $c_{1}, c_{2}, \ldots, c_{n}$ are scalars called translations.
- $\mathrm{f}(z)$ is a function, also called a unit function, that takes a scalar input and returns a scalar output. Here, $z$ is a scalar of the form $b_{i}(X-\bar{X})^{\mathrm{T}} Q+c_{i}$.


## Algorithms

customnet uses an iterative search technique for estimating parameters.

## Compatibility Considerations

Use of previous nonlinearity estimator properties is not recommended
Not recommended starting in R2021a
Starting in R2021a, the properties of the mapping objects, previously known as nonlinearity estimators, have been reorganized. These objects are wavenet (W), sigmoidnet (S), treepartition (T), customnet (C), and linear (L). The property changes do not apply to neuralnet. The use of the pre-R2021a properties in the following table is discouraged. However, the software still accepts commands that set these properties. There are no plans to exclude such commands at this time.

| Pre-R2021a Property | R2021a Property | Applicable Mapping Objects |
| :--- | :--- | :--- |
| NumberOfUnits | NonlinearFcn.NumberOfUni <br> ts | W,S,T,C |
| LinearTerm | LinearFcn.Use, Offset.Use | W,S,C |
| Parameters | Split into three pieces: <br> - LinearFcn.Value <br> - <br> Offset.Value <br> - NonlinearFcn.Parameter <br> s | W,S,T,C,L <br> linear (L) excludes <br> NonlinearFcn.Parameters. |
| Options | NonlinearFcn.Structure | W,T |

## See Also

deadzone |evaluate |idnlarx|idnlhw|linear|linearRegressor|neuralnet|nlarx| nlhw | polyld | pwlinear|saturation| sigmoidnet | treepartition|unitgain|wavenet

## Introduced in R2007a

## customreg

(Not recommended) Custom regressor for nonlinear ARX models

Note The customreg command is not recommended. For polynomial regressors, use polynomial Regressor instead. For other custom regressors, use customRegressor. For more information, see "Compatibility Considerations".

## Syntax

C=customreg(Function, Variables)
C=customreg(Function, Variables,Delays, Vectorized)

## Description

customreg class represents arbitrary functions of past inputs and outputs, such as products, powers, and other MATLAB expressions of input and output variables.

You can specify custom regressors in addition to or instead of standard regressors for greater flexibility in modeling your data using nonlinear ARX models. For example, you can define regressors like $\tan (u(t-1)), u(t-1)^{2}$, and $u(t-1) * y(t-3)$.

For simpler regressor expressions, specify custom regressors directly in the app or in the $n l a r x$ estimation command. For more complex expressions, create a customreg object for each custom regressor and specify these objects as inputs to the estimation. Regardless of how you specify custom regressors, the toolbox represents these regressors as customreg objects. Use getreg to list the expressions of all standard and custom regressors in your model.

A special case of custom regressors involves polynomial combinations of past inputs and outputs. For example, it is common to capture nonlinearities in the system using polynomial expressions like $y(t$ $-1) 2, u(t-1) 2, y(t-2) 2, y(t-1) * y(t-2), y(t-1) * u(t-1), y(t-2) * u(t-1)$. At the command line, use the polyreg command to generate polynomial-type regressors automatically by computing all combinations of input and output variables up to a specified degree. polyreg produces customreg objects that you specify as inputs to the estimation.

The nonlinear ARX model (idnlarx object) stores all custom regressors as the CustomRegressors property. You can list all custom regressors using $m$. CustomRegressors, where $m$ is a nonlinear ARX model. For MIMO models, to retrieve the rth custom regressor for output ky, use m.CustomRegressors\{ky (r).

Use the Vectorized property to specify whether to compute custom regressors using vectorized form during estimation. If you know that your regressor formulas can be vectorized, set Vectorized to 1 to achieve better performance. To better understand vectorization, consider the custom regressor function handle $z=@(x, y) x^{\wedge} 2^{*} y . x$ and $y$ are vectors and each variable is evaluated over a time grid. Therefore, z must be evaluated for each ( $\mathrm{xi}, \mathrm{yi}$ ) pair, and the results are concatenated to produce a $z$ vector:

```
for k = 1:length(x)
    z(k) = x(k)^2*y(k)
end
```

The above expression is a nonvectorized computation and tends to be slow. Specifying a Vectorized computation uses MATLAB vectorization rules to evaluate the regressor expression using matrices instead of the FOR-loop and results in faster computation:

```
% ".*" indicates element-wise operation
z=(x.^2).*y
```


## Construction

C=customreg (Function, Variables) specifies a custom regressor for a nonlinear ARX model. C is a customreg object that stores custom regressor. Function is a function of input and output variables. Variables represent the names of model inputs and outputs in the function Function. Each input and output name must coincide with the InputName and OutputName properties of the corresponding idnlarx object. The size of Variables must match the number of Function inputs. For multiple-output models with $p$ outputs, the custom regressor is a p-by-1 cell array or an array of customreg object, where the kyth entry defines the custom regressor for output ky. You must add these regressors to the model by assigning the CustomRegressors model property or by using addreg.

C=customreg(Function, Variables,Delays, Vectorized) create a custom regressor that includes the delays corresponding to inputs or outputs in Arguments. Delays is a vector of positive integers that represent the delays of Variables variables (default is 1 for each vector element). The size of Delays must match the size of Variables. Vectorized value of 1 uses MATLAB vectorization rules to evaluate the regressor expression Function. By default, Vectorized value is 0 (false).

## Properties

After creating the object, you can use get or dot notation to access the object property values. For example:

```
% List all property values
get(C)
% Get value of Arguments property
C.Arguments
```

You can also use the set function to set the value of particular properties. For example:

```
set(C,'Vectorized',1)
```

| Property Name | Description |
| :--- | :--- |
| Function | Function handle or character vector representing a function of standards <br> regressors. <br>  <br>  <br> For example: <br> cr $=@(x, y) \quad x^{*} y$ |


| Property Name | Description |
| :---: | :---: |
| Variables | Cell array of character vectors that represent the names of model input and output variables in the function Function. Each input and output name must coincide with the InputName and OutputName properties of the idnlarx object-the model for which you define custom regressors. The size of Variables must match the number of Function inputs. <br> For example, Variables correspond to \{'y1', 'u1'\} in: $C=\text { customreg(cr, \{'y1','u1'\},[2 3]) }$ |
| Delays | Vector of positive integers representing the delays of Variables. The size of Delays must match the size of Arguments. <br> Default: 1 for each vector element. <br> For example, Delays are [2 3] in: <br> C = customreg(cr,\{'y1','u1'\},[2 3]) |
| Vectorized | Assignable values: <br> - 0 (default)-Function is not computed in vectorized form. <br> - 1-Function is computed in vectorized form when called with vector arguments. |

## Examples

## Define Custom Regressors

Load estimation data.
load iddata1
Specify the regressors as a cell array of character vectors.
C = \{'ul(t-1)*sin(y1(t-3))','u1(t-2)^3'\};
u 1 and y 1 are input and output data, respectively.
Estimate a nonlinear ARX model using the custom regressors.
m = nlarx(z1,[2 2 1],'linear','CustomRegressors',C);

## Define Custom Regressors During Estimation

Load the estimation data.
load iddata1
Estimate a nonlinear ARX model with custom regressors.

```
m = nlarx(z1,[2 2 1],'linear','CustomRegressors',...
    {'u1(t-1)*sin(y1(t-3))','ul(t-2)^3'});
```


## Define Custom Regressors as Array of customreg Objects

Load the estimation data.
load iddatal
Construct a nonlinear ARX model.
m = idnlarx([2 2 1]);
Define the custom regressors.
cr1 $=@(x, y) x^{*} \sin (y) ;$
cr2 = @(x) x^3;
C = [customreg(cr1,\{'u','y'\},[1 3]), customreg(cr2,\{'u'\},2)];
Add custom regressors to the model.
$\mathrm{m} 2=\operatorname{addreg}(\mathrm{m}, \mathrm{C})$;

## Use Vectorization Rules to Evaluate Regressor Expression

Load the estimation data.
load iddatal
Specify the regressors.

```
C = customreg(@(x,y) x.*sin(y),{'u' 'y'},[1 3]);
set(C,'Vectorized',1);
```

Estimate a nonlinear ARX model with custom regressors.
m = nlarx(z1,[2 2 1],'sigmoidnet','CustomReg',C);

## Compatibility Considerations

## customreg is not recommended

Not recommended starting in R2021a
Starting in R2021a, create polynomial regressors using polynomialRegressor. To create other custom regressor forms, use customRegressor. Add the new regressor to the idnlarx
Regressors property by using the syntax by using the syntax model. Regressors(end+1) = new_regressor_object.

There are no plans to remove customreg at this time.

## See Also

addreg|getreg|idnlarx|nlarx|polyreg
Topics
"Identifying Nonlinear ARX Models"
Introduced in R2007a

## customRegressor

Specify custom regressor for nonlinear ARX model

## Description

A custom regressor represents a single user-provided formula that operates on delayed input and output variables. For example, $y(t-1) \mathrm{e}^{u(t-1)}$ is a custom regressor that you can construct using the formula @( $\mathrm{x}, \mathrm{y}) \mathrm{x} . * \exp (\mathrm{y})$. A customRegressor object encapsulates a set of custom regressors. Use customRegressor objects when you create nonlinear ARX models using idnlarx or nlarx. You can specify customRegressor objects along with linearRegressor and polynomialRegressor objects and combine them into a single combined regressor set.

## Creation

## Syntax

cReg = customRegressor(Variables,Lags,Fcn)
cReg = customRegressor(Variables,Lags,Fcn,Vectorized)

## Description

cReg = customRegressor(Variables,Lags,Fcn) creates a customRegressor object, with the output and input names in Variables, the corresponding lags in Lags, and the function handle in Fcn. Fcn sets the VariablesToRegressorFcn property. For example, if Variables contains ' y', lags contains the corresponding lag vector [24], and the custom function is @(x)sin(x), then the regressors that use ' $y$ ' are $\sin (y(t-2))$ and $\sin (y(t-4))$.
cReg = customRegressor(Variables,Lags,Fcn,Vectorized) specifies whether Fcn can process a vector of inputs to return a vector of output values, based on the value of Vectorized.

## Properties

## VariablesToRegressorFcn - Custom Function

function handle
Custom function that transforms a set of delayed variables into a numeric scalar output, specified as a function handle.

## Example: @(x)sin(x)

Example: @(x,y)x.*exp(y)

## Variables - Output and input variable names

cell array of strings | iddata object properties
Output and input variable names, specified as a cell array of strings or a cell array that references the OutputName and InputName properties of an iddata object. Each entry must be a string with no special characters other than white space.

Example: \{'y1', 'u1'\}
Example: [z.OutputName; z.InputName]

## Lags - Lags in each variable

cell array of non-negative integers
Lags in each variable, specified as a $1-b y-n_{v}$ cell array of non-negative integer row vectors, where $n_{v}$ is the total number of regressor variables. Each row vector contains $n_{r}$ integers that specify the $n_{r}$ regressor lags for the corresponding variable. When $n_{r}>1$ for at least one of the variables, then the software generates a regressor for every lag combination. For instance, suppose that you want to create the formula $r(t)=\sin \left(y_{1}(t-a)\right) \cos \left(u_{1}(t-b)\right)$, where lag $a$ can be 1 or 2 and lag $b$ can be 0 or 3 . Specify Lags as $\left\{\left[\begin{array}{ll}1 & 2\end{array}\right],\left[\begin{array}{ll}0 & 3\end{array}\right]\right\}$, which corresponds to the variables $\{' y 1 ', ' u 1 '\}$. This specification creates the following set of regressors:

- ' $\sin (y 1(t-1)) * \cos (u 1(t)) '$
- $\quad \sin (y 1(t-1)) * \cos (u 1(t-3))^{\prime}$
- ' $\sin (y 1(t-2)) * \cos (u 1(t)) '$
- $\quad \sin (y 1(t-2)) * \cos (u 1(t-3))^{\prime}$

If a lag corresponds to an output variable of an idnlarx model, the minimum lag must be greater than or equal to 1.

Example: \{1 1\}
Example: $\{[12],[1,3,4]\}$

## Vectorized - Vectorization indicator <br> true (default) | false

Vectorization indicator that determines whether VariablesToRegressorFcn is vectorized, specified as true or false.

For an example of setting this property, see "Use Absolute Value in Polynomial Regressor Set" on page 1-1091.

Example: [true,false]

## TimeVariable - Name of time variable

't' (default) | character array | string
Name of the time variable, specified as a valid MATLAB variable name that is distinct from values in Variables.

Example: 'ClockTime'

## Examples

## Create Custom Regressor

Create a custom regressor that represents the formula $x e^{y}$.
Specify the input variables as ' $\mathbf{u 1}$ ' and ' $y 1$ ' and corresponding lags of 1 and 3 delays.

```
vars = {'y1','u1'};
lags = {1 3};
```

Specify the custom function.

```
fcn = @(x,y)x.*exp(y);
```

Create the regressor.

```
cReg = customRegressor(vars,lags,fon)
cReg =
Custom regressor: y1(t-1).*exp(u1(t-3))
    VariablesToRegressorFcn: @(x,y)x.*exp(y)
            Variables: {'y1' 'ul'}
                            Lags: {[1] [3]}
                            Vectorized: 1
        TimeVariable: 't'
```

    Regressors described by this set
    
## Create Custom Regressors in Multiple Variables

Create a set of custom regressors in three variables, all based on the formula $x y+\sin (z)$.
Specify the variable names and the lags.

```
vars = {'a','b','c'};
lags = {[1 5],[0 8],7};
```

Specify the custom function.

```
fcn = @(x,y,z)x.*y+sin(z);
```

Create the custom regressor set.

```
cReg = customRegressor(vars,lags,fcn)
cReg =
Custom regressor: @(x,y,z)x.*y+sin(z)
    VariablesToRegressorFcn: @(x,y,z)x.*y+sin(z)
            Variables: {'a' 'b' 'c'}
                            Lags: {[1 5] [0 8] [7]}
            Vectorized: 1
        TimeVariable: 't'
    Regressors described by this set
```

cReg specifies regressors for all possible lag combinations.

## Specify Linear, Polynomial, and Custom Regressors

Load the estimation data $z 1$, which has one input and one output, and obtain the output and input names.

```
load iddatal z1;
names = [z1.OutputName z1.InputName]
names = 1\times2 cell
    {'y1'} {'u1'}
```

Specify $L$ as the set of linear regressors that represents $y_{1}(t-1), u_{1}(t-2)$, and $u_{1}(t-5)$.
$\mathrm{L}=$ linearRegressor(names,\{1,[2 5]\});
Specify P as the polynomial regressor $y_{1}(t-1)^{2}$.
$P=$ polynomialRegressor(names(1),1,2);
Specify $C$ as the custom regressor $y_{1}(t-2) u_{1}(t-3)$. Use an anonymous function handle to define this function.

```
C = customRegressor(names,{2 3},@(x,y)x.*y)
C =
Custom regressor: y1(t-2).*u1(t-3)
    VariablesToRegressorFcn: @(x,y)x.*y
            Variables: {'y1' 'u1'}
                Lags: {[2] [3]}
            Vectorized: 1
            TimeVariable: 't'
    Regressors described by this set
```

Combine the regressors in the column vector $R$.

```
R = [L;P;C]
```

$\mathrm{R}=3 \times 1$ object
[3 1] array of linearRegressor, polynomialRegressor, customRegressor objects.

1. Linear regressors in variables y1, u1
Variables: \{'y1' 'ul'\}
Lags: \{[1] [2 5]\}
UseAbsolute: [0 0]
TimeVariable: 't'
2. Order 2 regressors in variables y1
Order: 2
Variables: \{'yl'\}
Lags: \{[1]\}
UseAbsolute: 0
AllowVariableMix: 0
AllowLagMix: 0
TimeVariable: 't'
3. Custom regressor: y1(t-2).*u1(t-3)
VariablesToRegressorFcn: @(x,y)x.*y
Variables: \{'yl' 'ul'\}
Lags: \{[2] [3]\}
```
    Vectorized: 1
TimeVariable: 't'
Regressors described by this set
```

Estimate a nonlinear ARX model with R.

```
sys = nlarx(z1,R)
sys =
Nonlinear ARX model with 1 output and 1 input
    Inputs: u1
    Outputs: y1
Regressors:
    1. Linear regressors in variables y1, u1
    2. Order 2 regressors in variables y1
    3. Custom regressor: y1(t-2).*ul(t-3)
    List of all regressors
Output function: Wavelet Network with 1 unit
Sample time: 0.1 seconds
Status:
Estimated using NLARX on time domain data "z1".
Fit to estimation data: 59.73% (prediction focus)
FPE: 3.356, MSE: 3.147
```

View the full regressor set.

```
getreg(sys)
```

```
ans = 5\times1 cell
    {'y1(t-1)' }
    {'u1(t-2)' }
    {'ul(t-5)' }
    {'y1(t-1)^2' }
    {'y1(t-2).*u1(t-3)'}
```


## See Also

getreg|idnlarx|linearRegressor|nlarx|polynomialRegressor

Introduced in R2021a

## d2c

Convert model from discrete to continuous time

## Syntax

sysc $=\mathrm{d} 2 \mathrm{c}($ sysd)
sysc = d2c(sysd,method)
sysc = d2c(sysd,opts)
[sysc,G] = d2c( $\qquad$ )

## Description

sysc $=\mathrm{d} 2 \mathrm{c}($ sysd $)$ converts a the discrete-time dynamic system model sysd to a continuous-time model using zero-order hold on the inputs.
sysc $=d 2 c(s y s d, m e t h o d)$ specifies the conversion method.
sysc $=\mathrm{d} 2 \mathrm{c}($ sysd,opts) specifies conversion options for the discretization.
[sysc, G] = d2c (__ ), where sysd is a state-space model, returns a matrix $G$ that maps the states $\mathrm{xd}[\mathrm{k}]$ of the discrete-time state-space model to the states $\mathrm{xc}(\mathrm{t})$ of sysc.

## Examples

## Convert Discrete-Time Transfer Function to Continuous Time

Create the following discrete-time transfer function:
$H(z)=\frac{z-1}{z^{2}+z+0.3}$
$H=t f\left(\left[\begin{array}{ll}1 & -1\end{array}\right],\left[\begin{array}{lll}1 & 1 & 0.3\end{array}\right], 0.1\right)$;
The sample time of the model is $T_{S}=0.1 \mathrm{~s}$.
Derive a continuous-time, zero-order-hold equivalent model.
$\mathrm{Hc}=\mathrm{d} 2 \mathrm{c}(\mathrm{H})$
Hc =
121.7 s - 8.407e-13
$\mathrm{s}^{\wedge} 2+12.04 \mathrm{~s}+776.7$
Continuous-time transfer function.
Discretize the resulting model, Hc , with the default zero-order hold method and sample time 0.1 s to return the original discrete model, H .
c2d(Hc, 0.1)

```
ans =
    z - 1
    ------------
    z^2 + z + 0.3
Sample time: 0.1 seconds
Discrete-time transfer function.
```

Use the Tustin approximation method to convert H to a continuous time model.

```
Hc2 = d2c(H,'tustin');
```

Discretize the resulting model, Hc 2 , to get back the original discrete-time model, H .
c2d(Hc2,0.1,'tustin');

## Convert Identified Discrete-Time Transfer Function to Continuous Time

Estimate a discrete-time transfer function model, and convert it to a continuous-time model.
load iddatal
sys1d = tfest(z1,2,'Ts',0.1);
sys1c = d2c(sysld,'zoh');
Estimate a continuous-time transfer function model.
sys2c = tfest(z1,2);
Compare the response of sys1c and the directly estimated continuous-time model, sys2c. compare(z1,sys1c,sys2c)

Simulated Response Comparison


The two systems are almost identical.

## Regenerate Covariance Information After Converting to Continuous-Time Model

Convert an identified discrete-time transfer function model to continuous-time.

```
load iddatal
sysd = tfest(z1,2,'Ts',0.1);
sysc = d2c(sysd,'zoh');
```

sys1c has no covariance information. The d2c operation leads to loss of covariance data of identified models.

Regenerate the covariance information using a zero iteration update with the same estimation command and estimation data.

```
opt = tfestOptions;
opt.SearchOptions.MaxIterations = 0;
syslc = tfest(zl,sysc,opt);
```

Analyze the effect on frequency-response uncertainty.

```
h = bodeplot(sysd,sys1c);
```

showConfidence(h,3)


The uncertainties of sys1c and sysd are comparable up to the Nyquist frequency. However, sys1c exhibits large uncertainty in the frequency range for which the estimation data does not provide any information.

If you do not have access to the estimation data, use the translatecov command which is a Gaussapproximation formula based translation of covariance across model type conversion operations.

## Input Arguments

## sysd - Discrete-time dynamic system

dynamic system model
Discrete-time model, specified as a dynamic system model such as idtf, idss, or idpoly.
You cannot directly use an idgrey model whose FunctionType is ' d ' with d 2 c . Convert the model into idss form first.

## method - Discrete-to-continuous time conversion method

'zoh' (default)|'foh'|'tustin'|'matched'
Discrete-to-continuous time conversion method, specified as one of the following values:

- 'zoh' - Zero-order hold on the inputs. Assumes that the control inputs are piecewise constant over the sampling period.
- 'foh' - Linear interpolation of the inputs (modified first-order hold). Assumes that the control inputs are piecewise linear over the sampling period.
- 'tustin' - Bilinear (Tustin) approximation to the derivative. To specify this method with frequency prewarping (formerly known as the 'prewarp' method), use the PrewarpFrequency option of d2cOptions.
- 'matched ' - Zero-pole matching method (for SISO systems only). See [1] .

For information about the algorithms for each d2c conversion method, see "Continuous-Discrete Conversion Methods".

## opts - Discrete-to-continuous time conversion options

d2cOptions object
Discrete-to-continuous time conversion options, created using d2c0ptions. For example, specify the prewarp frequency or the conversion method as an option.

## Output Arguments

## sysc - Continuous-time model

dynamic system model
Continuous-time model, returned as a dynamic system model of the same type as the input system sysd.

When sysd is an identified (IDLTI) model, sysc:

- Includes both the measured and noise components of sysd. If the noise variance is $\lambda$ in sysd, then the continuous-time model sysc has an indicated level of noise spectral density equal to $T s * \lambda$.
- Does not include the estimated parameter covariance of sysd. If you want to translate the covariance while converting the model, use translatecov.


## G - Mapping of discrete-time states of state-space model to continuous-time states

 matrixMapping of the states $x d[k]$ of the state-space model sysd to the states $x c(t)$ of sysc, returned as a matrix. The mapping of the states is as follows:

$$
x_{C}\left(k T_{s}\right)=G\left[\begin{array}{l}
x_{d}[k] \\
u[k]
\end{array}\right]
$$

Given an initial condition $x 0$ for sysd and an initial input $u 0=u[0]$, the corresponding initial condition for sysc (assuming $u[k]=0$ for $k<0$ is:

$$
x_{C}(0)=G\left[\begin{array}{l}
x_{0} \\
u_{0}
\end{array}\right] .
$$

## References

[1] Franklin, G.F., Powell,D.J., and Workman, M.L., Digital Control of Dynamic Systems (3rd Edition), Prentice Hall, 1997.
[2] Kollár, I., G.F. Franklin, and R. Pintelon, "On the Equivalence of z-domain and s-domain Models in System Identification," Proceedings of the IEEE ${ }^{\circledR}$ Instrumentation and Measurement Technology Conference, Brussels, Belgium, June, 1996, Vol. 1, pp. 14-19.

## See Also

c2d | d2cOptions | d2d | logm | translatecov
Topics
"Dynamic System Models"
"Transforming Between Discrete-Time and Continuous-Time Representations"
"Continuous-Discrete Conversion Methods"
Introduced before R2006a

## d2cOptions

Create option set for discrete- to continuous-time conversions

## Syntax

opts = d2cOptions
opts = d2cOptions(Name,Value)

## Description

opts $=d 2 c 0 p t i o n s$ returns the default options for $d 2 c$.
opts = d2cOptions(Name, Value) creates an option set with the options specified by one or more Name, Value pair arguments.

## Input Arguments

Name-Value Pair Arguments
method
Discretization method, specified as one of the following values:

$$
\begin{array}{ll}
\text { 'zoh' } & \begin{array}{l}
\text { Zero-order hold, where d2c assumes that the control inputs are piecewise } \\
\text { constant over the sample time Ts. } \\
\text { Linear interpolation of the inputs (modified first-order hold). Assumes that the } \\
\text { control inputs are piecewise linear over the sampling period. }
\end{array} \\
\text { ' foh ' } & \begin{array}{l}
\text { Bilinear (Tustin) approximation. By default, d2c converts with no prewarp. To } \\
\text { include prewarp, use the PrewarpFrequency option. }
\end{array} \\
\text { 'matched ' } & \begin{array}{l}
\text { Zero-pole matching method. (See [1] on page 1-245, p. 224.) }
\end{array}
\end{array}
$$

For information about the algorithms for each d2c conversion method, see "Continuous-Discrete Conversion Methods".

Default: 'zoh'

## PrewarpFrequency

Prewarp frequency for 'tustin' method, specified in rad/TimeUnit, where TimeUnit is the time units, specified in the TimeUnit property, of the discrete-time system. Specify the prewarp frequency as a positive scalar value. A value of 0 corresponds to the 'tustin' method without prewarp.

Default: 0

## Output Arguments

## opts - Option set for d2c

d2c0ptions option set

Option set for d2c, returned as an d2c0ptions option set.

## Examples

## Specify Model Discretization Method

Consider the following discrete-time transfer function.

$$
H(z)=\frac{z+1}{z^{2}+z+1}
$$

Create the discrete-time transfer function with a sample time of 0.1 seconds.
Hd = tf([11],[111],0.1);
Specify the discretization method as bilinear (Tustin) approximation and the prewarp frequency as 20 rad/seconds.

```
opts = d2cOptions('Method','tustin','PrewarpFrequency',20);
```

Convert the discrete-time model to continuous-time using the specified discretization method.
Hc = d2c(Hd,opts);
You can use the discretization option set opts to discretize additional models using the same options.

## References

[1] Franklin, G.F., Powell,D.J., and Workman, M.L., Digital Control of Dynamic Systems (3rd Edition), Prentice Hall, 1997.

## See Also

d2c
Introduced in R2012a

## d2d

Resample discrete-time model

## Syntax

```
sys1 = d2d(sys, Ts)
sys1 = d2d(sys, Ts, 'method')
sys1 = d2d(sys, Ts, opts)
```


## Description

sys1 = d2d(sys, Ts) resamples the discrete-time dynamic system model sys to produce an equivalent discrete-time model sys1 with the new sample time Ts (in seconds), using zero-order hold on the inputs.
sys1 = d2d(sys, Ts, 'method') uses the specified resampling method 'method':

- 'zoh ' - Zero-order hold on the inputs
- 'tustin' - Bilinear (Tustin) approximation

For information about the algorithms for each d2d conversion method, see "Continuous-Discrete Conversion Methods".
sys1 = d2d(sys, Ts, opts) resamples sys using the option set with d2dOptions.

## Examples

## Resample a Discrete-Time Model

Create the following zero-pole-gain-model with sample time 0.1 seconds.
$H(z)=\frac{z-0.7}{z-0.5}$
H = zpk(0.7,0.5,1,0.1);
Resample the model at 0.05 s .
H2 $=d 2 d(H, 0.05)$
H2 =
(z-0.8243)
(z-0.7071)

```
Sample time: 0.05 seconds
Discrete-time zero/pole/gain model.
```

Resample H 2 at 0.1 seconds to obtain the original model H .

```
H3 = d2d(H2,0.1)
H3 =
    (z-0.7)
    (z-0.5)
Sample time: 0.1 seconds
Discrete-time zero/pole/gain model.
```


## Resample an Identified Discrete-Time Model

Suppose that you estimate a discrete-time output-error polynomial model with sample time commensurate with the estimation data ( 0.1 seconds). However, your deployment application requires a faster sampling frequency ( 0.01 seconds). You can use d2d to resample your estimated model.

Load the estimation data.

```
load iddata1 z1
```

z1.Ts
ans $=0.1000$
z1 is an iddata object containing the estimation input-output data with sample time 0.1 seconds.
Estimate an output-error polynomial model of order [lllll $\begin{array}{ll}2 & 2\end{array} 1$.
sys = oe(z1,[2 2 1]);
sys.Ts
ans $=0.1000$
Resample the model at sample time 0.01 seconds.
sys2 = d2d(sys,0.01);
sys2.Ts
ans $=0.0100$
d2d resamples the model using the zero-order hold method.

## Tips

- Use the syntax sys1 = d2d(sys,Ts,'method') to resample sys using the default options for 'method'. To specify tustin resampling with a frequency prewarp, use the syntax sys1 = d2d(sys,Ts,opts). For more information, see d2dOptions.
- When sys is an identified (IDLTI) model, sys1 does not include the estimated parameter covariance of sys. If you want to translate the covariance while converting the model, use translatecov.


## See Also

c2d | d2c | d2dOptions | translatecov | upsample
Introduced before R2006a

## d2dOptions

Create option set for discrete-time resampling

## Syntax

```
opts = d2dOptions
opts = d2dOptions('OptionName', OptionValue)
```


## Description

opts $=$ d2dOptions returns the default options for d2d.
opts = d2dOptions('OptionName', OptionValue) accepts one or more comma-separated name-value pairs that specify options for the d2d command. Specify OptionName inside single quotes.

This table summarizes the options that the d2d command supports.

## Input Arguments

## Name-Value Pair Arguments

## Method

Discretization method, specified as one of the following values:
'zoh ' Zero-order hold, where d2d assumes that the control inputs are piecewise constant over the sample time Ts.
'tustin' Bilinear (Tustin) approximation. By default, d2d resamples with no prewarp. To include prewarp, use the PrewarpFrequency option.

For information about the algorithms for each d2d conversion method, see "Continuous-Discrete Conversion Methods".

Default: 'zoh'

## PrewarpFrequency

Prewarp frequency for 'tustin' method, specified in rad/TimeUnit, where TimeUnit is the time units, specified in the TimeUnit property, of the resampled system. Takes positive scalar values. The prewarp frequency must be smaller than the Nyquist frequency before and after resampling. A value of 0 corresponds to the standard 'tustin' method without prewarp.

Default: 0

## Examples

## Specify Method for Resampling a Discrete-Time Model

Create the following discrete-time transfer function with sample time 0.1 seconds.

$$
H(z)=\frac{z+1}{z^{2}+z+1}
$$

h1 = tf([1 1],[1 1 1],0.1);
Specify the discretization method as bilinear Tustin method with a prewarping frequency of $20 \mathrm{rad} /$ seconds.
opts = d2dOptions('Method','tustin','PrewarpFrequency',20);
Resample the discrete-time model using the specified options.
h2 = d2d(h1, 0.05,opts);
You can use the option set opts to resample additional models using the same options.

## See Also

d2d

Introduced in R2012a

## damp

Natural frequency and damping ratio

## Syntax

```
damp(sys)
[wn,zeta] = damp(sys)
[wn,zeta,p] = damp(sys)
```


## Description

damp(sys) displays the damping ratio, natural frequency, and time constant of the poles of the linear model sys. For a discrete-time model, the table also includes the magnitude of each pole. The poles are sorted in increasing order of frequency values.
[wn,zeta] = damp(sys) returns the natural frequencies wn, and damping ratios zeta of the poles of sys.
[wn,zeta, p] = damp(sys) also returns the poles $p$ of sys.

## Examples

## Display Natural Frequency, Damping Ratio, and Poles of Continuous-Time System

For this example, consider the following continuous-time transfer function:

$$
\operatorname{sys}(s)=\frac{2 s^{2}+5 s+1}{s^{3}+2 s-3} .
$$

Create the continuous-time transfer function.

```
sys = tf([2,5,1],[1,0,2,-3]);
```

Display the natural frequencies, damping ratios, time constants, and poles of sys.

```
damp(sys)
```

| Pole | Damping | Frequency <br> (rad/seconds) | Time Constant <br> $($ seconds $)$ |
| :---: | :---: | :---: | :---: |
| $1.00 \mathrm{e}+00$ |  |  |  |
| $-5.00 \mathrm{e}-01+1.66 \mathrm{e}+00 \mathrm{i}$ | $-1.00 \mathrm{e}+00$ | $1.00 \mathrm{e}+00$ | $-1.00 \mathrm{e}+00$ |
| $-5.00 \mathrm{e}-01-1.66 \mathrm{e}+00 \mathrm{i}$ | $2.89 \mathrm{e}-01$ | $1.73 \mathrm{e}+00$ | $2.00 \mathrm{e}+00$ |
| $2.00 \mathrm{e}+00$ |  |  |  |

The poles of sys contain an unstable pole and a pair of complex conjugates that lie int he left-half of the s-plane. The corresponding damping ratio for the unstable pole is -1 , which is called a driving force instead of a damping force since it increases the oscillations of the system, driving the system to instability.

## Display Natural Frequency, Damping Ratio, and Poles of Discrete-Time System

For this example, consider the following discrete-time transfer function with a sample time of 0.01 seconds:
$\operatorname{sys}(z)=\frac{5 z^{2}+3 z+1}{z^{3}+6 z^{2}+4 z+4}$.
Create the discrete-time transfer function.

```
sys = tf([[5 3 1],[\begin{array}{llll}{1}&{6}&{4}&{4}\end{array}],0.01)
sys =
    5 z^2 + 3 z + 1
    z^3 + 6 (`^2 + 4 z + 4
Sample time: 0.01 seconds
Discrete-time transfer function.
```

Display information about the poles of sys using the damp command.

```
damp(sys)
```

| Pole | Magnitude | Damping | Frequency <br> (rad/seconds) | Time Constant <br> $($ seconds $)$ |
| :---: | :---: | :---: | :---: | ---: |
| $-3.02 \mathrm{e}-01+8.06 \mathrm{e}-01 \mathrm{i}$ | $8.61 \mathrm{e}-01$ | $7.74 \mathrm{e}-02$ | $1.93 \mathrm{e}+02$ | $6.68 \mathrm{e}-02$ |
| $-3.02 \mathrm{e}-01-8.06 \mathrm{e}-01 \mathrm{i}$ | $8.61 \mathrm{e}-01$ | $7.74 \mathrm{e}-02$ | $1.93 \mathrm{e}+02$ | $6.68 \mathrm{e}-02$ |
| $-5.40 \mathrm{e}+00$ | $5.40 \mathrm{e}+00$ | $-4.73 \mathrm{e}-01$ | $3.57 \mathrm{e}+02$ | $-5.93 \mathrm{e}-03$ |

The Magnitude column displays the discrete-time pole magnitudes. The Damping, Frequency, and Time Constant columns display values calculated using the equivalent continuous-time poles.

## Natural Frequency and Damping Ratio of Zero-Pole-Gain Model

For this example, create a discrete-time zero-pole-gain model with two outputs and one input. Use sample time of 0.1 seconds.

```
sys = zpk({0;-0.5},{0.3;[0.1+1i,0.1-1i]},[1;2],0.1)
sys =
    From input to output...
            z
        1: -------
        (z-0.3)
            2 (z+0.5)
    2: --------------------
```

```
    (z^2 - 0.2z + 1.01)
Sample time: 0.1 seconds
Discrete-time zero/pole/gain model.
```

Compute the natural frequency and damping ratio of the zero-pole-gain model sys.

```
[wn,zeta] = damp(sys)
```

$w n=3 \times 1$
12.0397
14.7114
14.7114
zeta $=3 \times 1$
1.0000
-0.0034
-0.0034

Each entry in wn and zeta corresponds to combined number of I/Os in sys. zeta is ordered in increasing order of natural frequency values in wn.

## Compute Natural Frequency, Damping Ratio and Poles of a State-Space Model

For this example, compute the natural frequencies, damping ratio and poles of the following statespace model:

$$
A=\left[\begin{array}{cc}
-2 & -1 \\
1 & -2
\end{array}\right], \quad B=\left[\begin{array}{cc}
1 & 1 \\
2 & -1
\end{array}\right], \quad C=\left[\begin{array}{ll}
1 & 0
\end{array}\right], \quad D=\left[\begin{array}{ll}
0 & 1
\end{array}\right]
$$

Create the state-space model using the state-space matrices.
$\mathrm{A}=\left[\begin{array}{lll}-2 & -1 ; 1 & -2\end{array}\right] ;$
$B=[11 ; 2-1]$;
$C=\left[\begin{array}{ll}1 & 0\end{array}\right] ;$
D = [0 1];
sys $=s s(A, B, C, D) ;$
Use damp to compute the natural frequencies, damping ratio and poles of sys.

```
[wn,zeta,p] = damp(sys)
wn = 2\times1
    2.2361
    2.2361
zeta = 2×1
    0.8944
```

```
p = 2x1 complex
    -2.0000 + 1.0000i
    -2.0000 - 1.0000i
```

The poles of sys are complex conjugates lying in the left half of the s-plane. The corresponding damping ratio is less than 1 . Hence, sys is an underdamped system.

## Input Arguments

## sys - Linear dynamic system

dynamic system model
Linear dynamic system, specified as a SISO, or MIMO dynamic system model. Dynamic systems that you can use include:

- Continuous-time or discrete-time numeric LTI models, such as tf, zpk, or ss models.
- Generalized or uncertain LTI models such as genss or uss models. (Using uncertain models requires Robust Control Toolbox software.)
damp assumes
- current values of the tunable components for tunable control design blocks.
- nominal model values for uncertain control design blocks.


## Output Arguments

## wn - Natural frequency of each pole

vector
Natural frequency of each pole of sys, returned as a vector sorted in ascending order of frequency values. Frequencies are expressed in units of the reciprocal of the TimeUnit property of sys.

If sys is a discrete-time model with specified sample time, wn contains the natural frequencies of the equivalent continuous-time poles. If the sample time is not specified, then damp assumes a sample time value of 1 and calculates wn accordingly. For more information, see "Algorithms" on page 1-255.

## zeta - Damping ratio of each pole

vector
Damping ratios of each pole, returned as a vector sorted in the same order as wn.
If sys is a discrete-time model with specified sample time, zeta contains the damping ratios of the equivalent continuous-time poles. If the sample time is not specified, then damp assumes a sample time value of 1 and calculates zeta accordingly. For more information, see "Algorithms" on page 1255.

## p - Poles of the dynamic system model

vector

Poles of the dynamic system model, returned as a vector sorted in the same order as wn. p is the same as the output of pole(sys), except for the order. For more information on poles, see pole.

## Algorithms

damp computes the natural frequency, time constant, and damping ratio of the system poles as defined in the following table:

|  | Continuous Time | Discrete Time with Sample <br> Time Ts |
| :--- | :--- | :--- |
| Pole Location | $s$ | $z$ |
| Equivalent Continuous-Time <br> Pole | Not applicable | $s=\frac{\ln (z)}{T_{s}}$ |
| Natural Frequency | $\omega_{n}=\|s\|$ | $\omega_{n}=\|s\|=\left\|\frac{\ln (z)}{T_{s}}\right\|$ |
| Damping Ratio | $\zeta=-\cos (\angle s)$ | $\zeta=-\cos (\angle s)=-\cos (\angle \ln (z))$ |
| Time Constant | $\tau=\frac{1}{\omega_{n} \zeta}$ | $\tau=\frac{1}{\omega_{n} \zeta}$ |

If the sample time is not specified, then damp assumes a sample time value of 1 and calculates zeta accordingly.

## See Also

dsort|eig|esort|pole|pzmap|zero
Introduced before R2006a

## data2state

Map past data to states of state-space and nonlinear ARX models

## Syntax

X = data2state(sys, PastData)
[X,XCov] = data2state(sys,PastData)

## Description

X = data2state(sys,PastData) maps the past data to the states of a state-space or a nonlinear ARX model sys. $X$ contains the state values at the time instant immediately after the most recent data sample in PastData. The software computes the state estimates by minimizing the 1 -step ahead prediction error between predicted response and output signal in PastData.
data2state is useful for continued model simulation. That is, suppose you have simulated a model up to a certain time instant and would like to then simulate the model for future inputs. Use data2state to estimate states of the model at the beginning of the second simulation.
[X,XCov] = data2state(sys,PastData) returns the estimated covariance, XCov, of the current states.

## Examples

## Compute Mapped States of Identified Model

Compute the mapped states of an identified model, and use the states as initial state values for model simulation.

Load estimation data.
load iddata3 z3
Estimate a second-order state-space model using the data.
sys $=\operatorname{ssest}(z 3,2)$;
Simulate the model using the entire input signal in z3.

```
Input = z3(:,[],:); % |iddata| object containing only the input signal
y_all = sim(sys,Input);
sim uses zero initial conditions to compute y_all.
```

Now simulate the model using only the first-half of the input signal.

```
Input1 = Input(1:150);
y_1 = sim(sys,Input1);
```

Continue the simulation with the second-half of the input signal such that the results show no discontinuity owing to initial-condition-induced transients. To do so, first construct a past data set
comprising of the input and simulated output from the first-half of the input signal. Then calculate the state values corresponding to the start of the second-half of the input signal $(\mathrm{t}=151)$.

```
PastData = [y_1,Input1];
X = data2state(sys,PastData);
```

$X$ contains the state values at the time instant immediately after the most recent data sample in PastData. This time point is also the start of the future data (second-half of the input signal).

FutureData $=$ Input(151:end);
Simulate the model using the second-half of the input signal and $X$ as initial conditions.

```
opt = simOptions('InitialCondition',X);
y_2 = sim(sys,FutureData,opt);
```

Verify that $y \_2$ matches the second half of $y \_a l l$.

```
plot(y_all,y_2,'r.')
```

legend('Simulation using all input data',...
'Separate simulation of second-half of input data')

## Input-Output Data



## Calculate Mapped States and Covariance of States

Load the past data.

```
load iddata1 z1
```

PastData = z1;

Estimate an ARX model.
sys = arx(PastData,[1 1 0]);
Convert the model to a state-space model.
sys2 = idss(sys);
Calculate the mapped states and covariance of states using PastData.
[X,XCov] = data2state(sys2,PastData);
X is the state value at the time instant immediately after the most recent data sample in PastData.

## Determine Mapped State of a Nonlinear ARX model

Load your data and create a data object.
load motorizedcamera;
z = iddata(y,u,0.02,'Name','Motorized Camera','TimeUnit','s');
Estimate a nonlinear ARX model.
mw1 = nlarx(z,[ones(2,2),ones(2,6),ones(2,6)],'wavenet');
The estimated model has six inputs and two outputs.
Determine the model order, nx .
$n x=\operatorname{order}(m w 1) ;$
Use the first nx samples of data to generate initial conditions.

```
PastData = struct('Input', z.u(1:nx,:),'Output',z.y(1:nx,:));
```

Compute the mapped states of the model.
X = data2state(mw1,PastData);
X is the state value at the time instant immediately after the most recent data sample in PastData.
Simulate the model using the remaining input data, and specify the initial conditions for simulation.

```
InputSignal = z.u(nx+1:end,:);
opt = simOptions;
opt.InitialCondition = X;
sim(mwl,InputSignal,opt)
```



## Input Arguments

## sys - Identified model

idss |idgrey|idnlgrey |idnlarx
Identified model whose current states are estimated, specified as one of the following:

- State-space model (idss, idgrey, or idnlgrey object)
- Nonlinear ARX model (idnlarx object) - For a definition of the states of idnlarx models, see "Definition of idnlarx States" on page 1-571.


## PastData - Past input-output data

iddata object | structure
Past input-output data, specified as one of the following:

- iddata object - The number of samples must be greater than or equal to the model order. To determine model order, use order.

X is the value of model states at time PastData. SamplingInstants(end) + PastData.Ts.
When sys is continuous-time, specify PastData as an iddata object. $X$ then corresponds to the discretized (c2d) version of the model, where the discretization method is stored in the InterSampleproperty of PastData.

- Structure - Specified as a structure with the following fields:
- Input - Past input data, specified as an $N$-by- $N u$ matrix, where $N$ is great than or equal to the model order.
- Output - Past output data, specified as an $N$-by-Ny matrix, where $N$ is great than or equal to the model order.

Specify PastData as a structure only when sys is a discrete-time model.
The data samples in PastData should be in the order of increasing time. That is, the last row in PastData should correspond to the latest time.

## Output Arguments

## X - Mapped states of model

row vector
Mapped states of model, returned as a row vector of size equal to the number of states. $X$ contains the state value at the time instant immediately after the most recent data sample in PastData. That is, if PastData is an iddata object, X is the state value at time $\mathrm{t}=$ PastData. SamplingInstants(end)+PastData.Ts.

## XCov - Estimated covariance of state values

matrix
Estimated covariance of state values, returned as a matrix of size $N x-$ by- $N x$, where $N x$ is the number of states.

XCov is empty if sys is a nonlinear ARX model.

## See Also

findstates|getDelayInfo|idnlarx/findop|order|sim
Introduced in R2008a

## db2mag

Convert decibels (dB) to magnitude

## Syntax

y = db2mag (ydb)

## Description

$y=d b 2 m a g(y d b)$ returns the magnitude measurements, $y$, that correspond to the decibel (dB) values specified in ydb. The relationship between magnitude and decibels is $y d b=20 * \log _{10}(y)$

## Examples

## Magnitude of Elements in an Array

For this example, generate a 2-by-3-by-4 array of Gaussian random numbers. Assume the numbers are expressed in decibels and compute the corresponding magnitudes.

```
rng('default');
ydb = randn(2,3,4);
y = db2mag(ydb)
y =
y(:,:,1) =
    1.0639 0.7710 1.0374
    1.2351 1.1044 0.8602
y(:,:,2) =
    0.9513 1.5098 0.8561
    1.0402 1.3755 1.4182
y(:,:,3) =
    1.0871 1.0858 0.9858
    0.9928 0.9767 1.1871
y(:,:,4) =
\(1.1761 \quad 1.0804 \quad 1.0861\)
\(1.1772 \quad 0.8702 \quad 1.2065\)
```

Use the definition to check the calculation.
chck $=10 . \wedge(y d b / 20)$

```
chck =
chck(:,:,1) =
    1.0639 0.7710 1.0374
    1.2351 1.1044 0.8602
chck(:,:,2) =
    0.9513 1.5098 0.8561
    1.0402 1.3755 1.4182
chck(:,:,3) =
    1.0871 1.0858 0.9858
    0.9928 0.9767 1.1871
chck(:,:,4) =
\(1.1761 \quad 1.0804 \quad 1.0861\)
    1.1772
    0.8702
    1.2065
```


## Input Arguments

ydb - Input array in decibels
scalar | vector | matrix | array
Input array in decibels, specified as a scalar, vector, matrix, or an array. When ydb is nonscalar, db2mag is an element-wise operation.

Data Types: single | double

## Output Arguments

## y - Magnitude measurements

scalar | vector | matrix | array
Magnitude measurements, returned as a scalar, vector, matrix, or an array of the same size as ydb.

## See Also

mag2db
Introduced in R2008a

## dcgain

Low-frequency (DC) gain of LTI system

## Syntax

$\mathrm{k}=$ dcgain(sys)

## Description

$\mathrm{k}=$ dcgain(sys) computes the DC gain k of the LTI model sys.

## Continuous Time

The continuous-time DC gain is the transfer function value at the frequency $s=0$. For state-space models with matrices ( $A, B, C, D$ ), this value is
$K=D-C A^{-1} B$

## Discrete Time

The discrete-time DC gain is the transfer function value at $z=1$. For state-space models with matrices $(A, B, C, D)$, this value is
$K=D+C(I-A)^{-1} B$

## Examples

## Compute the DC Gain of a MIMO Transfer Function

Create the following 2-input 2-output continuous-time transfer function.

$$
\begin{gathered}
H(s)=\left[\begin{array}{cc}
1 & \frac{s-1}{s^{2}+s+3} \\
\frac{1}{s+1} & \frac{s+2}{s-3}
\end{array}\right] \\
H=\left[\begin{array}{ll}
1 & \left.\operatorname{tf}\left(\left[\begin{array}{ll}
1 & -1
\end{array}\right],\left[\begin{array}{ll}
1 & 1
\end{array}\right]\right) ; \operatorname{tf}\left(1,\left[\begin{array}{ll}
1 & 1
\end{array}\right]\right) \operatorname{tf}\left(\left[\begin{array}{ll}
1 & 2
\end{array}\right],\left[\begin{array}{ll}
1 & -3
\end{array}\right]\right)\right] ;
\end{array}, \quad \begin{array}{c}
\end{array}\right]
\end{gathered}
$$

Compute the DC gain of the transfer function. For continuous-time models, the DC gain is the transfer function value at the frequency $s=0$.
$K=$ dcgain(H)
$\mathrm{K}=2 \times 2$

| 1.0000 | -0.3333 |
| :--- | :--- |
| 1.0000 | -0.6667 |

The DC gain for each input-output pair is returned. $\mathrm{K}(\mathrm{i}, \mathrm{j})$ is the DC gain from input j to output i .

## Compute DC Gain of Identified Model

Load the estimation data.
load iddatal z1
z 1 is an iddata object containing the input-output estimation data.
Estimate a process model from the data. Specify that the model has one pole and a time delay term.

```
sys = procest(z1,'P1D')
sys =
Process model with transfer function:
            Kp
    G(s) = --------- * exp(-Td*s)
            1+Tp1*s
        Kp = 9.0754
        Tp1 = 0.25655
            Td = 0.068
Parameterization:
        {'P1D'}
    Number of free coefficients: 3
    Use "getpvec", "getcov" for parameters and their uncertainties.
Status:
Estimated using PROCEST on time domain data "z1".
Fit to estimation data: 44.85%
FPE: 6.02, MSE: 5.901
```

Compute the DC gain of the model.
$K=$ dcgain(sys)
$K=9.0754$
This DC gain value is stored in the Kp property of sys.
sys.Kp
ans $=9.0754$

## Tips

The DC gain is infinite for systems with integrators.

## See Also

evalfr|norm
Introduced in R2012a

## deadzone

Create a dead-zone nonlinearity estimator object

## Syntax

NL = deadzone
NL = deadzone('ZeroInterval', [a, b])

## Description

$N L=$ deadzone creates a default dead-zone nonlinearity estimator object for estimating
Hammerstein-Wiener models. The interval in which the dead-zone exists (zero interval) is set to [ NaN $\mathrm{NaN}]$. The initial value of the zero interval is determined from the estimation data range, during estimation using $n l h w$. Use dot notation to customize the object properties, if needed.

NL = deadzone('ZeroInterval', [a,b]) creates a dead-zone nonlinearity estimator object initialized with zero interval, [a,b].

Alternatively, use NL = deadzone([a,b]).

## Object Description

deadzone is an object that stores the dead-zone nonlinearity estimator for estimating HammersteinWiener models.

Use deadzone to define a nonlinear function $y=F(x, \theta)$, where $y$ and $x$ are scalars, and $\theta$ represents the parameters $a$ and $b$, which define the zero interval.

The dead-zone nonlinearity function has the following characteristics:

$$
\begin{array}{lc}
a \leq x<b & F(x)=0 \\
x<a & F(x)=x-a \\
x \geq b & F(x)=x-b
\end{array}
$$

For example, in the following plot, the dead-zone is in the interval $[-4,4]$.


The value $F(x)$ is computed by evaluate $(N L, x)$, where $N L$ is the deadzone object.
For deadzone object properties, see "Properties" on page 1-269.

## Examples

## Create a Default Dead-Zone Nonlinearity Estimator

NL = deadzone;
Specify the zero interval.
NL.ZeroInterval = [-4,5];

## Estimate a Hammerstein-Wiener Model with Dead-zone Nonlinearity

Load estimation data.
load twotankdata;
z = iddata(y,u,0.2,'Name','Two tank system');
z1 = z(1:1000);
Create a deadzone object, and specify the initial guess for the zero-interval.

OutputNL = deadzone('ZeroInterval',[-0.1 0.1]);
Estimate model with no input nonlinearity.

```
m = nlhw(z1,[2 3 0],[],OutputNL);
```


## Estimate MIMO Hammerstein-Wiener Model

Load the estimation data.
load motorizedcamera;
Create an iddata object.
z = iddata(y,u,0.02,'Name','Motorized Camera','TimeUnit','s');
$z$ is an iddata object with 6 inputs and 2 outputs.
Specify the model orders and delays.
Orders $=[\operatorname{ones}(2,6)$, ones $(2,6)$, ones $(2,6)]$;
Specify the same nonlinearity estimator for each input channel.
InputNL = saturation;
Specify different nonlinearity estimators for each output channel.

```
OutputNL = [deadzone,wavenet];
```

Estimate the Hammerstein-Wiener model.
sys $=$ nlhw(z,Orders,InputNL,OutputNL);
To see the shape of the estimated input and output nonlinearities, plot the nonlinearities.
plot(sys)


Click on the input and output nonlinearity blocks on the top of the plot to see the nonlinearities.

## Input Arguments

## [a,b] - Zero interval

[NaN NaN] (default)|2-element row vector
Zero interval of the dead-zone, specified as a 2-element row vector of doubles.
The dead-zone nonlinearity is initialized at the interval [a,b]. The interval values are adjusted to the estimation data by nlhw. To remove the lower limit, set a to - Inf. The lower limit is not adjusted
during estimation. To remove the upper limit, set b to Inf. The upper limit is not adjusted during estimation.

When the interval is [ NaN NaN ], the initial value of the zero interval is determined from the estimation data range during estimation using nlhw.
Example: [-2 1]

## Properties

## ZeroInterval

Zero interval of the dead-zone, specified as a 2-element row vector of doubles.
Default: [NaN NaN]

## Output Arguments

NL - Dead-zone nonlinearity estimator object
deadzone object
Dead-zone nonlinearity estimator object, returned as a deadzone object.

## See Also

nlhw

Introduced in R2007a

## delayest

Estimate time delay (dead time) from data

## Syntax

```
nk = delayest(Data)
nk = delayest(Data, na, nb, nkmin, nkmax, maxtest)
```


## Description

$n k=$ delayest(Data) estimates time delay from data. Data is an iddata object containing the input-output data. It can also be an idfrd object defining frequency-response data. Only singleoutput data can be handled. nk is returned as an integer or a row vector of integers, containing the estimated time delay in samples from the input(s) to the output in Data.

The estimate is based on a comparison of ARX models with different delays:

$$
\begin{aligned}
& y(t)+a_{1} y(t-1)+\ldots+a_{n a} y(t-n a)= \\
& b_{1} u(t-n k)+\ldots+b_{n b} u(t-n b-n k+1)+e(t)
\end{aligned}
$$

nk = delayest(Data,na,nb,nkmin,nkmax,maxtest) specifies additional options. The integer na is the order of the A polynomial (default 2). nb is a row vector of length equal to the number of inputs, containing the order(s) of the B polynomial(s) (default all 2). nkmin and nkmax are row vectors of the same length as the number of inputs, containing the smallest and largest delays to be tested. Defaults are nkmin $=0$ and nkmax $=n k m i n+20$. If nb, nkmax, and/or nkmin are entered as scalars in the multiple-input case, all inputs will be assigned the same values. maxtest is the largest number of tests allowed (default 10,000 ).

## Introduced before R2006a

## detrend

Subtract offset or trend from time-domain signals contained in iddata objects

## Syntax

```
data_d = detrend(data)
data_d = detrend(data,Type)
[data_d,T_r] = detrend(___)
data_d = detrend(data,1,brkpt)
```


## Description

detrend subtracts offsets or linear trends from time-domain input-output data represented in iddata objects. det rend either computes the trend data to subtract, or subtracts the trend that you specify.

For a more general detrending function that does not require iddata objects, see detrend.
data_d = detrend(data) computes and subtracts the mean value from each time-domain signal in data. The iddata objects data_d and data each contain input and output data originating from SISO, MIMO, or multiexperiment datasets.
data_d = detrend(data,Type) subtracts the trend you specify in Type. You can specify a meanvalue, linear, or custom trend.
[data_d,T_r] = detrend ( $\quad$ _ $)$ also returns the subtracted trend as a TrendInfo object $T_{-} r$. You can obtain T_r with any of the input-argument combinations in previous syntaxes.
data_d = detrend(data, 1, brkpt) computes and subtracts the piecewise-linear trends for data with segmented trends, using the breakpoints that you define with brkpt.

The second argument, which corresponds to Type, must be 1.
With this syntax, you cannot retrieve the resulting piecewise-linear trend information as an output.

## Examples

## Remove Biases From Signals

Remove biases from steady-state signals in an iddata object by using det rend to compute and subtract the mean values of the input and output.

Load the input and output time series data y 2 and u 2 . Construct the iddata object data2, using the data and a sample time of 0.08 seconds.

```
load dryer2 y2 u2
data2 = iddata(y2,u2,0.08);
```

Use detrend to both compute the mean values and subtract them from input and output signals. Use the input argument Tr to store the computed trend information. Plot the original data and detrended data together.
[data2_d,Tr] = detrend(data2);
plot(data2,data2_d)
legend('Original Data','Detrended Data')

## Input-Output Data



The detrended data has shifted by about 5 units. Inspect $\operatorname{Tr}$ to obtain the precise mean values that detrend computed and subtracted. These values are returned in the InputOffset and OutputOffset properties.

Tr
Trend specifications for data "data2" with 1 input(s), 1 output(s), 1 experiment(s): DataName: 'data2' InputOffset: 5.0000
Output0ffset: 4.8901
InputSlope: 0
OutputSlope: 0
The mean of the original input is higher than the mean of the original output.

## Remove Linear Trend from a Signal

Remove the linear trend from a signal in an iddata object, and overlay the trendline on a before-and-after data plot.

Load and plot signal data from the file lintrend2. For this example, only output data is provided in iddata object dataL.
load lintrend2 dataL
plot(dataL,'b')
Input-Output Data


The plot shows a clear linear trend in the data. Use detrend linear option (Type $=1$ ) to subtract the trend from the data. detrend fits the data and determines the linear trend to subtract. Include the TrendInfo object Tr as an output argument so you can see what detrend subtracts.

```
[dataL_d,Tr] = detrend(dataL,1);
```

Plot the detrended data against the original data.

```
hold on
```

plot(dataL_d,'g')
legend('Oríginal','Detrended','Location','northwest')

## Input-Output Data



The linear trend has been removed. Inspect Tr to get more information about the removed trend.
Tr

```
Trend specifications for data "dataL" with 0 input(s), 1 output(s), 1 experiment(s):
            DataName: 'dataL'
    Input0ffset: [1x0 double]
    OutputOffset: 0.8888
    InputSlope: [1x0 double]
    OutputSlope: 19.3830
```

The OutputOffset and the OutputSlope properties provide the parameters of the line that detrend removed. You can reconstruct this line, and then overlay it on the before-and-after data plot. The SamplingInstants property of DataL provides the timepoints associated with the data.

```
m = Tr.OutputSlope
m = 19.3830
b = Tr.OutputOffset
b = 0.8888
t = dataL.SamplingInstants;
TrLn = m*t+b;
plot(t,TrLn,'r')
legend('Original','Detrended','Trendline','Location','northwest')
```


## Input-Output Data



## Remove Specified Offsets from Signals

Remove known offsets from an input-output signal pair contained in an iddata object.
Detrend can compute and subtract the mean values for input and output signals, resulting in zeromean detrended signals. However, if you already know you have specific data offsets beforehand, you can have detrend subtract these from your signals instead. Specifying the offsets also allows you to retain a non-zero operating point in the detrend result.

Load SISO data containing vectors $u 2$ and y 2 . For this example, suppose that you know both signals have an offset of 4 from the expected operating point of 1 . Combine these vectors into an iddata object, using a sample time of 0.08 seconds, and plot it.
load dryer2 u2 y2
data = iddata(y2,u2,0.08);
plot(data)

## Input-Output Data



The known offset of 4 (from operating point 1) is visible in the plots. You can construct a TrendInfo object to capture this offset, using the function getTrend.

Create the TrendInfo object, and then set its offset properties.
T = getTrend(data);
T. InputOffset = 4;
T.OutputOffset = 4

Trend specifications for data "data" with 1 input(s), 1 output(s), 1 experiment(s):
DataName: 'data'
InputOffset: 4
OutputOffset: 4
InputSlope: 0
OutputSlope: 0
Use detrend to subtract the offset from the data. Plot it alongside the original data.

```
data_d = detrend(data,T);
hold on
plot(data_d)
legend('Original','Detrended')
```


## Input-Output Data



The offset of 4 has been removed.

## Remove Segmented Linear Trends from Signals by using Breakpoints

Detrend data with segmented piecewise-linear trends by specifying breakpoints to delimit the segments.

Most of the det rend syntaxes assume and compute a single trend for each of the signals. However, in some cases there are discontinuities in the linear trends, caused by test configuration changes, environmental conditions, or other influences. When the signal displays such segmentation, you can have detrend operate on the test segments independently. To do so, specify breakpoints in the brkpt input argument. These are the indices to the timepoints in the signal at which linear trends change slope.

You may know these breakpoints up front, based on changes that you know occurred during data collection. Alternatively, you may need to approximate them by inspecting the data itself.

Load the data, inspect its structure and contents, and plot it. This data consists of output data only in the iddata object dataLb2.

```
load brkTrend dataLb2
dataLb2
dataLb2 =
```

Time domain data set with 512 samples.
Sample time: 0.00390625 seconds
Outputs Unit (if specified)
y1
plot(dataLb2)
Input-Output Data


For this example, the data has known breakpoints at indices [100 300]. Applying the sample time (property Ts), these breakpoints correspond to the actual timepoints as follows:

```
brkpt=[100 300];
brkpt_time = brkpt*dataLb2.Ts
brkpt_time = 1×2
    0.3906 1.1719
```

Detrend the data using brkpt.

```
dataLb2_d = detrend(dataLb2,1,brkpt);
```

Plot the original and detrended data.
plot (dataLb2,dataLb2_d)
legend('Original Data','Detrended Data')

## Input-Output Data



The linear trend segments have been removed.

## Detrend Multiexperiment Signals using Multiple-Breakpoint Sets

Apply a unique set of breakpoints to each experiment when you detrend a Multiexperiment dataset.
Experiments within a multiexperiment dataset may contain unique linear trending discontinuities. You can apply a unique set of breakpoints to each experiment by expressing them in a cell array.

Load the data, which consists of:

- datmult, a multiexperiment iddata object containing three experiments (output only)
- bpn vectors, which provide known breakpoints for each experiment in the form of indices to timepoints

```
load multiexpdt datmult bp1 bp2 bp3
datmult
datmult =
Time domain data set containing 3 experiments.
\begin{tabular}{ccc} 
Experiment & Samples & Sample Time \\
expl & 250 & 1
\end{tabular}
```

```
    exp3 350 1
Outputs Unit (if specified)
    y1
bp1,bp2,bp3
bp1 = 1\times2
    50 200
bp2 = 100
bp3 =
[]
```

Plot the data. There are significant differences among the streams, and they drift at different rates from zero mean.

```
plot(datmult)
```

legend
Input-Output Data


For this set of experiments, it is known that there is unique trending for each run and unique discontinuities indicated by the bp vectors.
detrend can incorporate these unique characteristics if the bp information is provided as a cell array.

Construct the cell array.
bpcell $=\{b p 1 ; b p 2 ; b p 3\}$

```
bpcell=3\times1 cell array
    {[ 50 200]}
    {[ [ 100]}
    {0x0 double}
```

Apply detrend and plot the result, using the same scale as the original plot.

```
datmult_d = detrend(datmult,1,bpcell);
figure
plot(datmult_d)
axis([0,inf,-15,30])
legend
```



The experimental data are now better aligned, and do not drift significantly away from zero mean.

## Detrend Input and Output Signals Separately

Apply different trend types to the input and output signals contained in an iddata object.

Detrend assumes that the same type of trend applies to both input and output signals. In some cases, there may be a trend type that is present in only one signal. You can perform det rend individually on each signal by extracting the signals into separate iddata objects. Apply detrend to each object using its individual signal trend type. Then reassemble the results back into a single detrended iddata object.

Load, examine, and plot the data in iodatab.

```
load septrend iodatab;
iodatab
iodatab =
Time domain data set with 1000 samples
Sample time: 0.08 seconds
Outputs Unit (if specified)
    y1
Inputs Unit (if specified)
    ul
plot(iodatab)
hold on
```


## Input-Output Data



Both input and output plots show a bias. However, the output plot also shows an inverted V-shape trend that is not present in the input data.

Separate the input data and the output data into separate objects for detrending, using the iddata general data-selection form (see "Representing Time- and Frequency-Domain Data Using iddata Objects"):

```
data(samples,outputchannels,inputchannels)
idatab = iodatab(:,[],:);
odatab = iodatab(:,:,[]);
```

Remove the bias from the input data, using det rend to calculate and subtract the mean.
idatab_d = detrend(idatab,0);
Remove the bias and the inverted-V trend from the output data, using the midpoint index 500 as a breakpoint.
odatab_d = detrend(odatab,1,500);
Combine the detrended input and output data into a single iddata object.
iodatab_d = [odatab_d,idatab_d];
Overlay the detrended data on the original data.
plot(iodatab_d)
legend('original','detrended')

## Input-Output Data



The input and output data now contain neither bias nor V-shape trend.

## Input Arguments

## data - Time-domain input-output data

iddata object
Time-domain input-output data, specified as an iddata object containing one or more sets of timedomain signals. The iddata object can contain SISO, MIMO, or multiexperiment data. The signal sets can contain either input and output data, or output data only.

## Type - Trend type to be subtracted

0 (default) | 1 | TrendInfo object
Trend type to be subtracted, specified as one of:

- 0 - Compute and subtract the mean value
- 1 - Compute and subtract the linear trend (least-squares fit)
- TrendInfo object - subtract the trend you specify in the TrendInfo object. Use getTrend to create a TrendInfo object. For an example, see "Remove Specified Offsets from Signals" on page 1-275.


## brkpt - Timepoint locations of trending discontinuities

integer row vector | cell array of integer vectors
Timepoint locations of trending discontinuities (breakpoints), specified as:

- An integer row vector - For single-experiment SISO and MIMO datasets. Doing so applies a single set of breakpoints to all input and output signals. For an example, see "Remove Segmented Linear Trends from Signals by using Breakpoints" on page 1-277.
- A cell array containing individually-sized integer row vectors - For multiple-experiment datasets. Doing so applies a unique set of breakpoints to the output and input signals for each experiment. For an example, see "Detrend Multiexperiment Signals using Multiple-Breakpoint Sets" on page 1279.


## Output Arguments

## data_d - Detrended signals

iddata object
Detrended signals, returned as an iddata object. Dimensions of the contents are the same as dimensions of the contents of data.

## T_r - Subtracted trend data <br> TrendInfo object

Trend data subtracted from data to produce data_d, returned as a TrendInfo object .
When you use brkpt to specify multiple trends, you cannot retrieve the computed trend data.

## See Also

getTrend|iddata|idfilt|retrend

Introduced before R2006a

## diff

Difference signals in iddata objects

## Syntax

zdi $=\operatorname{diff}(z)$
$z d i=\operatorname{diff}(z, n)$

## Description

zdi $=\operatorname{diff}(z)$ and $z d i=\operatorname{diff}(z, n)$ return the difference signals in iddata objects. $z$ is a timedomain iddata object. $\operatorname{diff}(z)$ and diff( $z, n$ ) apply this command to each of the input/output signals in z .

Introduced before R2006a

## Estimate Process Model

Estimate continuous-time process model for single-input, single-output (SISO) system in either time or frequency domain in the Live Editor

## Description

The Estimate Process Model task lets you interactively estimate and validate a process model for SISO systems. You can define and vary the model structure and specify optional parameters, such as initial condition handling and search methods. The task automatically generates MATLAB code for your live script. For more information about Live Editor tasks generally, see "Add Interactive Tasks to a Live Script".

Process models are simple continuous-time transfer functions that describe the linear system dynamics. Process model elements include static gain, time constants, time delays, integrator, and process zero.

Process models are popular for describing system dynamics in numerous industries and are applicable to various production environments. The advantages of these models are that they are simple, they support transport delay estimation, and the model coefficients are easy to interpret as poles and zeros. For more information about process model estimation, see "What Is a Process Model?"

The Estimate Process Model task is independent of the more general System Identification app. Use the System Identification app when you want to compute and compare estimates for multiple model structures.

To get started, load experiment data that contains input and output data into your MATLAB workspace and then import that data into the task. Then select a model structure to estimate. The task gives you controls and plots that help you experiment with different model structures and compare how well the output of each model fits the measurements.

## Related Functions

The code that Estimate Process Model generates uses the following functions.

- iddata
- idfrd
frd
- procest
- procestOptions
- compare

The task estimates an idproc process model.


## Description (Collapsed Portion)

## Related Functions

The code that Estimate Process Model generates uses the following functions.

- iddata
- idfrd
frd
- procest
- procestOptions
- compare

The task estimates an idproc process model.

## Open the Task

To add the Estimate Process Model task to a live script in the MATLAB Editor:

- On the Live Editor tab, select Task > Estimate Process Model.
- In a code block in your script, type a relevant keyword, such as process or estimate. Select Estimate Process Model from the suggested command completions.


## Examples

## Estimate Process Model with Live Editor Task

Use the Estimate Process Model Live Editor Task to estimate a state-space model and compare the model output to the measurement data.

Open this example to see a preconfigured script containing the task.

## Set Up Data

Load the measurement data iddatal into your MATLAB workspace.

```
load iddata1 z1
z1
z1 =
Time domain data set with 300 samples.
Sample time: 0.1 seconds
Outputs Unit (if specified)
    y1
Inputs Unit (if specified)
    u1
```


## Import Data into the Task

In the Select data section, set Data Type to Data Object and set Estimation Object to z1.

## Estimate Process Model

sys = Estimated continuous process model for z1 with One Pole

- Select data


The data object contains the input and output variable names as well as the sample time, so you do not have to specify them.

## Estimate the Model Using Default Settings

Examine the model structure and optional parameters.

- Specify model structure
One Pole $\quad$ v $\square$ Delay $\square$ Zero $\square$ Integrator

$$
\frac{K p}{\left(T_{p 1} s+1\right)}
$$

- Specify estimation initialization

- Specify optional parameters

$\checkmark$ Output Plot

In the Specify model structure section, the default option is One Pole with no delay, zero, or integrator. Equations below the parameters in this section display the specified structure.

In the Specify estimation initialization section, initialization parameters matching the parameters in your model structure allow you to set starting points for estimation. If you select Fix, the parameter remains fixed to the value you specify. For this example, do not specify initialization. The task then uses default values for starting points.

In the Specify optional parameters section, the default options for process estimation are set.
Execute the task from the Live Editor tab using Run. A plot displays the estimation data, the estimated model output, and the fit percentage.


## Experiment with Parameter Settings

Experiment with the parameter settings and see how they influence the fit.
For instance, add delay to the One Pole structure and run the task.

- Specify model structure
One Pole $\quad \checkmark$ Delay $\square$ Zero $\square$ Integrator

$$
\frac{K p}{\left(T_{p 1} s+1\right)} e^{-} T_{d} s
$$

[^0]

The estimation fit improves, although the fit percentage is still below $50 \%$.
Try a different model structure. In Specify model structure, select Underdamped Pair with no delay and run the task.

- Specify model structure
Underdamped Pair
vDelayZeroIntegrator

$$
\frac{K p}{\left(T_{\omega}^{2} s^{2}+2 \zeta T_{\omega} s+1\right)}
$$

- Specify estimation initialization
$K p$ $\square$
$\qquad$ Fix
$T \omega$ $\square$
$\square$ Fix $\zeta$ $\square$ $\square \mathrm{F}$ Fix


The fit results improve significantly.

## Generate Code

To display the code that the task generates, click $\nabla$ at the bottom of the parameter section. The code that you see reflects the current parameter configuration of the task.


## Use Validation Data Set to Validate Estimated Model

Use separate estimation and validation data so that you can validate the estimated process model.

Open this example to see a preconfigured script containing the task.

## Set Up Data

Load the measurement data iddatal into your MATLAB workspace and examine its contents.

```
load iddata1 z1
z1
z1 =
Time domain data set with 300 samples.
Sample time: 0.1 seconds
Outputs Unit (if specified)
    y1
Inputs Unit (if specified)
    ul
```

Extract the input and output measurements.
u = z1.u;
$y=z 1 . y ;$
Split the data into two sets, with one half for estimation and one half for validation. The original data set has 300 samples, so each new data set has 150 samples.
u_est = u(1:150);
$u_{-}^{-}$val $=u(151: 300)$;
y_est = y(1:150);
y_val = y(151:300);

## Import Data into Task

In the Select data section, set Data Type to Time. Set the sample time to 0.1 seconds, which is the sample time in the original iddata object $z 1$. Select the appropriate data sets for estimation and validation.

## Estimate Process Model

sys = Estimated continuous process model for input u_est and output y_est with One Pole


## Estimate and Validate the Model

The example "Estimate Process Model with Live Editor Task" on page 1-288 achieves the best results using the model structure Underdamped Pair. Choose the same option for this example.


Execute the task from the Live Editor tab using Run. Executing the task creates two plots.The first plot shows the estimation results and the second plot shows the validation results.

## Estimation




The fit to the estimation data is somewhat worse than in "Estimate Process Model with Live Editor Task" on page 1-288. Estimation in the current example has only half the data with which to estimate the model. The fit to the validation data, which represents the goodness of the model more generally, is better than the fit to the estimation data.

## Parameters

## Select Data

Data Type - Data type for input and output data
Time (default)|Frequency | Data Object
The task accepts single-channel numeric measurement values that are uniformly sampled in time. Data can be packaged as numeric arrays (Time or Frequency type) or in a data object, such as an iddata or idfrd object.

The data type you choose determines the additional parameters you must specify.

- Time - Specify Sample Time and Start Time in the time units that you select.
- Frequency - Specify Frequency by selecting the variable name of a frequency vector in your MATLAB workspace. Specify the units for this frequency vector. Specify Sample Time in seconds.
- Data Object - Specify no additional parameters, because the data object already contains information on time or frequency sampling.


## Estimation Input and Estimation Output - Variable names of input and output data for estimation <br> valid variable names

Select the input and output variable names from the MATLAB workspace choices. Use these parameters when Data Type is Time or Frequency.

## Estimation Object - Variable name of data object containing input and output data to be used for estimation <br> valid variable name

Select the data object variable name from the MATLAB workspace choices. Use this parameter when Data Type is Data Object.

Validation Input (u) and Validation Output (y) - Variable names of input and output data to be used for validation
valid variable names
Select the input and output variable names, or the data object name, from the workspace choices. Use these parameters when Data Type is Time or Frequency. Specifying validation data is optional but recommended.

## Validation Object - Variable name of data object containing input and output data for validation <br> valid variable name

Select the data object variable name from the MATLAB workspace choices. Use this parameter when Data Type is Data Object. Specifying validation data is optional but recommended.

## Specify Model Structure

## Structure - Zeros and poles in model

One Pole (default)|Two Real Poles|Underdamped Pair|Underdamped Pair + Real Pole
The task allows you to specify one of four basic structures. These structures range from a simple firstorder process to a more dynamic second-order or third-order process with complex conjugate (underdamped) poles.

- One Pole
- Two Real Poles
- Underdamped Pair
- Underdamped Pair + Real Pole


## Delay - Include transport delay

off (default) | on
Include transport delay, or input-to-output delay, of one sample. The transport delay is also known as dead time.

## Zero - Include process zero

off (default) | on
Include a process zero in the numerator.

## Integrator - Include integrator

off (default) | on
Include an integrator, represented by an additional $1 / s$ term. Including an integrator creates a selfregulating process.

## Specify Estimation Initialization

## Initial Values - Initial values of structure parameters <br> 0 | parameter values

Specify initial values for the estimation and whether these values are to be fixed or estimated. The values to specify depend on the model structure and your specifications for Delay and Zero. Below Specify model structure, the task displays the equation that represents the specified system. This equation contains all of the parameters that can be estimated, and that you can initialize or fix. The possible parameters are:

- Kp - Static gain
- $T_{p 1}$ - Time constant for first real pole
- $T_{p 2}$ - Time constant for second real pole
- T $\omega$ - Time constant for complex poles, equal to the inverse of the natural frequency
- $\zeta$ - Damping coefficient for complex poles
- $T_{d}$ - Transport delay
- $T_{z}$ - Time constant for the process zero

All time-based parameters are in the time units you select for Sample Time.

## Specify Optional Parameters

## Fit Focus - Minimize prediction error or simulation error Prediction (default) | Simulation

Fit focus specifies what error to minimize in the loss function during estimation.

- Prediction - Minimize the one-step-ahead prediction error between measured and predicted outputs. This estimation approach focuses on producing a good predictor model for the estimation inputs and outputs. Prediction focus generally produces the best estimation results because it uses both input and output measurements, thus accounting for disturbances.
- Simulation - Minimize the error between measured and simulation outputs. This estimation approach focuses on producing a simulated model response that has a good fit with the estimation inputs and outputs. Simulation focus is generally best for validation, especially with data sets not used for the original estimation.


## Initial Conditions - Handling of initial conditions

Auto (default) | Zero|Estimate | Backcast
Set this option when you want to choose a specific method for initializing the model. With the default setting of Auto, the software chooses the method based on the estimation data. Choices are:

- Zero - The initial state is set to zero.
- Estimate - The initial state is treated as an independent estimation parameter.
- Backcast - The initial state is estimated using the best least squares fit.


## Input Intersampling - Intersampling behavior for input signal <br> Zero-order hold (default)|Triangle approximation | Band-limited

Input intersampling is a property of the input data. The task uses this property when estimating process models. Specify Input Intersampling when your data type is Time or Frequency. If you are using an iddata object, the object already contains the intersampling information. Choices for this property are:

- Zero-order hold - Piecewise-constant input signal between samples
- Triangle approximation - Piecewise-linear input signal between samples, also known as first-order hold
- Band-limited - Input signal has zero power above the Nyquist frequency


## Search Method - Numerical search mode for iterative parameter estimation

Auto (default) | Gauss-Newton | Adaptive Gauss-Newton | Levenberg-Marquardt | Gradient Search

- Auto - For each iteration, the software cycles through the methods until it finds the first direction descent that leads to a reduction in estimation cost.
- Gauss-Newton - Subspace Gauss-Newton least-squares search.
- Levenberg-Marquardt - Levenberg-Marquardt least-squares search.
- Adaptive Gauss-Newton -Adaptive subspace Gauss-Newton search.
- Gradient Search - Steepest descent least-squares search.

Max. Iterations - Maximum number of iterations during error minimization 20 (default) | positive integer

Set the maximum number of iterations during error minimization. The iterations stop when Max. Iterations is reached or another stopping criterion is satisfied, such as Tolerance.

## Tolerance - Minimum percentage of expected improvement in error

### 0.01 (default) | positive integer

When the percentage of expected improvement is less than Tolerance, the iterations stop.

## Weighting Prefilter - Weighting prefilter for loss function

No filter (default) | Passband(s) | LTI Filter | Frequency weights vector
Set this option when you want to apply a weighting prefilter to the loss function that the task minimizes when you estimate the model. When you select an option, you must also select the associated variable in your workspace that contains the filter information. The available options depend on the domain of the data.

| Weighting Prefilter | Data Domain | Filter Information |
| :--- | :--- | :--- |
| No Filter | Time and frequency |  |
| Passbands | Time and frequency | Passband ranges, specified as a <br> 1 -by-2 row vector or an $n$-by-2 <br> matrix, where $n$ is the number <br> of passbands |
| LTI Filter | Time and frequency | SISO LTI model |
| Frequency Weights Vector | Frequency | Frequency weights, specified as <br> a column vector with the same <br> length as the frequency vector |

For instance, suppose that you are performing estimation with SISO frequency-domain data and that in your MATLAB workspace, you have a column vector $W$ that contains frequency weights for the prefilter. In the task, select Weighting prefilter > Frequency weights vector and the variable W.

## Visualize Results

## Output Plot - Plot comparison of model and measured outputs

on (default) | off
Plot a comparison of the model output and the original measured data, along with the fit percentage. If you have separate validation data, a second plot compares the model response to the validation input data with the measured output from the validation data set.

See Also<br>compare|frd|iddata|idfrd|idproc|procest|procestOptions<br>\section*{Introduced in R2019b}

## Estimate State-Space Model

Estimate state-space model using time or frequency data in the Live Editor

## Description

The Estimate State-Space Model task lets you interactively estimate and validate a state-space model using time or frequency data. You can define and vary the model structure and specify optional parameters, such as initial condition handling and search method. The task automatically generates MATLAB code for your live script. For more information about Live Editor tasks generally, see "Add Interactive Tasks to a Live Script".

State-space models are models that use state variables to describe a system by a set of first-order differential or difference equations, rather than by one or more $n$ th-order differential or difference equations. State variables can be reconstructed from the measured input-output data, but are not themselves measured during the experiment.

The state-space model structure is a good choice for quick estimation because it requires you to specify only one input, the model order. For more information about state-space estimation, see "What Are State-Space Models?"

The Estimate State-Space Model task is independent of the more general System Identification app. Use the System Identification app when you want to compute and compare estimates for multiple model structures.

To get started, load experiment data that contains input and output data into your MATLAB workspace and then import that data into the task. Then specify a model structure to estimate. The task gives you controls and plots that help you experiment with different model parameters and compare how well the output of each model fits the measurements.

## Related Functions

The code that Estimate State-Space Model generates uses the following functions.

- iddata
- idfrd
frd
- ssest
- ssestOptions
- compare

The task estimates an idss state-space model.

## Description (Collapsed Portion)

## Related Functions

The code that Estimate State-Space Model generates uses the following functions.

- iddata
- idfrd
frd
- ssest
- ssestOptions
- compare

The task estimates an idss state-space model.

## Open the Task

To add the Estimate State-Space Model task to a live script in the MATLAB Editor:

- On the Live Editor tab, select Task > Estimate State-Space Model.
- In a code block in your script, type a relevant keyword, such as state, space, or estimate. Select Estimate State Space Model from the suggested command completions.


## Examples

## Estimate State-Space Model with Live Editor Task

Use the Estimate State-Space Model Live Editor Task to estimate a state-space model and compare the model output to the measurement data.

Open this example to see a preconfigured script containing the task.

## Set Up Data

Load the measurement data iddatal into your MATLAB workspace.

```
load iddatal z1
z1
z1 =
Time domain data set with 300 samples.
Sample time: 0.1 seconds
Outputs Unit (if specified)
    y1
Inputs Unit (if specified)
    u1
```


## Import Data into the Task

In the Select data section, set Data Type to Data Object and set Estimation Object to z1.
v
Estimate State-Space Model
sys = Estimated continuous state-space model for z1 with plant order 4

- Select data
 not have to specify them.


## Estimate the Model Using Default Settings

Examine the model structure and optional parameters.

- Specify model structure


In the Specify model structure section, the plant order is set to its default value of 4 and the model is in the continuous-time domain. Equations below the parameters in this section display the specified structure.

In the Specify optional parameters section, parameters display the default options for state-space estimation.

Execute the task from the Live Editor tab using Run. A plot displays the estimation data, the estimated model output, and the fit percentage.


## Experiment with Parameter Settings

Experiment with the parameter settings and see how they influence the fit.
For instance, in Specify model structure, the Estimate disturbance box is selected, so the disturbance matrix $\mathbf{K}$ is present in the equations. If you clear the box, the $\mathbf{K}$ term disappears. Run the updated configuration, and see how the fit changes.



Change the Plant Order setting to Pick best value in range. The default setting is 1:10.


When you run the model, a Model Order Selection plot displays the contribution of each state to the model dynamic behavior. With the initial task settings for the other parameters, the plot displays a recommendation of 2 for the model order.


Accept this recommendation by clicking Apply, and see how this change affects the fit.


## Generate Code

To display the code that the task generates, click $\boldsymbol{v}$ at the bottom of the parameter section. The code that you see reflects the current parameter configuration of the task.


## Use Validation Data Set to Validate Estimated Model

Use separate estimation and validation data so that you can validate the estimated state-space model. Open this example to see a preconfigured script containing the task.

## Set Up Data

Load measurement data iddatal into your MATLAB workspace and examine its contents.

```
load iddata1 z1
z1
z1 =
Time domain data set with 300 samples.
Sample time: 0.1 seconds
Outputs Unit (if specified)
    y1
Inputs Unit (if specified)
    u1
```

Extract the input and output measurements.

```
u = z1.u;
y = z1.y;
```

Split the data into two sets, with one half for estimation and one half for validation. The original data set has 300 samples, so each new data set has 150 samples.

```
u_est = u(1:150);
u_val = u(151:300);
y_est = y(1:150);
y_val = y(151:300);
```


## Import Data into Task

In the Select data section, set Data Type to Time. Set Sample Time to 0.1 seconds, which is the sample time in the original iddata object zl . Select the appropriate data sets for estimation and validation.


## Estimate and Validate Model

The example "Estimate State-Space Model with Live Editor Task" on page 1-302 recommends a model order of 2. Use that value for Plant Order. Leave other parameters at their default values. Note that Input Channel refers not to the input data set, but to the channel index within the input data set, which for a single-input system is always u1.


Execute the task from the Live Editor tab using Run. Executing the task creates two plots. The first plot shows the estimation results and the second plot shows the validation results.



The fit to the estimation data is somewhat worse than in "Estimate State-Space Model with Live Editor Task" on page 1-302. Estimation in the current example has only half the data with which to estimate the model. The fit to the validation data, which represents the goodness of the model more generally, is better than the fit to the estimation data.

## Parameters

## Select Data

## Data Type - Data type for input and output data

Time (default)|Frequency|Data Object
The task accepts numeric measurement values that are uniformly sampled in time. Input and output signals can contain multiple channels. Data can be packaged as numeric arrays (for Time or Frequency) or in a data object, such as an iddata or idfrd object.

The data type you choose determines whether you must specify additional parameters.

- Time - Specify Sample Time and Start Time in the time units that you select.
- Frequency - Specify Frequency by selecting the variable name of a frequency vector in your MATLAB workspace. Specify the units for this frequency vector. Specify Sample Time in seconds.
- Data Object - Specify no additional parameters because the data object already contains information on time or frequency sampling.


## Estimation Input (u) and Estimation Output (y) - Variable names of input and output data for estimation

valid variable names
Select the input and output variable names from the MATLAB workspace choices. Use these parameters when Data Type is Time or Frequency.

## Estimation Object - Variable name of data object containing input and output data for estimation <br> valid variable name

Select the data object variable name from the MATLAB workspace choices. Use this parameter when Data Type is Data Object.

Validation Input (u) and Validation Output (y) - Variable names of input and output data for validation
valid variable names
Select the input and output variable names from the workspace choices. Use these parameters when Data Type is Time or Frequency. Specifying validation data is optional but recommended.

Validation Object - Variable name of data object containing input and output data for validation
valid variable name
Select the data object variable name from the MATLAB workspace choices. Use this parameter when Data Type is Data Object. Specifying validation data is optional but recommended.

## Specify Model Structure

## Plant Order - Order of model to estimate <br> 4 (default) | integer scalar | integer range

The task allows you to specify a single value or a range of values for the order of the model to estimate.

- Specify value - Specify the order of the model explicitly.
- Pick best value in range - Specify a range of values, such as $1: 10$. When you run the task, the Hankel singular-value plot visualizes the relative energy contribution of each state in the estimated model and recommends the lowest order that reproduces critical dynamic behavior. Proceed with this recommendation or select another order in Chosen Order. Click Apply to accept the model order and proceed.


## Time Domain - Continuous or discrete time domain

Continuous (default) | Discrete
Select a continuous-time or discrete-time model.

## Estimate Disturbance - Include disturbance in estimation model off (default) | on

Select this option to estimate the disturbance model. When you select this option, the model equations update to show the $K$ matrix and $e$ term.

## Input Channel - Set input channel delay and feedthrough options u1 (default) \| u2 \| ...

For each input channel, assign values for Input Delay and Feedthrough.

- Input Channel - Select an input channel. The input channel is always of the form ui, where $i$ is the ith channel of the input $u$.
- Input Delay - Enter the input delay in number of samples (discrete-time model) or number of time units (continuous-time model) for the channel. For instance, to specify a 0.2 -second input delay for a continuous-time system for which the time unit is milliseconds, enter 200.
- Feedthrough - Select this option to estimate channel feedthrough from input to output. When you select this option, the model equations update to show the $D u$ term.


## Specify Optional Parameters

## Fit Focus - Minimize prediction error or simulation error <br> Prediction (default) | Simulation

Fit focus specifies what error to minimize in the loss function during estimation.

- Prediction - Minimize the one-step-ahead prediction error between measured and predicted outputs. This estimation approach focuses on producing a good predictor model for the estimation inputs and outputs. Prediction focus generally produces the best estimation results because it uses both input and output measurements, thus accounting for disturbances.
- Simulation - Minimize the error between measured and simulated outputs. This estimation approach focuses on producing a simulated model response that has a good fit with the estimation inputs and outputs. Simulation focus is generally best for validation, especially with data sets not used for the original estimation.


## Initial Conditions - Handling of initial states <br> Auto (default)|Zero|Estimate | Backcast

Set this option when you want to choose a specific method for initializing the model states. With the default setting of Auto, the software chooses the method based on the estimation data. Choices are:

- Zero - The initial state is set to zero.
- Estimate - The initial state is treated as an independent estimation parameter.
- Backcast - The initial state is estimated using the best least-squares fit.


## Input Intersampling - Intersampling behavior for input signal

Zero-order hold (default)|Triangle approximation|Band-limited
Input intersampling is a property of the input data. The task uses this property when estimating continuous models. Specify Input Intersampling when your data type is Time or Frequency. If you are using an iddata object, the object already contains the intersampling information. Choices for this property are:

- Zero-order hold - Piecewise-constant input signal between samples
- Triangle approximation - Piecewise-linear input signal between samples, also known as first-order hold
- Band-limited - Input signal has zero power above the Nyquist frequency


## Search Method - Numerical search mode for iterative parameter estimation

Auto (default)|Gauss-Newton | Adaptive Gauss-Newton | Levenberg-Marquardt | Gradient Search

- Auto - For each iteration, the software cycles through the methods until it finds the first direction descent that leads to a reduction in estimation cost.
- Gauss-Newton - Subspace Gauss-Newton least-squares search.
- Levenberg-Marquardt - Levenberg-Marquardt least-squares search.
- Adaptive Gauss-Newton -Adaptive subspace Gauss-Newton search.
- Gradient Search - Steepest descent least-squares search.


## Max. Iterations - Maximum number of iterations during error minimization <br> 20 (default) | positive integer

Set the maximum number of iterations during error minimization. The iterations stop when Max. Iterations is reached or another stopping criterion is satisfied, such as Tolerance.

## Tolerance - Minimum percentage of expected improvement in error <br> 0.01 (default) | positive integer

When the percentage of expected improvement is less than Tolerance, the iterations stop.

## Weighting Prefilter - Weighting prefilter for loss function

No filter (default)|Passband(s)|LTI Filter|Frequency weights vector|Inverse of magnitude of the frequency response|Inverse of square root of magnitude of the frequency response

Set this option when you want to apply a weighting prefilter to the loss function that the task minimizes when you estimate the model. When you select an option, you must also select the
associated variable in your workspace that contains the filter information. The available options depend on the domain of the data.

| Weighting Prefilter | Data Domain | Filter Information |
| :--- | :--- | :--- |
| No Filter | Time and frequency |  |
| Passbands | Time and frequency | Passband ranges, specified as a <br> 1 -by-2 row vector or an $n$-by-2 <br> matrix, where $n$ is the number <br> of passbands. |
| LTI Filter | Time and frequency | SISO LTI model. |
| Frequency Weights Vector | Frequency | Frequency weights, specified as <br> a column vector with the same <br> length as the frequency vector. |
| Inverse of magnitude of <br> the frequency response | Frequency response | The weighting filter is1/\|G( $\omega) \mid$, <br> where $G(\omega)$ is the complex <br> frequency-response data. SISO <br> and SIMO systems only. |
| Inverse of square root <br> of magnitude of the <br> frequency response | Frequency response | The weighting filter is <br> $1 / \sqrt{\|G(\omega)\| . ~ S I S O ~ a n d ~ S I M O ~}$ <br> systems only. |

For instance, suppose that you are performing estimation with SISO frequency-domain data and that in your MATLAB workspace, you have a column vector $W$ that contains frequency weights for the prefilter. In the task, select Weighting prefilter > Frequency weights vector and the variable W.

## Visualize Results

## Output Plot - Plot comparison of model and measured outputs <br> on (default) | off

Plot a comparison of the model output and the original measured data, along with the fit percentage. If you have separate validation data, a second plot compares the model response to the validation input data with the measured output from the validation data set.

## See Also

compare|frd|iddata|idfrd|idss|ssest|ssestOptions

## Topics

"What Are State-Space Models?"

## Introduced in R2019b

## etfe

Estimate empirical transfer functions and periodograms

## Syntax

g = etfe(data)
$\mathrm{g}=\mathrm{etfe}(\mathrm{data}, \mathrm{M})$
$\mathrm{g}=\mathrm{etfe}(\mathrm{data}, \mathrm{M}, \mathrm{N})$

## Description

$\mathrm{g}=\mathrm{etfe}($ data) estimates a transfer function of the form:
$y(t)=G(q) u(t)+v(t)$
data contains time- or frequency-domain input-output data or time-series data:

- If data is time-domain input-output signals, g is the ratio of the output Fourier transform to the input Fourier transform for the data.

For nonperiodic data, the transfer function is estimated at 128 equally-spaced frequencies [1:128]/128*pi/Ts.

For periodic data that contains a whole number of periods (data.Period = integer), the response is computed at the frequencies $\mathrm{k}^{*} 2 * \mathrm{pi} /$ period for $\mathrm{k}=0$ up to the Nyquist frequency.

- If data is frequency-domain input-output signals, g is the ratio of output to input at all frequencies, where the input is nonzero.
- If data is time-series data (no input channels), g is the periodogram, that is the normed absolute square of the Fourier transform, of the data. The corresponding spectral estimate is normalized, as described in "Spectrum Normalization" and differs from the spect rum normalization in the Signal Processing Toolbox ${ }^{\text {TM }}$ product.
$\mathrm{g}=\mathrm{etfe}($ data, M$)$ applies a smoothing operation on the raw spectral estimates using a Hamming Window that yields a frequency resolution of about pi/M. The effect of $M$ is similar to the effect of $M$ in spa. M is ignored for periodic data. Use this syntax as an alternative to spa for narrowband spectra and systems that require large values of M .
$\mathrm{g}=\mathrm{etfe}($ data $, \mathrm{M}, \mathrm{N})$ specifies the frequency spacing for nonperiodic data.
- For nonperiodic time-domain data, N specifies the frequency grid [1:N]/N*pi/Ts rad/TimeUnit. When not specified, N is 128 .
- For periodic time-domain data, N is ignored.
- For frequency-domain data, the $N$ is fmin:delta_f:fmax, where [fmin fmax] is the range of frequencies in data, and delta_f is ( $f$ max- fmin ) / $(\mathrm{N}-1) \mathrm{rad} /$ TimeUnit. When not specified, the response is computed at the frequencies contained in data where input is nonzero.


## Examples

## Compare an Empirical Transfer Function to a Smoothed Spectral Estimate

Load estimation data.
load iddatal z1;
Estimate empirical transfer function and smoothed spectral estimate.
ge = etfe(z1);
gs = spa(z1);
Compare the two models on a Bode plot.
bode(ge,gs)


## Generate Empirical Transfer Function Using Periodic Input

Generate a periodic input, simulate a system with it, and compare the frequency response of the estimated model with the original system at the excited frequency points.

Generate a periodic input signal and output signal using simulation.
m = idpoly([1-1.5 0.7],[0 1 0.5]);
u = iddata([],idinput([50,1,10],'sine'));
u.Period = 50;
$y=\operatorname{sim}(m, u)$;

Estimate an empirical transfer function.
me = etfe([y u]);
Compare the empirical transfer function with the original model.
bode(me,'b*',m,'r')


## Apply Smoothing Operation on Empirical Transfer Function Estimate

Perform a smoothing operation on raw spectral estimates using a Hamming Window and compare the responses.

Load data.
load iddata1
Estimate empirical transfer functions with and without the smoothing operation.
ge1 = etfe(z1);
ge2 $=$ etfe(z1,32);
Compare the models on a Bode plot.
ge 2 is smoother than ge 1 because of the effect of the smoothing operation.
bode(ge1,ge2)


## Compare Effect of Frequency Spacing on Empirical Transfer Function Estimate

Estimate empirical transfer functions with low- and high-frequency spacings and compare the responses.

Load data.
load iddata9
Estimate empirical transfer functions with low and high frequency spacings.
gel $=$ etfe(z9,[],32);
ge2 $=$ etfe(z9,[],512);
Plot the output power spectrum of the two models.

```
spectrum(ge1,'b.-',ge2,'g')
```



## Input Arguments

## data - Estimation data

iddata
Estimation data, specified as an iddata object. The data can be time- or frequency-domain input/ output signals or time-series data.

## M - Frequency resolution

[] (default) | positive scalar
Frequency resolution, specified as a positive scalar.

## N - Frequency spacing

128 for nonperiodic time-domain data (default) | positive scalar
Frequency spacing, specified as a positive scalar. For frequency-domain data, the default frequency spacing is the spacing inherent in the estimation data.

## Output Arguments

## g - Transfer function estimate idfrd

Transfer function estimate, returned as an idfrd model.
Information about the estimation results and options used is stored in the model's Report property. Report has the following fields:

| Report Field | Description |  |
| :---: | :---: | :---: |
| Status | Summary of the model status, which indicates whether the model was created by construction or obtained by estimation. |  |
| Method | Estimation command used. |  |
| WindowSize | Size of the Hamming window. |  |
| DataUsed | Attributes of the data used for estimation, returned as a structure with the following fields: |  |
|  | Field | Description |
|  | Name | Name of the data set. |
|  | Type | Data type. |
|  | Length | Number of data samples. |
|  | Ts | Sample time. |
|  | InterSa mple | Input intersample behavior, returned as one of the following values: <br> - 'zoh' - Zero-order hold maintains a piecewise-constant input signal between samples. <br> - ' foh ' - First-order hold maintains a piecewise-linear input signal between samples. <br> - 'bl' - Band-limited behavior specifies that the continuous-time input signal has zero power above the Nyquist frequency. |
|  | Input0f fset | Offset removed from time-domain input data during estimation. For nonlinear models, it is [ ]. |
|  | Output0 ffset | Offset removed from time-domain output data during estimation. For nonlinear models, it is [ ]. |

For more information on using Report, see "Estimation Report".

## See Also

bode | freqresp|idfrd|impulseest|nyquist|spa|spafdr|spectrum

## Topics

"Estimate Frequency-Response Models at the Command Line"
"What is a Frequency-Response Model?"

## Introduced before R2006a

## evalfr

Evaluate frequency response at given frequency

## Syntax

frsp = evalfr(sys,f)

## Description

frsp = evalfr(sys,f) evaluates the transfer function of the TF, SS, or ZPK model sys at the complex number f . For state-space models with data $(A, B, C, D)$, the result is
$H(f)=D+C(f I-A)^{-1} B$
eval $f r$ is a simplified version of freqresp meant for quick evaluation of the response at a single point. Use freqresp to compute the frequency response over a set of frequencies.

## Examples

## Evaluate Discrete-Time Transfer Function

Create the following discrete-time transfer function.

$$
H(z)=\frac{z-1}{z^{2}+z+1}
$$

H = tf([1-1],[11 1],-1);
Evaluate the transfer function at $z=1+j$.

```
z = 1+j;
evalfr(H,z)
ans = 0.2308 + 0.1538i
```


## Evaluate Frequency Response of Identified Model at Given Frequency

Create the following continuous-time transfer function model:

$$
H(s)=\frac{1}{s^{2}+2 s+1}
$$

sys = idtf(1,[12 1]);
Evaluate the transfer function at frequency $0.1 \mathrm{rad} /$ second.

```
w = 0.1;
s = j*w;
evalfr(sys,s)
ans = 0.9705 - 0.1961i
```

Alternatively, use the freqresp command.
freqresp(sys,w)
ans $=0.9705-0.1961 i$

## Limitations

The response is not finite when $f$ is a pole of sys.

## See Also

bode | freqresp | sigma
Introduced in R2012a

## extendedKalmanFilter

Create extended Kalman filter object for online state estimation

## Syntax

```
obj = extendedKalmanFilter(StateTransitionFcn,MeasurementFcn,InitialState)
obj = extendedKalmanFilter(StateTransitionFcn,MeasurementFcn,InitialState,
Name,Value)
obj = extendedKalmanFilter(StateTransitionFcn,MeasurementFcn)
obj = extendedKalmanFilter(StateTransitionFcn,MeasurementFcn,Name,Value)
obj = extendedKalmanFilter(Name,Value)
```


## Description

obj = extendedKalmanFilter(StateTransitionFcn,MeasurementFcn,InitialState) creates an extended Kalman filter object for online state estimation of a discrete-time nonlinear system. StateTransitionFcn is a function that calculates the state of the system at time $k$, given the state vector at time $k-1$. MeasurementFcn is a function that calculates the output measurement of the system at time $k$, given the state at time $k$. InitialState specifies the initial value of the state estimates.

After creating the object, use the correct and predict commands to update state estimates and state estimation error covariance values using a first-order discrete-time extended Kalman filter algorithm and real-time data.
obj = extendedKalmanFilter(StateTransitionFcn,MeasurementFcn,InitialState, Name, Value) specifies additional attributes of the extended Kalman filter object using one or more Name, Value pair arguments.
obj = extendedKalmanFilter(StateTransitionFcn, MeasurementFcn) creates an extended Kalman filter object using the specified state transition and measurement functions. Before using the predict and correct commands, specify the initial state values using dot notation. For example, for a two-state system with initial state values $[1 ; 0]$, specify obj. State $=[1 ; 0]$.
obj = extendedKalmanFilter(StateTransitionFcn,MeasurementFcn,Name,Value) specifies additional attributes of the extended Kalman filter object using one or more Name, Value pair arguments. Before using the predict and correct commands, specify the initial state values using Name, Value pair arguments or dot notation.
obj = extendedKalmanFilter(Name,Value) creates an extended Kalman filter object with properties specified using one or more Name, Value pair arguments. Before using the predict and correct commands, specify the state transition function, measurement function, and initial state values using Name, Value pair arguments or dot notation.

## Object Description

extendedKalmanFilter creates an object for online state estimation of a discrete-time nonlinear system using the first-order discrete-time extended Kalman filter algorithm.

Consider a plant with states $x$, input $u$, output $y$, process noise $w$, and measurement noise $v$. Assume that you can represent the plant as a nonlinear system.


The algorithm computes the state estimates $\widehat{x}$ of the nonlinear system using state transition and measurement functions specified by you. The software lets you specify the noise in these functions as additive or nonadditive:

- Additive Noise Terms - The state transition and measurements equations have the following form:

$$
\begin{aligned}
& x[k]=f\left(x[k-1], u_{s}[k-1]\right)+w[k-1] \\
& y[k]=h\left(x[k], u_{m}[k]\right)+v[k]
\end{aligned}
$$

Here $f$ is a nonlinear state transition function that describes the evolution of states x from one time step to the next. The nonlinear measurement function $h$ relates $x$ to the measurements $y$ at time step k. w and v are the zero-mean, uncorrelated process and measurement noises, respectively. These functions can also have additional input arguments that are denoted by $u_{s}$ and $u_{m}$ in the equations. For example, the additional arguments could be time step $k$ or the inputs $u$ to the nonlinear system. There can be multiple such arguments.

Note that the noise terms in both equations are additive. That is, $x(k)$ is linearly related to the process noise $w(k-1)$, and $y(k)$ is linearly related to the measurement noise $v(k)$.

- Nonadditive Noise Terms - The software also supports more complex state transition and measurement functions where the state $x[k]$ and measurement $y[k]$ are nonlinear functions of the process noise and measurement noise, respectively. When the noise terms are nonadditive, the state transition and measurements equation have the following form:

$$
\begin{aligned}
& x[k]=f\left(x[k-1], w[k-1], u_{s}[k-1]\right) \\
& y[k]=h\left(x[k], v[k], u_{m}[k]\right)
\end{aligned}
$$

When you perform online state estimation, you first create the nonlinear state transition function $f$ and measurement function $h$. You then construct the extendedKalmanFilter object using these nonlinear functions, and specify whether the noise terms are additive or nonadditive. You can also specify the Jacobians of the state transition and measurement functions. If you do not specify them, the software numerically computes the Jacobians.

After you create the object, you use the predict command to predict state estimate at the next time step, and correct to correct state estimates using the algorithm and real-time data. For information about the algorithm, see "Extended and Unscented Kalman Filter Algorithms for Online State Estimation".

You can use the following commands with extendedKalmanFilter objects:

| Command | Description |
| :--- | :--- |
| correct | Correct the state and state estimation error <br> covariance at time step $k$ using measured data at <br> time step $k$. |
| residual | Predict the state and state estimation error <br> covariance at time the next time step. |
| clone | Return the difference between the actual and <br> predicted measurements. |
| Create another object with the same object <br> property values. <br> Do not create additional objects using syntax <br> obj2 = obj. Any changes made to the <br> properties of the new object created in this way <br> (obj2) also change the properties of the original <br> object (obj). |  |

For extendedKalmanFilter object properties, see "Properties" on page 1-331.

## Examples

## Create Extended Kalman Filter Object for Online State Estimation

To define an extended Kalman filter object for estimating the states of your system, you first write and save the state transition function and measurement function for the system.

In this example, use the previously written and saved state transition and measurement functions, vdpStateFcn.m and vdpMeasurementFcn.m. These functions describe a discrete-approximation to a van der Pol oscillator with nonlinearity parameter, mu, equal to 1 . The oscillator has two states.

Specify an initial guess for the two states. You specify the guess as an M-element row or column vector, where M is the number of states.
initialStateGuess = [1;0];
Create the extended Kalman filter object. Use function handles to provide the state transition and measurement functions to the object.
obj = extendedKalmanFilter(@vdpStateFcn,@vdpMeasurementFcn,initialStateGuess);
The object has a default structure where the process and measurement noise are additive.
To estimate the states and state estimation error covariance from the constructed object, use the correct and predict commands and real-time data.

## Specify Process and Measurement Noise Covariances in Extended Kalman Filter Object

Create an extended Kalman filter object for a van der Pol oscillator with two states and one output. Use the previously written and saved state transition and measurement functions, vdpStateFcn.m
and $v d p M e a s u r e m e n t F c n . m$. These functions are written for additive process and measurement noise terms. Specify the initial state values for the two states as [2;0].

Since the system has two states and the process noise is additive, the process noise is a 2 -element vector and the process noise covariance is a 2-by-2 matrix. Assume there is no cross-correlation between process noise terms, and both the terms have the same variance 0.01 . You can specify the process noise covariance as a scalar. The software uses the scalar value to create a 2 -by- 2 diagonal matrix with 0.01 on the diagonals.

Specify the process noise covariance during object construction.
obj = extendedKalmanFilter(@vdpStateFcn,@vdpMeasurementFcn,[2;0],...
'ProcessNoise',0.01);
Alternatively, you can specify noise covariances after object construction using dot notation. For example, specify the measurement noise covariance as 0.2 .
obj. MeasurementNoise $=0.2$;
Since the system has only one output, the measurement noise is a 1 -element vector and the MeasurementNoise property denotes the variance of the measurement noise.

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## Specify Jacobians for State and Measurement Functions

Create an extended Kalman filter object for a van der Pol oscillator with two states and one output. Use the previously written and saved state transition and measurement functions, vdpStateFcn.m and vdpMeasurementFcn.m. Specify the initial state values for the two states as [2;0].

```
obj = extendedKalmanFilter(@vdpStateFcn,@vdpMeasurementFcn,[2;0]);
```

The extended Kalman filter algorithm uses Jacobians of the state transition and measurement functions for state estimation. You write and save the Jacobian functions and provide them as function handles to the object. In this example, use the previously written and saved functions vdpStateJacobianFcn.m and vdpMeasurementJacobianFcn.m.
obj.StateTransitionJacobianFcn = @vdpStateJacobianFcn;
obj.MeasurementJacobianFcn = @vdpMeasurementJacobianFcn;
Note that if you do not specify the Jacobians of the functions, the software numerically computes the Jacobians. This numerical computation may result in increased processing time and numerical inaccuracy of the state estimation.

## Specify Nonadditive Measurement Noise in Extended Kalman Filter Object

Create an extended Kalman filter object for a van der Pol oscillator with two states and one output. Assume that the process noise terms in the state transition function are additive. That is, there is a linear relation between the state and process noise. Also assume that the measurement noise terms are nonadditive. That is, there is a nonlinear relation between the measurement and measurement noise.

```
obj = extendedKalmanFilter('HasAdditiveMeasurementNoise',false);
```

Specify the state transition function and measurement functions. Use the previously written and saved functions, vdpStateFcn.m and vdpMeasurementNonAdditiveNoiseFcn.m.

The state transition function is written assuming the process noise is additive. The measurement function is written assuming the measurement noise is nonadditive.

```
obj.StateTransitionFcn = @vdpStateFcn;
obj.MeasurementFcn = @vdpMeasurementNonAdditiveNoiseFcn;
```

Specify the initial state values for the two states as [2;0].

```
obj.State = [2;0];
```

You can now use the correct and predict commands to estimate the state and state estimation error covariance values from the constructed object.

## Specify State Transition and Measurement Functions with Additional Inputs

Consider a nonlinear system with input $u$ whose state $x$ and measurement $y$ evolve according to the following state transition and measurement equations:

$$
\begin{aligned}
& x[k]=\sqrt{x[k-1]+u[k-1]}+w[k-1] \\
& y[k]=x[k]+2 * u[k]+v[k]^{2}
\end{aligned}
$$

The process noise w of the system is additive while the measurement noise v is nonadditive.
Create the state transition function and measurement function for the system. Specify the functions with an additional input $u$.
$f=@(x, u)(\operatorname{sqrt}(x+u)) ;$
$h=@(x, v, u)\left(x+2 * u+v^{\wedge} 2\right)$;
f and h are function handles to the anonymous functions that store the state transition and measurement functions, respectively. In the measurement function, because the measurement noise is nonadditive, $v$ is also specified as an input. Note that $v$ is specified as an input before the additional input $u$.

Create an extended Kalman filter object for estimating the state of the nonlinear system using the specified functions. Specify the initial value of the state as 1 and the measurement noise as nonadditive.
obj = extendedKalmanFilter(f,h,1,'HasAdditiveMeasurementNoise',false);
Specify the measurement noise covariance.
obj. MeasurementNoise $=0.01$;
You can now estimate the state of the system using the predict and correct commands. You pass the values of $u$ to predict and correct, which in turn pass them to the state transition and measurement functions, respectively.

Correct the state estimate with measurement $\mathrm{y}[\mathrm{k}]=0.8$ and input $\mathrm{u}[\mathrm{k}]=0.2$ at time step k .
correct (obj, 0.8, 0.2)
Predict the state at the next time step, given $u[k]=0.2$.
predict(obj,0.2)
Retrieve the error, or residual, between the prediction and the measurement.
[Residual, ResidualCovariance] = residual(obj,0.8,0.2);

## Input Arguments

## StateTransitionFcn - State transition function

function handle
State transition function $f$, specified as a function handle. The function calculates the $N s$-element state vector of the system at time step $k$, given the state vector at time step $k-1$. Ns is the number of states of the nonlinear system.

You write and save the state transition function for your nonlinear system, and use it to construct the object. For example, if vdpStateFcn.m is the state transition function, specify
StateTransitionFcn as @vdpStateFcn. You can also specify StateTransitionFcn as a function handle to an anonymous function.

The inputs to the function you write depend on whether you specify the process noise as additive or nonadditive in the HasAdditiveProcessNoise property of the object:

- HasAdditiveProcessNoise is true - The process noise w is additive, and the state transition function specifies how the states evolve as a function of state values at the previous time step:
$x(k)=f(x(k-1), U s 1, \ldots, U s n)$
Where $x(k)$ is the estimated state at time $k$, and Us $1, \ldots$, Usn are any additional input arguments required by your state transition function, such as system inputs or the sample time. During estimation, you pass these additional arguments to the predict command, which in turn passes them to the state transition function.
- HasAdditiveProcessNoise is false - The process noise is nonadditive, and the state transition function also specifies how the states evolve as a function of the process noise:

$$
x(k)=f(x(k-1), w(k-1), U s 1, \ldots, U s n)
$$

To see an example of a state transition function with additive process noise, type edit vdpStateFcn at the command line.

## MeasurementFcn - Measurement function

function handle
Measurement function $h$, specified as a function handle. The function calculates the $N$-element output measurement vector of the nonlinear system at time step $k$, given the state vector at time step $k . N$ is the number of measurements of the system. You write and save the measurement function, and use it to construct the object. For example, if vdpMeasurementFcn.m is the measurement function, specify

MeasurementFcn as @vdpMeasurementFcn. You can also specify MeasurementFcn as a function handle to an anonymous function.

The inputs to the function depend on whether you specify the measurement noise as additive or nonadditive in the HasAdditiveMeasurementNoise property of the object:

- HasAdditiveMeasurementNoise is true - The measurement noise v is additive, and the measurement function specifies how the measurements evolve as a function of state values:
$y(k)=h(x(k), U m 1, \ldots, U m n)$
Where $y(k)$ and $x(k)$ are the estimated output and estimated state at time $k$, and Um1, ..., Umn are any optional input arguments required by your measurement function. For example, if you are using multiple sensors for tracking an object, an additional input could be the sensor position. During estimation, you pass these additional arguments to the correct command, which in turn passes them to the measurement function.
- HasAdditiveMeasurementNoise is false - The measurement noise is nonadditive, and the measurement function also specifies how the output measurement evolves as a function of the measurement noise:

$$
y(k)=h(x(k), v(k), U m 1, \ldots, U m n)
$$

To see an example of a measurement function with additive process noise, type edit vdpMeasurementFcn at the command line. To see an example of a measurement function with nonadditive process noise, type edit vdpMeasurementNonAdditiveNoiseFcn.

## InitialState - Initial state estimate value

vector
Initial state estimate value, specified as an $N s$-element vector, where $N s$ is the number of states in the system. Specify the initial state values based on your knowledge of the system.

The specified value is stored in the State property of the object. If you specify InitialState as a column vector, then State is also a column vector, and the predict and correct commands return state estimates as a column vector. Otherwise, a row vector is returned.

If you want a filter with single-precision floating-point variables, specify InitialState as a singleprecision vector variable. For example, for a two-state system with state transition and measurement functions vdpStateFcn.m and vdpMeasurementFcn.m, create the extended Kalman filter object with initial state estimates [1;2] as follows:
obj = extendedKalmanFilter(@vdpStateFcn,@vdpMeasurementFcn,single([1;2]))
Data Types: double | single

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Use Name, Value arguments to specify properties on page 1-331 of extendedKalmanFilter object during object creation. For example, to create an extended Kalman filter object and specify the process noise covariance as 0.01 :
obj = extendedKalmanFilter(StateTransitionFcn,MeasurementFcn,InitialState,'ProcessNoise',0.01);

## Properties

extendedKalmanFilter object properties are of three types:

- Tunable properties that you can specify multiple times, either during object construction using Name, Value arguments, or any time afterward during state estimation. After object creation, use dot notation to modify the tunable properties.

```
obj = extendedKalmanFilter(StateTransitionFcn,MeasurementFcn,InitialState);
obj.ProcessNoise = 0.01;
```

The tunable properties are State, StateCovariance, ProcessNoise, and MeasurementNoise.

- Nontunable properties that you can specify once, either during object construction or afterward using dot notion. Specify these properties before state estimation using correct and predict. The StateTransitionFcn, MeasurementFcn, StateTransitionJacobianFcn, and MeasurementJacobianFcn properties belong to this category.
- Nontunable properties that you must specify during object construction. The HasAdditiveProcessNoise and HasAdditiveMeasurementNoise properties belong to this category.


## HasAdditiveMeasurementNoise - Measurement noise characteristics

true (default) | false
Measurement noise characteristics, specified as one of the following values:

- true - Measurement noise $v$ is additive. The measurement function $h$ that is specified in MeasurementFcn has the following form:
$y(k)=h(x(k), U m 1, \ldots, U m n)$
Where $\mathrm{y}(\mathrm{k})$ and $\mathrm{x}(\mathrm{k})$ are the estimated output and estimated state at time k , and Um1, ..., Umn are any optional input arguments required by your measurement function.
- false - Measurement noise is nonadditive. The measurement function specifies how the output measurement evolves as a function of the state and measurement noise:

```
y(k) = h(x(k),v(k),Um1,\ldots..,Umn)
```

HasAdditiveMeasurementNoise is a nontunable property, and you can specify it only during object construction. You cannot change it using dot notation.

## HasAdditiveProcessNoise - Process noise characteristics

true (default) | false
Process noise characteristics, specified as one of the following values:

- true - Process noise w is additive. The state transition function $f$ specified in StateTransitionFcn has the following form:
$x(k)=f(x(k-1), U s 1, \ldots, U s n)$
Where $\mathrm{x}(\mathrm{k})$ is the estimated state at time k , and Us $1, \ldots$, Usn are any additional input arguments required by your state transition function.
- false - Process noise is nonadditive. The state transition function specifies how the states evolve as a function of the state and process noise at the previous time step:

```
x(k)=f(x(k-1),w(k-1),Us1,...,Usn)
```

HasAdditiveProcessNoise is a nontunable property, and you can specify it only during object construction. You cannot change it using dot notation.

## MeasurementFcn - Measurement function

function handle
Measurement function $h$, specified as a function handle. The function calculates the $N$-element output measurement vector of the nonlinear system at time step $k$, given the state vector at time step $k . N$ is the number of measurements of the system. You write and save the measurement function and use it to construct the object. For example, if vdpMeasurementFcn.m is the measurement function, specify MeasurementFcn as @vdpMeasurementFcn. You can also specify MeasurementFcn as a function handle to an anonymous function.

The inputs to the function depend on whether you specify the measurement noise as additive or nonadditive in the HasAdditiveMeasurementNoise property of the object:

- HasAdditiveMeasurementNoise is true - The measurement noise v is additive, and the measurement function specifies how the measurements evolve as a function of state values:
$y(k)=h(x(k), U m 1, \ldots, U m n)$
Where $y(k)$ and $x(k)$ are the estimated output and estimated state at time $k$, and Um1, ..., Umn are any optional input arguments required by your measurement function. For example, if you are using multiple sensors for tracking an object, an additional input could be the sensor position. During estimation, you pass these additional arguments to the correct command which in turn passes them to the measurement function.
- HasAdditiveMeasurementNoise is false - The measurement noise is nonadditive, and the measurement function also specifies how the output measurement evolves as a function of the measurement noise:

$$
y(k)=h(x(k), v(k), U m 1, \ldots, U m n)
$$

To see an example of a measurement function with additive process noise, type edit vdpMeasurementFcn at the command line. To see an example of a measurement function with nonadditive process noise, type edit vdpMeasurementNonAdditiveNoiseFcn.

MeasurementFcn is a nontunable property. You can specify it once before using the correct command either during object construction or using dot notation after object construction. You cannot change it after using the correct command.

## MeasurementJacobianFcn - Jacobian of measurement function

[ ] (default) | function handle
Jacobian of measurement function $h$, specified as one of the following:

- [ ] - The Jacobian is numerically computed at every call to the correct command. This may increase processing time and numerical inaccuracy of the state estimation.
- function handle - You write and save the Jacobian function and specify the handle to the function. For example, if vdpMeasurementJacobianFcn.m is the Jacobian function, specify MeasurementJacobianFcn as @vdpMeasurementJacobianFcn.

The function calculates the partial derivatives of the measurement function with respect to the states and measurement noise. The number of inputs to the Jacobian function must equal the
number of inputs to the measurement function and must be specified in the same order in both functions. The number of outputs of the Jacobian function depends on the HasAdditiveMeasurementNoise property:

- HasAdditiveMeasurementNoise is true - The function calculates the partial derivatives of the measurement function with respect to the states ( $\partial h / \partial x$ ). The output is as an $N$-by-Ns Jacobian matrix, where $N$ is the number of measurements of the system and $N s$ is the number of states.
- HasAdditiveMeasurementNoise is false - The function also returns a second output that is the partial derivative of the measurement function with respect to the measurement noise terms $(\partial h / \partial v)$. The second output is returned as an $N$-by- $V$ Jacobian matrix, where $V$ is the number of measurement noise terms.

To see an example of a Jacobian function for additive measurement noise, type edit vdpMeasurementJacobianFcn at the command line.

MeasurementJacobianFcn is a nontunable property. You can specify it once before using the correct command either during object construction or using dot notation after object construction. You cannot change it after using the correct command.

## MeasurementNoise - Measurement noise covariance

1 (default) | scalar | matrix
Measurement noise covariance, specified as a scalar or matrix depending on the value of the HasAdditiveMeasurementNoise property:

- HasAdditiveMeasurementNoise is true - Specify the covariance as a scalar or an $N$-by- $N$ matrix, where $N$ is the number of measurements of the system. Specify a scalar if there is no cross-correlation between measurement noise terms and all the terms have the same variance. The software uses the scalar value to create an $N$-by- $N$ diagonal matrix.
- HasAdditiveMeasurementNoise is false - Specify the covariance as a $V$-by- $V$ matrix, where $V$ is the number of measurement noise terms. MeasurementNoise must be specified before using correct. After you specify MeasurementNoise as a matrix for the first time, to then change MeasurementNoise you can also specify it as a scalar. Specify as a scalar if there is no crosscorrelation between the measurement noise terms and all the terms have the same variance. The software extends the scalar to a $V$-by- $V$ diagonal matrix with the scalar on the diagonals.

MeasurementNoise is a tunable property. You can change it using dot notation.

## ProcessNoise - Process noise covariance

1 (default) | scalar | matrix
Process noise covariance, specified as a scalar or matrix depending on the value of the HasAdditiveProcessNoise property:

- HasAdditiveProcessNoise is true - Specify the covariance as a scalar or an Ns-by-Ns matrix, where $N s$ is the number of states of the system. Specify a scalar if there is no cross-correlation between process noise terms, and all the terms have the same variance. The software uses the scalar value to create an Ns-by-Ns diagonal matrix.
- HasAdditiveProcessNoise is false - Specify the covariance as a $W$-by- $W$ matrix, where $W$ is the number of process noise terms. ProcessNoise must be specified before using predict. After you specify ProcessNoise as a matrix for the first time, to then change ProcessNoise you can also specify it as a scalar. Specify as a scalar if there is no cross-correlation between the
process noise terms and all the terms have the same variance. The software extends the scalar to a $W$-by- $W$ diagonal matrix.

ProcessNoise is a tunable property. You can change it using dot notation.

## State - State of nonlinear system

[] (default) | vector
State of the nonlinear system, specified as a vector of size $N s$, where $N s$ is the number of states of the system.

When you use the predict command, State is updated with the predicted value at time step $k$ using the state value at time step $k-1$. When you use the correct command, State is updated with the estimated value at time step $k$ using measured data at time step $k$.

The initial value of State is the value you specify in the InitialState input argument during object creation. If you specify InitialState as a column vector, then State is also a column vector, and the predict and correct commands return state estimates as a column vector. Otherwise, a row vector is returned. If you want a filter with single-precision floating-point variables, you must specify State as a single-precision variable during object construction using the InitialState input argument.

State is a tunable property. You can change it using dot notation.

## StateCovariance - State estimation error covariance <br> 1 (default) | scalar | matrix

State estimation error covariance, specified as a scalar or an Ns-by-Ns matrix, where Ns is the number of states of the system. If you specify a scalar, the software uses the scalar value to create an $N s$-by-Ns diagonal matrix.

Specify a high value for the covariance when you do not have confidence in the initial state values that you specify in the InitialState input argument.

When you use the predict command, StateCovariance is updated with the predicted value at time step $k$ using the state value at time step $k-1$. When you use the correct command, StateCovariance is updated with the estimated value at time step $k$ using measured data at time step $k$.

StateCovariance is a tunable property. You can change it using dot notation after using the correct or predict commands.

## StateTransitionFcn - State transition function

function handle
State transition function $f$, specified as a function handle. The function calculates the $N s$-element state vector of the system at time step $k$, given the state vector at time step $k-1$. Ns is the number of states of the nonlinear system.

You write and save the state transition function for your nonlinear system and use it to construct the object. For example, if vdpStateFcn.m is the state transition function, specify StateTransitionFcn as @vdpStateFcn. You can also specify StateTransitionFcn as a function handle to an anonymous function.

The inputs to the function you write depend on whether you specify the process noise as additive or nonadditive in the HasAdditiveProcessNoise property of the object:

- HasAdditiveProcessNoise is true - The process noise w is additive, and the state transition function specifies how the states evolve as a function of state values at previous time step:
$x(k)=f(x(k-1), U s 1, \ldots, U s n)$
Where $x(k)$ is the estimated state at time $k$, and Us $1, \ldots$, Usn are any additional input arguments required by your state transition function, such as system inputs or the sample time. During estimation, you pass these additional arguments to the predict command, which in turn passes them to the state transition function.
- HasAdditiveProcessNoise is false - The process noise is nonadditive, and the state transition function also specifies how the states evolve as a function of the process noise:

$$
x(k)=f(x(k-1), w(k-1), U s 1, \ldots, U s n)
$$

To see an example of a state transition function with additive process noise, type edit vdpStateFcn at the command line.

StateTransitionFcn is a nontunable property. You can specify it once before using the predict command either during object construction or using dot notation after object construction. You cannot change it after using the predict command.

## StateTransitionJacobianFcn - Jacobian of state transition function

[] (default) | function handle
Jacobian of state transition function $f$, specified as one of the following:

- [ ] - The Jacobian is numerically computed at every call to the predict command. This may increase processing time and numerical inaccuracy of the state estimation.
- function handle - You write and save the Jacobian function and specify the handle to the function. For example, if vdpStateJacobianFcn.m is the Jacobian function, specify StateTransitionJacobianFcn as @vdpStateJacobianFcn.

The function calculates the partial derivatives of the state transition function with respect to the states and process noise. The number of inputs to the Jacobian function must equal the number of inputs of the state transition function and must be specified in the same order in both functions. The number of outputs of the function depends on the HasAdditiveProcessNoise property:

- HasAdditiveProcessNoise is true - The function calculates the partial derivative of the state transition function with respect to the states ( $\partial f / \partial x$ ). The output is an Ns-by-Ns Jacobian matrix, where Ns is the number of states.
- HasAdditiveProcessNoise is false - The function must also return a second output that is the partial derivative of the state transition function with respect to the process noise terms $(\partial f / \partial w)$. The second output is returned as an $N s$-by- $W$ Jacobian matrix, where $W$ is the number of process noise terms.

The extended Kalman filter algorithm uses the Jacobian to compute the state estimation error covariance.

To see an example of a Jacobian function for additive process noise, type edit vdpStateJacobianFcn at the command line.

StateTransitionJacobianFcn is a nontunable property. You can specify it once before using the predict command either during object construction or using dot notation after object construction. You cannot change it after using the predict command.

## Output Arguments

## obj - Extended Kalman filter object for online state estimation <br> extendedKalmanFilter object

Extended Kalman filter object for online state estimation, returned as an extendedKalmanFilter object. This object is created using the specified properties on page 1-331. Use the correct and predict commands to estimate the state and state estimation error covariance using the extended Kalman filter algorithm.

When you use predict, obj. State and obj. StateCovariance are updated with the predicted value at time step $k$ using the state value at time step $k-1$. When you use correct, obj. State and obj. StateCovariance are updated with the estimated values at time step $k$ using measured data at time step $k$.

## Compatibility Considerations

## Numerical Changes

Behavior changed in R2020b
Starting in R2020b, numerical improvements in the extendedKalmanFilter algorithm might produce results that are different from the results you obtained in previous versions.

## Extended Capabilities

## C/C++ Code Generation

Generate C and $\mathrm{C}++$ code using MATLAB® $\mathrm{Coder}^{\mathrm{TM}}$.
Usage notes and limitations:
For more information, see "Generate Code for Online State Estimation in MATLAB".
Generated code uses an algorithm that is different from the algorithm that the extendedKalmanFilter function uses. You might see some numerical differences in the results obtained using the two methods.

Supports MATLAB Function block: No

## See Also

## Functions

clone | correct | predict | residual|unscentedKalmanFilter
Blocks
Extended Kalman Filter | Kalman Filter | Unscented Kalman Filter

## Topics

"Nonlinear State Estimation Using Unscented Kalman Filter and Particle Filter"
"Generate Code for Online State Estimation in MATLAB"
"What Is Online Estimation?"
"Extended and Unscented Kalman Filter Algorithms for Online State Estimation"
"Validate Online State Estimation at the Command Line"
"Troubleshoot Online State Estimation"
External Websites
Understanding Kalman Filters: Nonlinear State Estimators - MATLAB Video Series

Introduced in R2016b

## evaluate

Value of nonlinearity estimator at given input

## Syntax

value = evaluate( $n l, x$ )

## Arguments

nl
Nonlinearity estimator object.
x
Value at which to evaluate the nonlinearity.
If $n l$ is a single nonlinearity estimator, then x is a 1-by- nx row vector or an $n v-b y-n \mathrm{x}$ matrix, where nx is the dimension of the regression vector input to nl (size( nl )) and nv is the number of points where nl is evaluated.

If $n l$ is an array of ny nonlinearity estimators, then $x$ is a 1-by-ny cell array of $n v-b y-n x$ matrices.

## Description

value = evaluate( $n l, x$ ) computes the value of a nonlinear estimator object of type customnet, deadzone, linear, neuralnet, pwlinear, saturation, sigmoidnet, treepartition, or wavenet.

## Examples

The following syntax evaluates the nonlinearity of an estimated nonlinear ARX model m:

```
value = evaluate(m.Nonlinearity,x)
```

where m.Nonlinearity accesses the nonlinearity estimator of the nonlinear ARX model.

## See Also

idnlarx|idnlhw

## Introduced in R2007a

## fcat

Concatenate FRD models along frequency dimension

## Syntax

sys $=$ fcat(sys1,sys2,...)

## Description

sys = fcat(sys1,sys2,...) takes two or more frd models and merges their frequency responses into a single frd model sys. The resulting frequency vector is sorted by increasing frequency. The frequency vectors of sys1, sys2, .. should not intersect. If the frequency vectors do intersect, use fdel to remove intersecting data from one or more of the models.

## See Also

fdel|frd|fselect|idfrd|interp
Introduced before R2006a

## fdel

Delete specified data from frequency response data (FRD) models

## Syntax

sysout $=$ fdel(sys, freq)

## Description

sysout = fdel (sys, freq) removes from the frd model sys the data nearest to the frequency values specified in the vector freq.

## Input Arguments

## sys

frd model.
freq
Vector of frequency values.

## Output Arguments

sysout
frd model containing the data remaining in sys after removing the frequency points closest to the entries of freq.

## Examples

## Delete Specified Data from Frequency Response Data Model

Create a frequency response data (FRD) model at specified frequencies from a transfer function model.

```
w = logspace(0,1,10);
sys = frd(tf([1],[1 1]),w)
sys =
Frequency(rad/s) Response
    1.0000 0.5000 - 0.5000i
    1.2915 0.3748 - 0.4841i
    1.6681 0.2644 - 0.4410i
    2.1544 0.1773 - 0.3819i
    2.7826 0.1144 - 0.3183i
    3.5938 0.0719 - 0.2583i
```

```
    4.6416 0.0444 - 0.2059i
    5.9948 0.0271 - 0.1623i
    7.7426 0.0164 - 0.1270i
    10.0000 0.0099 - 0.0990i
Continuous-time frequency response.
```

w is a logarithmically-spaced grid of 10 frequency points between 1 and $10 \mathrm{rad} /$ second.
Remove the data nearest 2, 3.5, and $6 \mathrm{rad} / \mathrm{s}$ from sys.

```
freq = [2, 3.5, 6];
sys2 = fdel(sys,freq)
sys2 =
    Frequency(rad/s) Response
    ---------------------
        1.0000 0.5000 - 0.5000i
        1.2915 0.3748 - 0.4841i
        1.6681 0.2644 - 0.4410i
        2.7826 0.1144 - 0.3183i
        4.6416 0.0444 - 0.2059i
        7.7426 0.0164 - 0.1270i
        10.0000 0.0099 - 0.0990i
```

Continuous-time frequency response.

Note that you do not have to specify the exact frequency of the data to remove. The fdel command removes the data corresponding to frequencies that are nearest to the specified frequencies.

## Tips

- Use fdel to remove unwanted data (for example, outlier points) at specified frequencies.
- Use fdel to remove data at intersecting frequencies from frd models before merging them with fcat. fcat produces an error when you attempt to merge frd models that have intersecting frequency data.
- To remove data from an frd model within a range of frequencies, use fselect.


## See Also

fcat|frd|fselect|idfrd
Introduced in R2012a

## feedback

Identify possible feedback data

## Syntax

[fbck,fbck0,nudir] = feedback(Data)

## Description

Data is an iddata set with $N y$ outputs and $N u$ inputs.
fbck is an $N y$-by-Nu matrix indicating the feedback. The $k y, k u$ entry is a measure of feedback from output ky to input $k u$. The value is a probability $P$ in percent. Its interpretation is that if the hypothesis that there is no feedback from output ky to input $k u$ were tested at the level $P$, it would have been rejected. An intuitive but technically incorrect way of thinking about this is to see $P$ as "the probability of feedback." Often only values above $90 \%$ are taken as indications of feedback. When fbck is calculated, direct dependence at lag zero between $u(t)$ and $y(t)$ is not regarded as a feedback effect.
fbck0: Same as fbck, but direct dependence at lag 0 between $u(t)$ and $y(t)$ is viewed as feedback effect.
nudir: A vector containing those input numbers that appear to have a direct effect on some outputs, that is, no delay from input to output.

## See Also

advice|iddata

## Introduced before R2006a

## fft

Transform iddata object to frequency domain data

## Syntax

Datf $=\mathrm{fft}($ Data $)$
Datf $=\mathrm{fft}($ Data, N$)$
Datf = fft(Data, N, 'complex')

## Description

Datf $=f f t($ Data $)$ transforms time-domain data to frequency domain data. If Data is a timedomain iddata object with real-valued signals and with constant sample time Ts, Datf is returned as a frequency-domain iddata object with the frequency values equally distributed from frequency 0 to the Nyquist frequency. Whether the Nyquist frequency actually is included or not depends on the signal length (even or odd). Note that the FFTs are normalized by dividing each transform by the square root of the signal length. That is in order to preserve the signal power and noise level.

Datf $=\mathrm{fft}($ Data,N $)$ specifies the transformation length. In the default case, the length of the transformation is determined by the signal length. A second argument N will force FFT transformations of length N , padding with zeros if the signals in Data are shorter and truncating otherwise. Thus the number of frequencies in the real signal case will be ( $N / 2$ ) +1 or $(N+1) / 2$. If Data contains several experiments, N can be a row vector of corresponding length.

Datf $=f f t($ Data, $N, '$ complex') specifies to include negative frequencies. For real signals, the default is that Datf only contains nonnegative frequencies. For complex-valued signals, negative frequencies are also included. To enforce negative frequencies in the real case, add a last argument, 'Complex'.

## See Also

iddata|ifft|spa

## Topics

"Representing Data in MATLAB Workspace"

Introduced in R2007a

## idnlarx/findop

Compute operating point for Nonlinear ARX model

## Syntax

[X,U] = findop(sys,'steady',InputLevel,OutputLevel)
[X,U] = findop(sys,spec)
[X,U] = findop(__,Options)
[X,U,Report] = findop (___)
$[\mathrm{X}, \mathrm{U}]=$ findop(sys,'snapshot',T,Uin)
[X,U] = findop(sys,'snapshot',T,Uin,X0)

## Description

$[\mathrm{X}, \mathrm{U}]=$ findop(sys,'steady', InputLevel,OutputLevel) returns the operating-point state values, $X$, and input values, $U$, for theidnlarx model, sys, using steady-state input and output specifications.
[X,U] = findop(sys,spec) returns the steady-state operating point for sys using the operatingpoint specification, spec.
$[\mathrm{X}, \mathrm{U}]=\mathrm{findop}($ $\qquad$ , Options) specifies optimization search options for all of the previous syntaxes.
$[X, U$, Report $]=$ findop ( __ ) returns a summary report on the optimization search results for all of the previous syntaxes.
$[\mathrm{X}, \mathrm{U}]=\mathrm{findop}($ sys,'snapshot', $\mathrm{T}, \mathrm{Uin})$ returns the operating point for sys at a simulation snapshot at time, T , using the specified input, Uin. The initial states of sys are assumed to be zero.
$[X, U]=$ findop(sys,'snapshot', $T, U i n, X 0$ ) specifies the initial states of the simulation.

## Examples

## Find Steady-State Nonlinear ARX Operating Point Using Default Specifications

Estimate a nonlinear ARX model.

```
load iddata6;
M = nlarx(z6,[4 3 1]);
```

Find the steady-state operating point where the input level is fixed to 1 and the output is unknown.

```
[X,U] = findop(M,'steady',1,NaN);
```


## Find Nonlinear ARX Operating Point Using Additional Specifications

Estimate a nonlinear ARX model.
load iddata7;
M = nlarx(z7,[4 3*ones(1,2) 2*ones(1,2)]);
Create a default operating point specification object.
spec = operspec(M);
Set the values for the input signals.

```
spec.Input.Value(1) = -1;
spec.Input.Value(2) = 1;
```

Set the maximum and minimum values for the output signal.

```
spec.Output.Max = 10;
spec.Output.Min = -10;
```

Find the steady-state operating point using the given specifications.

```
[X,U] = findop(M,spec);
```


## Find Nonlinear ARX Operating Point Using Custom Options

Estimate a nonlinear ARX model.

```
load iddata6;
M = nlarx(z6,[4 3 2]);
```

Create a default findopOptions option set.
opt = findopOptions(M);
Modify the option set to specify a steepest descent gradient search method with a maximum of 50 iterations.

```
opt.SearchMethod = 'grad';
opt.SearchOptions.MaxIterations = 50;
```

Find the steady-state operating point using the specified options.
$[\mathrm{X}, \mathrm{U}]=$ findop( M, 'steady $^{\prime}, 1,1$, opt $)$;

## Retrieve Nonlinear ARX Operating Point Search Report

Estimate a nonlinear ARX model.

```
load iddata7;
M = nlarx(z7,[4 3*ones(1,2) 2*ones(1,2)]);
```

Find the steady-state operating point where input 1 is set to 1 and input 2 is unrestricted. The initial guess for the output value is 2 .

```
[X,U,R] = findop(M,'steady',[1 NaN],2);
```

Display the summary report.

```
disp(R);
```

    SearchMethod: 'auto'
            WhyStop: 'Near (local) minimum, (norm(g) < tol).'
        Iterations: 11
            FinalCost: 0
    FirstOrderOptimality: 0
    SignalLevels: [1x1 struct]
    
## Find Nonlinear ARX Simulation Snapshot Using Default Initial States

Load the estimation data and estimate a nonlinear ARX model.
load twotankdata;
z = iddata(y,u,1);
M = nlarx(z,[4 3 1]);
Find the simulation snapshot after 10 seconds, assuming initial states of zero.

```
[X,U] = findop(M,'snapshot',10,z);
```


## Find Nonlinear ARX Simulation Snapshot Using Initial State Specifications

Load the estimation data and estimate a nonlinear ARX model.

```
load twotankdata;
z = iddata(y,u,1);
M = nlarx(z,[4 3 1]);
```

Create an initial state vector. The first four states correspond to delayed output values and the final three states correspond to delayed inputs.

X0 $=[2 ; 2 ; 2 ; 2 ; 5 ; 5 ; 5] ;$
Find the simulation snapshot after 10 seconds using the specified initial states.

```
[X,U] = findop(M,'snapshot',10,z,X0);
```


## Input Arguments

## sys - Nonlinear ARX model

idnlarx object
Nonlinear ARX model, specified as an idnlarx object.

## InputLevel - Steady-state input level

vector
Steady-state input level for computing the operating point, specified as a vector. The length of InputLevel must equal the number of inputs specified in sys.

The optimization algorithm assumes that finite values in InputLevel are fixed input values. Use NaN to specify unknown input signals with initial guesses of 0 . The minimum and maximum bounds for all inputs have default values of - Inf and + Inf respectively.

## OutputLevel - Steady-state output level

vector
Steady-state output level for computing the operating point, specified as a vector. The length of OutputLevel must equal the number of outputs specified in sys.

The values in OutputLevel indicate initial guesses for the optimization algorithm. Use NaN to specify unknown output signals with initial guesses of 0 . The minimum and maximum bounds for all outputs have default values of -Inf and +Inf respectively.

## spec - Operating-point specifications

operspec object
Operating-point specifications, such as minimum and maximum input/output constraints and known inputs, specified as anoperspec object.

## T - Operating point snapshot time <br> positive scalar

Operating point snapshot time, specified as a positive scalar. The value of T must be in the range [ $T_{0}$, $\left.N^{*} T_{s}\right]$, where $N$ is the number of input samples, $T_{s}$ is the sample time and $T_{0}$ is the input start time (Uin.Tstart).

## Uin - Snapshot simulation input

iddata object | matrix
Snapshot simulation input, specified as one of the following:

- Time-domain iddata object with a sample time and input size that matches sys.
- Matrix with as many columns as there are input channels. If the matrix has $N$ rows, the input data is assumed to correspond to the time vector $(1: N) *$ sys.Ts.


## X0 - Initial states

column vector
Initial states of the simulation, specified as a column vector with size equal to the number of states in sys. X0 provides the initial conditions at the time corresponding to the first input sample (Uin.Start, if Uin is an iddata object, or sys.Ts if Uin is a double matrix).

For more information about the states of an idnlarx model, see "Definition of idnlarx States" on page 1-571.

## Options - Operating point search options

findop0ptions option set

Operating point search options, specified as a findop0ptions option set.

## Output Arguments

## $X$ - Operating point state values

column vector
Operating point state values, returned as a column vector of length equal to the number of model states.

## U - Operating point input values

column vector
Operating point input values, returned as a column vector of length equal to the number of inputs.

## Report - Search result summary

structure
Search result summary report, returned as a structure with the following fields:

| Field | Description |
| :--- | :--- |
| SearchMeth <br> od | Search method used for iterative parameter estimation. See SearchMethod in <br> findopOptions for more information. |
| WhyStop | Search algorithm termination condition. |
| Iterations | Number of estimation iterations performed. |
| FinalCost | Final value of the minimization objective function (sum of the squared errors). |
| FirstOrder <br> Optimality | $\infty$-norm of the search gradient vector when the search algorithm terminates. |
| SignalLeve <br> ls | Structure containing the fields Input and Output, which are the operating point <br> input and output signal levels respectively. |

## Algorithms

findop computes the operating point from steady-state operating point specifications or at a simulation snapshot.

## Computing the Operating Point from Steady-State Specifications

To compute the steady-state operating point, call findop using either of the following syntaxes:

```
[X,U] = findop(sys,'steady',InputLevel,OutputLevel)
[X,U] = findop(sys,spec)
```

To compute the states, $X$, and the input, $U$, of the steady-state operating point, findop minimizes the norm of the error $e(t)=y(t)-f(x(t), u(t))$, where:

- $f$ is the nonlinearity estimator.
- $u(t)$ is the input.
- $x(t)$ is the model state.
- $y(t)$ is the model output.

You can specify the search algorithm and search options using the findopOptions option set.
The algorithm uses the following independent variables for minimization:

- Unknown (unspecified) input signal levels
- Output signal levels

Because idnlarx model states are delayed samples of the input and output variables, the state values are the constant values of the corresponding steady-state inputs and outputs. For more information about the definition of nonlinear ARX model states, see "Definition of idnlarx States" on page 1-571.

## Computing the Operating Point at a Simulation Snapshot

When you use the syntax $[\mathrm{X}, \mathrm{U}]=$ findop(sys, 'snapshot' $, \mathrm{T}, \mathrm{Uin}, \mathrm{X} 0$ ), the algorithm simulates the model output until the snapshot time, T . At the snapshot time, the algorithm passes the input and output samples to the data2state command to map these values to the current state vector.

Note For snapshot-based computations, findop does not perform numerical optimization.

## Extended Capabilities

## Automatic Parallel Support

Accelerate code by automatically running computation in parallel using Parallel Computing Toolbox ${ }^{\mathrm{TM}}$.
Parallel computing support is available for estimation using the lsqnonlin search method (requires Optimization Toolbox). To enable parallel computing, use findopOptions, set SearchMethod to 'lsqnonlin', and set SearchOptions.Advanced.UseParallel to true.

For example:

```
opt = findopOptions(idnlarx);
opt.SearchMethod = 'lsqnonlin';
opt.SearchOptions.Advanced.UseParallel = true;
```


## See Also

data2state|findopOptions|idnlarx|idnlarx/operspec|idnlhw/findop|sim

Introduced in R2008a

## idnlhw/findop

Compute operating point for Hammerstein-Wiener model

## Syntax

```
[X,U] = findop(sys,'steady',InputLevel,OutputLevel)
[X,U] = findop(sys,spec)
[X,U] = findop( __ , Options)
[X,U,Report] = findop( ___)
\([\mathrm{X}, \mathrm{U}]=\) findop(sys,'snapshot', \(\mathrm{T}, \mathrm{Uin}\) )
[X,U] = findop(sys,'snapshot', T,Uin, X0)
```


## Description

$[X, U]=$ findop(sys,'steady',InputLevel,OutputLevel) returns the operating-point state values, X , and input values, U , for the idnlhw model, sys, using steady-state input and output specifications.
$[\mathrm{X}, \mathrm{U}]=\mathrm{findop}(\mathrm{sys}, \mathrm{spec})$ returns the steady-state operating point for sys using the operating point specification in spec.
[X,U] = findop( __ , Options) specifies optimization search options for all of the previous syntaxes.
[X,U,Report] = findop( $\qquad$ ) returns a summary report on the optimization search results for all of the previous syntaxes.
$[\mathrm{X}, \mathrm{U}]=$ findop(sys,'snapshot', $\mathrm{T}, \mathrm{Uin}$ ) returns the operating point for sys at a simulation snapshot at time, T , using the specified input, Uin. The initial states of sys are assumed to be zero.
$[X, U]=$ findop(sys,'snapshot',T,Uin,X0) specifies the initial states of the simulation.

## Examples

## Find Steady-State Hammerstein-Wiener Operating Point Using Default Specifications

Load the estimation data and estimate a Hammerstein-Wiener model.

```
load twotankdata;
z = iddata(y,u,1);
M = nlhw(z,[5 1 3]);
```

Find the steady-state operating point where the input level is set to 1 and the output is unknown.

```
[X,U] = findop(M,'steady',1,NaN);
```


## Find Hammerstein-Wiener Operating Point Using Additional Specifications

Estimate a Hammerstein-Wiener model.

```
load iddata7;
orders = [4*ones(1,2) 2*ones(1,2) 3*ones(1,2)];
M = nlhw(z7,orders,'unitgain','pwlinear');
```

Create a default operating point specification object.

```
spec = operspec(M);
```

Set the values for the input signals.

```
spec.Input.Value(1) = -1;
spec.Input.Value(2) = 1;
```

Set the maximum and minimum values for the output signal.

```
spec.Output.Max = 10;
spec.Output.Min = -10;
```

Find the steady-state operating point using the given specifications.

```
[X,U] = findop(M,spec);
```


## Find Hammerstein-Wiener Operating Point Using Custom Options

Load the estimation data and estimate a Hammerstein-Wiener model.

```
load twotankdata;
z = iddata(y,u,1);
M = nlhw(z,[5 1 3]);
```

Create a default findopOptions option set.

```
opt = findopOptions(M);
```

Modify the option set to specify a steepest descent gradient search method with a maximum of 50 iterations.

```
opt.SearchMethod = 'grad';
opt.Search0ptions.MaxIterations = 50;
```

Find the steady-state operating point using the specified options.

```
[X,U] = findop(M,'steady',1,NaN,opt);
```


## Retrieve Hammerstein-Wiener Operating Point Search Report

Load the estimation data and estimate a Hammerstein-Wiener model.

```
load iddata7;
orders = [4*ones(1,2) 2*ones(1,2) 3*ones(1,2)];
M = nlhw(z7,orders,'unitgain','pwlinear');
```

Find the steady-state operating point where input 1 is set to 1 and input 2 is unrestricted. The initial guess for the output value is 2 .
[X,U,R] = findop(M,'steady',[1 NaN],2);
Display the summary report.

```
disp(R);
```

```
    SearchMethod: 'auto'
            WhyStop: 'Near (local) minimum, (norm(g) < tol).'
            Iterations: 3
            FinalCost: 0
FirstOrderOptimality: 0
    SignalLevels: [1x1 struct]
```


## Find Hammerstein-Wiener Simulation Snapshot Using Default Initial States

Load the estimation data estimate a Hammerstein-Wiener model.

```
load twotankdata;
z = iddata(y,u,1);
M = nlhw(z,[5 1 3]);
```

Find the simulation snapshot after 10 seconds, assuming initial states of zero.

```
[X,U] = findop(M,'snapshot',10,z);
```


## Find Hammerstein-Wiener Simulation Snapshot Using Initial State Specifications

Load the estimation data and estimate a Hammerstein-Wiener model.

```
load twotankdata
z = iddata(y,u,1);
M = nlhw(z,[5 1 3]);
```

Create an initial state vector.
X0 $=[10 ; 10 ; 5 ; 5 ; 1 ; 1 ; 0] ;$
Find the simulation snapshot after 10 seconds using the specified initial states.

```
[X,U] = findop(M,'snapshot',10,z,X0);
```


## Input Arguments

## sys - Hammerstein-Wiener model

idnlhw object

Hammerstein-Wiener model, specified as an idnlhw object.

## InputLevel - Steady-state input level <br> vector

Steady-state input level for computing the operating point, specified as a vector. The length of InputLevel must equal the number of inputs specified in sys.

The optimization algorithm assumes that finite values in InputLevel are fixed input values. Use NaN to specify unknown input signals with initial guesses of 0 . The minimum and maximum bounds for all inputs have default values of $-\operatorname{Inf}$ and + Inf respectively.

## OutputLevel - Steady-state output level

vector
Steady-state output level for computing the operating point, specified as a vector. The length of OutputLevel must equal the number of outputs specified in sys.

The values in OutputLevel indicate initial guesses for the optimization algorithm. Use NaN to specify unknown output signals with initial guesses of 0 . The minimum and maximum bounds for all outputs have default values of - Inf and + Inf respectively.

## spec - Operating-point specifications

operspec object
Operating-point specifications, such as minimum and maximum input/output constraints and known inputs, specified as an operspec object.

## T-Operating point snapshot time <br> positive scalar

Operating point snapshot time, specified as a positive scalar. The value of T must be in the range [ $T_{0}$, $\left.N^{*} T_{s}\right]$, where $N$ is the number of input samples, $T_{s}$ is the sample time and $T_{0}$ is the input start time (Uin.Tstart).

## Uin - Snapshot simulation input

iddata object | matrix
Snapshot simulation input, specified as one of the following:

- Time-domain iddata object with a sample time and input size that matches sys.
- Matrix with as many columns as there are input channels. If the matrix has $N$ rows, the input data is assumed to correspond to the time vector ( $1: \mathrm{N}$ )*sys.Ts.


## X0 - Initial states

column vector
Initial states of the simulation, specified as a column vector with length equal to the number of states in sys. X0 provides the initial conditions at the time corresponding to the first input sample (Uin.Start, if Uin is an iddata object, or sys.Ts if Uin is a double matrix).

For more information about the states of an idnlhw model, see "Definition of idnlhw States" on page 1-601.

## Options - Operating point search options <br> findopOptions option set

Operating point search options, specified as a findopOptions option set.

## Output Arguments

## X - Operating point state values

column vector
Operating point state values, returned as a column vector of length equal to the number of model states.

## U - Operating point input values

column vector
Operating point input values, returned as a column vector of length equal to the number of inputs.

## Report - Search result summary

structure
Search result summary report, returned as a structure with the following fields:

| Field | Description |
| :--- | :--- |
| SearchMeth <br> od | Search method used for iterative parameter estimation. See SearchMethod in <br> findopOptions for more information. |
| WhyStop | Search algorithm termination condition. |
| Iterations | Number of estimation iterations performed. |
| FinalCost | Final value of the minimization objective function (sum of the squared errors). |
| First0rder <br> Optimality | $\infty$-norm of the search gradient vector when the search algorithm terminates. |
| SignalLeve <br> ls | Structure containing the fields Input and Output, which are the operating point <br> input and output signal levels respectively. |

## Algorithms

findop computes the operating point from steady-state operating point specifications or at a simulation snapshot.

## Computing the Operating Point from Steady-State Specifications

To compute the steady-state operating point, call findop using either of the following syntaxes:

```
[X,U] = findop(sys,'steady',InputLevel,OutputLevel)
[X,U] = findop(sys,spec)
```

findop uses a different approach to compute the steady-state operating point depending on how much information you provide for this computation:

- When you specify values for all input levels (no NaN values). For a given input level, $U$, the equilibrium state values are $X=\operatorname{inv}(I-A) * B * f(U)$, where $[A, B, C, D]=$ ssdata (model. LinearModel), and $f()$ is the input nonlinearity.
- When you specify known and unknown input levels. findop uses numerical optimization to minimize the norm of the error and compute the operating point. The total error is the union of contributions from $e_{1}$ and $e_{2}, e(t)=\left(e_{1}(t) e_{2}(t)\right)$, such that:
- $e_{1}$ applies for known outputs and the algorithm minimizes $e_{1}=y-g(L(x, f(u)))$, where $f$ is the input nonlinearity, $L(x, u)$ is the linear model with states $x$, and $g$ is the output nonlinearity.
- $e_{2}$ applies for unknown outputs and the error is a measure of whether these outputs are within the specified minimum and maximum bounds. If a variable is within its specified bounds, the corresponding error is zero. Otherwise, the error is equal to the distance from the nearest bound. For example, if a free output variable has a value $z$ and its minimum and maximum bounds are $L$ and $U$, respectively, then the error is $e_{2}=\max [z-U, L-z, 0]$.

The independent variables for the minimization problem are the unknown inputs. In the error definition $e$, both the input $u$ and the states $x$ are free variables. To get an error expression that contains only unknown inputs as free variables, the algorithm findop specifies the states as a function of inputs by imposing steady-state conditions: $x=\operatorname{inv}(I-A) * B^{*} f(U)$, where $A$ and $B$ are state-space parameters corresponding to the linear model $L(x, u)$. Thus, substituting $x=\operatorname{inv}(I-$ $A) * B^{*} f(U)$ into the error function results in an error expression that contains only unknown inputs as free variables computed by the optimization algorithm.

## Computing the Operating Point at a Simulation Snapshot

When you use the syntax $[\mathrm{X}, \mathrm{U}]=$ findop(sys, 'snapshot' $, \mathrm{T}, \mathrm{UIN}, \mathrm{X0}$ ), the algorithm simulates the model output until the snapshot time, T . At the snapshot time, the algorithm computes the inputs for the linear model block of the Hammerstein-Wiener model (LinearModel property of theidnlhw object) by transforming the given inputs using the input nonlinearity: $w=f(u)$. findop uses the resulting $w$ to compute $x$ until the snapshot time using the following equation: $x(t+1)=A x(t)+B w(t)$, where $[A, B, C, D]=$ ssdata(model.LinearModel).

Note For snapshot-based computations, findop does not perform numerical optimization.

## Extended Capabilities

## Automatic Parallel Support

Accelerate code by automatically running computation in parallel using Parallel Computing Toolbox ${ }^{\mathrm{TM}}$.
Parallel computing support is available for estimation using the lsqnonlin search method (requires Optimization Toolbox). To enable parallel computing, use findopOptions, set SearchMethod to 'lsqnonlin', and set SearchOptions.Advanced.UseParallel to true.

For example:

```
opt = findopOptions(idnlhw);
opt.SearchMethod = 'lsqnonlin';
opt.Search0ptions.Advanced.UseParallel = true;
```


## See Also

findop0ptions|idnlarx/findop|idnlhw|idnlhw/operspec|sim

## Introduced in R2008a

## findopOptions

Option set for findop

## Syntax

```
opt = findopOptions(model)
opt = findopOptions(model,Name,Value)
```


## Description

opt $=$ findopOptions(model) creates a default option set for computing the operating point of a specified nonlinear ARX or Hammerstein-Wiener model. Use dot notation to modify this option set for your specific application. Options that you do not modify retain their default values.
opt = findopOptions(model,Name, Value) creates an option set with options specified by one or more Name, Value pair arguments.

## Examples

## Create Default Option Set for Operating Point Search

Create a default option set for findop using an idnlarx model

```
opt = findopOptions(idnlarx);
```


## Create and Modify Default Operating Point Search Options

Create a default option set for findop using an idnlhw model.

```
opt = findopOptions(idnlhw);
```

Use dot notation to specify a subspace Gauss-Newton least squares search with a maximum of 25 iterations.
opt.SearchMethod = 'gn';
opt.SearchOptions.MaxIterations = 25;

## Specify Options for Operating Point Search

Create an option set for findop using an idnlarx model. Specify a steepest descent least squares search with default search options.

```
opt = findopOptions(idnlarx,'SearchMethod','grad');
```


## Input Arguments

## model - Estimated nonlinear model

idnlarx model | idnlhw model
Estimated nonlinear model, specified as one of the following:

- idnlarx model
- idnlhw model


## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

## Example: 'SearchMethod', 'grad' specifies a steepest descent least squares search method

## SearchMethod - Numerical search method used for iterative parameter estimation 'auto' (default)|'gn'|'gna'|'lm' | 'grad'|'lsqnonlin' |'fmincon'

Numerical search method used for iterative parameter estimation, specified as the comma-separated pair consisting of 'SearchMethod' and one of the following:

- 'auto' - A combination of the line search algorithms, 'gn', 'lm', 'gna', and 'grad' methods is tried in sequence at each iteration. The first descent direction leading to a reduction in estimation cost is used.
- 'gn' - Subspace Gauss-Newton least squares search. Singular values of the Jacobian matrix less than GnPinvConstant*eps*max(size(J))*norm(J) are discarded when computing the search direction. $J$ is the Jacobian matrix. The Hessian matrix is approximated as $J^{T} J$. If there is no improvement in this direction, the function tries the gradient direction.
- 'gna' - Adaptive subspace Gauss-Newton search. Eigenvalues less than gamma*max(sv) of the Hessian are ignored, where $s v$ contains the singular values of the Hessian. The Gauss-Newton direction is computed in the remaining subspace. gamma has the initial value
InitialGnaTolerance (see Advanced in 'Search0ptions' for more information). This value is increased by the factor LMStep each time the search fails to find a lower value of the criterion in fewer than five bisections. This value is decreased by the factor $2 *$ LMStep each time a search is successful without any bisections.
- 'lm' - Levenberg-Marquardt least squares search, where the next parameter value is -pinv (H $\left.+d^{*} \mathrm{I}\right) * \operatorname{grad}$ from the previous one. $H$ is the Hessian, $I$ is the identity matrix, and grad is the gradient. $d$ is a number that is increased until a lower value of the criterion is found.
- 'grad' - Steepest descent least squares search.
- 'lsqnonlin' - Trust-region-reflective algorithm of lsqnonlin. Requires Optimization Toolbox software.
- 'fmincon' - Constrained nonlinear solvers. You can use the sequential quadratic programming (SQP) and trust-region-reflective algorithms of the fmincon solver. If you have Optimization Toolbox software, you can also use the interior-point and active-set algorithms of the fmincon solver. Specify the algorithm in the SearchOptions.Algorithm option. The fmincon algorithms may result in improved estimation results in the following scenarios:
- Constrained minimization problems when there are bounds imposed on the model parameters.
- Model structures where the loss function is a nonlinear or non smooth function of the parameters.
- Multi-output model estimation. A determinant loss function is minimized by default for multioutput model estimation. fmincon algorithms are able to minimize such loss functions directly. The other search methods such as ' $l m$ ' and ' $g n$ ' minimize the determinant loss function by alternately estimating the noise variance and reducing the loss value for a given noise variance value. Hence, the fmincon algorithms can offer better efficiency and accuracy for multi-output model estimations.


## Search0ptions - Option set for the search algorithm

search option set
Option set for the search algorithm, specified as the comma-separated pair consisting of 'Search0ptions ' and a search option set with fields that depend on the value of SearchMethod.

## Search0ptions Structure When SearchMethod is Specified as 'gn', 'gna', 'lm', 'grad', or 'auto'

| Field <br> Name | Description | Default |
| :--- | :--- | :--- |
| Toleran <br> ce | Minimum percentage difference between the current value of the loss <br> function and its expected improvement after the next iteration, specified as <br> a positive scalar. When the percentage of expected improvement is less <br> than Tolerance, the iterations stop. The estimate of the expected loss- <br> function improvement at the next iteration is based on the Gauss-Newton <br> vector computed for the current parameter value. | 0.01 |
| MaxIter <br> ations | Maximum number of iterations during loss-function minimization, specified <br> as a positive integer. The iterations stop when MaxIterations is reached <br> or another stopping criterion is satisfied, such as Tolerance. | 20 |
| Setting MaxIterations = 0 returns the result of the start-up procedure. |  |  |
| Use sys. Report. Termination. Iterations to get the actual number |  |  |
| of iterations during an estimation, where sys is an idtf model. |  |  |$\quad$


| Field Name | Description |  | Default |
| :---: | :---: | :---: | :---: |
| Advance <br> d | Advanced search settings, specified as a structure with the following fields: |  |  |
|  | Field Name | Description | Default |
|  | GnPinvCons tant | Jacobian matrix singular value threshold, specified as a positive scalar. Singular values of the Jacobian matrix that are smaller than GnPinvConstant*max(size(J)*norm(J)*eps) are discarded when computing the search direction. Applicable when SearchMethod is 'gn'. | 10000 |
|  | InitialGna Tolerance | Initial value of gamma, specified as a positive scalar. Applicable when SearchMethod is 'gna'. | 0.0001 |
|  | LMStartVal ue | Starting value of search-direction length $d$ in the Levenberg-Marquardt method, specified as a positive scalar. Applicable when SearchMethod is ' 1 m '. | 0.001 |
|  | LMStep | Size of the Levenberg-Marquardt step, specified as a positive integer. The next value of the search-direction length $d$ in the Levenberg-Marquardt method is LMStep times the previous one. Applicable when SearchMethod is ' lm '. | 2 |
|  | MaxBisecti ons | Maximum number of bisections used for line search along the search direction, specified as a positive integer. | 25 |
|  | MaxFunctio nEvaluatio ns | Maximum number of calls to the model file, specified as a positive integer. Iterations stop if the number of calls to the model file exceeds this value. | Inf |
|  | MinParamet erChange | Smallest parameter update allowed per iteration, specified as a nonnegative scalar. | 0 |
|  | RelativeIm provement | Relative improvement threshold, specified as a nonnegative scalar. Iterations stop if the relative improvement of the criterion function is less than this value. | 0 |
|  | StepReduct ion | Step reduction factor, specified as a positive scalar that is greater than 1 . The suggested parameter update is reduced by the factor StepReduction after each try. This reduction continues until MaxBisections tries are completed or a lower value of the criterion function is obtained. <br> StepReduction is not applicable for SearchMethod ' lm' (Levenberg-Marquardt method). | 2 |

## SearchOptions Structure When SearchMethod is Specified as ' Isqnonlin'

| Field <br> Name | Description | Default |
| :--- | :--- | :--- |
| Function <br> Toleranc <br> e | Termination tolerance on the loss function that the software <br> minimizes to determine the estimated parameter values, <br> specified as a positive scalar. | $1 \mathrm{e}-5$ |
| The value of FunctionTolerance is the same as that of <br> opt.Search0ptions. Advanced. TolFun. | (e-6 |  |
| StepTole <br> rance | Termination tolerance on the estimated parameter values, <br> specified as a positive scalar. | The value of StepTolerance is the same as that of <br> opt. SearchOptions . Advanced. TolX. |

SearchOptions Structure When SearchMethod is Specified as 'fmincon'

| Field Name | Description | Default |
| :---: | :---: | :---: |
| Algorithm | fmincon optimization algorithm, specified as one of the following: <br> - 'sqp'-Sequential quadratic programming algorithm. The algorithm satisfies bounds at all iterations, and it can recover from NaN or Inf results. It is not a large-scale algorithm. For more information, see "Large-Scale vs. MediumScale Algorithms" (Optimization Toolbox). <br> - 'trust-regionreflective' - Subspace trust-region method based on the interior-reflective Newton method. It is a largescale algorithm. <br> - 'interior-point' -Large-scale algorithm that requires Optimization Toolbox software. The algorithm satisfies bounds at all iterations, and it can recover from NaN or Inf results. <br> - 'active-set' - Requires Optimization Toolbox software. The algorithm can take large steps, which adds speed. It is not a large-scale algorithm. <br> For more information about the algorithms, see "Constrained Nonlinear Optimization Algorithms" (Optimization Toolbox) and "Choosing the Algorithm" (Optimization Toolbox). | 'sqp' |


| Field Name | Description | Default |
| :--- | :--- | :--- |
| FunctionTolerance | Termination tolerance on the <br> loss function that the software <br> minimizes to determine the <br> estimated parameter values, <br> specified as a positive scalar. | $1 \mathrm{e}-6$ |
| StepTolerance | Termination tolerance on the <br> estimated parameter values, <br> specified as a positive scalar. | $1 \mathrm{e}-6$ |
| MaxIterations | Maximum number of iterations <br> during loss function <br> minimization, specified as a <br> positive integer. The iterations <br> stop when MaxIterations is <br> reached or another stopping <br> criterion is satisfied, such as <br> FunctionTolerance. | 100 |

To specify field values in Search0ptions, create a default findop0ptions set and modify the fields using dot notation. Any fields that you do not modify retain their default values.

```
opt = findopOptions;
opt.SearchOptions.MaxIterations = 15;
opt.SearchOptions.Advanced.RelImprovement = 0.5;
```


## Output Arguments

## opt - Option set for findop command

## findopOptions object

Option set for findop command, returned as a findop0ptions object.

## Compatibility Considerations

## Renaming of Estimation and Analysis Options

The names of some estimation and analysis options were changed in R2018a. Prior names still work. For details, see the R2018a release note "Renaming of Estimation and Analysis Options".

## See Also

idnlarx/findop|idnlhw/findop

Introduced in R2015a

## findstates

Estimate initial states of model

## Syntax

```
x0 = findstates(sys,Data)
x0 = findstates(sys,Data,Horizon)
x0 = findstates(sys,Data,Horizon,Options)
[x0,Report]= findstates(
```

$\qquad$

``` )
```


## Description

$x 0=$ findstates(sys,Data) estimates the initial states, $x 0$, of an identified model sys, to maximize the fit between the model response and the output signal in the estimation data.
x0 = findstates(sys,Data,Horizon) specifies the prediction horizon for computing the response of sys.
$x 0=$ findstates(sys,Data,Horizon,Options) specifies additional options for computation of $x 0$.
[x0, Report]= findstates ( __ ) delivers a report on the initial state estimation. Report is returned with any of the previous syntaxes.

## Examples

## Estimate Initial States of a Model

Create a nonlinear grey-box model. The model is a linear DC motor with one input (voltage), and two outputs (angular position and angular velocity). The structure of the model is specified by dcmotor_m.m file.

```
FileName = 'dcmotor m';
Order = [2 1 2];
Parameters = [0.24365;0.24964];
nlgr = idnlgrey(FileName,Order,Parameters);
nlgr = setinit(nlgr, 'Fixed', false(2,1)); % set initial states free
```

Load data for initial state estimation.

```
load(fullfile(matlabroot,'toolbox','ident',...
    'iddemos','data','dcmotordata'));
z = iddata(y,u,0.1);
```

Estimate the initial states such that the model's response using the estimated states X0 and measured input $u$ is as close as possible to the measured output $y$.

```
X0 = findstates(nlgr,z,Inf);
```


## Estimate Initial States of State-Space Model

Estimate an idss model and simulate it such that the response of the estimated model matches the estimation data's output signal as closely as possible.

Load sample data.
load iddatal z1;
Estimate a linear model from the data.
model $=$ ssest(z1,2);
Estimate the value of the initial states to best fit the measured output z1.y.
x0est = findstates(model,z1,Inf);
Simulate the model.

```
opt = simOptions('InitialCondition',x0est);
sim(model,z1(:,[],:),opt);
```

Simulated output \#1: y1


## Selectively Estimate Initial States of a Model

Estimate the initial states of a model selectively by fixing the first state and allowing the second state of the model to be estimated.

Create a nonlinear grey-box model.

```
FileName = 'dcmotor_m';
Order = [2 1 2];
Parameters = [0.24365;0.24964];
nlgr = idnlgrey(FileName,Order,Parameters);
```

The model is a linear DC motor with one input (voltage), and two outputs (angular position and angular velocity). The structure of the model is specified by dcmotor_m.m file.

Load the estimation data.

```
load(fullfile(matlabroot,'toolbox','ident',...
    'iddemos','data','dcmotordata'));
z = iddata(y,u,0.1);
```

Hold the first state fixed at zero, and estimate the value of the second.

```
x0spec = idpar('x0',[0;0]);
x0spec.Free(1) = false;
opt = findstates0ptions;
opt.InitialState = x0spec;
[X0,Report] = findstates(nlgr,z,Inf,opt)
X0 = 2×1
    0
    0.0061
Report =
            Status: 'Estimated by simulation error minimization'
            Method: 'lsqnonlin'
        Covariance: [2x2 double]
        DataUsed: [1x1 struct]
    Termination: [1x1 struct]
```


## Estimate Initial States by Specifying an Initial State Vector

Create a nonlinear grey-box model.

```
FileName = 'dcmotor_m';
Order = [2 1 2];
Parameters = [0.24365;0.24964];
nlgr = idnlgrey(FileName,Order,Parameters);
```

The model is a linear DC motor with one input (voltage), and two outputs (angular position and angular velocity). The structure of the model is specified by dcmotor_m.m file.

Load the estimation data.

```
load(fullfile(matlabroot,'toolbox','ident',...
    'iddemos','data','dcmotordata'));
z = iddata(y,u,0.1);
```

Specify an initial guess for the initial states.

```
x0spec = idpar('x0',[10;10]);
x0spec.Free is true by default
Estimate the initial states
opt = findstates0ptions;
opt.InitialState = x0spec;
x0 = findstates(nlgr,z,Inf,opt)
x0 = 2x1
    0.0362
    -0.1322
```


## Estimate Initial States Using Multi-Experiment Data

Create a nonlinear grey-box model.

```
FileName = 'dcmotor m';
Order = [2 1 2];
Parameters = [0.24365;0.24964];
nlgr = idnlgrey(FileName,Order,Parameters);
set(nlgr, 'InputName','Voltage','OutputName', ...
    {'Angular position','Angular velocity'});
```

The model is a linear DC motor with one input (voltage), and two outputs (angular position and angular velocity). The structure of the model is specified by dcmotor_m.m file.

Load the estimation data.

```
load(fullfile(matlabroot,'toolbox','ident',...
    'iddemos','data','dcmotordata'));
z = iddata(y,u,0.1,'Name','DC-motor',...
    'InputName','Voltage','OutputName',...
    {'Angular position','Angular velocity'});
```

Create a three-experiment data set.

```
z3 = merge(z,z,z);
```

Choose experiment for estimating the initial states:

- Estimate initial state 1 for experiments 1 and 3
- Estimate initial state 2 for experiment 1

The fixed initial states have zero values.

```
x0spec = idpar('x0',zeros(2,3));
x0spec.Free(1,2) = false;
x0spec.Free(2,[2 3]) = false;
opt = findstatesOptions;
opt.InitialState = x0spec;
Estimate the initial states
```

```
[X0,EstInfo] = findstates(nlgr,z3,Inf,opt);
```


## Input Arguments

## sys - Identified model

idss object|idgrey object|idnlarx object|idnlhw object|idnlgrey object
Identified model whose initial states are estimated, represented as a linear state-space (idss or idgrey) or nonlinear model (idnlarx, idnlhw, or idnlgrey).

## Data - Estimation data

iddata object
Estimation data, specified as an iddata object with input/output dimensions that match sys.
If sys is a linear model, Data can be a frequency-domain iddata object. For easier interpretation of initial conditions, make the frequency vector of Data be symmetric about the origin. For converting time-domain data into frequency-domain data, use fft with ' compl' input argument, and ensure that there is sufficient zero padding. Scale your data appropriately when you compare x 0 between the time-domain and frequency-domain. Since for an $N$-point fft, the input/output signals are scaled by $1 / \operatorname{sqrt}(N)$, the estimated $\times 0$ vector is also scaled by this factor.

## Horizon - Prediction horizon for computing model response

1 (default) | positive integer between 1 and Inf
Prediction horizon for computing the response of sys, specified as a positive integer between 1 and Inf. The most common values used are:

- Horizon = 1 - Minimizes the 1-step prediction error. The 1-step ahead prediction response of sys is compared to the output signals in Data to determine $x 0$. See predict for more information.
- Horizon = Inf - Minimizes the simulation error. The difference between measured output, Data.y, and simulated response of sys to the measured input data, Data. u is minimized. See sim for more information.

Specify Horizon as any positive integer between 1 and Inf, with the following restrictions:

| Scenario | Horizon |
| :--- | :--- |
| Continuous-time model with time-domain data | 1 or Inf |
| Continuous-time frequency-domain data (data. $\mathrm{Ts}=0$ ) | Inf |


| Scenario | Horizon |
| :--- | :--- |
| Output Error models (trivial noise component): | Irrelevant |
| - Nonlinear grey-box (idnlgrey) | Any value of Horizon <br> returns the same answer <br> - Hammerstein-Wiener (idnlhw) |
| - Linear state-space with disturbance matrix, K = 0 | 1 or Inf |
| Nonlinear ARX (idnlarx) |  |

## Options - Estimation options for findstates

findstates Option set
Estimation options for findstates, specified as an option set created using findstatesOptions

## Output Arguments

## x0 - Estimated initial states

vector | matrix
Estimated initial states of model sys, returned as a vector or matrix. For multi-experiment data, x 0 is a matrix with one column for each experiment.

## Report - Initial state estimation information

structure
Initial state estimation information, returned as a structure. Report contains information about the data used, state covariance, and results of any numerical optimization performed to search for the initial states. Report has the following fields:

| Report <br> Field | Description |
| :--- | :--- |
| Status | Summary of how the initial state were estimated. |
| Method | Search method used. |
| Covarianc <br> e | Covariance of state estimates, returned as a Ns-by-Ns matrix, where Ns is the number <br> of states. |


| Report <br> Field | Description |  |
| :---: | :---: | :---: |
| DataUsed | Attributes of the data used for estimation, returned as a structure with the following fields: |  |
|  | Field | Description |
|  | Name | Name of the data set. |
|  | Type | Data type. |
|  | Length | Number of data samples. |
|  | Ts | Sample time. |
|  | InterSam ple | Input intersample behavior, returned as one of the following values: <br> - 'zoh ' - Zero-order hold maintains a piecewise-constant input signal between samples. <br> - ' foh ' - First-order hold maintains a piecewise-linear input signal between samples. <br> - 'bl' - Band-limited behavior specifies that the continuous-time input signal has zero power above the Nyquist frequency. |
|  | Input0ff set | Offset removed from time-domain input data during estimation. For nonlinear models, it is [ ]. |
|  | Output0f fset | Offset removed from time-domain output data during estimation. For nonlinear models, it is []. |
| Terminati on | Termination conditions for the iterative search used for initial state estimation of nonlinear models. Structure with the following fields: |  |
|  | Field | Description |
|  | WhyStop | Reason for terminating the numerical search. |
|  | Iteratio <br> ns | Number of search iterations performed by the estimation algorithm. |
|  | First0rd erOptima lity | $\infty$-norm of the gradient search vector when the search algorithm terminates. |
|  | FcnCount | Number of times the objective function was called. |
|  | UpdateNo rm | Norm of the gradient search vector in the last iteration. Omitted when the search method is 'lsqnonlin' or 'fmincon'. |
|  | LastImpr ovement | Criterion improvement in the last iteration, expressed as a percentage. Omitted when the search method is 'lsqnonlin' or 'fmincon'. |
|  | Algorith | Algorithm used by ' lsqnonlin' or ' fmincon' search method. Omitted when other search methods are used. |
|  | Termination is empty for linear models. |  |

## Extended Capabilities

## Automatic Parallel Support

Accelerate code by automatically running computation in parallel using Parallel Computing Toolbox ${ }^{\mathrm{TM}}$.
Parallel computing support is available for estimation using the lsqnonlin search method (requires Optimization Toolbox). To enable parallel computing, use findstatesOptions, set SearchMethod to 'lsqnonlin', and set SearchOptions.Advanced.UseParallel to true.

For example:

```
opt = findstatesOptions;
```

opt.SearchMethod = 'lsqnonlin';
opt.SearchOptions.Advanced.UseParallel = true;

## See Also

findstates0ptions|predict|sim

Introduced in R2015a

## findstatesOptions

Option set for findstates

## Syntax

opt = findstatesOptions
opt $=$ findstatesOptions(Name,Value)

## Description

opt $=$ findstatesOptions creates the default option set for findstates. Use dot notation to customize the option set, if needed.
opt = findstatesOptions(Name,Value) creates an option set with options specified by one or more Name, Value pair arguments. The options that you do not specify retain their default value.

## Examples

## Identify Initial States Using Option Set

Create an option set for findstates by configuring a specification object for the initial states.
Identify a fourth-order state-space model from data.

```
load iddata8 z8;
sys = ssest(z8,4);
```

z8 is an iddata object containing time-domain system response data. sys is a fourth-order idss model that is identified from the data.

Configure a specification object for the initial states of the model.

```
x0obj = idpar([1;nan(3,1)]);
x0obj.Free(1) = false;
x0obj.Minimum(2) = 0;
x0obj.Maximum(2) = 1;
```

x 0 obj specifies estimation constraints on the initial conditions. The value of the first state is specified as 1 when x 0 obj is created. x 0 obj . Free (1) = false specifies the first initial state as a fixed estimation parameter. The second state is unknown. But, $x 00 \mathrm{obj}$. Minimum (2) $=0$ and x0obj $\cdot \operatorname{Maximum}(2)=1$ specify the lower and upper bounds of the second state as 0 and 1 , respectively.

Create an option set for findstates to identify the initial states of the model.

```
opt = findstatesOptions;
opt.InitialState = x0obj;
```

Identify the initial states of the model.

```
x0_estimated = findstates(sys,z8,Inf,opt);
```


## Specify Option Set for Initial States Estimation

Create an option set for findstates where:

- Initial states are estimated such that the norm of prediction error is minimized. The initial values of the states corresponding to nonzero delays are also estimated.
- Adaptive subspace Gauss-Newton search is used for estimation.
opt = findstatesOptions('InitialState','d','SearchMethod','gna');


## Input Arguments

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.
Example: findstatesOptions('InitialState','d')

## InitialState - Estimation of initial states

'e' (default) | 'd' | vector or matrix | idpar object x00bj
Estimation of initial states, specified as the comma-separated pair consisting of 'InitialState' and one of the following:

- 'e' - The initial states are estimated such that the norm of prediction error is minimized.

For nonlinear grey-box models, only those initial states i that are designated as free in the model (sys.InitialStates(i).Fixed = false) are estimated. To estimate all the states of the model, first specify all the Nx states of the idnlgrey model sys as free.

```
for i = 1:Nx
sys.InitialStates(i).Fixed = false;
end
```

Similarly, to fix all the initial states to values specified in sys.InitialStates, first specify all the states as fixed in the sys. InitialStates property of the nonlinear grey-box model.

- 'd' - Similar to 'e', but absorbs nonzero delays into the model coefficients. The delays are first converted to explicit model states, and the initial values of those states are also estimated and returned.

Use this option for discrete-time linear models only.

- Vector or Matrix - Initial guess for state values, when using nonlinear models. Specify a column vector of length equal to the number of states. For multi-experiment data, use a matrix with Ne columns, where Ne is the number of experiments.

Use this option for nonlinear models only.

- x0obj - Specification object created using idpar. Use x0obj to impose constraints on the initial states by fixing their value or specifying minimum or maximum bounds.

Use x0obj only for nonlinear grey-box models and linear state-space models (idss or idgrey). This option is applicable only for prediction horizon equal to 1 or Inf. See findstates for more details about the prediction horizon.

## InputOffset - Removal of offset from time-domain input data during estimation

[ ] (default) | vector of positive integers | matrix
Removal of offset from time-domain input data during estimation, specified as the comma-separated pair consisting of 'InputOffset' and one of the following:

- A column vector of positive integers of length $N u$, where $N u$ is the number of inputs.
- [ ] - Indicates no offset.
- $N u$-by-Ne matrix - For multi-experiment data, specify InputOffset as an $N u$-by-Ne matrix. $N u$ is the number of inputs, and $N e$ is the number of experiments.

Each entry specified by Input0ffset is subtracted from the corresponding input data.

## OutputOffset - Removal of offset from time-domain output data during estimation

[ ] (default) | vector | matrix
Removal of offset from time-domain output data during estimation, specified as the comma-separated pair consisting of 'OutputOffset' and one of the following:

- A column vector of length $N y$, where $N y$ is the number of outputs.
- [ ] - Indicates no offset.
- $N y$-by-Ne matrix - For multi-experiment data, specify OutputOffset as a $N y$-by-Ne matrix. Ny is the number of outputs, and $N e$ is the number of experiments.

Each entry specified by OutputOffset is subtracted from the corresponding output data.

## OutputWeight - Weighting of prediction errors when using multi-output data

[] (default)|'noise' | matrix
Weighting of prediction errors when using multi-output data, specified as the comma-separated pair consisting of ' OutputWeight ' and one of the following:

- [ ] - No weighting is used. Specifying as [ ] is the same as eye(Ny), where Ny is the number of outputs.
- 'noise' - Inverse of the noise variance stored with the model is used for weighting during estimation of initial states.
- Positive semidefinite matrix, W , of size Ny -by-Ny - This weighting minimizes trace (E'*E*W) for estimation of initial states, where E is the matrix of prediction errors.


## SearchMethod - Numerical search method used for iterative parameter estimation

 'auto' (default)|'gn'|'gna'|'lm'|'grad'|'lsqnonlin'|'fmincon'Numerical search method used for iterative parameter estimation, specified as the comma-separated pair consisting of 'SearchMethod' and one of the following:

- 'auto' - A combination of the line search algorithms, 'gn', 'lm', 'gna', and 'grad' methods is tried in sequence at each iteration. The first descent direction leading to a reduction in estimation cost is used.
- 'gn' - Subspace Gauss-Newton least squares search. Singular values of the Jacobian matrix less than GnPinvConstant*eps*max(size(J))*norm(J) are discarded when computing the search direction. $J$ is the Jacobian matrix. The Hessian matrix is approximated as $J^{T} J$. If there is no improvement in this direction, the function tries the gradient direction.
- 'gna' - Adaptive subspace Gauss-Newton search. Eigenvalues less than gamma*max(sv) of the Hessian are ignored, where $s v$ contains the singular values of the Hessian. The Gauss-Newton direction is computed in the remaining subspace. gamma has the initial value InitialGnaTolerance (see Advanced in 'Search0ptions' for more information). This value is increased by the factor LMStep each time the search fails to find a lower value of the criterion in fewer than five bisections. This value is decreased by the factor $2 *$ LMStep each time a search is successful without any bisections.
- ' lm' - Levenberg-Marquardt least squares search, where the next parameter value is - pinv (H $+\mathrm{d} * \mathrm{I}) * \mathrm{grad}$ from the previous one. $H$ is the Hessian, $I$ is the identity matrix, and grad is the gradient. $d$ is a number that is increased until a lower value of the criterion is found.
- 'grad' - Steepest descent least squares search.
- 'lsqnonlin' - Trust-region-reflective algorithm of lsqnonlin. Requires Optimization Toolbox software.
- 'fmincon' - Constrained nonlinear solvers. You can use the sequential quadratic programming (SQP) and trust-region-reflective algorithms of the fmincon solver. If you have Optimization Toolbox software, you can also use the interior-point and active-set algorithms of the fmincon solver. Specify the algorithm in the SearchOptions. Algorithm option. The fmincon algorithms may result in improved estimation results in the following scenarios:
- Constrained minimization problems when there are bounds imposed on the model parameters.
- Model structures where the loss function is a nonlinear or non smooth function of the parameters.
- Multi-output model estimation. A determinant loss function is minimized by default for multioutput model estimation. fmincon algorithms are able to minimize such loss functions directly. The other search methods such as ' lm ' and 'gn' minimize the determinant loss function by alternately estimating the noise variance and reducing the loss value for a given noise variance value. Hence, the fmincon algorithms can offer better efficiency and accuracy for multi-output model estimations.


## SearchOptions - Option set for the search algorithm

search option set
Option set for the search algorithm, specified as the comma-separated pair consisting of
'Search0ptions ' and a search option set with fields that depend on the value of SearchMethod.

| SearchOptions Structure When SearchMethod is Specified as 'gn ', 'gna', 'lm', 'grad', or |
| :--- |
| 'auto' |
| Field <br> Name Description Default <br> Toleran <br> ce Minimum percentage difference between the current value of the loss <br> function and its expected improvement after the next iteration, specified as <br> a positive scalar. When the percentage of expected improvement is less <br> than Tolerance, the iterations stop. The estimate of the expected loss- <br> function improvement at the next iteration is based on the Gauss-Newton <br> vector computed for the current parameter value. 0.01MaxIter <br> ationsMaximum number of iterations during loss-function minimization, specified <br> as a positive integer. The iterations stop when MaxIterations is reached <br> or another stopping criterion is satisfied, such as Tolerance. |
| Setting MaxIterations = 0 returns the result of the start-up procedure. <br> Use sys. Report. Termination. Iterations to get the actual number <br> of iterations during an estimation, where sys is an idtf model. |


| Field <br> Name | Description |  | Default |
| :---: | :---: | :---: | :---: |
| Advan d | Advanced search settings, specified as a structure with the following fields: |  |  |
|  | Field Name | Description | Default |
|  | GnPinvCons tant | Jacobian matrix singular value threshold, specified as a positive scalar. Singular values of the Jacobian matrix that are smaller than GnPinvConstant*max(size(J)*norm(J)*eps) are discarded when computing the search direction. Applicable when SearchMethod is 'gn'. | 10000 |
|  | InitialGna Tolerance | Initial value of gamma, specified as a positive scalar. Applicable when SearchMethod is 'gna'. | 0.0001 |
|  | LMStartVal ue | Starting value of search-direction length $d$ in the Levenberg-Marquardt method, specified as a positive scalar. Applicable when SearchMethod is ' lm '. | 0.001 |
|  | LMStep | Size of the Levenberg-Marquardt step, specified as a positive integer. The next value of the search-direction length $d$ in the Levenberg-Marquardt method is LMStep times the previous one. Applicable when SearchMethod is ' 1 m '. | 2 |
|  | MaxBisecti ons | Maximum number of bisections used for line search along the search direction, specified as a positive integer. | 25 |
|  | MaxFunctio nEvaluatio ns | Maximum number of calls to the model file, specified as a positive integer. Iterations stop if the number of calls to the model file exceeds this value. | Inf |
|  | MinParamet erChange | Smallest parameter update allowed per iteration, specified as a nonnegative scalar. | 0 |
|  | RelativeIm provement | Relative improvement threshold, specified as a nonnegative scalar. Iterations stop if the relative improvement of the criterion function is less than this value. | 0 |
|  | StepReduct ion | Step reduction factor, specified as a positive scalar that is greater than 1. The suggested parameter update is reduced by the factor StepReduction after each try. This reduction continues until MaxBisections tries are completed or a lower value of the criterion function is obtained. <br> StepReduction is not applicable for SearchMethod <br> ' lm' (Levenberg-Marquardt method). | 2 |

SearchOptions Structure When SearchMethod is Specified as ' Isqnonlin'

| Field Name | Description | Default |
| :---: | :---: | :---: |
| Function Toleranc e | Termination tolerance on the loss function that the software minimizes to determine the estimated parameter values, specified as a positive scalar. <br> The value of FunctionTolerance is the same as that of opt.SearchOptions.Advanced.TolFun. | 1e-5 |
| StepTole rance | Termination tolerance on the estimated parameter values, specified as a positive scalar. <br> The value of StepTolerance is the same as that of opt.SearchOptions.Advanced.TolX. | 1e-6 |
| MaxItera tions | Maximum number of iterations during loss-function minimization, specified as a positive integer. The iterations stop when MaxIterations is reached or another stopping criterion is satisfied, such as FunctionTolerance. <br> The value of MaxIterations is the same as that of opt.Search0ptions.Advanced.MaxIter. | 20 |
| Advanced | Advanced search settings, specified as an option set for lsqnonlin. <br> For more information, see the Optimization Options table in "Optimization Options" (Optimization Toolbox). | Use optimset('lsqnonl in') to create a default option set. |

SearchOptions Structure When SearchMethod is Specified as 'fmincon'

| Field Name | Description | Default |
| :---: | :---: | :---: |
| Algorithm | fmincon optimization algorithm, specified as one of the following: <br> 'sqp ' - Sequential quadratic programming algorithm. The algorithm satisfies bounds at all iterations, and it can recover from NaN or Inf results. It is not a large-scale algorithm. For more information, see "Large-Scale vs. MediumScale Algorithms" (Optimization Toolbox). <br> - 'trust-regionreflective' - Subspace trust-region method based on the interior-reflective Newton method. It is a largescale algorithm. <br> - 'interior-point' -Large-scale algorithm that requires Optimization Toolbox software. The algorithm satisfies bounds at all iterations, and it can recover from NaN or Inf results. <br> - 'active-set' - Requires Optimization Toolbox software. The algorithm can take large steps, which adds speed. It is not a large-scale algorithm. <br> For more information about the algorithms, see "Constrained Nonlinear Optimization Algorithms" (Optimization Toolbox) and "Choosing the Algorithm" (Optimization Toolbox). | 'sqp ' |


| Field Name | Description | Default |
| :--- | :--- | :--- |
| FunctionTolerance | Termination tolerance on the <br> loss function that the software <br> minimizes to determine the <br> estimated parameter values, <br> specified as a positive scalar. | $1 \mathrm{e}-6$ |
| StepTolerance | Termination tolerance on the <br> estimated parameter values, <br> specified as a positive scalar. | $1 \mathrm{e}-6$ |
| MaxIterations | Maximum number of iterations <br> during loss function <br> minimization, specified as a <br> positive integer. The iterations <br> stop when MaxIterations is <br> reached or another stopping <br> criterion is satisfied, such as <br> FunctionTolerance. | 100 |

To specify field values in SearchOptions, create a default findstatesOptions set and modify the fields using dot notation. Any fields that you do not modify retain their default values.

```
opt = findstatesOptions;
opt.SearchOptions.Tolerance = 0.02;
opt.Search0ptions.Advanced.MaxBisections = 30;
```


## Output Arguments

## opt - Option set for findstates

findstatesOptions option set
Option set for findstates, returned as an findstatesOptions option set.

## Compatibility Considerations

## Renaming of Estimation and Analysis Options

The names of some estimation and analysis options were changed in R2018a. Prior names still work. For details, see the R2018a release note "Renaming of Estimation and Analysis Options".

## See Also

findstates |idpar
Introduced in R2012a

## fnorm

Pointwise peak gain of FRD model

## Syntax

```
fnrm = fnorm(sys)
fnrm = fnorm(sys,ntype)
```


## Description

fnrm $=$ fnorm(sys) computes the pointwise 2-norm of the frequency response contained in the FRD model sys, that is, the peak gain at each frequency point. The output fnrm is an FRD object containing the peak gain across frequencies.
fnrm = fnorm(sys, ntype) computes the frequency response gains using the matrix norm specified by ntype. See norm for valid matrix norms and corresponding NTYPE values.

## See Also

abs | norm
Introduced in R2006a

## forecast

Forecast identified model output

## Syntax

```
yf = forecast(sys,PastData,K)
yf = forecast(sys,PastData,K,FutureInputs)
yf = forecast(
```

$\qquad$

``` ,opts)
[yf,x0,sysf] = forecast(___)
[yf,x0,sysf,yf_sd,x,x_sd] = forecast(___) )
forecast(sys, PastData, K, _)
forecast(sys,Linespec, PastData,K,
``` \(\qquad\)
```

forecast(sys1,...,sysN,PastData,K,

``` \(\qquad\)
```

forecast(sys1,Linespec1,...,sysN,LinespecN,PastData,K,

``` \(\qquad\)
``` )
```


## Description

yf = forecast(sys,PastData,K) forecasts the output of an identified time series model sys, K steps into the future using past measured data, PastData.
forecast performs prediction into the future, in a time range beyond the last instant of measured data. In contrast, the predict command predicts the response of an identified model over the time span of measured data. Use predict to determine if the predicted result matches the observed response of an estimated model. If sys is a good prediction model, consider using it with forecast.
yf = forecast(sys,PastData,K,FutureInputs) uses the future values of the inputs, FutureInputs, to forecast the response of an identified model with input channels.
yf = forecast $\qquad$ , opts) uses the option set, opts, to specify additional forecast options. Use opts with any of the previous input argument combinations.
[yf,x0, sysf] = forecast (__ ) also returns the estimated values for initial states, $x 0$, and a forecasting model, sysf, and can include any of the previous input argument combinations.
[yf,x0,sysf,yf_sd,x,x_sd] = forecast( ___ ) also returns estimated standard deviation of the output, yf_sd, state trajectory, x , and standard deviation of the trajectory, x _sd. Use with any of the previous input argument combinations.
forecast(sys,PastData,K, $\qquad$ ) plots the forecasted output. Use with any of the previous input argument combinations.

To change display options, right-click the plot to access the context menu. For example, to view the estimated standard deviation of the forecasted output, select Confidence Region from the context menu. For more details about the menu, see "Tips" on page 1-394.
forecast(sys,Linespec, PastData,K, $\qquad$ ) uses Linespec to specify the line type, marker symbol, and color.
forecast (sys1,..., sysN, PastData, K, __ ) plots the forecasted outputs for multiple identified models. forecast automatically chooses colors and line styles.
forecast(sys1,Linespec1,..., sysN, LinespecN, PastData, K, $\qquad$ ) uses the line type, marker symbol, and color specified for each system.

## Examples

## Forecast Future Values of a Sinusoidal Signal

Forecast the values of a sinusoidal signal using an AR model.
Generate and plot data.

```
data = iddata(sin(0.1*[1:100])',[]);
plot(data)
```

Input-Output Data


Fit an AR model to the sine wave.
sys = ar(data,2);
Forecast the values into the future for a given time horizon.
$K=100 ;$
p = forecast(sys,data,K);

K specifies the forecasting time horizon as 100 samples. p is the forecasted model response. Plot the forecasted data.
plot(data,'b',p,'r'), legend('measured','forecasted')
Input-Output Data


Alternatively, plot the forecasted output using the syntax forecast(sys,data, K).

## Forecast Response of Time Series Model

Obtain past data, and identify a time series model.

```
load iddata9 z9
past_data = z9.OutputData(1:50);
mode\overline{l}= ar(z9,4);
\(z 9\) is an iddata object that contains measured output only.
```

model is an idpoly time series model.
Specify initial conditions for forecasting.
opt $=$ forecastOptions('InitialCondition','e');
Plot the forecasted system response for a given time horizon.

K = 100;
forecast(model,past_data,K,opt);
legend('Measured', 'Forecasted')

100-Step Ahead Forecast


## Plot Forecasted Output With Specified Line Type

Obtain past data, and identify a time series model.

```
load iddata9 z9
past_data = z9.OutputData(1:50);
mode\overline{l}}=\operatorname{ar(z9,4);
z9 is an iddata object that contains measured output only.
```

Plot the forecasted system response for a given time horizon as a red dashed line.
K = 100;
forecast(model, 'r--', past_data,K);

## 100-Step Ahead Forecast



The plot also displays the past data by default. To change display options, right-click the plot to access the context menu. For example, to view the estimated standard deviation of the forecasted output, select ConfidenceRegion from the context menu. To specify number of standard deviations to plot, double-click the plot and open the Property Editor dialog box. In the dialog box, in the Options tab, specify the number of standard deviations in Confidence Region for Identified Models. The default value is 1 standard deviation.

## Forecast Model Response for Known Future Inputs

Obtain past data, future inputs, and an identified linear model.

```
load iddata1 z1
z1 = iddata(cumsum(z1.y),cumsum(z1.u),z1.Ts,'InterSample','foh');
past_data = z1(1:100);
future_inputs = z1.u(101:end);
sys = \overline{polyest(z1,[2 2 2 0 0 1],'IntegrateNoise',true);}
```

z1 is an iddata object that contains integrated data. sys is an idpoly model. past_data contains the first 100 data points of $z 1$.
future_inputs contains the last 200 data points of $z 1$.
Forecast the system response into the future for a given time horizon and future inputs.
$K=200 ;$
[yf,x0,sysf,yf_sd,x,x_sd] = forecast(sys,past_data, K,future_inputs);
$y f$ is the forecasted model response, and yf_sd is the standard deviation of the output. $x 0$ is the estimated value for initial states, and sysf is the forecasting state-space model. Also returned are the state trajectory, x , and standard deviation of the trajectory, x _sd.

Plot the forecasted response.

```
UpperBound = iddata(yf.OutputData+3*yf_sd,[],yf.Ts,'Tstart',yf.Tstart);
LowerBound = iddata(yf.OutputData-3*yf_sd,[],yf.Ts,'Tstart',yf.Tstart);
plot(past_data(:,:,[]),yf(:,:,[]),UpperBound,'k--',LowerBound,'k--')
legend({'Measured','Forecasted','3 sd uncertainty'},'Location','best')
```


## Input-Output Data



Plot the state trajectory.

```
t = z1.SamplingInstants(101:end);
subplot(3,1,1)
plot(t,x(:,1),t,x(:,1)+3*x_sd(:,1),'k--',t,x(:,1)-3*x_sd(:,1),'k--'')
title('X_1')
subplot(3,1,2)
plot(t, x(:,2),t,x(:,2)+3*x_sd(:,2),'k--',t, x(:,2)-3*x_sd(:,2),'k--')
title('X 2')
subplot(3,1,3)
plot(t,x(:,3),t,x(:,3)+3*x_sd(:,3),'k--',t, x(:,3)-3*x_sd(:,3),'k--')
title('X_3')
```



The response uncertainty does not grow over the forecasting time span because of the specification of future inputs.

## Forecast Response of Multi-Output Nonlinear Time Series Model

Load data.

```
load(fullfile(matlabroot,'toolbox','ident','iddemos','data','predprey2data'));
z = iddata(y,[],0.1);
set(z,'Tstart',0,'OutputUnit',{'Population (in thousands)',...
    'Population (in thousands)'},'TimeUnit','Years');
```

z is a two output time-series data set (no inputs) from a 1-predator 1-prey population. The population exhibits a decline in predator population due to crowding. The data set contains 201 data samples covering 20 years of evolution.

The changes in the predator ( y 1 ) and prey ( y 2 ) population can be represented as:

$$
\begin{aligned}
& y_{1}(t)=p_{1} * y_{1}(t-1)+p_{2} * y_{1}(t-1) * y_{2}(t-1) \\
& y_{2}(t)=p_{3} * y_{2}(t-1)-p_{4} * y_{1}(t-1) * y_{2}(t-1)-p_{5} * y_{2}(t-1)^{2}
\end{aligned}
$$

The nonlinearity in the predator and prey populations can be fit using a nonlinear ARX model with custom regressors.

Use part of the data as past data.
past_data $=$ z(1:100);
Specify the standard regressors.
na $=$ [1 0; 0 1];
nb = [];
nk = [];

Specify the custom regressors.
$C=\left\{\left\{{ }^{\prime} y 1(t-1) * y 2(t-1)^{\prime}\right\} ;\left\{' y 1(t-1) * y 2(t-1)^{\prime}, ' y 2(t-1)^{\wedge} 2 '\right\}\right\} ;$
Estimate a nonlinear ARX model using past_data as estimation data.
sys = nlarx(past_data,[na nb nk],'wavenet','CustomRegressors',C);
Compare the simulated output of sys with measured data to ensure it is a good fit.
compare(past_data,sys);


Plot the forecasted output of sys.
forecast(sys, past_data,101);
legend('Measured','Forecasted');

## 101-Step Ahead Forecast



## Reproduce Forecasting Results by Simulation

Obtain past data, future inputs, and identified linear model.

```
load iddata3 z3
past_data = z3(1:100);
future_inputs = z3.u(101:end);
sys = polyest(z3,[2 2 2 0 0 1]);
```

Forecast the system response into the future for a given time horizon and future inputs.
K = size(future_inputs,1);
[yf,x0,sysf] = forecast(sys,past_data,K,future_inputs);
$y f$ is the forecasted model response, $x 0$ is the estimated value for initial states, and sys $f$ is the forecasting state-space model.

Simulate the forecasting state-space model with inputs, future_inputs, and initial conditions, x0.
opt = simOptions;
opt.InitialCondition = x0;
ys = sim(sysf,future_inputs(1:K),opt);
Plot the forecasted and simulated outputs.
$\mathrm{t}=\mathrm{yf}$.SamplingInstants;
plot(t,yf.OutputData,'b',t,ys,'.r');
legend('Forecasted Output','Simulated Output')


Simulation of forecasting model, sysf, with inputs, future_inputs, and initial conditions, $x 0$, yields the forecasted output, yf.

## Input Arguments

sys - Identified model
linear model | nonlinear model
Identified model whose output is to be forecasted, specified as one of the following:

- Linear model - idpoly, idproc, idss, idtf, or idgrey
- Nonlinear model - idnlgrey, idnlhw, or idnlarx

If a model is unavailable, estimate sys from PastData using commands such as ar, arx, armax, nlarx, and ssest.

## PastData - Past input-output time-domain data

iddata object | matrix of doubles
Past input-output time-domain data, specified as one of the following:

- iddata object - Use observed input and output signals to create an iddata object. For timeseries data (no inputs), specify as an iddata object with no inputs iddata(output, []).
- Matrix of doubles - For discrete-time models only. Specify as an $N$-by- $N y$ matrix for time-series data. Here, $N$ is the number of observations and $N y$ is the number of outputs.

For models with $N u$ inputs, specify PastData as an $N$-by- $(N y+N u)$ matrix.

## K - Time horizon of forecasting

positive integer
Time horizon of forecasting, specified as a positive integer. The output, $y f$, is calculated $K$ steps into the future, such that the prediction time horizon is $\mathrm{Ts} * \mathrm{~K}$.

## FutureInputs - Future input values

[ ] | matrix of doubles | iddata object | cell array of matrices
Future input values, specified as one of the following:

- [ ] - Future input values are assumed to be zero, or equal to input offset levels (if they are specified in opts). For time series models, specify as [].
- iddata object - Specify as an iddata object with no outputs.
- $K$-by- $N u$ matrix of doubles $-K$ is the forecast horizon, and $N u$ is the number of inputs.

If you have data from multiple experiments, you can specify a cell array of matrices, one for each experiment in PastData.

## opts - Forecast options

forecast0ptions option set
Forecast options, specified as a forecastOptions option set.

## Linespec - Line style, marker, and color

character vector
Line style, marker, and color, specified as a character vector. For example, 'b' or 'b+: '.
For more information about configuring Linespec, see the Linespec argument of plot.

## Output Arguments

## yf - Forecasted response

iddata object
Forecasted response, returned as an iddata object. yf is the forecasted response at times after the last sample time in PastData. yf contains data for the time interval T0+( $\mathrm{N}+1: \mathrm{N}+\mathrm{K}$ ) *T1, where T0 $=$ PastData.Tstart and T1 = PastData.Ts. $N$ is the number of samples in PastData.

## x0 - Estimated initial states

column vector | cell array
Estimated initial states at the start of forecasting, returned as a column vector of size equal to the number of states. Use $x 0$ with the forecasting model sysf to reproduce the result of forecasting by pure simulation.

If PastData is multi-experiment, x 0 is a cell array of size $N e$, where $N e$ is the number of experiments.

When sys is not a state-space model (idss, idgrey, or idnlgrey), the definition of states depends on if sys is linear or nonlinear:

- Linear model (idpoly, idproc, idtf) - sys is converted to a discrete-time state-space model, and $x 0$ is returned as the states of the converted model at a time-point beyond the last data in PastData.

If conversion of sys to idss is not possible, $x 0$ is returned empty. For example, if sys is a MIMO continuous-time model with irreducible internal delays.

- Nonlinear model (idnlhw or idnlarx) - For a definition of the states of idnlarx and idnlhw models, see "Definition of idnlarx States" on page 1-571, and "Definition of idnlhw States" on page 1-601.


## sysf - Forecasting model

discrete-time idss | idnlarx | idnlhw | idnlgrey | cell array of models
Forecasting model, returned as one of the following:

- Discrete-time idss - If sys is a discrete-time idss model, sysf is the same as sys. If sys is a linear model that is not a state-space model (idpoly, idproc, idtf), or is a continuous-time state-space model (idss, idgrey), sys is converted to a discrete-time idss model. The converted model is returned in sysf.
- idnlarx, idnlhw, or idnlgrey- If sys is a nonlinear model, sysf is the same as sys.
- Cell array of models - If PastData is multiexperiment, sysf is an array of Ne models, where Ne is the number of experiments.

Simulation of sysf using sim, with inputs, FutureInputs, and initial conditions, x0, yields yf as the output. For time-series models, FutureInputs is empty.

## yf_sd - Estimated standard deviations of forecasted response <br> matrix | cell array

Estimated standard deviations of forecasted response, returned as a $K$-by-Ny matrix, where K is the forecast horizon, and $N y$ is the number of outputs. The software computes the standard deviation by taking into account the model parameter covariance, initial state covariance, and additive noise covariance. The additive noise covariance is stored in the NoiseVariance property of the model.

If PastData is multiexperiment, yf_sd is a cell array of size $N e$, where $N e$ is the number of experiments.
yf_sd is empty if sys is a nonlinear ARX (idnlarx) or Hammerstein-Wiener model (idnlhw). yf_sd is also empty if sys does not contain parameter covariance information, that is if getcov(sys) is empty. For more information, see getcov.

## x - Forecasted state trajectory

matrix | cell array
Forecasted state trajectory, returned as a $K$-by- $N x$ matrix, where K, the forecast horizon and $N x$ is the number of states. x are the states of the forecasting model.

If PastData is multiexperiment, x is a cell array of size $N e$, where $N e$ is the number of experiments.

If sys is linear model other than a state-space model (not idss or idgrey), then it is converted to a discrete-time state-space model, and the states of the converted model are calculated. If conversion of sys to idss is not possible, $x$ is returned empty. For example, if sys is a MIMO continuous-time model with irreducible internal delays.
$x$ is empty if sys is a nonlinear ARX (idnlarx) or Hammerstein-Wiener model (idnlhw).

## x_sd - Estimated standard deviations of forecasted states

matrix | cell array
Estimated standard deviations of forecasted states x , returned as a $K$-by- $N s$ matrix, where K , the forecast horizon and $N s$ is the number of states. The software computes the standard deviation by taking into account the model parameter covariance, initial state covariance, and additive noise covariance. The additive noise covariance is stored in the NoiseVariance property of the model.

If PastData is multiexperiment, $\mathrm{x} \_$sd is a cell array of size $N e$, where $N e$ is the number of experiments.

If sys is linear model other than a state-space model (not idss or idgrey), then it is converted to a discrete-time state-space model, and the states and standard deviations of the converted model are calculated. If conversion of sys to idss is not possible, $x$ sd is returned empty. For example, if sys is a MIMO continuous-time model with irreducible internal delays.
$x$ _sd is empty if sys is a nonlinear ARX (idnlarx) or Hammerstein-Wiener model (idnlhw).

## Tips

- Right-clicking the plot opens the context menu, where you can access the following options:
- Systems - Select systems to view forecasted output. By default, the forecasted output of all systems is plotted.
- Data Experiment - For multi-experiment data only. Toggle between data from different experiments.
- Characteristics - View the following data characteristics:
- Peak Value - View peak value of the data.
- Mean Value - View mean value of the data.
- Confidence Region - View the estimated standard deviation of the forecasted output. To specify number of standard deviations to plot, double-click the plot and open the Property Editor dialog box. Specify the number of standard deviations in the Options tab, in Confidence Region for Identified Models. The default value is 1 standard deviation.

The confidence region is not generated for nonlinear ARX and Hammerstein-Wiener models and models that do not contain parameter covariance information.

- Show Past Data - Plot the past output data used for forecasting. By default, the past output data is plotted.
- I/O Grouping - For datasets containing more than one input or output channel. Select grouping of input and output channels on the plot.
- None - Plot input-output channels in their own separate axes.
- All - Group all input channels together and all output channels together.
- I/O Selector - For datasets containing more than one input or output channel. Select a subset of the input and output channels to plot. By default, all output channels are plotted.
- Grid - Add grids to the plot.
- Normalize - Normalize the y-scale of all data in the plot.
- Full View - Return to full view. By default, the plot is scaled to full view.
- Properties - Open the Property Editor dialog box to customize plot attributes.


## See Also

ar|arx|compare|forecastOptions|iddata|predict|sim|ssest
Topics
"Forecast Output of Dynamic System"
"Forecast Multivariate Time Series"
"Time Series Prediction and Forecasting for Prognosis"
"Introduction to Forecasting of Dynamic System Response"
Introduced in R2012a

## forecastOptions

Option set for forecast

## Syntax

opt = forecastOptions
opt $=$ forecastOptions(Name,Value)

## Description

opt = forecastOptions creates the default option set for forecast. Use dot notation to modify this option set. Any options that you do not modify retain their default values.
opt = forecastOptions(Name,Value) creates an option set with the options specified by one or more Name, Value pair arguments.

## Examples

## Specify Input Signal Offset for Model Forecasting

Create a default option set for forecast.
opt = forecastOptions;
Specify the input offset for a single-input data set as 5 .
opt. Input0ffset = 5;
You can now use this option set for forecasting. Before forecasting model response, the forecast command subtracts this offset value from the past input data signal.

## Specify Handling of Initial Conditions During Model Forecasting

Create an option set for forecast using zero initial conditions.

```
opt = forecastOptions('InitialCondition','z');
```


## Specify Output Offset for Forecasting Multi-Experiment Data

Load past measured data from two experiments.
load iddata1
load iddata2
$z 1$ and $z 2$ are iddata objects that store SISO input-output data. Create a two-experiment data set from $z 1$ and $z 2$.
z = merge(z1,z2);
Estimate a transfer function model with 2 poles using the multi-experiment data.
sys = tfest(z,2);
Specify the offset as -1 and 1 for the output signals of the two experiments.
opt = forecastOptions('OutputOffset',[-1 1]);
OutputOffset is specified as an $N y$-by-Ne matrix where $N y$ is the number of outputs in each experiment, and $N e$ is the number of experiments. In this example, $N y$ is 1 and $N e$ is 2 .

Using the option set opt, forecast the response of the model 10 time steps into the future. The software subtracts the offset value 0utput0ffset (i,j) from the output signal $i$ of experiment $j$ before using the data in the forecasting algorithm. The removed offsets are added back to generate the final result.

```
y = forecast(sys,z,10,opt)
y =
Time domain data set containing 2 experiments.
Experiment Samples Sample Time
    Exp1 10 0.1
    Exp2 10 0.1
Outputs Unit (if specified)
    y1
Inputs Unit (if specified)
    u1
```

$y$ is an iddata object that returns the forecasted response corresponding to each set of past experimental data.

## Input Arguments

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.
Example: forecast0ptions('InitialCondition', 'e') specifies that the software estimates the initial conditions of the measured input-output data such that the 1-step prediction error for observed output is minimized.

## InitialCondition - Handling of initial conditions

```
'e' (default)| 'z' | idpar object x00bj
```

Handling of initial conditions, specified as the comma-separated pair consisting of
' InitialCondition ' and one of the following values:

- 'z' - Zero initial conditions.
- 'e' - Estimate initial conditions such that the 1-step prediction error is minimized for the observed output.

For nonlinear grey-box models, only those initial states $i$ that are designated as free in the model (sys.InitialStates(i).Fixed $=$ false) are estimated. To estimate all the states of the model, first specify all the Nx states of the idnlgrey model sys as free.

```
for i = 1:Nx
sys.InitialStates(i).Fixed = false;
end
```

Similarly, to fix all the initial states to values specified in sys. InitialStates, first specify all the states as fixed in the sys. InitialStates property of the nonlinear grey-box model.

- x0obj - Specification object created using idpar. Use this object for discrete-time state-space models only (idss, idgrey, and idnlgrey). Use x0obj to impose constraints on the initial states by fixing their value or specifying minimum or maximum bounds.


## Input0ffset - Input signal offset

[] (default) | column vector | matrix
Input signal offset for time-domain data, specified as the comma-separated pair consisting of
' InputOffset ' and one of the following values:

- [ ] - No input offsets.
- A column vector of length $N u$, where $N u$ is the number of inputs. When you use the forecast command, the software subtracts the offset value Input0ffset (i) from the ith input signals in the past and future input values. You specify these values in the PastData and FutureInputs arguments of forecast. The software then uses the offset subtracted inputs to forecast the model response.
- $N u$-by- $N e$ matrix - For multi-experiment data, specify InputOffset as an $N u$-by- $N e$ matrix, where $N e$ is the number of experiments. The software subtracts the offset value InputOffset (i,j) from the ith input signal of the $j$ th experiment in the PastData and FutureInputs arguments of forecast before forecasting.


## OutputOffset - Output signal offset

[ ] (default) | column vector | matrix
Output signal offset for time-domain data, specified as the comma-separated pair consisting of 'OutputOffset' and one of the following values:

- [ ] - No output offsets.
- A column vector of length $N y$, where $N y$ is the number of outputs. When you use the forecast command, the software subtracts the offset value Output0ffset (i) from the ith past output signal that you specify in the PastData argument of forecast. The software then use the offset subtracted output to compute the detrended forecasts. The removed offsets are added back to the detrended forecasts to generate the final result.
- Ny-by-Ne matrix - For multi-experiment data, specify Output0ffset as an $N y$-by-Ne matrix, where $N e$ is the number of experiments. Before forecasting, the software subtracts the offset value Output0ffset ( $i, j$ ) from the ith output signal of the jth experiment in the PastData argument of forecast. For an example, see "Specify Output Offset for Forecasting MultiExperiment Data" on page 1-396.


## Output Arguments

opt - Option set for forecast
forecastOptions option set
Option set for forecast, retuned as a forecastOptions option set.

## See Also

forecast |idpar

## Topics

"Introduction to Forecasting of Dynamic System Response"
Introduced in R2012a

## fpe

Akaike's Final Prediction Error for estimated model

## Syntax

```
value = fpe(model)
value = fpe(modell,...,modeln)
```


## Description

value $=$ fpe(model) returns the Final Prediction Error (FPE) value for the estimated model.
value $=$ fpe(model1,..., modeln) returns the FPE value for multiple estimated models.

## Examples

## Compute Final Prediction Error of Estimated Model

Estimate a transfer function model.

```
load iddatal z1;
np = 2;
sys = tfest(z1,np);
```

Compute the Final Prediction Error (FPE) value.
value $=$ fpe(sys)
value $=1.7252$
Alternatively, use the Report property of the model to access the value.
sys.Report.Fit.FPE
ans $=1.7252$

Pick Model with Optimal Tradeoff Between Accuracy and Complexity Using FPE Criterion
Estimate multiple Output-Error (OE) models and use Akaike's Final Prediction Error (FPE) value to pick the one with optimal tradeoff between accuracy and complexity.

Load the estimation data.
load iddata2
Specify model orders varying in 1:4 range.

```
nf = 1:4;
nb = 1:4;
nk = 0:4;
```

Estimate OE models with all possible combinations of chosen order ranges.

```
NN = struc(nf,nb,nk);
models = cell(size(NN,1),1);
for ct = 1:size(NN,1)
    models{ct} = oe(z2, NN(ct,:));
end
```

Compute the small sample-size corrected AIC values for the models, and return the smallest value.
$V=$ fpe(models\{:\});
[Vmin, I] = min(V);
Return the optimal model that has the smallest AICc value.

```
models{I}
ans =
Discrete-time OE model: y(t) = [B(z)/F(z)]u(t) + e(t)
    B(z) = 1.067 z^-2
    F(z) = 1 - 1.824 z^-1 + 1.195 z^-2 - 0.2307 z^-3
Sample time: 0.1 seconds
Parameterization:
    Polynomial orders: nb=1 nf=3 nk=2
    Number of free coefficients: 4
    Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.
Status:
Estimated using OE on time domain data "z2".
Fit to estimation data: 86.53%
FPE: 0.9809, MSE: 0.9615
```


## Input Arguments

## model - Identified model

idtf|idgrey|idpoly|idproc|idss|idnlarx, |idnlhw|idnlgrey
Identified model, specified as one of the following model objects:

- idtf
- idgrey
- idpoly
- idproc
- idss
- idnlarx, except nonlinear ARX model that includes a binary-tree or neural network nonlinearity estimator
- idnlhw
- idnlgrey


## Output Arguments

## value - Final Prediction Error (FPE) value

scalar | vector
Final Prediction Error (FPE) value, returned as a scalar or vector. For multiple models, value is a row vector where value(k) corresponds to the kth estimated model modelk.

## More About

## Akaike's Final Prediction Error (FPE)

Akaike's Final Prediction Error (FPE) criterion provides a measure of model quality by simulating the situation where the model is tested on a different data set. After computing several different models, you can compare them using this criterion. According to Akaike's theory, the most accurate model has the smallest FPE.

If you use the same data set for both model estimation and validation, the fit always improves as you increase the model order and, therefore, the flexibility of the model structure.

Akaike's Final Prediction Error (FPE) is defined by the following equation:

$$
F P E=\operatorname{det}\left(\frac{1}{N} \sum_{1}^{N} e\left(t, \widehat{\theta}_{N}\right)\left(e\left(t, \widehat{\theta}_{N}\right)\right)^{T}\right)\left(\frac{1+d / N}{1-d / N}\right)
$$

where:

- $N$ is the number of values in the estimation data set.
- $e(t)$ is a ny-by- 1 vector of prediction errors.
- $\theta_{N}$ represents the estimated parameters.
- $d$ is the number of estimated parameters.

If number of parameters exceeds the number of samples, FPE is not computed when model estimation is performed (model. Report. FPE is empty). The fpe command returns NaN.

## Tips

- The software computes and stores the FPE value during model estimation. If you want to access this value, see the Report. Fit. FPE property of the model.


## References

[1] Ljung, L. System Identification: Theory for the User, Upper Saddle River, NJ, Prentice-Hall PTR, 1999. See sections 7.4 and 16.4.

## See Also

aic|goodness0fFit

## Topics

"Loss Function and Model Quality Metrics"
Introduced before R2006a

## frdata

Access data for frequency response data (FRD) object

## Syntax

[response,freq] = frdata(sys)
[response,freq, covresp] = frdata(sys)
[response,freq,Ts,covresp] = frdata(sys,'v')
[response,freq,Ts] = frdata(sys)

## Description

[response,freq] = frdata(sys) returns the response data and frequency samples of the FRD model sys. For an FRD model with Ny outputs and Nu inputs at Nf frequencies:

- response is an Ny-by-Nu-by-Nf multidimensional array where the ( $i, j$ ) entry specifies the response from input $j$ to output $i$.
- freq is a column vector of length $N f$ that contains the frequency samples of the FRD model.

See the frd reference page for more information on the data format for FRD response data.
[response,freq, covresp] = frdata(sys) also returns the covariance covresp of the response data resp for idfrd model sys. The covariance covresp is a 5D-array where $\operatorname{covH}(i, j, k,:,:)$ contains the 2 -by-2 covariance matrix of the response resp $(i, j, k)$. The ( 1,1 ) element is the variance of the real part, the $(2,2)$ element the variance of the imaginary part and the $(1,2)$ and $(2,1)$ elements the covariance between the real and imaginary parts.

For SISO FRD models, the syntax

```
[response,freq] = frdata(sys,'v')
```

forces frdata to return the response data as a column vector rather than a 3-dimensional array (see example below). Similarly
[response,freq,Ts,covresp] = frdata(sys,'v') for an IDFRD model sys returns covresp as a 3-dimensional rather than a 5 -dimensional array.
[response,freq,Ts] = frdata(sys) also returns the sample time Ts.
Other properties of sys can be accessed with get or by direct structure-like referencing (e.g., sys.Frequency).

## Arguments

The input argument sys to frdata must be an FRD model.

## Examples

## Extract Data from Frequency Response Data Model

Create a frequency response data model by computing the response of a transfer function on a grid of frequencies.
$H=\operatorname{tf}([-1.2,-2.4,-1.5],[1,20,9.1])$;
w = logspace(-2,3,101);
sys = frd(H,w);
sys is a SISO frequency response data ( frd ) model containing the frequency response at 101 frequencies.

Extract the frequency response data from sys.
[response,freq] = frdata(sys);
response is a 1-by-1-by-101 array. response ( $1,1, k$ ) is the complex frequency response at the frequency freq(k).

## See Also

frd | freqresp | get | idfrd | set | spectrum

## Introduced before R2006a

## freqresp

Frequency response over grid

## Syntax

[H, wout] = freqresp(sys)
H = freqresp(sys,w)
H = freqresp(sys,w, units)
[H, wout, covH] = freqresp(idsys,...)

## Description

[H,wout] = freqresp(sys) returns the frequency response on page 1-409 of the dynamic system model sys at frequencies wout. The freqresp command automatically determines the frequencies based on the dynamics of sys.
$\mathrm{H}=\mathrm{freqresp}(\mathrm{sys}, \mathrm{w})$ returns the frequency response on page 1-409 on the real frequency grid specified by the vector $w$.
$H=$ freqresp(sys,w,units) explicitly specifies the frequency units of $w$ with units.
[H, wout, covH] = freqresp(idsys, ...) also returns the covariance covH of the frequency response of the identified model idsys.

## Input Arguments

## sys

Any dynamic system model or model array.

## w

Vector of real frequencies at which to evaluate the frequency response. Specify frequencies in units of rad/TimeUnit, where TimeUnit is the time units specified in the TimeUnit property of sys.

## units

Units of the frequencies in the input frequency vector $w$, specified as one of the following values:

- 'rad/TimeUnit' - radians per the time unit specified in the TimeUnit property of sys
- 'cycles/TimeUnit ' - cycles per the time unit specified in the TimeUnit property of sys
- 'rad/s'
- 'Hz'
- 'kHz'
- 'MHz'
- 'GHz'
- 'rpm'

Default: 'rad/TimeUnit'
idsys
Any identified model.

## Output Arguments

## H

Array containing the frequency response values.
If sys is an individual dynamic system model having Ny outputs and Nu inputs, H is a 3D array with dimensions Ny -by-Nu-by-Nw, where Nw is the number of frequency points. Thus, $\mathrm{H}(:,:, k)$ is the response at the frequency $w(k)$ or wout ( $k$ ).

If sys is a model array of size [ $\mathrm{Ny} \mathrm{Nu} \mathrm{S} 1 \ldots \mathrm{Sn}$ ], H is an array with dimensions Ny -by-Nu-by-Nw-by-S1-by-...-by-Sn] array.

If sys is a frequency response data model (such as $f r d$, genfrd, or idfrd), freqresp(sys,w) evaluates to NaN for values of $w$ falling outside the frequency interval defined by sys.frequency. The freqresp command can interpolate between frequencies in sys.frequency. However, freqresp cannot extrapolate beyond the frequency interval defined by sys.frequency.

## wout

Vector of frequencies corresponding to the frequency response values in H. If you omit w from the inputs to freqresp, the command automatically determines the frequencies of wout based on the system dynamics. If you specify $w$, then wout $=w$

## covH

Covariance of the response $H$. The covariance is a 5D array where $\operatorname{covH}(i, j, k,:,:)$ contains the 2-by-2 covariance matrix of the response from the ith input to the $j$ th output at frequency $w(k)$. The $(1,1)$ element of this 2 -by-2 matrix is the variance of the real part of the response. The $(2,2)$ element is the variance of the imaginary part. The $(1,2)$ and $(2,1)$ elements are the covariance between the real and imaginary parts of the response.

## Examples

## Compute Frequency Response of System

Create the following 2 -input, 2 -output system:

$$
\text { sys }=\left[\begin{array}{cc}
0 & \frac{1}{s+1} \\
\frac{s-1}{s+2} & 1
\end{array}\right]
$$

sys11 = 0;
sys22 = 1;
sys12 = tf(1,[lll);

```
sys21 = tf([1 -1],[1 2]);
sys = [sys11,sys12;sys21,sys22];
```

Compute the frequency response of the system.
[H,wout] = freqresp(sys);
H is a 2-by-2-by-45 array. Each entry $\mathrm{H}(:,:, \mathrm{k})$ in H is a 2-by-2 matrix giving the complex frequency response of all input-output pairs of sys at the corresponding frequency wout(k). The 45 frequencies in wout are automatically selected based on the dynamics of sys.

## Compute Frequency Response on Specified Frequency Grid

Create the following 2-input, 2-output system:

$$
\text { sys }=\left[\begin{array}{cc}
0 & \frac{1}{s+1} \\
\frac{s-1}{s+2} & 1
\end{array}\right]
$$

sys11 = 0;
sys22 = 1;
sys12 = tf(1,[11]);
sys21 = tf([1-1],[1 2]);
sys $=$ [sys 11, sys $12 ;$ sys 21, sys 22$]$;
Create a logarithmically-spaced grid of 200 frequency points between 10 and 100 radians per second. $\mathrm{w}=$ logspace $(1,2,200)$;

Compute the frequency response of the system on the specified frequency grid.

```
H = freqresp(sys,w);
```

H is a 2-by-2-by-200 array. Each entry $\mathrm{H}(:,:, \mathrm{k})$ in H is a 2-by-2 matrix giving the complex frequency response of all input-output pairs of sys at the corresponding frequency $\mathrm{w}(\mathrm{k})$.

## Compute Frequency Response and Associated Covariance

Compute the frequency response and associated covariance for an identified process model at its peak response frequency.

Load estimation data $z 1$.
load iddatal z1
Estimate a SISO process model using the data.
model = procest(z1,'P2UZ');
Compute the frequency at which the model achieves the peak frequency response gain. To get a more accurate result, specify a tolerance value of 1e-6.

```
[gpeak,fpeak] = getPeakGain(model,1e-6);
```

Compute the frequency response and associated covariance for model at its peak response frequency.

```
[H,wout,covH] = freqresp(model,fpeak);
```

$H$ is the response value at fpeak frequency, and wout is the same as fpeak.
covH is a 5-dimensional array that contains the covariance matrix of the response from the input to the output at frequency fpeak. Here $\operatorname{covH}(1,1,1,1,1)$ is the variance of the real part of the response, and $\operatorname{covH}(1,1,1,2,2)$ is the variance of the imaginary part. The $\operatorname{covH}(1,1,1,1,2)$ and $\operatorname{covH}(1,1,1,2,1)$ elements are the covariance between the real and imaginary parts of the response.

## More About

## Frequency Response

In continuous time, the frequency response at a frequency $\omega$ is the transfer function value at $s=j \omega$. For state-space models, this value is given by

$$
H(j \omega)=D+C(j \omega I-A)^{-1} B
$$

In discrete time, the frequency response is the transfer function evaluated at points on the unit circle that correspond to the real frequencies. freqresp maps the real frequencies $w(1), \ldots, w(N)$ to points on the unit circle using the transformation $z=e^{j \omega T_{s}} . T_{s}$ is the sample time. The function returns the values of the transfer function at the resulting $z$ values. For models with unspecified sample time, freqresp uses $T_{s}=1$.

## Algorithms

For transfer functions or zero-pole-gain models, freqresp evaluates the numerator(s) and denominator(s) at the specified frequency points. For continuous-time state-space models $(A, B, C, D)$, the frequency response is

$$
D+C(j \omega-A)^{-1} B, \omega=\omega_{1}, \ldots, \omega_{N}
$$

For efficiency, $A$ is reduced to upper Hessenberg form and the linear equation $(j \omega-A) X=B$ is solved at each frequency point, taking advantage of the Hessenberg structure. The reduction to Hessenberg form provides a good compromise between efficiency and reliability. See [1] for more details on this technique.

## Alternatives

Use evalfr to evaluate the frequency response at individual frequencies or small numbers of frequencies. freqresp is optimized for medium-to-large vectors of frequencies.

## References

[1] Laub, A.J., "Efficient Multivariable Frequency Response Computations," IEEE Transactions on Automatic Control, AC-26 (1981), pp. 407-408.

## See Also

bode |evalfr|interp|nichols|nyquist| sigma| spectrum
Introduced before R2006a

## fselect

Select frequency points or range in FRD model

## Syntax

```
subsys = fselect(sys,fmin,fmax)
subsys = fselect(sys,index)
```


## Description

subsys = fselect(sys,fmin,fmax) takes an FRD model sys and selects the portion of the frequency response between the frequencies fmin and fmax. The selected range [fmin, fmax] should be expressed in the FRD model units. For an IDFRD model, the SpectrumData, CovarianceData and NoiseCovariance values, if non-empty, are also selected in the chosen range.
subsys = fselect(sys,index) selects the frequency points specified by the vector of indices index. The resulting frequency grid is
sys.Frequency(index)

## See Also

fcat|fdel|frd|idfrd|interp
Introduced before R2006a

## get

Access model property values

## Syntax

Value = get(sys,'PropertyName')
Struct = get(sys)

## Description

Value = get(sys,'PropertyName') returns the current value of the property PropertyName of the model object sys. 'PropertyName' can be the full property name (for example, 'UserData') or any unambiguous case-insensitive abbreviation (for example, 'user'). See reference pages for the individual model object types for a list of properties available for that model.

Struct $=$ get (sys) converts the TF, SS, or ZPK object sys into a standard MATLAB structure with the property names as field names and the property values as field values.

Without left-side argument,

```
get(sys)
```

displays all properties of sys and their values.

## Examples

## Display Model Property Values

Create the following discrete-time SISO transfer function model:

$$
H(z)=\frac{1}{z+2}
$$

Specify the sample time as 0.1 seconds and input channel name as Voltage.

```
h = tf(1,[1 2],0.1,'InputName','Voltage')
h =
    From input "Voltage" to output:
        1
    ----
    z + 2
```

Sample time: 0.1 seconds
Discrete-time transfer function.

Display all the properties of the transfer function.
get (h)

```
        Numerator: {[[0 1]}
Denominator: {[1 2]}
            Variable: 'z'
                IODelay: 0
        InputDelay: 0
OutputDelay: 0
                    Ts: 0.1000
            TimeUnit: 'seconds'
            InputName: {'Voltage'}
            InputUnit: {''}
    InputGroup: [1x1 struct]
    OutputName: {''}
    OutputUnit: {''}
OutputGroup: [1x1 struct]
            Notes: [0x1 string]
    UserData: []
            Name: '1
SamplingGrid: [1x1 struct]
```

Display the numerator of the transfer function.

```
num = get(h,'Numerator')
num = 1x1 cell array
    {[0 1]}
```

The numerator data is stored as a cell array, thus the Numerator property is a cell array containing the row vector [01].

```
num{1}
```

ans $=1 \times 2$
$0 \quad 1$

Display the sample time Ts of the transfer function.
get(h,'Ts')
ans $=0.1000$
Alternatively, use dot notation to access the property value.
h.Ts
ans $=0.1000$

## Tips

An alternative to the syntax
Value = get(sys,'PropertyName')
is the structure-like referencing

```
Value = sys.PropertyName
```

For example,
sys.Ts
sys.A
sys.user
return the values of the sample time, $A$ matrix, and UserData property of the (state-space) model sys.

## See Also <br> frdata|getcov|getpvec|idssdata|polydata|set|ssdata|tfdata Introduced before R2006a

## getcov

Parameter covariance of identified model

## Syntax

```
cov_data = getcov(sys)
cov_data = getcov(sys,cov_type)
cov_data = getcov(sys,cov_type,'free')
```


## Description

cov_data $=$ getcov(sys) returns the raw covariance of the parameters of an identified model.

- If sys is a single model, then cov_data is an $n p$-by-np matrix. $n p$ is the number of parameters of sys.
- If sys is a model array, then cov_data is a cell array of size equal to the array size of sys. cov_data(i,j,k,...) contains the covariance data for sys(:,:,i,j,k,...).
cov_data = getcov(sys,cov_type) returns the parameter covariance as either a matrix or a structure, depending on the covariance type that is specified.
cov_data $=$ getcov(sys,cov_type,'free') returns the covariance data of only the free model parameters.


## Examples

## Obtain Raw Parameter Covariance for Identified Model

Obtain the identified model.

```
load iddatal z1
sys = tfest(z1,2);
```

Get the raw parameter covariance for the model.

```
cov_data = getcov(sys)
cov_data = 5×5
\begin{tabular}{rrrrr}
1.2131 & -4.3949 & -0.0309 & -0.5531 & 0 \\
-4.3949 & 115.0838 & 1.8598 & 10.6660 & 0 \\
-0.0309 & 1.8598 & 0.0636 & 0.1672 & 0 \\
-0.5531 & 10.6660 & 0.1672 & 1.2433 & 0 \\
0 & 0 & 0 & 0 & 0
\end{tabular}
```

sys.Denominator(1) is fixed to 1 and not treated as a parameter. The covariance matrix entries corresponding to the delay parameter (fifth row and column) are zero because the delay was not estimated.

## Obtain Raw Parameter Covariance for Identified Model Array

Obtain the identified model array.

```
load iddatal z1;
sys1 = tfest(z1,2);
sys2 = tfest(z1,3);
sysarr = stack(1,sys1,sys2);
```

sysarr is a 2-by-1 array of continuous-time, identified transfer functions.
Get the raw parameter covariance for the models in the array.

```
cov_data = getcov(sysarr)
cov_data=2\times1 cell array
    {5x5 double}
    {7x7 double}
```

cov_data is a 2-by-1 cell array. cov_data\{1\} and cov_data\{2\} are the raw parameter covariance matrices for sys1 and sys2.

## Obtain Raw Covariance of Estimated Parameters of Identified Model

Load the estimation data.
load iddatal z1
z1.y = cumsum(z1.y);
Estimate the model.

```
init_sys = idtf([100 1500],[1 10 10 0]);
init_sys.Structure.Numerator.Minimum = eps;
init_sys.Structure.Denominator.Minimum = eps;
init_sys.Structure.Denominator.Free(end) = false;
opt = tfestOptions('SearchMethod','lm');
sys = tfest(z1,init_sys,opt);
```

sys is an idtf model with six parameters, four of which are estimated.
Get the covariance matrix for the estimated parameters.

```
cov_type = 'value';
cov_data = getcov(sys,cov_type,'free')
cov_data = 4×4
105
```

| 0.0269 | -0.1237 | -0.0001 | -0.0017 |
| ---: | ---: | ---: | ---: |
| -0.1237 | 1.0221 | 0.0016 | 0.0133 |
| -0.0001 | 0.0016 | 0.0000 | 0.0000 |
| -0.0017 | 0.0133 | 0.0000 | 0.0002 |

cov_data is a $4 \times 4$ covariance matrix, with entries corresponding to the four estimated parameters.

## Obtain Factored Parameter Covariance for Identified Model

Obtain the identified model.

```
load iddatal z1
sys = tfest(z1,2);
```

Get the factored parameter covariance for the model.

```
cov_type = 'factors';
cov_data = getcov(sys,cov_type);
```


## Obtain Factored Parameter Covariance for Identified Model Array

Obtain the identified model array.

```
load iddata1 z1
sys1 = tfest(z1,2);
sys2 = tfest(z1,3);
sysarr = stack(1,sys1,sys2);
```

sysarr is a 2-by-1 array of continuous-time, identified transfer functions.
Get the factored parameter covariance for the models in the array.

```
cov_type = 'factors';
cov_data = getcov(sysarr,cov_type)
cov_data=2\times1 struct array with fields:
    R
    T
    Free
```

cov_data is a 2-by-1 structure array. cov_data (1) and cov_data(2) are the factored covariance structures for sys1 and sys2.

## Obtain Factored Covariance of Estimated Parameters of Identified Model

Load the estimation data.
load iddatal z1
z1.y = cumsum(z1.y);

Estimate the model.

```
init sys = idtf([100 1500],[1 10 10 0]);
init_sys.Structure.Numerator.Minimum = eps;
init_sys.Structure.Denominator.Minimum = eps;
init_sys.Structure.Denominator.Free(end) = false;
opt = tfestOptions('SearchMethod','lm');
sys = tfest(zl,init_sys,opt);
```

sys, an idtf model, has six parameters, four of which are estimated.
Get the factored covariance for the estimated parameters.

```
cov_type = 'factors';
cov_data = getcov(sys,cov_type,'free');
```


## Input Arguments

## sys - Identified model

idtf, idss, idgrey, idpoly, idproc, idnlarx, idnlhw, or idnlgrey object | model array
Identified model, specified as an idtf, idss, idgrey, idpoly, idproc, idnlarx, idnlhw, or idnlgrey model or an array of such models.

The getcov command returns cov_data as [] for idnlarx and idnlhw models because these models do not store parameter covariance data.

## cov_type - Covariance type <br> 'vāue' (default)|'factors'

Covariance return type, specified as either 'value' or 'factors'.

- If cov_type is 'value', then cov_data is returned as a matrix (raw covariance).
- If cov_type is 'factors', then cov_data is returned as a structure containing the factors of the covariance matrix.

Use this option for fetching the covariance data if the covariance matrix contains nonfinite values, is not positive definite, or is ill conditioned. You can calculate the response uncertainty using the covariance factors instead of the numerically disadvantageous covariance matrix.

This option does not offer a numerical advantage in the following cases:

- sys is estimated using certain instrument variable methods, such as iv4.
- You have explicitly specified the parameter covariance of sys using the deprecated CovarianceMatrix model property.

Data Types: char

## Output Arguments

## cov_data - Parameter covariance of sys

matrix or cell array of matrices | structure or cell array of structures

Parameter covariance of sys, returned as a matrix, cell array of matrices, structure, or cell array of structures. cov_data is [] for idnlarx and idnlhw models.

- If sys is a single model and cov_type is 'value', then cov_data is an $n p-b y-n p$ matrix. $n p$ is the number of parameters of sys.

The value of the nonzero elements of this matrix is equal to sys.Report.Parameters.FreeParCovariance when sys is obtained via estimation. The row and column entries that correspond to fixed parameters are zero.

- If sys is a single model and cov_type is 'factors', then cov_data is a structure with fields:
- $\quad \mathrm{R}$ - Usually an upper triangular matrix.
- T-Transformation matrix.
- Free - Logical vector of length $n p$, indicating if a model parameter is free (estimated) or not. $n p$ is the number of parameters of sys.

To obtain the covariance matrix using the factored form, enter:

```
Free = cov_factored.Free;
T = cov factored.T;
R = cov_factored.R;
np = nparams(sys);
cov_matrix = zeros(np);
cov_matrix(Free, Free) = T*inv(R'*R)*T';
```

For numerical accuracy, calculate $T^{*} \operatorname{inv}\left(R^{\prime} * R\right) * T^{\prime}$ as $X^{*} X^{\prime}$, where $X=T / R$.

- If sys is a model array, then cov_data is a cell array of size equal to the array size of sys. cov_data(i,j,k,...) contains the covariance data for sys(:, : i, j,k,...).


## See Also

getpvec|nparams|rsample|setcov|sim|simsd

## Topics

"What Is Model Covariance?"
"Types of Model Uncertainty Information"

## Introduced in R2012a

## getDelayInfo

Get input/output delay information for idnlarx model structure

## Syntax

DELAYS = getDelayInfo(MODEL)
DELAYS = getDelayInfo(MODEL,TYPE)

## Description

DELAYS = getDelayInfo(MODEL) obtains the maximum delay in each input and output variable of an idnlarx model.

DELAYS = getDelayInfo(MODEL,TYPE) lets you choose between obtaining maximum delays across all input and output variables or maximum delays for each output variable individually. When delays are obtained for each output variable individually a matrix is returned, where each row is a vector containing $n_{y}+n_{u}$ maximum delays for each output variable, and:

- $n_{y}$ is the number of outputs of MODEL.
- $n_{u}$ is the number of inputs of MODEL.

Delay information is useful for determining the number of states in the model. For nonlinear ARX models, the states are related to the set of delayed input and output variables that define the model structure (regressors). For example, if an input or output variable $p$ has a maximum delay of $D$ samples, then it contributes $D$ elements to the state vector:
$p(t-1), p(t-2), \ldots p(t-D)$
The number of states of a nonlinear ARX model equals the sum of the maximum delays of each input and output variable. For more information about the definition of states for idnlarx models, see "Definition of idnlarx States" on page 1-571

## Input Arguments

getDelayInfo accepts the following arguments:

- MODEL: idnlarx model.
- TYPE: (Optional) Specifies whether to obtain channel delays ' channelwise' or 'all' as follows:
- 'all ': Default value. DELAYS contains the maximum delays across each output (vector of $n_{y}$ $+n_{u}$ entries, where [ny, nu] = size(MODEL)).
- 'channelwise ': DELAYS contains delay values separated for each output $\left(n_{y}-\right.$ by- $\left(n_{y}+n_{u}\right)$ matrix).


## Output Arguments

- DELAYS: Contains delay information in a vector of length $n_{y}+n_{u}$ arranged with output channels preceding the input channels, i.e., $[y 1, y 2, \ldots, u 1, u 2, .$.$] .$


## Examples

## Get Input and Output Delay Information for Nonlinear ARX Model

Create a two-output, three-input nonlinear ARX model.
$M=i d n l a r x\left([20221100 ; 10150112], ' l i n e a r^{\prime}\right) ;$
Compute the maximum delays for each output variable individually.

```
Del = getDelayInfo(M,'channelwise')
```

Del $=2 \times 5$

| 2 | 0 | 2 | 1 | 0 |
| :--- | :--- | :--- | :--- | :--- |
| 1 | 0 | 1 | 5 | 0 |

The matrix Del contains the maximum delays for the first and second output of model M. You can interpret the contents of matrix Del as follows:

- In the dynamics for output $1\left(y_{1}\right)$, the maximum delays in channels $y_{1}, y_{2}, u_{1}, u_{2}, u_{3}$ are $2,0,2,1$, and 0 respectively.
- Similarly, in the dynamics for output $2\left(y_{2}\right)$ of the model, the maximum delays in channels $y_{1}, y_{2}$, $u_{1}, u_{2}, u_{3}$ are $1,0,1,5$, and 0 respectively.

Find maximum delays for all the input and output variables in the order $y_{1}, y_{2}, u_{1}, u_{2}, u_{3}$.

```
Del = getDelayInfo(M,'all')
Del = 1\times5
    2 0 2 5 5 0
```

Note, The maximum delay across all output equations can be obtained by executing MaxDel = $\max (\operatorname{Del},[], 1)$. Since input $u_{2}$ has 5 delays (the fourth entry in Del), there are 5 terms corresponding to $u_{2}$ in the state vector. Applying this definition to all I/O channels, the complete state vector for model $M$ becomes:

$$
X(t)=\left[y_{1}(t-1), y_{1}(t-2), u_{1}(t-1), u_{1}(t-2), u_{2}(t-1), u_{2}(t-2), u_{2}(t-3), u_{2}(t-4), u_{2}(t-5)\right]
$$

## See Also

data2state| getreg|idnlarx
Introduced in R2008b

## getexp

Specific experiments from multiple-experiment data set

## Syntax

```
d1 = getexp(data,ExperimentNumber)
d1 = getexp(data,ExperimentName)
```


## Description

d1 = getexp(data,ExperimentNumber) retrieves specific experiments from multiple-experiment data set. data is an iddata object that contains several experiments. d1 is another iddata object containing the indicated experiment(s). ExperimentNumber is the experiment number as in d1 = getexp(data,3) or d1 = getexp(data,[4 2]).
d1 = getexp(data,ExperimentName) specifies the experiment name as in d1 = getexp(data,'Period1') or d1 = getexp(data,\{'Day1','Day3'\}).

See merge (iddata) and iddata for how to create multiple-experiment data objects.
You can also retrieve the experiments using a fourth subscript, as in d1 = data(:,:,:,ExperimentNumber). Type help iddata/subsref for details on this.

Introduced before R2006a

## getinit

Values of idnlgrey model initial states

## Syntax

getinit(model)
getinit(model, prop)

## Arguments

model
Name of the idnlgrey model object.
Property
Name of the InitialStates model property field, such as 'Name', 'Unit', 'Value',
'Minimum', 'Maximum', and 'Fixed'.
Default: 'Value'.

## Description

getinit (model) gets the initial-state values in the 'Value' field of the InitialStates model property.
getinit (model, prop) gets the initial-state values of the prop field of the InitialStates model property. prop can be 'Name', 'Unit', 'Value', 'Minimum', 'Maximum', and 'Fixed'.

The returned values are an $N x$-by- 1 cell array of values, where $N x$ is the number of states.

## See Also

getpar|idnlgrey|setinit|setpar
Introduced in R2007a

## getoptions

Return @PlotOptions handle or plot options property

## Syntax

$\mathrm{p}=$ getoptions(h)
$\mathrm{p}=$ getoptions(h, propertyname)

## Description

$\mathrm{p}=$ getoptions(h) returns the plot options handle associated with plot handle $\mathrm{h} . \mathrm{p}$ contains all the settable options for a given response plot.
$\mathrm{p}=$ getoptions( $\mathrm{h}, \mathrm{propertyname)} \mathrm{returns} \mathrm{the} \mathrm{specified} \mathrm{options} \mathrm{property}, \mathrm{propertyname}$, the plot with handle $h$. You can use this to interrogate a plot handle. For example,
p = getoptions(h,'Grid')
returns 'on' if a grid is visible, and 'off' when it is not.
For a list of the properties and values available for each plot type, see "Properties and Values Reference" (Control System Toolbox).

## See Also

setoptions

Introduced in R2012a

## getpar

Obtain attributes such as values and bounds of linear model parameters

## Syntax

```
value = getpar(sys,'value')
free = getpar(sys,'free')
bounds = getpar(sys,'bounds')
label = getpar(sys,'label')
getpar(sys)
```


## Description

value = getpar(sys,'value') returns the parameter values of the model sys. If sys is a model array, the returned value is a cell array of size equal to the model array.
free = getpar(sys,'free') returns the free or fixed status of the parameters.
bounds = getpar(sys,'bounds') returns the minimum and maximum bounds on the parameters.
label = getpar(sys,'label') returns the labels for the parameters.
getpar(sys) prints a table of parameter values, labels, free status and minimum and maximum bounds.

## Examples

## Get Parameter Values

Get the parameter values of an estimated ARMAX model.
Estimate an ARMAX model.

```
load iddata8
init_data = z8(1:100);
na = 1;
nb = [1 1 1];
nc = 1;
nk = [0 0 0];
sys = armax(init_data,[na nb nc nk]);
```

Get the parameter values.

```
val = getpar(sys,'value')
val = 5×1
    -0.7519
    -0.4341
        0.4442
        0.0119
```

To set parameter values, use sys = setpar(sys,'value', value).

## Get Free Parameters and Their Bounds

Get the free parameters and their bounds for a process model.
Construct a process model, and set its parameter values and free status.

```
m = idproc('P2DUZI');
m.Kp = 1;
m.Tw = 100;
m.Zeta = .3;
m.Tz = 10;
m.Td = 0.4;
m.Structure.Td.Free = 0;
```

Here, the value of Td is fixed.
Get the parameter values.

```
Val = getpar(m,'Value')
Val = 5 <1
    1.0000
    100.0000
        0.3000
        0.4000
    10.0000
```

Get the free statuses of the parameters.
Free $=$ getpar(m,'Free')
Free $=5 x 1$ logical array
1
1
1
0
1

The output indicates that Td is a fixed parameter and the remaining parameters are free.
Get the default bounds on the parameters.

```
MinMax = getpar(m,'bounds')
MinMax = 5×2
    -Inf Inf
```

```
    0 Inf
    0 Inf
    0 Inf
-Inf Inf
```

Extract the values of the free parameters.

```
FreeValues = Val(Free)
FreeValues = 4×1
    1.0000
    100.0000
    0.3000
    10.0000
```

Extract the bounds on the free parameters.

```
FreeValBounds = MinMax(Free,:)
FreeValBounds = 4×2
    -Inf Inf
            0 Inf
            0 Inf
    -Inf Inf
```


## Get Parameter Labels

Get the parameter labels of an estimated ARMAX model.

Estimate an ARMAX model.

```
load iddata8;
init_data = z8(1:100);
na = 1;
nb = [1 1 1];
nc = 1;
nk = [0 0 0];
sys = armax(init_data,[na nb nc nk]);
```

Assign parameter labels.
sys.Structure.A.Info(2).Label = 'a2';
Get the parameter labels.

```
label = getpar(sys,'label')
label = 5x1 cell
    {'a2' }
    {0x0 char}
    {0x0 char}
```

```
{0x0 char}
{0x0 char}
```


## Obtain a Table of Model Parameter Attributes

Obtain a table of all model parameter attributes of an ARMAX model.
Estimate an ARMAX model.

```
load iddata8;
init_data = z8(1:100);
na = 4;
nb = [3 2 3];
nc = 2;
nk = [0 0 0];
sys = armax(init_data,[na nb nc nk]);
```

Get all parameter attributes.

```
getpar(sys)
```

| \# | Label | Value | Free | Min. | Max. |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1. |  | -1.4328 | 1 | - Inf | Inf |
| 2. |  | 0.497 | 1 | - Inf | Inf |
| 3. |  | 0.22904 | 1 | - Inf | Inf |
| 4. |  | -0.09849 | 1 | - Inf | Inf |
| 5. |  | -0.10246 | 1 | - Inf | Inf |
| 6. |  | 1.1671 | 1 | - Inf | Inf |
| 7. |  | 0.39579 | 1 | - Inf | Inf |
| 8. |  | 0.97219 | 1 | - Inf | Inf |
| 9. |  | 0.026995 | 1 | - Inf | Inf |
| 10. |  | -0.17113 | 1 | - Inf | Inf |
| 11. |  | 0.16155 | 1 | - Inf | Inf |
| 12. |  | 0.48468 | 1 | - Inf | Inf |
| 13. |  | -1.8871 | 1 | - Inf | Inf |
| 14. |  | 0.97391 | 1 | - Inf | Inf |

## Input Arguments

sys - Identified linear model
idss | idproc | idgrey | idtf | idpoly | array of model objects
Identified linear model, specified as an idss, idpoly, idgrey, idtf, or idfrd model object or an array of model objects.

## Output Arguments

value - Parameter values
vector of doubles

Parameter values, returned as a double vector of length nparams (sys).

## free - Free or fixed status of parameters

vector of logical values
Free or fixed status of parameters, returned as a logical vector of length nparams (sys).

## bounds - Minimum and maximum bounds on parameters

matrix of doubles
Minimum and maximum bounds on parameters, returned as a double matrix of size nparams (sys)-by- 2 . The first column contains the minimum bound, and the second column the maximum bound.

## label - Parameter labels

cell array of character vectors
Parameter labels, returned as a cell array of character vectors of length nparams ( sys). For example, \{'a2', 'a3'\}, if nparams (sys) is two.

## See Also

getcov | getpvec|idssdata|polydata| setpar|tfdata

## Introduced in R2013b

## getpar

Parameter values and properties of idnlgrey model parameters

## Syntax

getpar(model)
getpar(model, prop)

## Arguments

model
Name of the idnlgrey model object.
Property
Name of the Parameters model property field, such as 'Name', 'Unit', 'Value', 'Minimum',
'Maximum', or 'Fixed'.
Default: 'Value'.

## Description

getpar(model) gets the model parameter values in the 'Value' field of the Parameters model property.
getpar(model, prop) gets the model parameter values in the prop field of the Parameters model property. prop can be 'Name', 'Unit', 'Value', 'Minimum', and 'Maximum'.

The returned values are an Np -by- 1 cell array of values, where Np is the number of parameters.

## See Also

getinit|getpvec|idnlgrey|setinit|setpar
Introduced in R2007a

## getpvec

Obtain model parameters and associated uncertainty data

## Syntax

```
pvec = getpvec(sys)
[pvec,pvec_sd] = getpvec(sys)
[___] = getpvec(sys,'free')
```


## Description

pvec $=$ getpvec(sys) returns a vector, pvec, containing the values of all the parameters of the identified model sys.
[pvec,pvec_sd] = getpvec(sys) also returns the 1 standard deviation value of the uncertainty associated with the parameters of sys. If the model covariance information for sys is not available, pvec_sd is [].
[___] = getpvec(sys,'free') returns data for only the free parameters of sys, using any of the output arguments in previous syntaxes. For idnlarx and idnlhw models, all parameters are treated as free.

## Input Arguments

## sys

Identified model.
Identified model, specified as an idtf, idss, idgrey, idpoly, idproc, idnlarx, idnlhw, or idnlgrey model or an array of such models.

## Output Arguments

## pvec

Values of the parameters of sys.
If sys is an array of models, then pvec is a cell array with parameter value vectors corresponding to each model in sys. pvec is [] for idnlarx and idnlhw models that have not been estimated.
pvec_sd
1 standard deviation value of the parameters of sys.
If the model covariance information for sys is not available, pvec_sd is []. Thus, pvec_sd is always [] for idnlarx and idnlhw models because these models do not store parameter covariance information.

If sys is an array of models, then pvec_sd is a cell array with standard deviation vectors corresponding to each model in sys.

## Examples

## Retrieve Parameter Values from Estimated Model

Load the estimation data.
load iddatal z1;
Estimate a transfer function model.
sys = tfest(z1,3);
Retrieve the parameter values from the estimated model.
pvec $=$ getpvec(sys);

## Retrieve Parameter Values and Standard Deviations from Estimated Model

Load the estimation data
load iddata2 z2;
Estimate a state-space model.
sys = ssest(z2,3);
Retrieve the model parameters, pvec, and associated standard deviations, pvec_sd, from the estimated model.
[pvec, pvec_sd] = getpvec(sys);

## Retrieve Values of Free Parameters from Estimated Model

Load the estimation data.
load iddata2 z2;
Estimate a state-space model.
sys = ssest(z2,3);
Retrieve the values of the free parameters from the estimated model.
pvec = getpvec(sys,'free');

## See Also

getcov|idssdata|setpvec|tfdata|zpkdata
Introduced in R2012a

## getreg

Regressor expressions and numerical values in nonlinear ARX model

## Syntax

```
Rs = getreg(model)
Rm = getreg(model,data)
Rm = getreg(model,data,init)
Rm = getreg(___,'Type',regressorType)
```


## Description

Rs = getreg(model) returns expressions for computing regressors in the nonlinear ARX model. model is an idnlarx object. A typical use of the regression matrices built by getreg is to generate input data when you want to evaluate the output of a mapping function such as wavenet using evaluate. For example, the following pair of commands evaluates the output of a mapping function model.

Regressor Value = getreg(model,data,'z')
y = evaluāte(model.OutputFcn,RegressorValue)
These commands are equivalent to the command:

```
y = predict(model,data,1,predictOptions('InitialCondition','z'))
```

$\mathrm{Rm}=$ getreg(model, data) returns regressor values as a timetable for the specified input/ output data set data.

Rm = getreg(model, data,init) uses the initial conditions that are specified in init. The first $N$ rows of each regressor matrix depend on the initial states init, where $N$ is the maximum delay in the regressors (see getDelayInfo).

Rm = getreg (__, 'Type', regressorType) returns the names of the regressors of the specified regressorType. For example, use the command Rm = getreg(model,'Type', 'input') to return the names of only the input regressors.

## Input Arguments

## data

iddata object containing measured data or numeric matrix that contains the values of the output and input variables in the order [model.OutputName model.InputName].
init
Initial conditions of your data:

- 'z' (default) specifies zero initial state.
- NaN denotes unknown initial conditions.
- Real column vector containing the initial state values. For more information on initial states, see Definition of idnlarx States in idnlarx. For multiple-experiment data, this is a matrix where each column specifies the initial state of the model corresponding to that experiment.
- iddata object containing input and output samples at time instants before to the first sample in data. When the iddata object contains more samples than the maximum delay in the model, only the most recent samples are used. The number of samples required is equal to max(getDelayInfo(model)).
model
iddata object representing nonlinear ARX model.
regressorType
Type of regressor to return, specified as one of the following:
- 'all' (default) - All regressors
- 'input' - Only input regressors
- 'output' - Only output regressors
- 'standard ' - Only linear and polynomial regressors
- 'custom' - Only custom regressors


## Output Arguments

Rm
timetable of regressor values for all or a specified subset of regressors. Each column in Rm contains as many rows as there are data samples. For a model with nr regressors, Rm contains one column for each regressor. When data contains multiple experiments, Rm is a cell array where each element corresponds to a timetable of regressor values for an experiment.

Rs
Regressor expressions represented as a cell array of character vectors. For example, the expression ' $u 1(t-2)^{\prime}$ computes the regressor by delaying the input signal ul by two time samples. Similarly, the expression 'y2(t-1)' computes the regressor by delaying the output signal y2 by one time sample.

The order of regressors in Rs corresponds to regressor indices in the idnlarx object property model. RegressorUsage.

## Examples

## Get Regressor Expressions and Values, and Evaluate Predicted Model Output

Load sample data $u$ and $y$.

```
load twotankdata;
Ts = 0.2;
```

Sample time is 0.2 sec.
Create data object and use first 1000 samples for estimation.

```
z = iddata(y,u,Ts);
ze = z(1:1000);
```

Estimate nonlinear ARX model.

```
model = nlarx(ze,[3 2 1]);
```

Get regressor expressions.
Rs = getreg(model);

Get regressor values.

```
Rm = getreg(model,ze);
```

Evaluate model output for one-step-prediction.

```
Y = evaluate(model.OutputFcn,Rm.Variables);
```

The previous result is equivalent to.

```
Y_p = predict(model,ze,1,'z');
Y- = Y_p.OutputData;
```


## Compatibility Considerations

getreg returns regressor values in timetable
Behavior changed in R2021a
Starting in R2021a, getreg returns single-experiment data in a timetable of regressor values instead of a matrix or a cell array of values as in previous versions.

## See Also

customRegressor|evaluate|idnlarx|linearRegressor|polynomialRegressor

## Topics

"Identifying Nonlinear ARX Models"
Introduced in R2007a

## getStateEstimate

Extract best state estimate and covariance from particles

## Syntax

State = getStateEstimate(pf)
[State,StateCovariance] = getStateEstimate(pf)

## Description

State = getStateEstimate(pf) returns the best state estimate based on the current set of particles. The estimate is extracted based on the StateEstimationMethod property from the particleFilter object, pf.
[State, StateCovariance] = getStateEstimate(pf) also returns the covariance of the state estimate. The covariance is a measure of the uncertainty of the state estimate. Not all state estimation methods support covariance output. In this case, getStateEstimate returns StateCovariance as [].

The State and StateCovariance information can directly be accessed as properties of the particle filter object pf, as pf.State and pf.StateCovariance. However, when both these quantities are needed, using the getStateEstimation method with two output arguments is more computationally efficient.

## Examples

## State Estimation using Particle Filter

Create a particle filter, and set the state transition and measurement likelihood functions.

```
myPF = particleFilter(@vdpParticleFilterStateFcn,@vdpMeasurementLikelihoodFcn);
```

Initialize the particle filter at state [2; 0] with unit covariance, and use 1000 particles.

```
initialize(myPF, 1000, [2;0], eye(2));
```

Pick the mean state estimation and systematic resampling methods.

```
myPF.StateEstimationMethod = 'mean';
myPF.ResamplingMethod = 'systematic';
myPF
myPF =
    particleFilter with properties:
                NumStateVariables: 2
                    NumParticles: 1000
                StateTransitionFcn: @vdpParticleFilterStateFcn
        MeasurementLikelihoodFcn: @vdpMeasurementLikelihoodFcn
        IsStateVariableCircular: [0 0]
```

```
    ResamplingPolicy: [1x1 particleResamplingPolicy]
    ResamplingMethod: 'systematic'
StateEstimationMethod: 'mean'
    StateOrientation: 'column'
            Particles: [2x1000 double]
        Weights: [1x1000 double]
            State: 'Use the getStateEstimate function to see the value.'
StateCovariance: 'Use the getStateEstimate function to see the value.'
```

Assuming a measurement 2.1, run one predict and correct step.

```
[PredictedState, PredictedStateCovariance] = predict(myPF);
[CorrectedState, CorrectedStateCovariance] = correct(myPF,2.1);
```

Get the best state estimate and covariance based on the StateEstimationMethod property.
[State, StateCovariance] = getStateEstimate(myPF)
State = 2×1
2.1018
-0.1413

StateCovariance $=2 \times 2$
$0.0175-0.0096$
$-0.0096 \quad 0.5394$

## Input Arguments

## pf - Particle filter

particleFilter object
Particle filter, specified as an object. See particleFilter for more information.

## Output Arguments

## State - Best state estimate

[ ] (default) | vector
Best state estimate, defined as a vector based on the condition of the StateOrientation property:

- If StateOrientation is 'row' then State is a 1-by-NumStateVariables vector
- If StateOrientation is 'column' then State is a NumStateVariables-by-1 vector


## StateCovariance - Current estimate of state estimation error covariance

NumStateVariables-by-NumStateVariables array (default)|[]| array
Current estimate of state estimation error covariance, defined as an NumStateVariables-byNumStateVariables array. StateCovariance is calculated based on the StateEstimationMethod. If you specify a state estimation method that does not support covariance, then the function returns StateCovariance as [ ].

## See Also

correct|initialize|particleFilter|predict

## Topics

"Nonlinear State Estimation Using Unscented Kalman Filter and Particle Filter"
"What Is Online Estimation?"
Introduced in R2017b

## getTrend

Create trend information object to store offset, mean, and trend information for time-domain signals stored in iddata object

## Syntax

$\mathrm{T}=$ getTrend(data)
$\mathrm{T}=$ getTrend(data, 0 )
$\mathrm{T}=\operatorname{get} T \mathrm{rend}($ data, 1 )

## Description

$T=$ getTrend(data) constructs a TrendInfo object to store offset, mean, or linear trend information for detrending or retrending data. You can assign specific offset and slope values to T . You can then apply the trend information in T to either data or to other iddata objects by using detrend or retrend.

T = getTrend(data, 0) computes the means of input and output signals and stores them as the InputOffset and OutputOffset properties of T, respectively.
$\mathrm{T}=$ getTrend(data, 1 ) computes a best-fit straight line for both input and output signals and stores them as properties of T . The following code represents the straight line:

ULine $=$ Tr.InputOffset $+($ time-t0)*Tr.InputSlope
YLine $=$ Tr.OutputOffset $+($ time-t0)*Tr.OutputSlope
Here, time is equal to $Z$. SamplingInstants and t 0 is equal to data.Tstart.

## Examples

## Remove Offsets from Data

Remove a specified offset from input and output signals.
Load SISO data containing vectors u2 and y2.
load dryer2
Create a data object with a sample time of 0.08 seconds and plot it.
data = iddata(y2, u2, 0.08);
plot(data)

## Input-Output Data



The data has a nonzero mean value.
Store the data offset and trend information in a TrendInfo object.
T = getTrend(data);
Assign offset values to the TrendInfo object.
T. InputOffset = 5;
T.OutputOffset = 5;

Subtract the offsets from the data.
data_d = detrend(data, T );
Plot the detrended data on the same plot.
hold on
plot(data_d)

## Input-Output Data



View the mean value removed from the data.

```
get(T)
ans = struct with fields:
    DataName: 'data'
    InputOffset: 5
    Output0ffset: 5
        InputSlope: 0
        OutputSlope: 0
```


## Compute and Store Means of Input and Output Signals

Compute input-output signal means, store them, and detrend the data.
Load SISO data containing vectors u 2 and y 2 .
load dryer2
Create a data object with a sample time of 0.08 seconds.
data $=$ iddata(y2,u2,0.08);
Compute the mean of the data.

T = getTrend(data, 0);
Remove the mean from the data.
data_d = detrend(data, T );
Plot the original and detrended data on the same plot.
plot(data,data_d)


## Combine Computed and Customized Trend Information for MISO Data

Load and plot data that contains two input channels and one output channel.
load z7lintrend z7L
plot(z7L)

## Input-Output Data



The output channel of $\mathbf{z 7 L}$ contains a linear trend that is not present in the input channels. Compute the trend information.

```
T = getTrend(z7L,1)
Trend specifications for data "z7L" with 2 input(s), 1 output(s), 1 experiment(s):
        DataName: 'z7L'
    InputOffset: [-0.0764 -0.0683]
    OutputOffset: -0.2642
        InputSlope: [4.8338e-04 3.1642e-04]
    OutputSlope: 0.0268
```

Limit the trend information to the output channel only by setting the input trend values to 0 .

```
T.Input0ffset = [0 0];
T.InputSlope = [0 0];
T
Trend specifications for data "z7L" with 2 input(s), 1 output(s), 1 experiment(s):
    DataName: 'z7L'
    InputOffset: [0 0]
    Output0ffset: -0.2642
    InputSlope: [0 0]
    OutputSlope: 0.0268
```

Remove the linear trend from the data.

```
z7d = detrend(z7L,T);
plot(z7d)
```


## Input-Output Data



The trend is no longer in the output data and the input data is unchanged.

## Input Arguments

## data - Time-domain input-output data

iddata object
Time-domain input-output data, specified as an iddata object containing one or more sets of timedomain signals. The iddata object can contain SISO, MIMO, or multiexperiment data. The signal sets can contain either input and output data or output data only.

## Output Arguments

## T - Trend information

trendInfo object
Trend information, returned as a TrendInfo object.

## See Also

TrendInfo | detrend | retrend

## Topics

"Handling Offsets and Trends in Data"

Introduced in R2009a

## goodnessOfFit

Goodness of fit between test and reference data for analysis and validation of identified models

## Syntax

fit = goodnessOfFit(x,xref,cost_func)

## Description

goodnessOfFit returns fit values that represent the error norm between test and reference data sets. If you want to compare and visualize simulated model output with measurement data, see also compare.
fit = goodness0fFit(x,xref,cost_func) returns the goodness of fit between the test data $x$ and the reference data xref using the cost function cost_func. fit is a quantitative representation of the closeness of $x$ to xref. To perform multiple test-to-reference fit comparisons, you can specify $x$ and xref as cell arrays of equal size that contain multiple test and reference data sets. With cell array inputs, fit returns an array of fit values.

## Examples

## Calculate Goodness of Fit Between Estimated and Measured Data

Find the goodness of fit between measured output data and the simulated output of an estimated model.

Obtain the measured output.
load iddatal z1
yref = zl.y;
z 1 is an iddata object containing measured input-output data. $\mathrm{z} 1 . \mathrm{y}$ is the measured output.
Estimate a second-order transfer function model and simulate the model output y_est.

```
sys = tfest(z1,2);
y_est = sim(sys,zl(:,[],:));
```

Calculate the goodness of fit, or error norm, between the measured and estimated outputs. Specify the normalized root mean squared error (NRMSE) as the cost function.

```
cost_func = 'NRMSE';
y = y_est.y;
fit = goodness0fFit(y,yref,cost_func)
fit = 0.2943
```

Alternatively, you can use compare to calculate the fit. compare uses the NRMSE cost function, and expresses the fit percentage using the one's complement of the error norm. The fit relationship
between compare and goodnessOfFit is therefore fit compare $=\left(1-\right.$ fit $\left._{\text {gof }}\right) * 100$. A compare result of $100 \%$ is equivalent to a goodness0fFit result of 0 .

Specify an initial condition of zero to match the initial condition that goodnessOfFit assumes.

```
opt = compareOptions('InitialCondition','z');
```

compare(z1,sys,opt);

Simulated Response Comparison


The fit results are equivalent.

## Goodness of Fit for Multiple Data Sets

Find the goodness of fit between measured and estimated outputs for two models.
Obtain the input-output measurements $z 2$ from iddata2. Copy the measured output into reference output yref.
load iddata2 z2
yref = z2.y;
Estimate second-order and fourth-order transfer function models using z2.

```
sys2 = tfest(z2,2);
sys4 = tfest(z2,4);
```

Simulate both systems to get estimated outputs.

```
y_sim2 = sim(sys2,z2(:,[],:));
y2 = y_sim2.y;
y_sim4 = sim(sys4,z2(:,[],:));
y4 = y_sim4.y;
```

Create cell arrays from the reference and estimated outputs. The reference data set is the same for both model comparisons, so create identical reference cells.

```
yrefc = {yref yref};
```

$\mathrm{yc}=\{\mathrm{y} 2 \mathrm{y} 4\}$;

Compute fit values for the three cost functions.

```
fit_nrmse = goodnessOfFit(yc,yrefc,'NRMSE')
fit_nrmse = 1×2
    0.1429 0.1439
fit_nmse = goodnessOfFit(yc,yrefc,'NMSE')
fit nmse = 1\times2
    0.0204 0.0207
fit_mse = goodnessOfFit(yc,yrefc,'MSE')
fit_mse = 1×2
    1.0811 1.0967
```

A fit value of 0 indicates a perfect fit between reference and estimated outputs. The fit value rises as fit goodness decreases. For all three cost functions, the fourth-order model produces a better fit than the second-order model.

## Input Arguments

## x - Data to test

matrix (default) | cell array
Data to test, specified as a matrix or cell array.

- For a single test data set, specify an $N_{s}$-by- $N$ matrix, where $N_{s}$ is the number of samples and $N$ is the number of channels. You must specify cost_fun as 'NRMSE' or 'NMSE ' to use multiplechannel data.
- For multiple test data sets, specify a cell array of length $N_{d}$, where $N_{d}$ is the number of test-toreference pairs and each cell contains one data matrix.
x must not contain any NaN or Inf values.


## xref - Reference data

matrix (default) | cell array

Reference data with which to compare x , specified as a matrix or cell array.

- For a single reference data set, specify an $N_{s}$-by- $N$ matrix, where $N_{s}$ is the number of samples and $N$ is the number of channels. xref must be the same size as x . You must specify cost_fun as 'NRMSE ' or 'NMSE' to use multiple-channel data.
- For multiple reference data sets, specify a cell array of length $N_{d}$, where $N_{d}$ is the number of test-to-reference pairs and each cell contains one reference data matrix. As with the individual data matrices, the cell array sizes for $x$ and xref must match. Each ith element of fit corresponds to the pairs of the $i$ th cells of $x$ and $x r e f$.
xref must not contain any NaN or Inf values.

```
cost_func - Cost function
```

'MSE' | 'NRMSE' | 'NMSE'
Cost function to determine goodness of fit, specified as one of the following values. In the equations, the value fit applies to a single pairing of test and reference data sets.

| Value | Description | Equation | Notes |
| :---: | :---: | :---: | :---: |
| 'MSE' | Mean squared error | $\text { fit }=\frac{\\|x-x r e f\\|^{2}}{N s}$ <br> where $N_{s}$ is the number of samples and \\|| indicates the 2-norm of a vector. | fit is a scalar. |
| 'NRMSE' | Normalized root mean squared error | $\begin{aligned} & \text { fit( }(\mathrm{i}) \\ & =\frac{\\| x r e f(:, i)-x}{\\| x r e f(:, i)-\text { mean }} \end{aligned}$ <br> where \|| indicates the 2norm of a vector. fit is a row vector of length $N$ and $i=1, \ldots, N$, where $N$ is the number of channels. | fit is a row vector. 'NRMSE' is the cost funidtion used by <br>  |
| 'NMSE' | Normalized mean squared error | $\begin{aligned} & f i t(i) \\ & =\frac{\\| \operatorname{xref}(:, i)-x}{\\| \operatorname{xref}(:, i)-\text { mean }} \end{aligned}$ | fit is a row vector. $\frac{(:, i) \\|^{2}}{(x r e f(:, i)) \\|^{2}}$ |

## Output Arguments

## fit - Goodness of fit

scalar | row vector | cell array
Goodness of fit between test and reference data pairs, returned as a scalar, a row vector, or a cell array.

- For a single test and reference data set pair, fit is returned as a scalar or row vector.
- If cost_fun is 'MSE', then fit is a scalar.
- If cost_fun is 'NRMSE' or 'NMSE', then fit is a column vector of length $N$, where $N$ is the number of channels.
- For multiple test and data set and reference pairs, where x and xref are cell arrays of length $N_{D}$, fit is returned as a vector or a matrix.
- If cost_fun is 'MSE', then fit is a row vector of length $N_{D}$.
- If cost_fun is 'NRMSE' or 'NMSE', then fit is a matrix of size $N$-by- $N_{d}$, where $N$ is the number of channels (data columns) and $N_{d}$ represents the number of test pairs. Each element of fit contains the goodness of fit values for the corresponding test data and reference pair.

Each element of fit contains the goodness of fit values for the corresponding test data and reference pair.

Possible values for individual fit elements depend on the selection of cost_func.

- If cost_func is 'MSE', each fit value is a positive scalar that grows with the error between test and reference data. A fit value of 0 indicates a perfect match between test and reference data.
- If cost_func is 'NRMSE' or 'NMSE', fit values vary between -Inf and 1.
- 0 - Perfect fit to reference data (zero error)
- -Inf - Bad fit
- $1-x$ is no better than a straight line at matching $x$ ref


## Compatibility Considerations

goodnessOfFit: Fit result represents the error norm for all three cost functions, with a value of zero indicating a perfect fit
Behavior changed in R2020a
goodnessOfFit now returns the error norm $E$ as the fit value for all three cost functions (MSE, NRMSE, and NMSE). Previously, goodnessOfFit returned the one's complement of the error norm, $1-E$, for fit values that used the NRMSE or NMSE cost functions. This change allows consistent fitvalue interpretation across the three cost functions, with the ideal fit value of zero representing a perfect fit.

Previously computed NRMSE and NMSE fit values are the one's complements of the fit values computed with the current software. Similarly, the NRMSE fit value is now the one's complement of the fit used in the percentage value that compare computes. For example, if the previous goodnessOfFit fit value was 0.8 , the current fit value is 0.2 . A goodnessOfFit fit value of 0.2 is equivalent to a compare fit percentage of $80 \%$.

## See Also

aic|compare|fpe|resid

## Introduced in R2012a

## greyest

Linear grey-box model estimation

## Syntax

sys = greyest(data,init_sys)
sys = greyest(data,init_sys,opt)
[sys,x0] = greyest( $\qquad$

## Description

sys = greyest(data,init_sys) estimates a linear grey-box model, sys, using time or frequency domain data, data. The dimensions of the inputs and outputs of data and init_sys, an idgrey model, must match. sys is an identified idgrey model that has the same structure as init_sys.
sys = greyest(data,init_sys,opt) estimates a linear grey-box model using the option set, opt, to configure the estimation options.
[sys,x0] = greyest (__ ) returns the value of the initial states computed during estimation. You can use this syntax with any of the previous input-argument combinations.

## Input Arguments

## data

Estimation data.
The dimensions of the inputs and outputs of data and init_sys must match.
For time-domain estimation, data is an iddata object containing the input and output signal values.
For frequency domain estimation, data can be one of the following:

- Recorded frequency response data (frd or idfrd)
- iddata object with its Domain property set to 'Frequency '


## init_sys

Identified linear grey-box model that configures the initial parameterization of sys.
init_sys, an idgrey model, must have the same input and output dimensions as data.
opt
Estimation options.
opt is an option set, created using greyestOptions, which specifies options including:

- Estimation objective
- Initialization choice
- Disturbance model handling
- Numerical search method to be used in estimation


## Output Arguments

## sys

Estimated grey-box model, returned as an idgrey model. This model is created using the specified initial system, and estimation options.

Information about the estimation results and options used is stored in the Report property of the model. Report has the following fields:

| Report Field | Description |
| :---: | :---: |
| Status | Summary of the model status, which indicates whether the model was created by construction or obtained by estimation. |
| Method | Estimation command used. |
| InitialSt ate | Handling of initial states during estimation, returned as one of the following: <br> - 'model ' - The initial state is parameterized by the ODE file used by the idgrey model. <br> - 'zero' - The initial state is set to zero. <br> - 'estimate' - The initial state is treated as an independent estimation parameter. <br> - 'backcast' - The initial state is estimated using the best least squares fit. <br> - Vector of doubles of length $N x$, where $N x$ is the number of states. For multiexperiment data, a matrix with $N e$ columns, where $N e$ is the number of experiments. <br> This field is especially useful to view how the initial states were handled when the InitialState option in the estimation option set is 'auto'. |
| Disturban ceModel | Handling of the disturbance component ( $K$ ) during estimation, returned as one of the following values: <br> - 'model ' $-K$ values are parameterized by the ODE file used by the idgrey model. <br> - 'fixed ' - The value of the K property of the idgrey model is fixed to its original value. <br> - 'none' $-K$ is fixed to zero. <br> - 'estimate ' $-K$ is treated as an independent estimation parameter. <br> This field is especially useful to view the how the disturbance component was handled when the DisturbanceModel option in the estimation option set is 'auto'. |


| Report <br> Field | Description |  |
| :--- | :--- | :--- |
| Fit | Quantitative assessment of the estimation, returned as a structure. See "Loss Function <br> and Model Quality Metrics" for more information on these quality metrics. The <br> structure has the following fields: |  |
|  | Field | Description |
|  | FitPerce <br> nt | Normalized root mean squared error (NRMSE) measure of how well the <br> response of the model fits the estimation data, expressed as the <br> percentage fit = 100(1-NRMSE). |
|  | LossFcn | Value of the loss function when the estimation completes. |
|  | MSE <br> Mean squared error (MSE) measure of how well the response of the <br> model fits the estimation data. |  |
|  | FPE | Final prediction error for the model. |
|  | AIC | Raw Akaike Information Criteria (AIC) measure of model quality. |
|  | AICc | Small sample-size corrected AIC. |
| nAIC | Normalized AIC. |  |
|  | BIC | Bayesian Information Criteria (BIC). |
| Parameter <br> s | Estimated values of model parameters. <br> OptionsUs <br> edOption set used for estimation. If no custom options were configured, this is a set of <br> default options. See greyest0ptions for more information. |  |
| RandState | State of the random number stream at the start of estimation. Empty, [ ], if <br> randomization was not used during estimation. For more information, see rng. |  |


| Report Field | Description |  |
| :---: | :---: | :---: |
| DataUsed | Attributes of the data used for estimation, returned as a structure with the following fields: |  |
|  | Field | Description |
|  | Name | Name of the data set. |
|  | Type | Data type. |
|  | Length | Number of data samples. |
|  | Ts | Sample time. |
|  | InterSam ple | Input intersample behavior, returned as one of the following values: <br> - 'zoh ' - Zero-order hold maintains a piecewise-constant input signal between samples. <br> - ' foh ' - First-order hold maintains a piecewise-linear input signal between samples. <br> - 'bl' - Band-limited behavior specifies that the continuous-time input signal has zero power above the Nyquist frequency. |
|  | InputOff set | Offset removed from time-domain input data during estimation. For nonlinear models, it is [ ]. |
|  | Output0f fset | Offset removed from time-domain output data during estimation. For nonlinear models, it is []. |
| Terminati on | Termination conditions for the iterative search used for prediction error minimization, returned as a structure with the following fields: |  |
|  | Field | Description |
|  | WhyStop | Reason for terminating the numerical search. |
|  | Iteratio ns | Number of search iterations performed by the estimation algorithm. |
|  | First0rd erOptima lity | $\infty$-norm of the gradient search vector when the search algorithm terminates. |
|  | FcnCount | Number of times the objective function was called. |
|  | UpdateNo rm | Norm of the gradient search vector in the last iteration. Omitted when the search method is 'lsqnonlin' or 'fmincon'. |
|  | LastImpr ovement | Criterion improvement in the last iteration, expressed as a percentage. Omitted when the search method is 'lsqnonlin' or 'fmincon'. |
|  | Algorith \|lm | Algorithm used by 'lsqnonlin' or 'fmincon' search method. Omitted when other search methods are used. |
|  | For estimation methods that do not require numerical search optimization, the Termination field is omitted. |  |

For more information on using Report, see "Estimation Report".

## x0

Initial states computed during the estimation, returned as a matrix containing a column vector corresponding to each experiment.

This array is also stored in the Parameters field of the model Report property.

## Examples

## Estimate Grey-Box Model

Estimate the parameters of a DC motor using the linear grey-box framework.
Load the measured data.

```
load(fullfile(matlabroot, 'toolbox', 'ident', 'iddemos', 'data', 'dcmotordata'));
data = iddata(y, u, 0.1, 'Name', 'DC-motor');
data.InputName = 'Voltage';
data.InputUnit = 'V';
data.OutputName = {'Angular position', 'Angular velocity'};
data.OutputUnit = {'rad', 'rad/s'};
data.Tstart = 0;
data.TimeUnit = 's';
```

data is an iddata object containing the measured data for the outputs, the angular position, the angular velocity. It also contains the input, the driving voltage.

Create a grey-box model representing the system dynamics.
For the DC motor, choose the angular position (rad) and the angular velocity (rad/s) as the outputs and the driving voltage $(\mathrm{V})$ as the input. Set up a linear state-space structure of the following form:

$$
\begin{aligned}
& \dot{x}(t)=\left[\begin{array}{cc}
0 & 1 \\
0 & -\frac{1}{\tau}
\end{array}\right] x(t)+\left[\begin{array}{l}
0 \\
\frac{G}{\tau}
\end{array}\right] u(t) \\
& y(t)=\left[\begin{array}{ll}
1 & 0 \\
0 & 1
\end{array}\right] x(t) .
\end{aligned}
$$

$\tau$ is the time constant of the motor in seconds, and $G$ is the static gain from the input to the angular velocity in rad/(V*s).
$\mathrm{G}=0.25$;
tau = 1;
init_sys = idgrey('motorDynamics',tau,'cd',G,0);
The governing equations in state-space form are represented in the MATLAB® file motorDynamics.m. To view the contents of this file, enter edit motorDynamics.m at the MATLAB command prompt.
$G$ is a known quantity that is provided to motorDynamics.m as an optional argument.
$\tau$ is a free estimation parameter.
init_sys is an idgrey model associated with motor.m.
Estimate $\tau$.
sys = greyest(data,init_sys);
sys is an idgrey model containing the estimated value of $\tau$.
To obtain the estimated parameter values associated with sys, use getpvec (sys).
Analyze the result.

```
opt = compareOptions('InitialCondition','zero');
```

compare(data,sys,Inf,opt)

sys provides a $98.35 \%$ fit for the angular position and an $84.42 \%$ fit for the angular velocity.

## Estimate Grey-Box Model Using Regularization

Estimate the parameters of a DC motor by incorporating prior information about the parameters when using regularization constants.

The model is parameterized by static gain G and time constant $\tau$. From prior knowledge, it is known that G is about 4 and $\tau$ is about 1 . Also, you have more confidence in the value of $\tau$ than G and would like to guide the estimation to remain close to the initial guess.

Load estimation data.
load regularizationExampleData.mat motorData
The data contains measurements of motor's angular position and velocity at given input voltages.
Create an idgrey model for DC motor dynamics. Use the function DCMotorODE that represents the structure of the grey-box model.

```
mi = idgrey(@DCMotorODE,{'G', 4; 'Tau', 1},'cd',{}, 0);
mi = setpar(mi, 'label', 'default');
```

If you want to view the DCMotorODE function, type:

```
type DCMotorODE.m
function [A,B,C,D] = DCMotorODE(G,Tau,Ts)
%DCMOTORODE ODE file representing the dynamics of a DC motor parameterized
%by gain G and time constant Tau.
%
% [A,B,C,D,K,X0] = DCMOTORODE(G,Tau,Ts) returns the state space matrices
% of the DC-motor with time-constant Tau and static gain G. The sample
% time is Ts.
% This file returns continuous-time representation if input argument Ts
% is zero. If Ts>0, a discrete-time representation is returned.
% See also IDGREY, GREYEST.
% Copyright 2013 The MathWorks, Inc.
A = [0 1;0 -1/Tau];
B = [0; G/Tau];
C = eye(2);
D = [0;0];
if Ts>0 % Sample the model with sample time Ts
    s = expm([[A B]*Ts; zeros(1,3)]);
    A = s(1:2,1:2);
    B = s(1:2,3);
end
```

Specify regularization options Lambda.
opt = greyestOptions;
opt.Regularization.Lambda = 100;

Specify regularization options R.

```
opt.Regularization.R = [1, 1000];
```

You specify more weighting on the second parameter because you have more confidence in the value of $\tau$ than G.

Specify the initial values of the parameters as regularization option $\theta^{*}$.
opt.Regularization.Nominal = 'model';
Estimate the regularized grey-box model.
sys = greyest(motorData, mi, opt);

## Extended Capabilities

## Automatic Parallel Support

Accelerate code by automatically running computation in parallel using Parallel Computing Toolbox ${ }^{\mathrm{TM}}$.
Parallel computing support is available for estimation using the lsqnonlin search method (requires Optimization Toolbox). To enable parallel computing, use greyestOptions, set SearchMethod to 'lsqnonlin', and set SearchOptions.Advanced.UseParallel to true.

For example:
opt = greyestOptions;
opt.SearchMethod = 'lsqnonlin';
opt.SearchOptions.Advanced.UseParallel = true;

## See Also

greyest0ptions|iddata|idfrd|idgrey|idnlgrey|pem|ssest
Topics
"Estimate Model Using Zero/Pole/Gain Parameters"
"Regularized Estimates of Model Parameters"
Introduced in R2012a

## greyestOptions

Option set for greyest

## Syntax

```
opt = greyestOptions
opt = greyestOptions(Name,Value)
```


## Description

opt $=$ greyestOptions creates the default options set for greyest.
opt = greyestOptions(Name, Value) creates an option set with the options specified by one or more Name, Value pair arguments.

## Input Arguments

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, ... , NameN, ValueN.

## InitialState - Handling of initial states <br> 'auto' (default)|'model'|'zero'|'estimate' | 'backcast'

Handling of initial states during estimation, specified as one of the following values:

- 'model ' - The initial state is parameterized by the ODE file used by the idgrey model. The ODE file must return 6 or more output arguments.
- 'zero' - The initial state is set to zero. Any values returned by the ODE file are ignored.
- 'estimate' - The initial state is treated as an independent estimation parameter.
- 'backcast' - The initial state is estimated using the best least squares fit.
- 'auto' - The software chooses the method to handle initial states based on the estimation data.
- Vector of doubles - Specify a column vector of length $N x$, where $N x$ is the number of states. For multiexperiment data, specify a matrix with $N e$ columns, where $N e$ is the number of experiments. The specified values are treated as fixed values during the estimation process.


## DisturbanceModel - Handling of disturbance component

'auto' (default)|'model'|'fixed'|'none'|'estimate'
Handling of the disturbance component ( $K$ ) during estimation, specified as one of the following values:

- 'model' $-K$ values are parameterized by the ODE file used by the idgrey model. The ODE file must return 5 or more output arguments.
- ' fixed ' - The value of the K property of the idgrey model is fixed to its original value.
- 'none ' $-K$ is fixed to zero. Any values returned by the ODE file are ignored.
- 'estimate' $-K$ is treated as an independent estimation parameter.
- 'auto' - The software chooses the method to handle how the disturbance component is handled during estimation. The software uses the 'model ' method if the ODE file returns 5 or more output arguments with a finite value for $K$. Else, the software uses the 'fixed ' method.

Note Noise model cannot be estimated using frequency domain data.

## Focus - Error to be minimized

'prediction' (default)|'simulation'
Error to be minimized in the loss function during estimation, specified as the comma-separated pair consisting of 'Focus' and one of the following values:

- 'prediction' - The one-step ahead prediction error between measured and predicted outputs is minimized during estimation. As a result, the estimation focuses on producing a good predictor model.
- 'simulation' - The simulation error between measured and simulated outputs is minimized during estimation. As a result, the estimation focuses on making a good fit for simulation of model response with the current inputs.

The Focus option can be interpreted as a weighting filter in the loss function. For more information, see "Loss Function and Model Quality Metrics".

## WeightingFilter - Weighting prefilter

## [ ] (default) | vector | matrix | cell array | linear system

Weighting prefilter applied to the loss function to be minimized during estimation. To understand the effect of WeightingFilter on the loss function, see "Loss Function and Model Quality Metrics".

Specify WeightingFilter as one of the following values:

- [] - No weighting prefilter is used.
- Passbands - Specify a row vector or matrix containing frequency values that define desired passbands. You select a frequency band where the fit between estimated model and estimation data is optimized. For example, [ $\mathrm{wl} \mathrm{l}, \mathrm{wh}$ ] where wl and $w h$ represent lower and upper limits of a passband. For a matrix with several rows defining frequency passbands,
[w1l,w1h;w2l,w2h;w3l,w3h;..], the estimation algorithm uses the union of the frequency ranges to define the estimation passband.

Passbands are expressed in rad/TimeUnit for time-domain data and in FrequencyUnit for frequency-domain data, where TimeUnit and FrequencyUnit are the time and frequency units of the estimation data.

- SISO filter - Specify a single-input-single-output (SISO) linear filter in one of the following ways:
- A SISO LTI model
- $\{A, B, C, D\}$ format, which specifies the state-space matrices of a filter with the same sample time as estimation data.
- \{numerator, denominator\} format, which specifies the numerator and denominator of the filter as a transfer function with same sample time as estimation data.

This option calculates the weighting function as a product of the filter and the input spectrum to estimate the transfer function.

- Weighting vector - Applicable for frequency-domain data only. Specify a column vector of weights. This vector must have the same length as the frequency vector of the data set, Data. Frequency. Each input and output response in the data is multiplied by the corresponding weight at that frequency.


## EnforceStability - Control whether to enforce stability of model

false (default)|true
Control whether to enforce stability of estimated model, specified as the comma-separated pair consisting of 'EnforceStability' and either true or false.

Data Types: logical

## EstimateCovariance - Control whether to generate parameter covariance data

true (default) | false
Controls whether parameter covariance data is generated, specified as true or false.
If EstimateCovariance is true, then use getcov to fetch the covariance matrix from the estimated model.

## Display - Specify whether to display the estimation progress

'off' (default)|'on'
Specify whether to display the estimation progress, specified as one of the following values:

- 'on' - Information on model structure and estimation results are displayed in a progress-viewer window.
- 'off' - No progress or results information is displayed.


## InputOffset - Removal of offset from time-domain input data during estimation

[ ] (default) | vector of positive integers | matrix
Removal of offset from time-domain input data during estimation, specified as the comma-separated pair consisting of 'Input0ffset' and one of the following:

- A column vector of positive integers of length $N u$, where $N u$ is the number of inputs.
- [ ] - Indicates no offset.
- $N u$-by-Ne matrix - For multi-experiment data, specify Input0ffset as an $N u$-by- $N e$ matrix. $N u$ is the number of inputs, and $N e$ is the number of experiments.

Each entry specified by Input0ffset is subtracted from the corresponding input data.

## OutputOffset - Removal of offset from time-domain output data during estimation

[ ] (default) | vector | matrix
Removal of offset from time-domain output data during estimation, specified as the comma-separated pair consisting of 'OutputOffset' and one of the following:

- A column vector of length $N y$, where $N y$ is the number of outputs.
- [ ] - Indicates no offset.
- Ny -by-Ne matrix - For multi-experiment data, specify OutputOffset as a Ny -by- Ne matrix. Ny is the number of outputs, and $N e$ is the number of experiments.

Each entry specified by OutputOffset is subtracted from the corresponding output data.

## OutputWeight - Weighting of prediction errors in multi-output estimations

[] (default) | 'noise' | positive semidefinite symmetric matrix
Weighting of prediction errors in multi-output estimations, specified as one of the following values:

- 'noise' $-\operatorname{Minimize} \operatorname{det}\left(E^{*} E / N\right)$, where $E$ represents the prediction error and $N$ is the number of data samples. This choice is optimal in a statistical sense and leads to maximum likelihood estimates if nothing is known about the variance of the noise. It uses the inverse of the estimated noise variance as the weighting function.

Note OutputWeight must not be 'noise' if SearchMethod is 'lsqnonlin'.

- Positive semidefinite symmetric matrix (W) - Minimize the trace of the weighted prediction error matrix trace (E'*E*W/N) where:
- $E$ is the matrix of prediction errors, with one column for each output, and $W$ is the positive semidefinite symmetric matrix of size equal to the number of outputs. Use $W$ to specify the relative importance of outputs in multiple-output models, or the reliability of corresponding data.
- $N$ is the number of data samples.
- [ ] - The software chooses between the ' noise' or using the identity matrix for W.

This option is relevant for only multi-output models.

## Regularization - Options for regularized estimation of model parameters

structure
Options for regularized estimation of model parameters. For more information on regularization, see "Regularized Estimates of Model Parameters".

Regularization is a structure with the following fields:

- Lambda - Constant that determines the bias versus variance tradeoff.

Specify a positive scalar to add the regularization term to the estimation cost.
The default value of zero implies no regularization.
Default: 0

- R - Weighting matrix.

Specify a vector of nonnegative numbers or a square positive semi-definite matrix. The length must be equal to the number of free parameters of the model.

For black-box models, using the default value is recommended. For structured and grey-box models, you can also specify a vector of np positive numbers such that each entry denotes the confidence in the value of the associated parameter.

The default value of 1 implies a value of eye(npfree), where npfree is the number of free parameters.

## Default: 1

- Nominal - The nominal value towards which the free parameters are pulled during estimation.

The default value of zero implies that the parameter values are pulled towards zero. If you are refining a model, you can set the value to 'model' to pull the parameters towards the parameter values of the initial model. The initial parameter values must be finite for this setting to work.

Default: 0

## SearchMethod - Numerical search method used for iterative parameter estimation

'auto' (default)|'gn'|'gna'|'lm'|'grad'|'lsqnonlin'|'fmincon'
Numerical search method used for iterative parameter estimation, specified as the comma-separated pair consisting of 'SearchMethod' and one of the following:

- 'auto' - A combination of the line search algorithms, 'gn', 'lm', 'gna', and 'grad' methods is tried in sequence at each iteration. The first descent direction leading to a reduction in estimation cost is used.
- 'gn' - Subspace Gauss-Newton least squares search. Singular values of the Jacobian matrix less than GnPinvConstant*eps*max(size(J))*norm(J) are discarded when computing the search direction. $J$ is the Jacobian matrix. The Hessian matrix is approximated as $J^{T} J$. If there is no improvement in this direction, the function tries the gradient direction.
- 'gna' - Adaptive subspace Gauss-Newton search. Eigenvalues less than gamma*max(sv) of the Hessian are ignored, where $s v$ contains the singular values of the Hessian. The Gauss-Newton direction is computed in the remaining subspace. gamma has the initial value InitialGnaTolerance (see Advanced in 'Search0ptions' for more information). This value is increased by the factor LMStep each time the search fails to find a lower value of the criterion in fewer than five bisections. This value is decreased by the factor $2 *$ LMStep each time a search is successful without any bisections.
- 'lm' - Levenberg-Marquardt least squares search, where the next parameter value is - pinv (H $+\mathrm{d} * \mathrm{I}) * \mathrm{grad}$ from the previous one. $H$ is the Hessian, $I$ is the identity matrix, and grad is the gradient. $d$ is a number that is increased until a lower value of the criterion is found.
- 'grad' - Steepest descent least squares search.
- 'lsqnonlin' - Trust-region-reflective algorithm of lsqnonlin. Requires Optimization Toolbox software.
- 'fmincon' - Constrained nonlinear solvers. You can use the sequential quadratic programming (SQP) and trust-region-reflective algorithms of the fmincon solver. If you have Optimization Toolbox software, you can also use the interior-point and active-set algorithms of the fmincon solver. Specify the algorithm in the SearchOptions.Algorithm option. The fmincon algorithms may result in improved estimation results in the following scenarios:
- Constrained minimization problems when there are bounds imposed on the model parameters.
- Model structures where the loss function is a nonlinear or non smooth function of the parameters.
- Multi-output model estimation. A determinant loss function is minimized by default for multioutput model estimation. fmincon algorithms are able to minimize such loss functions directly. The other search methods such as ' lm ' and 'gn' minimize the determinant loss function by alternately estimating the noise variance and reducing the loss value for a given noise variance value. Hence, the fmincon algorithms can offer better efficiency and accuracy for multi-output model estimations.


## SearchOptions - Option set for the search algorithm

search option set
Option set for the search algorithm, specified as the comma-separated pair consisting of 'Search0ptions ' and a search option set with fields that depend on the value of SearchMethod.

## Search0ptions Structure When SearchMethod is Specified as 'gn', 'gna', 'lm', 'grad', or 'auto'

| Field <br> Name | Description | Default |
| :--- | :--- | :--- |
| Toleran <br> ce | Minimum percentage difference between the current value of the loss <br> function and its expected improvement after the next iteration, specified as <br> a positive scalar. When the percentage of expected improvement is less <br> than Tolerance, the iterations stop. The estimate of the expected loss- <br> function improvement at the next iteration is based on the Gauss-Newton <br> vector computed for the current parameter value. | 0.01 |
| MaxIter <br> ations | Maximum number of iterations during loss-function minimization, specified <br> as a positive integer. The iterations stop when MaxIterations is reached <br> or another stopping criterion is satisfied, such as Tolerance. | 20 |
| Setting MaxIterations = 0 returns the result of the start-up procedure. |  |  |
| Use sys. Report. Termination. Iterations to get the actual number |  |  |
| of iterations during an estimation, where sys is an idtf model. |  |  |$\quad$


| Field Name | Description |  | Default |
| :---: | :---: | :---: | :---: |
| Advance <br> d | Advanced search settings, specified as a structure with the following fields: |  |  |
|  | Field Name | Description | Default |
|  | GnPinvCons tant | Jacobian matrix singular value threshold, specified as a positive scalar. Singular values of the Jacobian matrix that are smaller than GnPinvConstant*max(size(J)*norm(J)*eps) are discarded when computing the search direction. Applicable when SearchMethod is 'gn'. | 10000 |
|  | InitialGna Tolerance | Initial value of gamma, specified as a positive scalar. Applicable when SearchMethod is 'gna'. | 0.0001 |
|  | LMStartVal ue | Starting value of search-direction length $d$ in the Levenberg-Marquardt method, specified as a positive scalar. Applicable when SearchMethod is ' 1 m '. | 0.001 |
|  | LMStep | Size of the Levenberg-Marquardt step, specified as a positive integer. The next value of the search-direction length $d$ in the Levenberg-Marquardt method is LMStep times the previous one. Applicable when SearchMethod is ' lm '. | 2 |
|  | MaxBisecti ons | Maximum number of bisections used for line search along the search direction, specified as a positive integer. | 25 |
|  | MaxFunctio nEvaluatio ns | Maximum number of calls to the model file, specified as a positive integer. Iterations stop if the number of calls to the model file exceeds this value. | Inf |
|  | MinParamet erChange | Smallest parameter update allowed per iteration, specified as a nonnegative scalar. | 0 |
|  | RelativeIm provement | Relative improvement threshold, specified as a nonnegative scalar. Iterations stop if the relative improvement of the criterion function is less than this value. | 0 |
|  | StepReduct ion | Step reduction factor, specified as a positive scalar that is greater than 1 . The suggested parameter update is reduced by the factor StepReduction after each try. This reduction continues until MaxBisections tries are completed or a lower value of the criterion function is obtained. <br> StepReduction is not applicable for SearchMethod ' lm' (Levenberg-Marquardt method). | 2 |

## SearchOptions Structure When SearchMethod is Specified as ' Isqnonlin'

| Field <br> Name | Description | Default |
| :--- | :--- | :--- |
| Function <br> Toleranc <br> e | Termination tolerance on the loss function that the software <br> minimizes to determine the estimated parameter values, <br> specified as a positive scalar. | $1 \mathrm{e}-5$ |
| The value of FunctionTolerance is the same as that of <br> opt.Search0ptions. Advanced. TolFun. | (e-6 |  |
| StepTole <br> rance | Termination tolerance on the estimated parameter values, <br> specified as a positive scalar. | The value of StepTolerance is the same as that of <br> opt. SearchOptions . Advanced. TolX. |

SearchOptions Structure When SearchMethod is Specified as 'fmincon'

| Field Name | Description | Default |
| :---: | :---: | :---: |
| Algorithm | fmincon optimization algorithm, specified as one of the following: <br> - 'sqp' - Sequential quadratic programming algorithm. The algorithm satisfies bounds at all iterations, and it can recover from NaN or Inf results. It is not a large-scale algorithm. For more information, see "Large-Scale vs. MediumScale Algorithms" (Optimization Toolbox). <br> - 'trust-regionreflective' - Subspace trust-region method based on the interior-reflective Newton method. It is a largescale algorithm. <br> - 'interior-point' -Large-scale algorithm that requires Optimization Toolbox software. The algorithm satisfies bounds at all iterations, and it can recover from NaN or Inf results. <br> - 'active-set' - Requires Optimization Toolbox software. The algorithm can take large steps, which adds speed. It is not a large-scale algorithm. <br> For more information about the algorithms, see "Constrained Nonlinear Optimization Algorithms" (Optimization Toolbox) and "Choosing the Algorithm" (Optimization Toolbox). | 'sqp' |


| Field Name | Description | Default |
| :--- | :--- | :--- |
| FunctionTolerance | Termination tolerance on the <br> loss function that the software <br> minimizes to determine the <br> estimated parameter values, <br> specified as a positive scalar. | $1 \mathrm{e}-6$ |
| StepTolerance | Termination tolerance on the <br> estimated parameter values, <br> specified as a positive scalar. | $1 \mathrm{e}-6$ |
| MaxIterations | Maximum number of iterations <br> during loss function <br> minimization, specified as a <br> positive integer. The iterations <br> stop when MaxIterations is <br> reached or another stopping <br> criterion is satisfied, such as <br> FunctionTolerance. | 100 |

## Advanced - Additional advanced options

structure
Additional advanced options, specified as a structure with the following fields:

- ErrorThreshold - Specifies when to adjust the weight of large errors from quadratic to linear.

Errors larger than ErrorThreshold times the estimated standard deviation have a linear weight in the loss function. The standard deviation is estimated robustly as the median of the absolute deviations from the median of the prediction errors, divided by 0.7 . For more information on robust norm choices, see section 15.2 of [2].

ErrorThreshold $=0$ disables robustification and leads to a purely quadratic loss function. When estimating with frequency-domain data, the software sets ErrorThreshold to zero. For time-domain data that contains outliers, try setting ErrorThreshold to 1.6.

Default: 0

- MaxSize - Specifies the maximum number of elements in a segment when input-output data is split into segments.

MaxSize must be a positive integer.
Default: 250000

- StabilityThreshold - Specifies thresholds for stability tests.

StabilityThreshold is a structure with the following fields:

- $s$ - Specifies the location of the right-most pole to test the stability of continuous-time models. A model is considered stable when its right-most pole is to the left of $s$.


## Default: 0

- $z$ - Specifies the maximum distance of all poles from the origin to test stability of discretetime models. A model is considered stable if all poles are within the distance $z$ from the origin.

Default: $1+$ sqrt(eps)

- AutoInitThreshold - Specifies when to automatically estimate the initial state.

The initial state is estimated when

$$
\frac{\left\|y_{p, z}-y_{\text {meas }}\right\|}{\left\|y_{p, e}-y_{\text {meas }}\right\|}>\text { AutoInitThreshold }
$$

- $y_{\text {meas }}$ is the measured output.
- $y_{p, z}$ is the predicted output of a model estimated using zero initial states.
- $y_{p, e}$ is the predicted output of a model estimated using estimated initial states.

Applicable when InitialState is 'auto'.
Default: 1.05

## Output Arguments

```
opt - Options set for greyest
```

greyest0ptions option set
Option set for greyest, returned as an greyestOptions option set.

## Examples

## Create Default Options Set for Linear Grey Box Estimation

```
opt = greyestOptions;
```


## Specify Options for Linear Grey Box Estimation

Create an options set for greyest using the 'backcast' algorithm to initialize the state. Specify Display as 'on'.
opt = greyestOptions('InitialState','backcast','Display','on');
Alternatively, use dot notation to set the values of opt.

```
opt = greyestOptions;
opt.InitialState = 'backcast';
opt.Display = 'on';
```


## Compatibility Considerations

## Renaming of Estimation and Analysis Options

The names of some estimation and analysis options were changed in R2018a. Prior names still work. For details, see the R2018a release note "Renaming of Estimation and Analysis Options".

## References

[1] Wills, Adrian, B. Ninness, and S. Gibson. "On Gradient-Based Search for Multivariable System Estimates". Proceedings of the 16th IFAC World Congress, Prague, Czech Republic, July 3-8, 2005. Oxford, UK: Elsevier Ltd., 2005.
[2] Ljung, L. System Identification: Theory for the User. Upper Saddle River, NJ: Prentice-Hall PTR, 1999.

## See Also

greyest|idgrey|idnlgrey|pem|ssest
Topics
"Loss Function and Model Quality Metrics"

Introduced in R2012a

## hasdelay

True for linear model with time delays

## Syntax

B = hasdelay(sys)
$B=$ hasdelay(sys,'elem')

## Description

B = hasdelay(sys) returns 1 (true) if the model sys has input delays, output delays, I/O delays, or internal delays, and 0 (false) otherwise. If sys is a model array, then B is true if least one model in sys has delays.

B = hasdelay(sys, 'elem') returns a logical array of the same size as the model array sys. The logical array indicates which models in sys have delays.

## See Also

absorbDelay|totaldelay
Introduced in R2012a

## iddata

Input-output data and its properties for system identification in the time or frequency domain

## Description

Use the iddata object to encapsulate input and output measurement data for the system you want to identify. System identification functions use these measurements to estimate a model. Model validation functions use the input measurements to provide the input for simulations, and the output measurements to compare how well the estimated model response fits the original data.
iddata objects can contain a single set of measurements or multiple sets. Each set of data corresponds to an experiment. The objects have the following characteristics, which are encoded in the object properties on page 1-475:

- Data can be in the frequency domain or the time domain. You can convert objects from one domain to the other.
- In the time domain, the data can be uniformly or nonuniformly sampled. To use the iddata object for estimation, however, the data must be uniformly sampled, and the input and output data for each experiment must be recorded at the same time instants.
- You can specify data properties, such as the sample time, start time, time points, frequency sample points, and intersample behavior.
- You can provide labels and comments to differentiate and annotate data components, experiments, and the object as a whole.


## Creation

## Syntax

```
data = iddata(y,u,Ts)
data = iddata(y,[],Ts)
data = iddata(y,u,Ts,'Frequency',W)
data = iddata(___,Name,Value)
```


## Description

data $=$ iddata $(\mathrm{y}, \mathrm{u}, \mathrm{Ts})$ creates an iddata object containing a time-domain output signal y and input signal $u$. Ts specifies the sample time of the experimental data.

You can use iddata to create a multiexperiment iddata object by specifying y and $u$ as cell arrays. Alternatively, you can create single-experiment iddata objects and use merge (iddata) to combine the objects into one multiexperiment iddata object. For more information on multiexperiment iddata objects, see "Create Multiexperiment Data at the Command Line".
data $=$ iddata( $\mathrm{y},[\mathrm{]}, \mathrm{Ts}$ ) creates an iddata object for time-series data. The object contains a time-domain output signal y and an empty input signal [ ]. Ts specifies the sample time of the experimental data.
 data. W sets the iddata property Frequency to a vector of frequencies. Typically, y and u are the discrete Fourier transform of time-domain signals.
data = iddata( __ , Name, Value) sets additional properties using name-value pair arguments. Specify Name, Value after any of the input argument combinations in the previous syntaxes.

## Input Arguments

## y - Output signal from system

column vector | matrix | cell array | [ ]
Output signal from a system, specified as one of the following:

- An $N$-by-1 vector for a single output system, where $N$ is the number of observations
- An $N$-by-Ny matrix for a multiple-output system, where $N y$ is the number of output channels
- An $N_{e}$ element cell array for a multiexperiment data set, where $N_{e}$ is the number of experiments and each cell contains the output signals for one experiment
- [ ] for a system that has no output signal, such as when only the input signal is recorded
y must be in the same domain as the input data u . If the data is in the time domain, y and u must be recorded at the same time instants.

If you use the iddata object for estimation, y and u must be uniformly sampled. If the nonuniformity is small, you may be able to able to convert your data into a uniformly sampled set with enough integrity that the converted data supports estimation. For more information on techniques you can try, see interpl and "Missing Data in MATLAB".
y sets the OutputData property of the iddata object.

## u - Input signal to system

column vector | matrix | cell array | [ ]
Input signal to a system, specified as one of the following:

- An $N$-by-1 vector for a single input system, where $N$ is the number of observations
- An $N$-by- $N_{u}$ matrix for a multiple-input system, where $N_{u}$ is the number of input channels
- An $N_{e}$ element cell array for a multiexperiment data set, where $N_{e}$ is the number of experiments and each cell contains the input signals for one experiment
- [] for a system that has no input signal, such as a time series
$u$ must be in the same domain as output data $y$. If the data is in the time domain, $y$ and $u$ must be recorded at the same time instants.

If you use the iddata object for estimation, y and u must be uniformly sampled. If the nonuniformity is small, you may be able to able to convert your data into a uniformly sampled set with enough integrity that the converted data supports estimation. For more information on techniques you can try, see interp1 and "Missing Data in MATLAB".
u sets the InputData property of the iddata object.

## Ts - Sample time

1 (default) | scalar | 0 | [ ]
Sample time in the units specified by the property TimeUnit, specified as one of the following:

- A scalar when y and $u$ are uniformly sampled.
- 0 for continuous-time data in the frequency domain.
- [ ] when $y$ and $u$ are not uniformly sampled and you specify the time values in the property SamplingInstants. For nonuniform sampling, y and u must be in the time domain.

Ts sets the Ts property of the iddata object.

## Properties

## Domain - Data time or frequency domain

## 'Time' (default)| 'Frequency'

Data time or frequency domain, specified as either:

- 'Time' - Data is in the time domain
- 'Frequency ' - Data is in the frequency domain


## ExperimentName - Name of each data set

\{'Exp1';'Exp2';...\} (default)| cell array
Name of each data set contained in the iddata object, specified as an $N_{e}$-by-1 cell array of character vectors, where $N_{e}$ is the number of experiments. Each cell contains the name of the corresponding experiment. For instance, \{'MyMeas1';'MyMeas2'; 'MyMeas3'\} contains experiment names for a three-experiment iddata object.

## Frequency - Frequency values

column vector | cell array
Frequency values for frequency-domain data, specified as either:

- An $N$-by- 1 vector, where $N$ is the number of frequency values in a single experiment
- A 1-by- $N_{e}$ cell array, where $N_{e}$ is the number of experiments and each cell contains the frequency vector for the corresponding experiment. The frequency vectors must all be in the same units.


## FrequencyUnit - Frequency units for frequency-domain data

'rad/TimeUnit' (default)|'cycles/TimeUnit'|'rad/s'|'Hz'|'kHz'|'MHz'|'GHz'| 'rpm'

Frequency units for frequency-domain data, specified as one of the following:
1 A scalar for a single experiment.
A 1-by- $N_{e}$ cell array, where $N_{e}$ is the number of experiments. Because all Frequency vectors must have the same units, all values of FrequencyUnit must be the same.

Changing this property does not scale or convert the data. Modifying the property changes only the interpretation of the existing data.

## InputData - Input signal values

vector | matrix | cell array of matrices
Input signal values to the system, specified as one of the following:

- For a single experiment, an $N$-by- $N_{u}$ matrix, where $N$ is the number of data samples and $N_{u}$ is the number of input channels
- For multiple experiments, a cell array containing $N_{e}$ single-experiment matrices, where $N_{e}$ is the number of experiments

When accessing InputData from the command line, you can use the shorthand form $u$. For example, $\mathrm{u} 1=$ data. InputData is equivalent to $\mathrm{ul}=$ data. u .

## InputName - Input channel names

\{'u1';'u2'; ...\} (default)|cell array of character vectors
Input channel names, specified as an $N_{u}$-by- 1 cell array, where $N_{u}$ is the number of input channels.

## InputUnit - Input channel units <br> cell array

Input channel units, specified as an $N_{u}$-by- 1 cell array, where $N_{u}$ is the number of input channels. Each cell contains the units of the corresponding input channel.

```
Example: {'rad';'rad/s'}
```


## InterSample - Intersample behavior

'zoh ' (default) | 'foh ' | 'bl' | cell array of character vectors
Intersample behavior for transformations between discrete time and continuous time, specified as a character vector or as a cell array of character vectors. For each experiment, the possible values for each input channel are:

- zoh - Zero-order hold maintains a piecewise-constant input signal between samples.
- foh - First-order hold maintains a piecewise-linear input signal between samples.
- bl - Band-limited behavior specifies that the continuous-time input signal has zero power above the Nyquist frequency.

For a single experiment with a single input channel, InterSample contains one of the values in the previous list. For multiple experiments, InterSample is an $N_{u}$-by- $N_{e}$ cell array, where $N_{u}$ is the number of input channels and $N_{e}$ is the number of experiments. Each cell contains the behavior value associated with the experiment and input channel that the cell represents.

## Name - Name of data set

' ' (default) | character vector
Name of the data set, specified as a character vector.
Example: 'dryer data'

## Notes - Comments about data set

' ' (default) | character vector | cell array
Comments about the data set, specified as a character vector or, for multiexperiment data sets, an $N_{e^{-}}$ by- 1 cell array of character vectors, where $N_{e}$ is the number of experiments.

Example: \{'data from experiment 1';data from experiment 2'\}

## OutputData - Output signal values

## vector | matrix

Output signal values from the system, specified as one of the following:

- For a single experiment, an $N$-by- $N_{y}$ matrix, where $N$ is the number of data samples and $N_{y}$ is the number of output channels
- For multiple experiments, a cell array containing $N_{e}$ single-experiment matrices, where $N_{e}$ is the number of experiments

When accessing OutputData from the command line, you can use the shorthand form y. For example, $\mathrm{y} 1=$ data. InputData is equivalent to $\mathrm{y} 1=$ data. y .

## OutputName - Output channel names

\{'y1';'y2'; ...\} (default)| cell array of character vectors
Output channel names, specified as an $N_{y}$-by-1 cell array, where $N_{y}$ is the number of output channels.

## OutputUnit - Output channel units

cell array
Output channel units, specified as an $N_{y}$-by- 1 cell array, where $N_{u}$ is the number of output channels. Each cell contains the units of the corresponding input channel.

Example: \{'rad';'rad/s'\}

## Period - Period of input signal

Inf (default) | double | vector | cell array | cell array of character vectors
Period of the input signal, specified as a double for each experiment. The value is either Inf for nonperiodic input signals or the period in the units specified by the property TimeUnit for periodic input signals.

- For a single experiment with a single input channel, Period contains a single value.
- For a multiple-input system, Period is an $N_{u}$-by-1 vector, where $N_{u}$ is the number of input channels and the $k$ th entry contains the period of the $k$ th input.
- For multiple-experiment data, Period is a 1 -by- $N_{e}$ cell array, where $N_{e}$ is the number of experiments and each cell contains a scalar or vector of periods for the corresponding experiment.


## SamplingInstants - Time values for time-domain data

vector | cell array
Time values for time-domain data in units specified by TimeUnit, specified as:

- An $N$-by- 1 vector, where $N$ is the number of data points
- A 1-by- $N_{e}$ cell array, where $N_{e}$ is the number of experiments and each cell contains the sampling instants for the corresponding experiment.

The values in SamplingInstants can be uniform or nonuniform. If you specify the Ts property, the software computes uniform time values in SamplingInstants from Ts and Tstart. If you have nonuniform sample points, specify the time values in SamplingInstants. The software then sets the Ts property to empty. Estimation functions do not support nonuniform sampling.

## TimeUnit - Units for time variable and sample time Ts

'seconds' (default)|'nanoseconds'|'microseconds'|'milliseconds'|'minutes'|
'hours'|'days'|'weeks'|'months'|'years'
Units for the time variable and the sample time, specified as a scalar. This property applies to all experiments in the data set.

Changing this property does not resample or convert the data. Modifying the property changes only the interpretation of the existing data.

## Ts - Sample time

1 (default) | positive scalar | 0 | [] | cell array
Sample time in units specified by TimeUnit, specified as a scalar or a cell array. For each experiment, the value is one of the following:

- A scalar, when y and $u$ are uniformly sampled
- 0 for continuous-time data in the frequency domain
- [ ] when $y$ and $u$ are non uniformly sampled and in the time domain, because the SamplingInstants property sets the time values for such data.

For a single experiment, Ts is a scalar. For multiexperiment data, Ts is a 1-by- $N_{e}$ cell array, where $N_{e}$ is the number of experiments and each cell contains the sample time for the corresponding experiment.

For frequency-domain data, the software uses Ts to interpret the data.

- If Ts is 0 , the software interprets inputs and outputs as continuous-time Fourier transforms (CTFTs) of the corresponding signals.
- If Ts is a scalar, the software interprets inputs and outputs as discrete-time Fourier transforms (DTFTs) of the corresponding signals with Ts as sample time.


## Tstart - Start time for time-domain data

0 (default) | scalar | cell array
Start time for time-domain data, specified as:

- A scalar for a single experiment
- A 1-by- $N_{e}$ cell array for multiple experiments, where $N_{e}$ is the number of experiments and each cell contains the start time for the corresponding experiment


## UserData - Additional comments

[ ] (default) | any MATLAB data type
Additional comments on the data set, specified as any MATLAB data type.

## Object Functions

In general, any function applicable to system identification data is applicable to an iddata object. These functions are of three general types.

1 Functions that both operate on and return iddata objects enable you to manipulate and process iddata objects.

- Use fft and ifft to transform existing iddata objects to and from the time and frequency domains. For example:

```
datafd = fft(Data);
datatd = ifft(Dataf);
```

- Use merge (iddata) to merge iddata objects into a single iddata object containing multiple experiments. To extract an experiment from a multiexperiment iddata object, use getexp. For example:

```
data123 = merge(data1,data2,data3);
data2 = getexp(datal23,2);
```

For a more detailed example, see "Extract and Model Specific Data Segments".

- Use preprocessing functions such as detrend or idfilt to filter data in iddata objects and to remove bad data. For example:

```
data_d = detrend(data);
data_f = idfilt(data,filter);
```

2 Functions that perform analytical processing on iddata objects and create plots or return specific parameters or values let you analyze data and determine inputs to use for estimation.

- Use analysis functions such as delayest and spa to compute variables such as time delay and frequency spectrum.
3 Functions that use the data in iddata objects to estimate, simulate, and validate models let you create dynamic models and evaluate how closely the model response matches validation data.
- Use estimation functions such as ssest and tfest to estimate models with specific structures.
- Use validation functions such as compare and sim to simulate estimated models and compare the simulated outputs with validation data and with other models.

The following lists contain a representative subset of the functions you can use with iddata objects.

## Data Visualization

plot Plot input and output channels of iddata object

## Data Selection

getexp Specific experiments from multiple-experiment data set merge (iddata) Merge data sets into iddata object

## Data Preprocessing

detrend Subtract offset or trend from time-domain signals contained in iddata objects
retrend Add offsets or trends to time-domain data signals stored in iddata objects
idfilt Filter data using user-defined passbands, general filters, or Butterworth filters
diff Difference signals in iddata objects
misdata Reconstruct missing input and output data
idresamp Resample time-domain data by decimation or interpolation

## Data Transformation

fft Transform iddata object to frequency domain data
ifft Transform iddata objects from frequency to time domain

## Data Analysis

| realdata | Determine whether iddata is based on real-valued signals |
| :--- | :--- |
| delayest | Estimate time delay (dead time) from data |
| isreal | Determine whether model parameters or data values are real |
| impulseest | Nonparametric impulse response estimation |
| pexcit | Level of excitation of input signals |
| feedback | Identify possible feedback data |
| etfe | Estimate empirical transfer functions and periodograms |
| spafdr | Estimate frequency response and spectrum using spectral analysis with |
| spa | frequency-dependent resolution |
| Estimate frequency response with fixed frequency resolution using spectral |  |
| iddataPlotOptions | analysis |
|  | Option set for plot when plotting data contained in an iddata object |

## Model Estimation, Simulation, and Validation

ssest Estimate state-space model using time-domain or frequency-domain data
tfest Estimate transfer function
ar Estimate parameters of AR model or ARI model for scalar time series
sim $\quad$ Simulate response of identified model
findstates
compare
predict
goodnessOfFit
Estimate initial states of model
Compare identified model output and measured output
Predict K-step-ahead model output
Goodness of fit between test and reference data for analysis and validation of identified models
procest Estimate process model using time or frequency data
resid Compute and test residuals

## Examples

## Time-Domain Data

Create an iddata object using single-input/single-output (SISO) time-domain data. The input and output each contain 1000 samples with a sample time of 0.08 seconds.

```
load dryer2_data output input;
data = iddata(output,input,0.08)
data =
Time domain data set with 1000 samples.
Sample time: 0.08 seconds
Outputs Unit (if specified)
    y1
Inputs Unit (if specified)
    u1
```

The software assigns the default channel name ' y 1 ' to the first and only output channel. When the output signal contains several channels, the software assigns the default names 'y1','y2',...,'yn'. Similarly, the software assigns the default channel name 'u1' to the first and only input channel. For more information about naming channels, see "Naming, Adding, and Removing Data Channels".

Plot the data.
plot(data)

## Input-Output Data



Adjacent plots display output data and input data.

## Time-Series Data

Create an iddata object from time-series data. Time-series data has no input channel.
Load the output channel of a data set, and create an iddata object that has a sample time of 0.08 seconds.

```
load dryer2_data output
data = idda\overline{ta(output,[],0.08)}
data =
```

```
Time domain data set with 1000 samples.
Sample time: 0.08 seconds
Outputs Unit (if specified)
    y1
```

Plot the data.
plot(data)


You can use data for time-series model estimation.

## Frequency-Domain Data

Create and examine an iddata object from complex-valued frequency-domain input-output data. Convert the object into the time domain.

Input and output data is sometimes expressed in the form of the Fourier transforms of time-domain input-output signals. You can encapsulate this data in a frequency-domain iddata object.

Load the data, which consists of the complex-valued input-output frequency-domain data U and Y , frequency vector $W$, and sample time Ts.

```
load demofr1 U Y W Ts
```

```
Create the frequency-domain iddata object data_fr.
data_fr = iddata(Y,U,Ts,'Frequency',W)
data_fr =
Frequency domain data set with responses at 501 frequencies.
Frequency range: 0 to 31.416 rad/seconds
Sample time: 0.1 seconds
Outputs Unit (if specified)
    y1
Inputs Unit (if specified)
    ul
```

Examine the properties. Frequency-domain iddata objects include frequency-specific properties, such as Frequency for the frequency vector and FrequencyUnit for frequency units. In contrast, time-domain iddata objects include time-specific properties such as Tstart and SamplingInstants for time-domain data.

```
get(data_fr)
ans = struct with fields:
            Domain: 'Frequency'
                Name: ''
        OutputData: [501x1 double]
                    y: 'Same as OutputData'
        OutputName: {'yl'}
        OutputUnit: {''}
            InputData: [501x1 double]
            u: 'Same as InputData'
        InputName: {'ul'}
        InputUnit: {''}
            Period: Inf
        InterSample: 'zoh'
            Ts: 0.1000
    FrequencyUnit: 'rad/TimeUnit'
            Frequency: [501x1 double]
            TimeUnit: 'seconds'
    ExperimentName: 'Exp1'
            Notes: {}
        UserData: []
```

Assign the contents of the frequency property to the variable F.
$F=$ data_fr.Frequency;
Get the frequency units of the data. The property TimeUnit sets the units of the sample time.

```
frequ = data_fr.FrequencyUnit
frequ =
'rad/TimeUnit'
timeu = data_fr.TimeUnit
```

```
timeu =
'seconds'
```

Convert data_fr back into the time domain by using the inverse Fourier transform function ifft.
data_t = ifft(data_fr)
data_t =
Time domain data set with 1000 samples.
Sample time: 0.1 seconds
Outputs Unit (if specified)
y1
Inputs Unit (if specified)
u1
get(data_t)
ans = struct with fields:
Domain: 'Time'
Name: ''
OutputData: [1000x1 double]
y: 'Same as OutputData'
OutputName: \{'y1'\}
OutputUnit: \{''\}
InputData: [1000x1 double]
u: 'Same as InputData'
InputName: \{'u1'\}
InputUnit: \{''\}
Period: Inf
InterSample: 'zoh'
Ts: 0.1000
Tstart: []
SamplingInstants: []
TimeUnit: 'seconds'
ExperimentName: 'Exp1'
Notes: \{\}
UserData: []

## View and Modify Properties

View properties of an iddata object. Modify the properties both during and after object creation.
Load input and output data.
load dryer2_data input output
Create an iddata object.
data $=$ iddata(output,input, 0.08 )
data $=$

```
Time domain data set with 1000 samples.
Sample time: 0.08 seconds
Outputs Unit (if specified)
    y1
Inputs Unit (if specified)
    ul
```

View all properties of the iddata object.

```
get(data)
ans = struct with fields:
            Domain: 'Time'
                Name: ''
        OutputData: [1000x1 double]
            y: 'Same as OutputData'
        OutputName: {'y1'}
        OutputUnit: {''}
            InputData: [1000x1 double]
            u: 'Same as InputData'
        InputName: {'u1'}
        InputUnit: {''}
            Period: Inf
        InterSample: 'zoh'
            Ts: 0.0800
            Tstart: []
    SamplingInstants: [1000x0 double]
            TimeUnit: 'seconds'
        ExperimentName: 'Exp1'
            Notes: {}
        UserData: []
```

You can specify properties when you create an iddata object using name-value pair arguments.
Create an iddata object from the same data inputs, but change the experiment name from its default setting to Dryer2.

```
data = iddata(output,input,0.08,'ExperimentName','Dryer2')
data =
Experiment Dryer2.Time domain data set with 1000 samples.
Sample time: 0.08 seconds
Outputs Unit (if specified)
    y1
Inputs Unit (if specified)
    u1
```

To change property values for an existing iddata object, use dot notation. Change the sample time property Ts to 0.05 seconds.
data.Ts $=0.05$

```
data =
Experiment Dryer2.Time domain data set with 1000 samples.
Sample time: 0.05 seconds
Outputs Unit (if specified)
    y1
Inputs Unit (if specified)
    u1
```

Property names are not case sensitive. Also, if the first few letters uniquely identify the property, you do not need to type the entire property name.

```
data.exp = "Dryer2 January 2015"
data =
Experiment Dryer2 January 2015.Time domain data set with }1000\mathrm{ samples.
Sample time: 0.05 seconds
Outputs Unit (if specified)
    y1
Inputs Unit (if specified)
    u1
```

You can use data. y as a shorthand for data. OutputData to access the output values, or use data.u as a shorthand for data. InputData to access the input values.
y_data = data.y;
u_data = data.u;

## See Also

idfrd|idinput

## Topics

"Representing Data in MATLAB Workspace"
"Create Multiexperiment Data at the Command Line"
"Representing Time- and Frequency-Domain Data Using iddata Objects"
"Managing iddata Objects"

## Introduced before R2006a

## iddataPlotOptions

Option set for plot when plotting data contained in an iddata object

## Syntax

```
opt = iddataPlotOptions('time')
opt = iddataPlotOptions('frequency')
opt = iddataPlotOptions(
```

$\qquad$

``` ,'identpref')
```


## Description

opt = iddataPlotOptions('time') creates the default option set for plotting time-domain data. Use dot notation to customize the option set, if needed.
opt = iddataPlotOptions('frequency') creates a default option set for plotting frequencydomain data. Use dot notation to customize the option set, if needed.
opt = iddataPlotOptions( __ ,'identpref') initializes the plot options with the System Identification Toolbox preferences. This syntax can include any of the input argument combinations in the previous syntaxes. Use this syntax to change a few plot options but otherwise use your toolbox preferences.

## Examples

## Create Option Set for Plotting Time-Domain Data

Create an options set with default options for time-domain data.

```
opt = iddataPlotOptions('time');
```

Specify plot properties, such as time units and grid. View the plot in minutes

```
opt.TimeUnits = 'minutes';
% Turn grid on
opt.Grid = 'on';
```

Create a plot using the specified options.
load iddatal z1
h = plot(z1, opt);


## Change Orientation of Input-Output Data Axes

Generate data with two inputs and one output.
$z=i d d a t a(r a n d n(100,1), r a n d(100,2)) ;$
Configure a time plot.
opt = iddataPlotOptions('time');
Plot the data.
h = plot(z,opt);


Change the orientation of the plots such that all inputs are plotted in one column, and all outputs are in a second column.

```
opt.Orientation = 'two-column';
```

h = plot(z,opt);


Alternatively, use setoptions.
setoptions(h,'Orientation','two-column')
You can also change the orientation by right-clicking the plot and choosing Orientation in the context menu.

## Create Option Set for Plotting Frequency-Domain Data

Create an option set with default options for frequency-domain data.

```
opt = iddataPlotOptions('frequency');
```

Specify plot properties, such as phase visibility and frequency units.

```
opt.PhaseVisible = 'off';
```

opt.FreqUnits = 'Hz';

Create a plot with the specified options.

```
load iddata7 z7
zf = fft(z7);
h = plot(zf,opt);
```



Initialize a Plot Using Toolbox Preferences
opt = iddataPlotOptions('time','identpref');

## Output Arguments

## opt - Option set for iddata/plot

iddataPlotOptions option set
Option set containing the specified options for iddata/plot. The structure has the following fields:


| Field | Description |
| :---: | :---: |
|  | Default: 'tex' |
| TickLabel | Tick label style, specified as a structure array with the following fields: <br> - FontSize - Font size, specified as scalar value greater than 0. <br> Default: 8 <br> - FontWeight - Thickness of text, specified as one of the following values: 'Normal ' \| 'Bold' <br> Default: 'Normal' <br> - Font Angle - Text character angle, specified as one of the following values: 'Normal ' \| 'Italic' <br> Default: 'Normal' <br> - Color - Color of text, specified as vector of RGB values between 0 to $1 \mid$ character vector of color name \| 'none '. For example, for yellow color, specify as one of the following: [1 1 0], 'yellow', or 'y'. <br> Default: [0,0,0] |
| Grid | Show or hide the grid, specified as one of the following values: 'off' \|'on' <br> Default: 'off' |
| GridColor | Color of the grid lines, specified as one of the following values: vector of RGB values in the range $[0,1] \mid$ character vector of color name \| 'none'. For example, for yellow color, specify as one of the following: [1 1 0 1 ], 'yellow', or 'y'. <br> Default: [0.15, 0. 15, 0.15] |
| XlimMode, YlimMode | Axes limit modes, specified as one of the following values: <br> - 'auto ' - The axis limits are based on the data plotted <br> - 'manual' - The values explicitly set with Xlim, Ylim <br> Default: 'auto ' |
| Xlim, Ylim | Axes limits, specified as maximum and minimum values. <br> Default: [0 1] |


| Field | Description |
| :--- | :--- |
| IOG rouping | Grouping of input-output pairs, specified as one of <br> the following values: 'none' $\mid$ 'inputs' $\mid$ <br> 'outputs ' $\mid$ 'all' <br> Default: ' none' |


| Field | Description |
| :---: | :---: |
| InputLabels, OutputLabels | Input and output label styles on individual plot axes, specified as a structure array with the following fields: <br> - FontSize - Font size, specified as data type scalar. <br> Default: 8 <br> - FontWeight - Thickness of text, specified as one of the following values: 'Normal ' \| 'Bold' <br> Default: 'Normal' <br> - Font Angle - Text character angle, specified as one of the following values: 'Normal' \| 'Italic' <br> Default: 'Normal' <br> - Color - Color of text, specified as a vector of RGB values between 0 to 1 \| character vector of color name | 'none '. For example, for yellow color, specify as one of the following: [1 1 0], 'yellow', or 'y'. Default: [0.4, 0.4, 0.4] <br> - Interpreter - Interpretation of text characters, specified as one of the following values: 'tex'\|'latex'|'none' Default: 'tex' |
| InputVisible, OutputVisible | Visibility of input and output channels, specified as one of the following values: 'off' \| 'on' <br> Default: ' on ' |


| Field | Description |
| :---: | :---: |
| Orientation | Orientation of the input and output data plots, specified as one of the following values: <br> - 'two-row' - Plot all outputs in one row and all inputs in a second row <br> - 'two-column ' - Plot all outputs in one column and all inputs in a second column <br> - 'single-row' - Plot all inputs and outputs in one row <br> - 'single-column' - Plot all inputs and outputs in one column <br> Default: 'two-row'. |
| For time-domain data plots only: |  |
| Field | Description |
| TimeUnits | Time units, specified as one of the following values: <br> - 'nanoseconds' <br> - 'microseconds' <br> - 'milliseconds' <br> - 'seconds' <br> - 'minutes' <br> - 'hours' <br> - 'days' <br> - 'weeks' <br> - 'months' <br> - 'years' <br> You can also specify 'auto' which uses time units specified in the TimeUnit property of the data. For multiple systems with different time units, the units of the first system is used. |
| Normalize | Normalize responses, specified as one of the following values: 'on'\|'off' <br> Default: 'off' |


| Field | Description |
| :---: | :---: |
| For frequency-domain data plots only: |  |
| Field | Description |
| FreqUnits | Frequency units, specified as one of the following values: <br> - 'Hz' <br> - 'rad/second' <br> - 'rpm' <br> - 'kHz' <br> - 'MHz' <br> - 'GHz' <br> - 'rad/nanosecond' <br> - 'rad/microsecond' <br> - 'rad/millisecond' <br> - 'rad/minute' <br> - 'rad/hour' <br> - 'rad/day' <br> - 'rad/week' <br> - 'rad/month' <br> - 'rad/year' <br> - 'cycles/nanosecond' <br> - 'cycles/microsecond' <br> - 'cycles/millisecond' <br> - 'cycles/hour' <br> - 'cycles/day' <br> - 'cycles/week' <br> - 'cycles/month' <br> - 'cycles/year' <br> Default: 'rad/s' <br> You can also specify 'auto ' which uses frequency units rad/TimeUnit relative to system time units specified in the TimeUnit property. For multiple systems with different time units, the units of the first system are used. |
| FreqScale | Frequency scale, specified as one of the following values: 'linear'\|'log' <br> Default: ' $\log$ ' |


| Field | Description <br> Field <br> MagUnitsMagnitude units, specified as one of the <br> following values: 'dB' $~$ <br> ' 'abs ' |
| :--- | :--- |
| Default: 'dB' |  |

## See Also

identpref|plot

Introduced in R2014a

## identpref

Set System Identification Toolbox preferences

## Syntax

identpref

## Description

identpref opens a Graphical User Interface (GUI) which allows you to change the System Identification Toolbox preferences. Preferences set in this GUI affect future plots only (existing plots are not altered).

Your preferences are stored to disk (in a system-dependent location) and will be automatically reloaded in future MATLAB sessions using the System Identification Toolbox software.

## See Also

## Topics

"Toolbox Preferences Editor"

Introduced in R2012a

## idfilt

Filter data using user-defined passbands, general filters, or Butterworth filters

## Syntax

Zf = idfilt(Z,filter)
Zf = idfilt(Z,filter, causality)
Zf = idfilt(Z,filter,'FilterOrder',NF)

## Description

Zf = idfilt(Z,filter) filters data using user-defined passbands, general filters, or Butterworth filters. $Z$ is the data, defined as an iddata object. $Z f$ contains the filtered data as an iddata object. The filter can be defined in three ways:

- As an explicit system that defines the filter.
filter $=$ idm or filter $=\{n u m, d e n\}$ or filter $=\{A, B, C, D\}$
idm can be any SISO identified linear model or LTI model object. Alternatively the filter can be defined as a cell array $\{A, B, C, D\}$ of SISO state-space matrices or as a cell array \{num, den\} of numerator/denominator filter coefficients.
- As a vector or matrix that defines one or several passbands.
filter=[[wp1l,wp1h];[ wp2l,wp2h]; ....; [wpnl,wpnh]]
The matrix is $n$-by- 2 , where each row defines a passband. A filter is constructed that gives the union of these passbands. For time-domain data, it is computed as cascaded Butterworth filters or order NF. The default value of NF is 5 .
- For time-domain data - The passbands are in units of rad/TimeUnit, where TimeUnit is the time units of the estimation data.
- For frequency-domain data - The passbands are in the frequency units (FrequencyUnit property) of the estimation data.

For example, to define a stopband between ws 1 and ws2, use
filter = [0 ws1; ws2,Nyqf]
where Nyqf is the Nyquist frequency.

- For frequency-domain data, only the frequency response of the filter can be specified.
filter = Wf
Here Wf is a vector of possibly complex values that define the filter's frequency response, so that the inputs and outputs at frequency Z. Frequency (kf) are multiplied by Wf (kf). Wf is a column vector of length $=$ number of frequencies in $Z$. If the data object has several experiments, Wf is a cell array of length $=\#$ of experiments in $Z$.

Zf = idfilt(Z,filter, causality) specifies causality. For time-domain data, the filtering is carried out in the time domain as causal filtering as default. This corresponds to a last argument
causality = 'causal'. With causality = 'noncausal', a noncausal, zero-phase filter is used for the filtering (corresponding to filtfilt in the Signal Processing Toolbox product).

For frequency-domain data, the signals are multiplied by the frequency response of the filter. With the filters defined as passband, this gives ideal, zero-phase filtering ("brickwall filters"). Frequencies that have been assigned zero weight by the filter (outside the passband, or via the frequency response) are removed from the iddata object Zf .

Zf = idfilt(Z,filter,'FilterOrder',NF) specifies the filter order. The time domain filters in the pass-band case are calculated as cascaded Butterworth pass-band and stop-band filters. The orders of these filters are 5 by default, which can be changed to an arbitrary integer NF.

It is common practice in identification to select a frequency band where the fit between model and data is concentrated. Often this corresponds to bandpass filtering with a passband over the interesting breakpoints in a Bode diagram. For identification where a disturbance model is also estimated, it is better to achieve the desired estimation result by using the 'WeightingFilter' option of the estimation command than just to prefilter the data. The values for 'WeightingFilter' are the same as the argument filter in idfilt.

## Algorithms

The Butterworth filter is the same as butter in the Signal Processing Toolbox product. Also, the zero-phase filter is equivalent to filtfilt in that toolbox.

## References

Ljung (1999), Chapter 14.

## See Also

iddata|resample

Introduced before R2006a

## idfrd

Frequency response data or model

## Description

An idfrd object stores frequency response data over a range of frequency values. You can use an idfrd object in two ways. You can use the object as estimation data for estimating a time-domain or frequency-domain model, similarly to an iddata object. Or, you can use the object as a linear model, similarly to how you use an idss state-space model or any other identified linear model. Use the idfrd command to encapsulate frequency response data or to convert a linear time-domain or frequency-domain dynamic model into a frequency response model.

Commands that accept iddata objects, such as the model estimation command ssest, generally also accept idfrd objects. However, an idfrd object can contain data from only one experiment. It does not have the multiexperiment capability that an iddata object has.

Commands that accept identified linear models, such as the analysis and validation commands compare, sim, and bode, generally also accept idfrd models.

For a model of the form

$$
y(t)=G(q) u(t)+H(q) e(t)
$$

the transfer function estimate is $G\left(e^{i \omega}\right)$ and the additive noise spectrum $\Phi_{v}$ at the output is

$$
\Phi_{v}(\omega)=\lambda T|H(e i \omega T)|^{2}
$$

Here, $\lambda$ is the estimated variance of $e(t)$ and $T$ is the sample time.
For a continuous-time system, the noise spectrum is

$$
\Phi_{v}(\omega)=\lambda|H(e i \omega)|^{2}
$$

An idfrd object stores $G\left(e^{i \omega}\right)$ and $\Phi_{v}$.

## Creation

You can obtain an idfrd model in one of three ways.

- Create the model from frequency response data using the idfrd command. For example, create an idfrd model that encapsulates frequency response data taken at specific frequencies using the sample time Ts.

```
sysfr = idfrd(ResponseData,Freq,Ts)
```

For an example, see "Create idfrd Object from Frequency Response Data" on page 1-512.

- Estimate the model using a frequency response estimation command such as spa, using timedomain, frequency-domain, or frequency response data.

```
sysfr = spa(data)
```

For more information about frequency response estimation commands, see spa, spafdr, and etfe.

- Convert a linear model such as an idss model into an idfrd model by computing the frequency response of the model.
sysfr = idfrd(sys)
For an example of linear model conversion, see "Convert Time-Domain Model to Frequency Response Model" on page 1-515.

For information on functions you can use to extract information from or transform idfrd model objects, see "Object Functions" on page 1-512.

## Syntax

sysfr $=$ idfrd(ResponseData,Frequency,Ts)
sysfr $=$ idfrd( $\qquad$ , Name, Value)
sysfr $=$ idfrd(sys)
sysfr $=$ idfrd(sys,Frequency)
sysfr $=$ idfrd(sys,Frequency,FrequencyUnits)

## Description

## Create Frequency Response Object

sysfr $=$ idfrd(ResponseData,Frequency,Ts) creates a discrete-time idfrd object that stores the frequency response ResponseData of a linear system at frequency values Frequency. Ts is the sample time. For a continuous-time system, set Ts to 0 .
sysfr $=$ idfrd(__, Name, Value) sets additional properties using one or more name-value arguments. Specify the name-value arguments after the first three arguments. For instance, to specify the frequency units as MHz , use sysfr = idfrd(ResponseData, Frequency, Ts,'FrequencyUnits', 'MHz').

## Convert Linear Identified Model to Frequency Response Model

sysfr $=$ idfrd(sys) converts a System Identification Toolbox or Control System Toolbox linear model to frequency response data at default frequencies, including the output noise spectra and spectra covariance.
sysfr $=$ idfrd(sys,Frequency) computes the frequency response at frequencies Frequency, where Frequency is expressed in radians/TimeUnit.
sysfr $=$ idfrd(sys,Frequency,FrequencyUnits) interprets frequencies in the Frequency vector in the units specified by FrequencyUnit.

## Input Arguments

sys - Linear dynamic system model
linear dynamic system model
Linear dynamic system model, specified as a System Identification Toolbox or Control System Toolbox linear model.

## Properties

## ResponseData - Frequency response data

3-D array of complex numbers
Frequency response data, specified as a 3-D array of complex numbers.

- For SISO systems, ResponseData is a vector of frequency response values at the frequency points specified in the Frequency property.
- For MIMO systems with $N_{u}$ inputs and $N_{y}$ outputs, ResponseData is an $N_{y}$-by- $N_{u}$-by- $N_{f}$ array, where $N_{f}$ is the number of frequency points.

ResponseData ( $\mathrm{ky}, \mathrm{ku}, \mathrm{kf}$ ) represents the frequency response from the input ku to the output ky at the frequency Frequency (kf).

## Frequency - Frequency points

column vector
Frequency points corresponding to ResponseData, specified as a column vector that contains $N_{f}$ points in the units specified by FrequencyUnit.

## FrequencyUnit - Units for frequency vector

'rad/TimeUnit' (default) | 'cycles/TimeUnit'|'rad/s'| 'Hz'|'kHz'|'MHz'|'GHz'| 'rpm'

Units of the frequency vector in the Frequency property, specified as one of the following values:

- 'rad/TimeUnit'
- 'cycles/TimeUnit'
- 'rad/s'
- 'Hz'
- ' kHz'
- 'MHz'
- 'GHz'
- 'rpm'

The units 'rad/TimeUnit' and 'cycles/TimeUnit' are relative to the time units specified in the TimeUnit property.

Changing this property does not resample or convert the data. Modifying the property changes only the interpretation of the existing data. Use chgTimeUnit to convert the data to different frequency units.

## SpectrumData - Power spectra and cross spectra

vector of complex numbers | 3-D array of complex numbers
Power spectra and cross spectra of the system output disturbances (noise), specified as a vector (single-output system) or a 3-D array of complex numbers (multiple-output system). For response data with $N_{y}$ outputs and $N_{f}$ frequency points, specify SpectrumData as an $N_{y}$-by- $N_{y}$-by- $N_{w}$ array.

SpectrumData(ky1,ky2,kf) is the cross spectrum between the noise at output ky1 and the noise at output ky2 at the frequency Frequency (kf). The power spectrum is the subset of the cross spectrum where ky1 and ky2 are equal.

## CovarianceData - Covariance of response

5-D numeric array
Covariance of SpectrumData, specified as a 5-D array with dimensions $N_{y}$-by- $N_{u}$-by- $N_{f}$-by-2-by-2, where $N_{y}$ is the number of outputs, $N_{u}$ is the number of inputs, and $N_{f}$ is the number of frequency points.

CovarianceData(ky, ku, kf,: ,: ) is the 2-by-2 covariance matrix of SpectrumData(ky, ku,kf). The $(1,1)$ element is the variance of the real part, the $(2,2)$ element is the variance of the imaginary part, and the $(1,2)$ and $(2,1)$ elements are the covariance between the real and imaginary parts. squeeze(CovarianceData(ky,ku,kf,:,:)) thus gives the covariance matrix of the corresponding response.

If you obtain sysfr by converting a model sys, the value of CovarianceData depends on how you obtained sys.

- If you obtained sys by identification, the software computes the estimated covariance for sysfr from the uncertainty information in sys. The software uses Gauss' approximation formula for this calculation for all model types, except grey-box models. For grey-box models (idgrey), the software applies numerical differentiation.
- If you created sys by using commands such as idss, idtf, idproc, idgrey, or idpoly, then the software sets CovarianceData for sysfr to [].


## NoiseCovariance - Power spectra variance

numeric vector | 3-D numeric array | 0
Power spectra variance, specified as a vector (single-output system) or a 3-D array (multiple-output system). For response data with $N_{y}$ outputs and $N_{w}$ frequency points, specify NoiseCovariance as an $N_{y}$-by- $N_{y}$-by- $N_{w}$ array. NoiseCovariance (ky1, ky2, kw) is the variance of the corresponding power spectrum.

To eliminate the influence of the noise component from the model, specify NoiseCovariance as 0 . With zero covariance, the predicted output is the same as the simulated output.

## InterSample - Intersample behavior

'zoh'|'foh'|'bl'| cell array of character vectors
Intersample behavior of the input signal for transformations between discrete time and continuous time, specified as a character vector or as an $N_{u}$-by- 1 cell array of character vectors, where $N_{u}$ is the number of input channels. This property is meaningful only when you are estimating continuous-time models (sample time $\mathrm{T} s>0$ ) from discrete-time data.

For each input channel, the possible values of InterSample are:

- 'zoh ' - Zero-order hold maintains a piecewise-constant input signal between samples.
- 'foh ' - First-order hold maintains a piecewise-linear input signal between samples.
- 'bl' - Band-limited behavior specifies that the continuous-time input signal has zero power above the Nyquist frequency (pi/sys.Ts rad/s). This behavior typically occurs when the input signal is measured experimentally using an antialiasing filter and a sampler. Ideally, treat the data
as continuous-time. That is, if the signals used for the estimation of the frequency response were subject to anti-aliasing filters, set sys. Ts to zero.

If you obtain sysfr by conversion of a model sys, then InterSample is equal to the Intersample property of the iddata object that you used to estimate sys.

For more information on this property, see "Effect of Input Intersample Behavior on Continuous-Time Models".

## IODelay - Transport delays

0 (default) | numeric array
Transport delays, specified as a numeric array containing a separate transport delay for each inputoutput pair.

For continuous-time systems, transport delays are expressed in the time unit stored in the TimeUnit property. For discrete-time systems, transport delays are expressed as integers denoting delays of a multiple of the sample time Ts.

For a MIMO system with $N_{y}$ outputs and $N_{u}$ inputs, set IODelay as an $N_{y}$-by- $N_{u}$ array. Each entry of this array is a numerical value representing the transport delay for the corresponding input-output pair. You can set IODelay to a scalar value to apply the same delay to all input-output pairs.

## InputDelay - Input delay for each input channel

0 (default) | scalar | vector
Input delay for each input channel, specified as a scalar value or numeric vector. For continuous-time systems, specify input delays in the time unit stored in the TimeUnit property. For discrete-time systems, specify input delays in integer multiples of the sample time Ts. For example, setting InputDelay to 3 specifies a delay of three sample times.

For a system with $N_{u}$ inputs, set InputDelay to an $N_{u}$-by-1 vector. Each entry of this vector is a numerical value that represents the input delay for the corresponding input channel.

You can also set InputDelay to a scalar value to apply the same delay to all channels.

## OutputDelay - Output delay for each output channel

0 (default)
For identified systems such as idfrd, OutputDelay is fixed to zero.

## Ts - Sample time

1 (default) | 0 | positive scalar | -1
Sample time, specified as one of the following.

- Discrete-time model with a specified sampling time - a positive scalar representing the sampling period expressed in the unit specified by the TimeUnit property of the model
- Continuous-time model - 0
- Discrete-time model with an unspecified sample time - - 1

Changing this property does not discretize or resample the model. Use c2d and d2c to convert between continuous- and discrete-time representations. Use d2d to change the sample time of a discrete-time system.

## TimeUnit - Units for time variable

```
'seconds' (default)|'nanoseconds'|'microseconds' | 'milliseconds'|'minutes'|
'hours'|'days'| 'weeks'|'months'| 'years'
```

Units for the time variable, the sample time Ts, and any time delays in the model, specified as one of the following values.

- 'nanoseconds'
- 'microseconds'
- 'milliseconds'
- 'seconds'
- 'minutes'
- 'hours'
- 'days'
- 'weeks'
- 'months'
- 'years'

Changing this property does not resample or convert the data. Modifying the property changes only the interpretation of the existing data. Use chgTimeUnit to convert data to different time units

## InputName - Input channel names

' ' (default) | character vector | cell array
Input channel names, specified as a character vector or cell array.

- Single-input model - Character vector. For example, ' controls'.
- Multi-input model - Cell array of character vectors.

Alternatively, use automatic vector expansion to assign input names for multi-input models. For example, if sys is a two-input model, enter:
sys.InputName = 'controls';
The input names automatically expand to \{'controls(1)'; 'controls(2)'\}.
When you estimate a model using an iddata object data, the software automatically sets InputName to data. InputName.

You can use the shorthand notation $u$ to refer to the InputName property. For example, sys. $u$ is equivalent to sys.InputName.

You can use input channel names in several ways, including:

- To identify channels on model display and plots
- To extract subsystems of MIMO systems
- To specify connection points when interconnecting models


## InputUnit - Input channel units

' ' (default) | character vector | cell array
Input channel units, specified as a character vector or cell array:

- Single-input model - Character vector
- Multi-input Model - Cell array of character vectors

Use InputUnit to keep track of input signal units. InputUnit has no effect on system behavior.

## InputGroup - Input channel groups

structure with no fields (default)| structure
Input channel groups, specified as a structure. The InputGroup property lets you divide the input channels of MIMO systems into groups so that you can refer to each group by name. In the InputGroup structure, set field names to the group names, and field values to the input channels belonging to each group.

For example, create input groups named controls and noise that include input channels 1 and 2 and channels 3 and 5, respectively.

```
sys.InputGroup.controls = [1 2];
sys.InputGroup.noise = [3 5];
```

You can then extract the subsystem from the controls inputs to all outputs using the following syntax:

```
sys(:,'controls')
```


## OutputName - Output channel names

' ' (default) | character vector | cell array
Output channel names, specified as a character vector or cell array.

- Single-input model - Character vector. For example, 'measurements '
- Multi-input model - Cell array of character vectors

Alternatively, use automatic vector expansion to assign output names for multi-output models. For example, if sys is a two-output model, enter:
sys.OutputName = 'measurements';
The output names automatically expand to \{'measurements(1)';'measurements(2)'\}.
When you estimate a model using an iddata object data, the software automatically sets OutputName to data. OutputName.

You can use the shorthand notation y to refer to the OutputName property. For example, sys.y is equivalent to sys.OutputName.

You can use output channel names in several ways, including:

- To identify channels on model display and plots
- To extract subsystems of MIMO systems
- To specify connection points when interconnecting models


## OutputUnit - Output channel units

' ' (default) | character vector | cell array
Output channel units, specified as a character vector or cell array.

- Single-input model - Character vector. For example, ' seconds ' .
- Multi-input model - Cell array of character vectors.

Use OutputUnit to keep track of output signal units. OutputUnit has no effect on system behavior.

## OutputGroup - Output channel groups

structure with no fields (default) | structure
Output channel groups, specified as a structure. The OutputGroup property lets you divide the output channels of MIMO systems into groups and refer to each group by name. In the OutputGroup structure, set field names to the group names, and field values to the output channels belonging to each group.

For example, create output groups named temperature and measurement that include output channel 1, and channels 3 and 5, respectively.

```
sys.OutputGroup.temperature = [1];
sys.OutputGroup.measurement = [3 5];
```

You can then extract the subsystem from all inputs to the measurement outputs using the following syntax.
sys('measurement',:)

## Name - System name

' ' (default) | character vector
System name, specified as a character vector. For example, 'system_1'.

## Notes - Notes on system

0 -by-1 string (default) | string | character vector
Any text that you want to associate with the system, specified as a string or a cell array of character vectors. The property stores whichever data type you provide. For instance, if sys1 and sys2 are dynamic system models, you can set their Notes properties as follows.

```
sys1.Notes = "sys1 has a string.";
sys2.Notes = 'sys2 has a character vector.';
sys1.Notes
sys2.Notes
ans =
    "sysl has a string."
ans =
    'sys2 has a character vector.'
```

UserData - Data to associate with system
[ ] (default) | any MATLAB data type

Data to associate with the system, specified as any MATLAB data type.

## SamplingGrid - Sampling grid

[] (default) | struct
Sampling grid for model arrays, specified as a structure.
For arrays of identified linear (IDLTI) models that you derive by sampling one or more independent variables, this property tracks the variable values associated with each model. This information appears when you show or plot the model array. Use this information to trace results back to the independent variables.

Set the field names of the data structure to the names of the sampling variables. Set the field values to the sampled variable values associated with each model in the array. All sampling variables must be numeric and scalar valued, and all arrays of sampled values must match the dimensions of the model array.

For example, suppose that you collect data at various operating points of a system. You can identify a model for each operating point separately and then stack the results together into a single system array. You can tag the individual models in the array with information regarding the operating point.

```
nominal_engine_rpm = [1000 5000 10000];
sys.SamplingGrid = struct('rpm', nominal_engine_rpm)
```

Here, sys is an array containing three identified models obtained at 1000,5000 , and $10,000 \mathrm{rpm}$, respectively.

For model arrays that you generate by linearizing a Simulink ${ }^{\circledR}$ model at multiple parameter values or operating points, the software populates SamplingGrid automatically with the variable values that correspond to each entry in the array.

## Report - Summary report

report field values
This property is read-only.
Summary report that contains information about the estimation options and results when the frequency-response model is obtained using estimation commands, such as spa, spafdr, and etfe. Use Report to query a model for how it was estimated, including its:

- Estimation method
- Estimation options

The contents of Report are irrelevant if the model was created by construction.

```
f = logspace(-1,1,100);
[mag,phase] = bode(idtf([1 .2],[1 2 1 1]),f);
response = mag.*exp(lj*phase*pi/180);
sysfr = idfrd(response,f,0.08);
sysfr.Report.Method
ans =
```

If you obtain the frequency-response model using estimation commands, the fields of Report contain information on the estimation data, options, and results.

```
load iddata3;
sysfr = spa(z3);sysfr.Report.Method
ans =
SPA
```

For more information on this property and how to use it, see the Output Arguments section of the corresponding estimation command reference page and "Estimation Report".

## Object Functions

Many functions applicable to "Dynamic System Models" are also applicable to an idfrd model object. These functions are of three general types.

- Functions that operate on and return idfrd model objects enable you to convert and manipulate idfrd models.
- Functions that perform analytical and simulation functions on idfrd objects, such as bode and sim
- Functions that retrieve or interpret model information, such as getcov

Unlike other identified linear models, you cannot directly convert an idfrd model into another model type using commands such as idss or idtf. Instead, use the estimation command for the model you want, using the idfrd object as the estimation data. For instance, use sys $=$ ssest (sysfr,2) to estimate a second-order state-space model from the frequency response data in idfrd model sysfr. For an example of using an idfrd object as estimation data, see "Estimate Time-Domain Model Using Frequency Response Data" on page 1-517.

The following lists contain a representative subset of the functions that you can use with idss models.

## Transformation and Manipulation

chgTimeUnit Change time units of dynamic system
chgFreqUnit Change frequency units of frequency-response data model
fselect Select frequency points or range in FRD model
frdata Access data for frequency response data (FRD) object
fcat Concatenate FRD models along frequency dimension

## Analysis and Simulation

bode Bode plot of frequency response, or magnitude and phase data
spectrum Plot or return output power spectrum of time series model or disturbance spectrum of linear input-output model

## Information Extraction and Interpretation

get Access model property values
getcov Parameter covariance of identified model

## Examples

## Create idfrd Object from Frequency Response Data

Create an idfrd object from frequency response data.
Load the magnitude data AMP, the phase data PHA, and the frequency vector W. Set sample time Ts to 0.1.
load demofr AMP PHA W
Ts = 0.1;
Use the values of AMP and PHA to compute the complex-valued response response.

```
response = AMP.*exp(1j*PHA*pi/180);
```

Create an idfrd object to store response in the idfrd object frdata.
frdata $=$ idfrd(response, W,Ts)
frdata =
IDFRD model.
Contains Frequency Response Data for 1 output(s) and 1 input(s).
Response data is available at 1000 frequency points, ranging from $0.03142 \mathrm{rad} / \mathrm{s}$ to $31.42 \mathrm{rad} / \mathrm{s}$.
Sample time: 0.1 seconds
Status:
Created by direct construction or transformation. Not estimated.
Plot the data.
bode(frdata)

## Bode Diagram


frdata is a complex idfrd object with object properties that you can access using dot notation. For example, confirm the value of Ts.
tsproperty = frdata.Ts
tsproperty $=0.1000$
You can also set property values. Set the Name property to 'DC_Converter '.
frdata.Name = 'DC_Converter';
If you import frdata into the System Identification app, the app names this data DC_Converter, and not the variable name frdata.

Use get to obtain the full set of property settings.
get(frdata)

```
    FrequencyUnit: 'rad/TimeUnit'
            Report: [1x1 idresults.frdest]
        SpectrumData: []
    CovarianceData: []
NoiseCovariance: []
        InterSample: {'zoh'}
        ResponseData: [1\times1\times1000 double]
            IODelay: 0
        InputDelay: 0
        OutputDelay: 0
```

```
            Ts: 0.1000
        TimeUnit: 'seconds'
        InputName: {''}
        InputUnit: {''}
    InputGroup: [1x1 struct]
    OutputName: {''}
    OutputUnit: {''}
OutputGroup: [1x1 struct]
        Notes: [0x1 string]
    UserData: []
            Name: 'DC Converter'
SamplingGrid: [1x1 struct]
    Frequency: [1000x1 double]
```


## Convert Time-Domain Model to Frequency Response Model

Convert a state-space model to a frequency response model using the idfrd command.
Load the data $z 2$ and estimate a second-order state-space model sys.
load iddata2 z2
sys = ssest(z2,2);
Convert sys to the idfrd model frsys.
frsys = idfrd(sys)
frsys =
IDFRD model.
Contains Frequency Response Data for 1 output(s) and 1 input(s), and the spectra for disturbance
Response data and disturbance spectra are available at 68 frequency points, ranging from 0.1 rad,
Output channels: 'y1'
Input channels: 'u1'
Status:
Created by conversion from idss model
Plot frsys.
bode(frsys)

frsys is an idfrd model that you can use as a dynamic system model or as estimation data for a time-domain or frequency-domain model.

## Create idfrd Object from Frequency Response of Time-Domain Model

Obtain the frequency response of a transfer function model and convert the response into an idfrd object.

Construct a transfer function model with one zero and three poles.
systf = idtf([1 .2],[1 2 1 1]);
Use bode to obtain the frequency response of systf, in terms of magnitude and phase, for the frequency vector $f$.
f = logspace(-1,1,100);
[mag,phase] = bode(systf,f);
Use the values of mag and phase to compute the complex-valued response response.
response $=$ mag. $* \exp (1 j *$ phase*pi/180);
Create an idfrd object frdata to store response, specifying a sample rate Ts of 0.8.

```
Ts = 0.8;
frdata = idfrd(response,f,Ts)
frdata =
IDFRD model.
Contains Frequency Response Data for 1 output(s) and 1 input(s).
Response data is available at }100\mathrm{ frequency points, ranging from 0.1 rad/s to 10 rad/s.
Sample time: 0.8 seconds
Status:
Created by direct construction or transformation. Not estimated.
```

Plot the data.
bode(frdata)

frdata is a complex idfrd object.

## Estimate Time-Domain Model Using Frequency Response Data

Estimate a transfer function model from time-domain data and convert the resulting idtf model to an idfrd model. Estimate a new transfer function model from the frequency response data in the idfrd model. Compare the model responses with the original data.

Load time-domain data $z 2$ and use it to estimate a transfer function sys that has two poles and one zero.
load iddata2 z2
sys = tfest(z2,2,1);
Convert sys to an idfrd model and plot the frequency response.
frsys = idfrd(sys);
bode(sys)


Estimate a new transfer function sys1 using the data from frsys as the estimation data.
sys1 = tfest(frsys,2,1);
Compare the responses of sys and sys1 with the original estimation data $z 2$.
compare(z2,sys,sys1)


The model responses are identical.

## See Also

bode |etfe|freqresp|nyquist|spa|spafdr|tfest

## Topics

"Representing Frequency-Response Data Using idfrd Objects"
"Estimating Models Using Frequency-Domain Data"
"Frequency Domain Identification: Estimating Models Using Frequency Domain Data"

## Introduced before R2006a

## idgrey

Linear ODE (grey-box model) with identifiable parameters

## Syntax

sys = idgrey(odefun, parameters,fcn_type)
sys = idgrey(odefun,parameters,fcn_type,optional_args)
sys = idgrey(odefun, parameters,fcn_type,optional_args,Ts)
sys = idgrey(odefun, parameters,fcn_type,optional_args,Ts,Name,Value)

## Description

sys = idgrey(odefun, parameters,fcn_type) creates a linear grey-box model with identifiable parameters, sys. odefun specifies the user-defined function that relates the model parameters, parameters, to its state-space representation.
sys = idgrey(odefun, parameters,fcn_type,optional_args) creates a linear grey-box model with identifiable parameters using the optional arguments required by odefun.
sys = idgrey(odefun, parameters,fcn_type,optional_args,Ts) creates a linear grey-box model with identifiable parameters with the specified sample time, Ts.
sys = idgrey(odefun, parameters,fcn_type,optional_args,Ts, Name, Value) creates a linear grey-box model with identifiable parameters with additional options specified by one or more Name, Value pair arguments.

## Object Description

An idgrey model represents a system as a continuous-time or discrete-time state-space model with identifiable (estimable) coefficients.

A state-space model of a system with input vector, $u$, output vector, $y$, and disturbance, $e$, takes the following form in continuous time:

$$
\begin{aligned}
& \dot{x}(t)=A x(t)+B u(t)+K e(t) \\
& y(t)=C x(t)+D u(t)+e(t)
\end{aligned}
$$

In discrete time, the state-space model takes the form:

$$
\begin{aligned}
& x[k+1]=A x[k]+B u[k]+K e[k] \\
& y[k]=C x[k]+D u[k]+e[k]
\end{aligned}
$$

For idgrey models, the state-space matrices $A, B, C$, and $D$ are expressed as a function of userdefined parameters using a MATLAB function. You access estimated parameters using sys.Structures.Parameters, where sys is an idgrey model.

Use an idgrey model when you know the system of equations governing the system dynamics explicitly. You should be able to express these dynamics in the form of ordinary differential or difference equations. You specify complex relationships and constraints among the parameters that cannot be done through structured state-space models (idss).

You can create an idgrey model using the idgrey command. To do so, write a MATLAB function that returns the $A, B, C$, and $D$ matrices for given values of the estimable parameters and sample time. The MATLAB function can also return the $K$ matrix and accept optional input arguments. The matrices returned may represent a continuous-time or discrete-time model, as indicated by the sample time.

Use the estimating functions pem or greyest to obtain estimated values for the unknown parameters of an idgrey model.

You can convert an idgrey model into other dynamic systems, such as idpoly, idss, tf, ss etc. You cannot convert a dynamic system into an idgrey model.

## Examples

## Create Grey-Box Model with Estimable Parameters

Create an idgrey model to represent a DC motor. Specify the motor time-constant as an estimable parameter and that the ODE function can return continuous- or discrete-time state-space matrices.

Create the idgrey model.

```
odefun = 'motorDynamics';
parameters = 1;
fcn_type = 'cd';
optional_args = 0.25;
Ts = 0;
sys = idgrey(odefun,parameters,fcn_type,optional_args,Ts);
```

sys is an idgrey model that is configured to use the shipped file motorDynamics.m to return the $A$, $B, C, D$, and $K$ matrices. motorDynamics.m also returns the initial conditions, $X 0$. The motor constant, $\tau$, is defined in motorDynamics. m as an estimable parameter, and parameters $=1$ specifies its initial value as 1 .

You can use pem or greyest to refine the estimate for $\tau$.

## Configure Estimable Parameter of Grey-Box Model

Specify the known parameters of a grey-box model as fixed for estimation. Also specify a minimum bound for an estimable parameter.

Create an ODE file that relates the pendulum model coefficients to its state-space representation. Save this function as LinearPendulum.m such that it is in the MATLAB® search path.

```
function [A,B,C,D] = LinearPendulum(m,g,l,b,Ts)
A = [0 1; -g/l, -b/m/l^2];
B = zeros(2,0);
C = [1 0];
D = zeros(1,0);
end
```

In this function:

- $m$ is the pendulum mass.
- g is the gravitational acceleration.
- $l$ is the pendulum length.
- $b$ is the viscous friction coefficient.
- Ts is the model sample time.

Create a linear grey-box model associated with the ODE function.

```
odefun = 'LinearPendulum';
m = 1;
g = 9.81;
l = 1;
b = 0.2;
parameters = {'mass',m;'gravity',g;'length',l;'friction',b};
fcn_type = 'c';
sys = idgrey(odefun,parameters,fcn_type);
sys has four parameters.
```

Specify the known parameters, $m, g$, and $l$, as fixed for estimation.
sys.Structure.Parameters(1).Free = false;
sys.Structure.Parameters(2).Free = false;
sys.Structure.Parameters(3).Free = false;
$\mathrm{m}, \mathrm{g}$, and l are the first three parameters of sys.
Specify a zero lower bound for $b$, the fourth parameter of sys.
sys.Structure.Parameters(4).Minimum = 0;
Similarly, to specify an upper bound for an estimable parameter, use the Maximum field of the parameter.

## Specify Additional Attributes of Grey-Box Model

Create a grey-box model with identifiable parameters. Name the input and output channels of the model, and specify seconds for the model time units.

Use Name, Value pair arguments to specify additional model properties on model creation.

```
odefun = 'motorDynamics';
parameters = 1;
fon type = 'cd';
optional_args = 0.25;
Ts = 0;
sys = idgrey(odefun,parameters,fcn_type,optional_args,Ts,'InputName','Voltage',...
    'OutputName',{'Angular Position','Angular Velocity'});
```

To change or specify more attributes of an existing model, you can use dot notation. For example:

```
sys.TimeUnit = 'seconds';
```


## Create Array of Grey-Box Models

Use the stack command to create an array of linear grey-box models.

```
odefun1 = @motorDynamics;
parameters1 = [1 2];
fcn_type = 'cd';
optional_args1 = 1;
sys1 = idgrey(odefun1,parameters1,fcn_type,optional_args1);
odefun2 = 'motorDynamics';
parameters2 = {[1 2]};
optional_args2 = 0.5;
sys2 = i\overline{dgrey(odefun2,parameters2,fcn_type,optional_args2);}
sysarr = stack(1,sys1,sys2);
```

stack creates a 2-by-1 array of idgrey models, sysarr.

## Input Arguments

## odefun

MATLAB function that relates the model parameters to its state-space representation.
odefun specifies the name of a MATLAB function (.m, .p, a function handle or .mex* file). This function establishes the relationship between the model parameters, parameters, and its statespace representation. The function may optionally relate the model parameters to the disturbance matrix and initial states.

If the function is not on the MATLAB path, then specify the full file name, including the path.
The syntax for odefun must be as follows:

```
[A,B,C,D] = odefun(par1,par2,...,parN,Ts,optional_arg1,optional_arg2,...)
```

The function outputs describe the model in the following linear state-space innovations form:

$$
\begin{gathered}
x n(t)=A x(t)+B u(t)+K e(t) ; x(0)=x_{0} \\
y(t)=C x(t)+D u(t)+e(t)
\end{gathered}
$$

In discrete time $x n(t)=\chi(t+T s)$ and in continuous time, $x n(t)=\dot{x}(t)$.
par1, par2, ..., parN are model parameters. Each entry may be a scalar, vector or matrix.
Ts is the sample time.
optional_arg1,optional_arg2,... are the optional inputs that odefun may require. The values of the optional input arguments are unchanged through the estimation process. However, the values of par1, par2, . . . parN are updated during estimation to fit the data. Use optional input arguments to vary the constants and coefficients used by your model without editing odefun.

The disturbance matrix, $K$, and the initial state values, $x 0$, are not parametrized. Instead, these values are determined separately, using the DisturbanceModel and InitialState estimation options, respectively. For more information regarding the estimation options, see greyestOptions.

A good choice for achieving the best simulation results is to set the DisturbanceModel option to ' none', which fixes $K$ to zero.
(Optional) Parameterizing Disturbance: odefun can also return the disturbance component, $K$, using the syntax:

```
[A,B,C,D,K] = odefun(par1,par2,...,parN,Ts,optional_arg1,optional_arg2,...)
```

If odefun returns a value for $K$ that contains $N a N$ values, then the estimating function assumes that $K$ is not parameterized. In this case, the value of the DisturbanceModel estimation option determines how $K$ is handled.
(Optional) Parameterizing Initial State Values: To make the model initial states, $X 0$, dependent on the model parameters, use the following syntax for odefun:

```
[A,B,C,D,K,X0] = odefun(par1,par2,...,parN,Ts,optional_arg1,optional_arg2,...)
```

If odefun returns a value for $X 0$ that contains NaN values, then the estimating function assumes that $X 0$ is not parameterized. In this case, $X 0$ may be fixed to zero or estimated separately, using the InitialStates estimation option.

## parameters

Initial values of the parameters required by odefun.
Specify parameters as a cell array containing the parameter initial values. If your model requires only one parameter, which may itself be a vector or a matrix, you may specify parameters as a matrix.

You may also specify parameter names using an $N$-by- 2 cell array, where $N$ is the number of parameters. The first column specifies the names, and the second column specifies the values of the parameters.

For example:
parameters = \{'mass',par1;'stiffness',par2;'damping',par3\}

## fcn_type

Indicates whether the model is parameterized in continuous-time, discrete-time, or both.
fcn_type requires one of the following values:

- ' c' - odefun returns matrices corresponding to a continuous-time system, regardless of the value of Ts.
- 'd' - odefun returns matrices corresponding to a discrete-time system, whose values may or may not depend on the value of Ts.
- 'cd' - odefun returns matrices corresponding to a continuous-time system, if Ts=0.

Otherwise, if Ts>0, odefun returns matrices corresponding to a discrete-time system. Select this option to sample your model using the values returned by odefun, rather than using the software's internal sample time conversion routines.

## optional_args

Optional input arguments required by odefun.
Specify optional_args as a cell array.
If odefun does not require optional input arguments, specify optional_args as \{\}.

## Ts

Model sample time.
If Ts is unspecified, it is assumed to be:

-     - 1 - If fcn_type is ' $d$ ' or 'cd'.

Ts $=-1$ indicates a discrete-time model with unknown sample time.

- 0 - If fon_type is ' c '.

Ts = 0 indicates a continuous-time model.

## Name, Value

Specify optional comma-separated pairs of Name, Value arguments, where Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, ... , NameN, ValueN.

Use Name, Value arguments to specify additional properties of idgrey models during model creation. For example, idgrey (odefun, parameters,fcn_type,'InputName', 'Voltage') creates an idgrey model with the InputName property set to Voltage.

## Properties

idgrey object properties include:

## A,B,C,D

Values of state-space matrices.

- A - State matrix $A$, an $N x$-by-Nx matrix, as returned by the ODE function associated with the idgrey model. $N x$ is the number of states.
- B - Input-to-state matrix $B$, an $N x$-by-Nu matrix, as returned by the ODE function associated with the idgrey model. $N u$ is the number of inputs and $N x$ is the number of states.
- C - State-to-output matrix $C$, an $N y$-by-Nx matrix, as returned by the ODE function associated with the idgrey model. $N x$ is the number of states and $N y$ is the number of outputs.
- D - Feedthrough matrix $D$, an $N y$-by-Nu matrix, as returned by the ODE function associated with the idgrey model. $N y$ is the number of outputs and $N u$ is the number of inputs.

The values $A, B, C, D$ are returned by the ODE function associated with the idgrey model. Thus, you can only read these matrices; you cannot set their values.

## K

Value of state disturbance matrix, $K$

K is $N x$-by- $N y$ matrix, where $N x$ is the number of states and $N y$ is the number of outputs.

- If odefun parameterizes the $K$ matrix, then $K$ has the value returned by odefun. odefun parameterizes the $K$ matrix if it returns at least five outputs and the value of the fifth output does not contain NaN values.
- If odefun does not parameterize the $K$ matrix, then K is a zero matrix of size $N x$-by- $N y . N x$ is the number of states and $N y$ is the number of outputs. The value is treated as a fixed value of the $K$ matrix during estimation. To make the value estimable, use the DisturbanceModel estimation option.
- Regardless of whether the $K$ matrix is parameterized by odefun or not, you can set the value of the K property explicitly as an $N x$-by- $N y$ matrix. $N x$ is the number of states and $N y$ is the number of outputs. The specified value is treated as a fixed value of the $K$ matrix during estimation. To make the value estimable, use the DisturbanceModel estimation option.

To create an estimation option set for idgrey models, use greyestOptions.

## StateName

State names, specified as one of the following:

- Character vector - For first-order models, for example, 'velocity '.
- Cell array of character vectors - For models with two or more states
- ' ' - For unnamed states.

Default: ' ' for all states

## StateUnit

State units, specified as one of the following:

- Character vector - For first-order models, for example, ' velocity '
- Cell array of character vectors - For models with two or more states
- ' ' - For states without specified units

Use StateUnit to keep track of the units each state is expressed in. StateUnit has no effect on system behavior.

Default: ' ' for all states

## Structure

Information about the estimable parameters of the idgrey model.
Structure stores information regarding the MATLAB function that parameterizes the idgrey model.

- Structure. Function - Name or function handle of the MATLAB function used to create the idgrey model.
- Structure.FunctionType - Indicates whether the model is parameterized in continuous-time, discrete-time, or both.
- Structure. Parameters - Information about the estimated parameters.

Structure. Parameters contains the following fields:

- Value - Parameter values. For example, sys.Structure.Parameters(2).Value contains the initial or estimated values of the second parameter.

NaN represents unknown parameter values.

- Minimum - Minimum value that the parameter can assume during estimation. For example, sys.Structure.Parameters(1).Minimum $=0$ constrains the first parameter to be greater than or equal to zero.
- Maximum - Maximum value that the parameter can assume during estimation.
- Free - Boolean value specifying whether the parameter is estimable. If you want to fix the value of a parameter during estimation, set Free $=$ false for the corresponding entry.
- Scale - Scale of the parameter's value. Scale is not used in estimation.
- Info - Structure array for storing parameter units and labels. The structure has Label and Unit fields.

Specify parameter units and labels as character vectors. For example, 'Time '.

- Structure. ExtraArguments - Optional input arguments required by the ODE function.
- Structure. StateName - Names of the model states.
- Structure.StateUnit - Units of the model states.


## NoiseVariance

The variance (covariance matrix) of the model innovations, $e$.
An identified model includes a white, Gaussian noise component, $e(t)$. NoiseVariance is the variance of this noise component. Typically, the model estimation function (such as greyest or pem) determines this variance.

For SISO models, NoiseVariance is a scalar. For MIMO models, NoiseVariance is a Ny -by-Ny matrix, where $N y$ is the number of outputs in the system.

## Report

Summary report that contains information about the estimation options and results when the greybox model is obtained using the greyest estimation command. Use Report to query a model for how it was estimated, including its:

- Estimation method
- Estimation options
- Search termination conditions
- Estimation data fit and other quality metrics

The contents of Report are irrelevant if the model was created by construction.

```
odefun = 'motorDynamics';
m = idgrey(odefun,1,'cd',0.25,0);
m.Report.OptionsUsed
ans =
```

If you obtain the grey-box model using estimation commands, the fields of Report contain information on the estimation data, options, and results.

```
load(fullfile(matlabroot,'toolbox','ident','iddemos','data','dcmotordata'));
data = iddata(y,u,0.1,'Name','DC-motor');
odefun = 'motorDynamics';
init_sys = idgrey('motorDynamics',1,'cd',0.25,0);
m = greyest(data,init_sys);
m.Report.OptionsUsed
InitialState: 'auto'
    DisturbanceModel: 'auto'
                            Focus: 'prediction'
    EstimateCovariance: 1
            Display: 'off'
            InputOffset: []
        OutputOffset: []
        Regularization: [1x1 struct]
        OutputWeight: []
        SearchMethod: 'auto'
        SearchOptions: [1x1 idoptions.search.identsolver]
            Advanced: [1x1 struct]
```

Report is a read-only property.
For more information on this property and how to use it, see the Output Arguments section of the corresponding estimation command reference page and "Estimation Report".

## InputDelay

Input delay for each input channel, specified as a scalar value or numeric vector. For continuous-time systems, specify input delays in the time unit stored in the TimeUnit property. For discrete-time systems, specify input delays in integer multiples of the sample time Ts. For example, InputDelay = 3 means a delay of three sample times.

For a system with Nu inputs, set InputDelay to an Nu-by-1 vector. Each entry of this vector is a numerical value that represents the input delay for the corresponding input channel.

You can also set InputDelay to a scalar value to apply the same delay to all channels.

## Default: 0

## OutputDelay

Output delays.
For identified systems, like idgrey, OutputDelay is fixed to zero.

## Ts

Sample time.
For continuous-time models, $\mathrm{Ts}=0$. For discrete-time models, Ts is a positive scalar representing the sample time expressed in the unit specified by the TimeUnit property of the model. To denote a discrete-time model with unspecified sample time, set Ts $=-1$.

Changing this property does not discretize or resample the model.

For idgrey models, there is no unique default value for Ts. Ts depends on the value of fcn_type.

## TimeUnit

Units for the time variable, the sample time Ts, and any time delays in the model, specified as one of the following values:

- 'nanoseconds'
- 'microseconds'
- 'milliseconds'
- 'seconds'
- 'minutes'
- 'hours'
- 'days'
- 'weeks'
- 'months'
- 'years'

Changing this property has no effect on other properties, and therefore changes the overall system behavior. Use chgTimeUnit to convert between time units without modifying system behavior.

## Default: 'seconds'

## InputName

Input channel names, specified as one of the following:

- Character vector - For single-input models, for example, ' controls '.
- Cell array of character vectors - For multi-input models.

Alternatively, use automatic vector expansion to assign input names for multi-input models. For example, if sys is a two-input model, enter:
sys.InputName = 'controls';
The input names automatically expand to \{'controls(1)'; 'controls(2)' \}.
When you estimate a model using an iddata object, data, the software automatically sets InputName to data. InputName.

You can use the shorthand notation $u$ to refer to the InputName property. For example, sys. $u$ is equivalent to sys. InputName.

Input channel names have several uses, including:

- Identifying channels on model display and plots
- Extracting subsystems of MIMO systems
- Specifying connection points when interconnecting models

Default: ' ' for all input channels

## InputUnit

Input channel units, specified as one of the following:

- Character vector - For single-input models, for example, ' seconds '.
- Cell array of character vectors - For multi-input models.

Use InputUnit to keep track of input signal units. InputUnit has no effect on system behavior.
Default: ' ' for all input channels

## InputGroup

Input channel groups. The InputGroup property lets you assign the input channels of MIMO systems into groups and refer to each group by name. Specify input groups as a structure. In this structure, field names are the group names, and field values are the input channels belonging to each group. For example:
sys.InputGroup.controls = [12];
sys.InputGroup.noise = [3 5];
creates input groups named controls and noise that include input channels 1, 2 and 3,5, respectively. You can then extract the subsystem from the controls inputs to all outputs using:
sys(:,'controls')
Default: Struct with no fields

## OutputName

Output channel names, specified as one of the following:

- Character vector - For single-output models. For example, ' measurements '.
- Cell array of character vectors - For multi-output models.

Alternatively, use automatic vector expansion to assign output names for multi-output models. For example, if sys is a two-output model, enter:
sys.OutputName = 'measurements';
The output names automatically expand to \{'measurements(1)';'measurements(2)'\}.
When you estimate a model using an iddata object, data, the software automatically sets OutputName to data.OutputName.

You can use the shorthand notation $y$ to refer to the OutputName property. For example, sys.y is equivalent to sys.OutputName.

Output channel names have several uses, including:

- Identifying channels on model display and plots
- Extracting subsystems of MIMO systems
- Specifying connection points when interconnecting models

Default: ' ' for all output channels

## OutputUnit

Output channel units, specified as one of the following:

- Character vector - For single-output models. For example, ' seconds ' .
- Cell array of character vectors - For multi-output models.

Use OutputUnit to keep track of output signal units. OutputUnit has no effect on system behavior.
Default: ' ' for all output channels

## OutputGroup

Output channel groups. The OutputGroup property lets you assign the output channels of MIMO systems into groups and refer to each group by name. Specify output groups as a structure. In this structure, field names are the group names, and field values are the output channels belonging to each group. For example:

```
sys.OutputGroup.temperature = [1];
sys.InputGroup.measurement = [3 5];
```

creates output groups named temperature and measurement that include output channels 1 , and 3,5 , respectively. You can then extract the subsystem from all inputs to the measurement outputs using:

```
sys('measurement',:)
```

Default: Struct with no fields

## Name

System name, specified as a character vector. For example, 'system_1'.

## Default: ''

## Notes

Any text that you want to associate with the system, stored as a string or a cell array of character vectors. The property stores whichever data type you provide. For instance, if sys1 and sys2 are dynamic system models, you can set their Notes properties as follows:

```
sys1.Notes = "sys1 has a string.";
sys2.Notes = 'sys2 has a character vector.';
sysl.Notes
sys2.Notes
ans =
    "sys1 has a string."
ans =
    'sys2 has a character vector.'
```

Default: [ $0 \times 1$ string]
UserData

Any type of data you want to associate with system, specified as any MATLAB data type.
Default: []

## SamplingGrid

Sampling grid for model arrays, specified as a data structure.
For arrays of identified linear (IDLTI) models that are derived by sampling one or more independent variables, this property tracks the variable values associated with each model. This information appears when you display or plot the model array. Use this information to trace results back to the independent variables.

Set the field names of the data structure to the names of the sampling variables. Set the field values to the sampled variable values associated with each model in the array. All sampling variables should be numeric and scalar valued, and all arrays of sampled values should match the dimensions of the model array.

For example, if you collect data at various operating points of a system, you can identify a model for each operating point separately and then stack the results together into a single system array. You can tag the individual models in the array with information regarding the operating point:

```
nominal_engine_rpm = [1000 5000 10000];
sys.SamplingGr\overline{i}d= struct('rpm', nominal_engine_rpm)
```

where sys is an array containing three identified models obtained at rpms 1000, 5000 and 10000, respectively.

For model arrays generated by linearizing a Simulink model at multiple parameter values or operating points, the software populates SamplingGrid automatically with the variable values that correspond to each entry in the array. For example, the Simulink Control Design ${ }^{\mathrm{TM}}$ commands linearize and slLinearizer populate SamplingGrid in this way.

Default: []

## See Also

getpvec|greyest|greyest0ptions|idnlgrey|idss|pem|setpvec|ssest

## Topics

"Estimate Coefficients of ODEs to Fit Given Solution"
"Estimate Model Using Zero/Pole/Gain Parameters"
"Estimate Discrete-Time Grey-Box Model with Parameterized Disturbance"
"Specifying the Linear Grey-Box Model Structure"

## Introduced before R2006a

## idinput

Generate input signals

## Syntax

$\mathrm{u}=$ idinput( N )
$\mathrm{u}=$ idinput([ $\mathrm{N}, \mathrm{Nu}])$
u = idinput([Period,Nu,NumPeriod])
u = idinput ( $\qquad$ ,Type)
u = idinput ( ,Type, Band)
u = idinput ( ,Type, Band,Range)
[u,freq] = idinput( $\qquad$ ,'sine',Band,Range,SineData)

## Description

The idinput command generates an input signal with specified characteristics for your system. You can use the generated input, and simulate the response of your system to study system behavior. For example, you can study the system response to periodic inputs. The system can be an actual physical system or a model such as a Simulink model. You can also design optimal experiments. For example, you can determine which input signals isolate faults or nonlinearities in your system. You can also use idinput to design an input that has sufficient bandwidth to excite the dynamic range of your system.
$\mathrm{u}=$ idinput ( N ) returns a single-channel random binary input signal u of length N . The generated signal values are either -1 or 1 .
$\mathrm{u}=$ idinput([ $\mathrm{N}, \mathrm{Nu}]$ ) returns an Nu-channel random binary input signal, where each channel signal has length $N$. The signals in each channel differ from each other.
$\mathrm{u}=$ idinput([Period, Nu,NumPeriod]) returns an Nu-channel periodic random binary input signal with specified period and number of periods. Each input channel signal is of length NumPeriod*Period.
u = idinput( $\qquad$ ,Type) specifies the type of input to be generated as one of the following:

- 'rbs' - Random binary signal
- 'rgs' - Random Gaussian signal
- 'prbs' - Pseudorandom binary signal
- 'sine' - Sum-of-sinusoids signal

Use with any of the previous input argument combinations.
u = idinput ( $\qquad$ ,Type, Band) specifies the frequency band of the signal. For pseudorandom binary signals (PRBS), Band specifies the inverse of the clock period of the signal.
u = idinput ( _ _ , Type, Band, Range) specifies the amplitude-range of the signal.
[u,freq] = idinput(__, 'sine',Band,Range,SineData) specifies the Type as a sum-ofsinusoids signal and specifies the characteristics of the sine waves used to generate the signal in

SineData. You can specify characteristics such as the number of sine waves and their frequency separation. The frequencies of the sine waves are returned in freq.

## Examples

## Generate a Random Binary Input Signal

Generate a single-channel random binary input signal with 200 samples.
$\mathrm{N}=200$;
u = idinput(N);
$u$ is a column vector of length 200 . The values in $u$ are either -1 or 1 .
Create an iddata object from the generated signal. For this example, specify the sample time as 1 second.
u = iddata([], u,1);
To examine the signal, plot it.
plot(u)


The generated signal is a random binary input signal with values -1 or 1 . You can use the generated input signal to simulate the output of your system using the sim command.

## Generate a Multichannel Random Binary Input Signal

Generate a two-channel random binary input signal with 200 samples.
$\mathrm{N}=200 ;$
u = idinput([N,2]);
u is a 200 -by- 2 matrix with values -1 or 1 .
Create an iddata object from the generated signal. For this example, specify the sample time as 1 second.
u = iddata([],u,1);
Plot the signals for the two channels, and examine the signals.
plot(u)
Input-Output Data


The plot shows the two generated random binary signals with values -1 or 1 .

## Generate a Periodic Random Binary Input Signal

Generate a single-channel periodic random binary input signal with a period of 10 samples and 5 periods in the signal.

NumChannel = 1;
Period = 10;
NumPeriod = 5;
u = idinput([Period,NumChannel,NumPeriod]);
u is a column vector of length $50(=$ Period*NumPeriod). The values in u are either -1 or 1 .
Create an iddata object from the generated signal. Specify the sample time as 1 second.
u = iddata([], u,1);
Plot the signal.
plot(u)
Input-Output Data


As specified, the generated single-channel periodic random binary input signal has a period of 10 seconds, and there are 5 whole periods in the signal.

## Generate a Periodic Random Gaussian Input Signal in Specified Frequency Range

Generate a single-channel periodic random Gaussian input signal with a period of 50 samples and 5 periods in the signal. First generate the signal using the entire frequency range, then specify a passband.

NumChannel = 1;
Period = 50;
NumPeriod = 5;
u = idinput([Period,NumChannel,NumPeriod],'rgs');
u is a column vector of length 250 (= Period*NumPeriod).
Create an iddata object from the generated signal, and plot the signal. For this example, specify the sample time as 0.01 seconds.

```
u = iddata([],u,0.01);
```

plot(u)

Input-Output Data


The plot shows that u contains a random segment of 50 samples, repeated 5 times. The signal is a Gaussian white noise signal with zero mean and variance one.

Since the sample time is 0.01 seconds, the generated signal has a period of 0.5 seconds. The frequency content of the signal spans the entire available range $(0-50 \mathrm{~Hz})$.

Now specify a passband between 0 and 25 Hz ( $=0.5$ times the Nyquist frequency).

Band = [0 0.5];
u2 = idinput([Period,NumChannel,NumPeriod],'rgs',Band);
Create an iddata object, and plot the signal.
u2 = iddata([],u2,0.01);
plot(u2)


The frequency content of the generated signal u2 is limited to $0-25 \mathrm{~Hz}$.

## Generate a Nonperiodic Pseudorandom Binary Input Signal

A pseudorandom binary input signal (PRBS) is a deterministic signal whose frequency properties mimic white noise. A PRBS is inherently periodic with a maximum period length of $2^{n}-1$, where integer $n$ is the order of the PRBS. For more information, see "Pseudorandom Binary Signals" on page 1-550.

Specify that the single-channel PRBS value switches between -2 and 2 .
Range = [-2,2];
Specify the clock period of the signal as 1 sample. That is, the signal value can change at each time step. For PRBS signals, the clock period is specified in Band $=[0 \mathrm{~B}]$, where $B$ is the inverse of the required clock period.

Band = [0 1];
Generate a nonperiodic PRBS of length 100 samples.
u = idinput(100,'prbs',Band,Range);
Warning: The PRBS signal delivered is the 100 first values of a full sequence of length 127.
A PRBS is inherently periodic. To generate a nonperiodic signal, the software generates a maximum length PRBS of length 127 that has a period greater than the required number of samples, 100. The software returns the first 100 samples of the generated PRBS. This action ensures that the generated signal is not periodic, as indicated in the generated warning.

Create an iddata object from the generated signal. For this example, specify the sample time as 1 second.
$\mathrm{u}=$ iddata([], $\mathrm{u}, 1)$;
Plot, and examine the generated signal.
plot(u);
title('Non-Periodic Signal')
Non-Periodic Signal


The generated signal is a nonperiodic PRBS of length 100 that switches between -2 and 2 .

## Generate a Periodic Pseudorandom Binary Input Signal

Specify that the pseudorandom binary input signal (PRBS) switches between -2 and 2 .
Range = [-2,2];
Specify the clock period of the signal as 1 sample. That is, the signal value can change at each time step. For PRBS signals, the clock period is specified in Band $=[0 \mathrm{~B}]$, where B is the inverse of the required clock period.

Band $=[01]$;
Generate a single-channel, periodic PRBS with a period of 100 samples and 3 periods in the signal.
u1 = idinput([100,1,3],'prbs',Band,Range);
Warning: The period of the PRBS signal was changed to 63. Accordingly, the length of the generate
A PRBS is inherently periodic with a maximum period length of $2^{n}-1$, where integer $n$ is the order of the PRBS. If the period you specify is not equal to a maximum length PRBS, the software adjusts the period of the generated signal to obtain an integer number of maximum length PRBS, and issues a warning. For more information about maximum length PRBS, see "Pseudorandom Binary Signals" on page 1-550. In this example, the desired period, 100, is not equal to a maximum length PRBS, thus the software instead generates a maximum length PRBS of order $n=f l o o r(\log 2(P e r i o d))=6$. Thus, the period of the PRBS signal is $63\left(=2^{6}-1\right)$, and the length of the generated signal is 189 (= NumPeriod*63). This result is indicated in the generated warning.

Create an iddata object from the generated signal, and plot the signal. Specify the period of the signal as 63 samples.
u1 = iddata([],u1,1,'Period',63);
plot(u1)
title('Periodic Signal')


The generated signal is a periodic PRBS with three periods.

## Generate Pseudorandom Binary Input Signal with Specified Clock Period

Generate periodic and nonperiodic pseudorandom binary input signals (PRBS) with specified clock period.

Generate a single-channel PRBS that switches between -2 and 2. Specify the clock period of the signal as 4 samples. That is, the signal has to stay constant for at least 4 consecutive samples before it can change. For PRBS signals, the clock period is specified in Band $=[0 B]$, where $B$ is the inverse of the required clock period.

Range $=[-2,2] ;$
Band $=[01 / 4] ;$
First generate a nonperiodic signal of length 100 .
ul = idinput(100,'prbs',Band,Range);
Warning: The PRBS signal delivered is the 100 first values of a full sequence of length 124.
To understand the generated warning, first note that the code is equivalent to generating a singlechannel PRBS with a 100-sample period and 1 period.
u1 = idinput([100,1,1],'prbs',Band,Range);

The generated PRBS signal has to remain constant for at least 4 samples before the value can change. To satisfy this requirement, the software first computes the order of the smallest possible maximum length $\operatorname{PRBS}$ as $n=f l o o r(\log 2(\operatorname{Period} * B))=4$ and period $2^{n}-1=15$. For information about maximum length PRBS, see "Pseudorandom Binary Signals" on page 1-550. The software then stretches this PRBS such that the period of the stretched signal is
$P=(1 / B)\left(2^{n}-1\right)=60$.
However, since this period is less than the specified length, 100, the software computes instead a maximum length PRBS of order $m=n+1=5$. The software then stretches this PRBS such that the period is now $P 2=(1 / B)\left(2^{m}-1\right)=124$. The software returns the first 100 samples of this signal as u1. This result ensures that the generated signal is not periodic but is constant for every 4 samples.

Create an iddata object from the generated signal. For this example, specify the sample time as 1 second.

```
u1 = iddata([],u1,1);
```

Plot, and examine the signal.

```
plot(u1);
title('Nonperiodic Signal')
```

Nonperiodic Signal


The generated signal is a nonperiodic PRBS of length 100. The signal remains constant for at least 4 samples before each change in value. Thus, the signal satisfies the clock period specified in Band.

Now generate a periodic signal with a 100 -sample period and 3 periods.
u2 = idinput([100,1,3],'prbs',Band,Range);
Warning: The period of the PRBS signal was changed to 60. Accordingly, the length of the generat
To generate a periodic signal with specified clock period, the software generates u2 as 3 repetitions of the original stretched signal of period $P=60$. Thus, the length of $u 2$ is $P *$ NumPeriod $=60 * 3=$ 180. This change in period and length of the generated signal is indicated in the generated warning.

Create an iddata object from the generated signal, and plot the signal. Specify the period of the signal as 60 seconds.

```
u2 = iddata([],u2,1,'Period',60);
plot(u2)
title('Periodic Signal')
```

Periodic Signal


The generated signal is a periodic PRBS with a 60 -second period and 3 periods. The signal remains constant for at least 4 samples before each change in value. Thus, the signal satisfies the specified clock period.

## Generate a Sum-of-Sinusoids Signal

You can generate a sum-of-sinusoids signal using default characteristics for the sine waves. Alternatively, you configure the number of sine waves, and the frequencies and phases of the sine waves. This example shows both approaches.

Specify that the signal has 50 samples in each period and 3 periods. Also specify that the signal amplitude range is between -1 and 1 .

```
Period = 50;
NumPeriod = 3;
Range = [-1 1];
```

Specify the frequency range of the signal. For a sum-of-sinusoids signal, you specify the lower and upper frequencies of the passband in fractions of the Nyquist frequency. In this example, use the entire frequency range between 0 and Nyquist frequency.

```
Band = [0 1];
```

First generate the signal using default characteristics for the sine waves. By default, the software uses 10 sine waves to generate the signal. The software assigns a random phase to each sinusoid, and then changes these phases 10 times to get the smallest signal spread. The signal spread is the difference between the minimum and the maximum value of the signal over all samples.

```
[u,freq] = idinput([Period 1 NumPeriod],'sine',Band,Range);
```

The software returns the sum-of-sinusoids signal in $u$ and the frequencies of the sinusoids in freq. The values in freq are scaled assuming that the sample time is 1 time unit. Suppose that the sample time is 0.01 hours. To retrieve the actual frequencies in rad/hours, divide the values by the sample time.

```
Ts = 0.01; % Sample time in hours
freq = freq/Ts;
freq(1)
ans = 12.5664
```

freq(1) is the frequency of the first sine wave. To see how the software chooses the frequencies, see the SineData argument description on the idinput reference page.

To verify that 10 sine waves were used to generate the signal, you can view the frequency content of the signal. Perform a Fourier transform of the signal, and plot the single-sided amplitude spectrum of the signal.

```
ufft = fft(u);
Fs = 2*pi/Ts; % Sampling frequency in rad/hour
L = length(u);
w = (0:L-1)*Fs/L;
stem(w(1:L/2),abs(ufft(1:L/2))) % Plot until Nyquist frequency
title('Single-Sided Amplitude Spectrum of u(t)')
xlabel('Frequency (rad/hour)')
ylabel('Amplitude')
```



The generated plot shows the frequencies of the 10 sine waves used to generate the signal. For example, the plot shows that the first sine wave has a frequency of $12.57 \mathrm{rad} / \mathrm{hour}$, the same as freq(1).

Convert the generated signal into an iddata object, and plot the signal. Specify the sample time as 0.01 hours.
u = iddata([],u,Ts,'TimeUnit','hours');
plot(u)

## Input-Output Data



The signal $u$ is generated using 10 sinusoids and has a period of 0.5 hours and 3 periods.
Now modify the number, frequency, and phase of the sinusoids that are used to generate the sum-ofsinusoids signal. Use 12 sinusoids and try 15 different sets of phases. To set the frequencies of the sinusoids, specify GridSkip $=2$. The software selects the frequencies of the sinusoids from the intersection of the frequency grid 2*pi*[1:GridSkip:fix(Period/2)]/Period and the passband pi*Band.

NumSinusoids = 12;
NumTrials = 15;
GridSkip = 2;
SineData = [NumSinusoids,NumTrials,GridSkip];
u2 = idinput([Period 1 NumPeriod],'sine',Band,Range,SineData);
Convert the generated signal into an iddata object, and plot the signal.
u2 = iddata([],u2,Ts,'TimeUnit','hours');
plot(u2)

## Input-Output Data



The signal $u 2$ is generated using 12 sinusoids and has a period of 0.5 hours and 3 periods.

## Input Arguments

## N - Number of generated input data samples

real positive integer
Number of generated input data samples, specified as a real positive integer. For a single-channel input data, the generated input $u$ has $N$ rows. For an Nu-channel input data, $u$ is returned as an $N$-byNu matrix, where each channel signal has length $N$.

## Nu - Number of input channels

1 (default) | real positive integer
Number of input channels in generated signal, specified as a real positive integer.

## Period - Number of samples in each period

real positive integer
Number of samples in each period of generated signal, specified as a real positive integer. Use this input to specify a periodic signal. Also specify the number of periods in NumPeriod. Each generated input channel signal has NumPeriod*Period samples.

## NumPeriod - Number of periods in generated signal

1 (default) | real positive integer

Number of periods in generated signal, specified as a real positive integer. Use this input to specify a periodic signal. Also specify the signal Period. Each generated input channel signal has
NumPeriod*Period samples.

## Type - Type of generated signal <br> 'rbs' (default)|'rgs'|'prbs'|'sine'

Type of generated signal, specified as one of the following values:

- 'rbs ' - Generates a random binary signal. A random binary signal is a random process that assumes only two values. You can specify these values using Range. To generate a band-limited signal, specify the passband in Band. To generate a periodic signal, specify Period and NumPeriod.
- 'rgs ' - Generates a random Gaussian signal. The generated Gaussian signal has mean $\mu$ and standard deviation $\sigma$ such that $[\mu-\sigma, \mu+\sigma]$ equals Range. To generate a band-limited Gaussian signal, specify the passband in Band. To generate a periodic Gaussian signal with an n samples period that repeats itself $m$ times, specify Period as $n$ and NumPeriod as $m$.
- 'prbs ' - Generates a pseudorandom binary signal (PRBS). A PRBS is a periodic, deterministic signal with white-noise-like properties that shifts between two values. You can specify these two values using Range. You can also specify the clock period, the minimum number of sampling intervals for which the value of the signal does not change. You specify the inverse of the clock period in Band.

The length of the generated signal is not always the same as what you specify. The length depends on whether you require a periodic or nonperiodic signal and also on the clock period you specify. For more information, see "Pseudorandom Binary Signals" on page 1-550.

- 'sine'-Generates a signal that is a sum-of-sinusoids. The software selects the frequencies of the sinusoids to be equally spread over a chosen grid and assigns each sinusoid a random phase. The software then tries several random phases for each sinusoid and selects the phases that give the smallest signal spread. The signal spread is the difference between the minimum and the maximum value of the signal over all samples. The amplitude of the generated sum-of-sinusoids signal is scaled to satisfy the Range you specify.

You can specify the characteristics of the sine waves used to generate the signal, such as the number of sine waves and their frequency separation, in the SineData argument.

## Band - Frequency range of generated signal

[0 1] (default) | 1-by-2 row vector
Frequency range of generated signal, specified as a 1-by-2 row vector containing minimum and maximum frequency values.

- If Type is 'rgs', 'rbs', or 'sine' - Specify Band as a passband [wlow whigh]. Where, wlow and whigh are the lower and upper frequencies of the passband, expressed in fractions of the Nyquist frequency. For example, to generate an input with white noise characteristics, use Band $=\left[\begin{array}{ll}0 & 1\end{array}\right]$.

The software achieves the frequency contents for a random Gaussian signal ('rgs ') using idfilt with an eighth-order Butterworth, noncausal filter. For generating a random binary signal ('rbs '), the software uses the same filter and then makes the signal binary. Thus, the frequency content in the generated random binary signal may not match the specified passband.

For 'sine' signals, the frequencies of the sinusoids are selected to be equally spread over a chosen grid in the specified passband. For more information, see the SineData argument description.

- If Type is 'prbs' - Specify Band as [0 B], where B is the inverse of the clock period of the signal. The clock period is the minimum number of sampling intervals for which the value of the signal does not change. Thus, the generated signal is constant over intervals of length 1/B samples. If $1 / B$ is not an integer, the software uses $f l o o r(1 / B)$ as the clock period.


## Range - Generated input signal range

[-1,1] (default) | two-element row vector
Generated input signal range, specified as a two-element row vector of the form [umin, umax].

- If Type is 'rbs' or 'prbs'- The generated signal u has values umin or umax.
- If Type is 'sine' - The generated signal $u$ has values between umin and umax.
- If Type is 'rgs ' - The generated Gaussian signal has mean $\mu$ and standard deviation $\sigma$ such that umin and umax are equal to $\mu-\sigma$ and $\mu+\sigma$, respectively. For example, Range $=[-1,1]$ returns a Gaussian white noise signal with zero mean and variance one.


## SineData - Characterization of sinusoids

[10, 10, 1] (default)| three-element row vector [NumSinusoids,NumTrials, GridSkip]
Characterization of sinusoids used to generate a sum-of-sinusoids signal, specified as a three-element row vector [NumSinusoids, NumTrials, GridSkip]. Where,

- NumSinusoids is the number of sinusoids used to generate the signal. The default value is 10.
- NumTrials is the number of different random relative phases of the sinusoids that the software tries to find the lowest signal spread. The signal spread is the difference between the minimum and the maximum value of the signal over all samples.

The maximum amplitude of the sum-of-sinusoids signal depends on the relative phases of the different sinusoids. To find the phases that give the smallest signal spread, the software tries NumTrials different random choices of phases to find the best phase values. For example, suppose that NumSinusoids is 20 and NumTrials is 5 . The software tries 5 different sets of relative phases for the 20 sinusoids, and selects the phases that give the smallest signal spread. The default value for NumTrials is 10 .

- GridSkip is used to characterize the frequency of the sinusoids. The software selects the frequency of the sinusoids from the intersection of the frequency grid
2*pi*[1:GridSkip:fix(Period/2)]/Period and the pass band pi*[Band(1) Band(2)]. For multichannel input signals, the software uses different frequencies from this frequency grid to generate the different input channels. You can use GridSkip for controlling odd and even frequency multiples, for example, to detect nonlinearities of different kinds.

To extract the frequencies freq that are selected by the software to generate the signal, use the following syntax.
[u,freq] = idinput(__)

## Output Arguments

## u - Generated input signal

column vector | matrix

Generated input signal, returned as a column vector of length N for a single-channel input or an N -byNu matrix for an Nu-channel signal. You use the generated signal to simulate the response of your system using sim.

You can create an iddata object from $u$ by specifying output data as [].
u = iddata([],u);
In the iddata object, you can also specify the properties of the signal such as sample time, input names, and periodicity.

## freq - Frequencies of sine waves

column vector | matrix
Frequencies of sine waves used for sum-of-sinusoids signal, returned as a column vector of length equal to the number of sinusoids, NumSinusoids. You specify NumSinusoids in the SineData argument. The frequency values are scaled assuming the sample time is 1 time unit. To retrieve the actual frequencies, divide the values by the sample time. For an example, see "Generate a Sum-ofSinusoids Signal" on page 1-543.

For multichannel input signals, freq is an Nu-by-NumSinusoids matrix where the $k$ th row contains the frequencies corresponding to the $k$ th channel. For information about how the software selects the frequencies, see the SineData argument description.

## More About

## Pseudorandom Binary Signals

A pseudorandom binary signal (PRBS) is a periodic, deterministic signal with white-noise-like properties that shifts between two values.

A PRBS is generated as:

$$
u(t)=\operatorname{rem}\left(a_{1} u(t-1)+\ldots+a_{n} u(t-n), 2\right)
$$

Here, $u(t-1), \ldots u(t-n)$ is the vector of past inputs, $n$ is the PRBS order, and rem denotes the remainder when $\left(a_{1} u(t-1)+\ldots a_{n} u(t-n)\right)$ is divided by 2 . Thus, a PRBS can only take the values 0 and 1. The software scales these values according to the Range you specify. In addition, the vector of past inputs $u(t-1), \ldots u(t-n)$ can only take $2^{n}$ values. Out of these values, the state with all zeros is ignored because it will result in future signals equal to zero. Thus, a PRBS is an inherently periodic signal with a maximum period length of $2^{n}-1$. The following table lists the maximum length possible for different orders $n$ of the PRBS.

| Order $\boldsymbol{n}$ | Maximum length PRBS (2 $\mathbf{n}^{\mathbf{n}} \mathbf{1} \mathbf{)}$ |
| :--- | :--- |
| 2 | 3 |
| 3 | 7 |
| 4 | 15 |
| 5 | 31 |
| 6 | 63 |
| 7 | 127 |


| Order $\boldsymbol{n}$ | Maximum length PRBS $\left(\mathbf{2}^{\boldsymbol{n}} \mathbf{- 1}\right)$ |
| :--- | :--- |
| $\vdots$ | $\vdots$ |
| 32 | 4294967295 |

Note The software does not generate signals with period greater than $2^{32}-1$.

## Length of Generated PRBS

Since PRBS are inherently periodic, the length and period of the generated signal depends on the clock period that you specify and whether you require a periodic or nonperiodic signal. The clock period is the minimum number of sampling intervals for which the value of the signal does not change. You specify the clock period in Band.

Clock period $=1$ sample (Band $=\left[\begin{array}{ll}0 & B\end{array}\right]=\left[\begin{array}{ll}0 & 1\end{array}\right]$ :

- To generate a nonperiodic signal of length N, (NumPeriod = 1), the software first computes a maximum length PRBS with a period greater than N. The software then returns the first N samples of the PRBS as $u$. This action ensures that $u$ is not periodic. For example, if $N$ is 100 , the software creates a maximum length PRBS of period 127 (order 7), and returns the first 100 samples as $u$.

For an example, see "Generate a Nonperiodic Pseudorandom Binary Input Signal" on page 1-538.

- To generate a periodic signal (NumPeriod $>1$ ), the software adjusts the period of the signal to obtain an integer number of maximum length PRBS. To do so, the software computes a PRBS of order $n=f \operatorname{loor}(\log 2(P e r i o d))$ and period $P=2^{n}-1$. The signal $u$ is then generated as NumPeriod repetitions of this PRBS signal of period P. Thus, the length of $u$ is $P^{*}$ NumPeriod.

For an example, see "Generate a Periodic Pseudorandom Binary Input Signal" on page 1-539.
In the multiple-input channel case, the signals are maximally shifted. That is, the overlap between the different inputs is minimized. This means Period/NumPeriod is an upper bound for the model orders that you can estimate using such a signal.

Clock period $>1$ sample (Band $=[0 B]$, where $B<1$ ):
The generated signal has to remain constant for at least $1 / B$ samples. To satisfy this requirement, the software first computes the order of the smallest possible maximum length PRBS as $\mathrm{n}=$ floor $(\log 2(\operatorname{Period} * B))$ and period $2^{n}-1$. The software then stretches the PRBS such that period of the stretched signal is $P=B^{-1}\left(2^{n}-1\right)$.

- To generate a nonperiodic signal of length $N$, if the period $P$ of the stretched signal is greater than or equal to $N$, the software returns the first $N$ samples of the stretched signal as $u$. This ensures that $u$ is nonperiodic but constant for every $1 / B$ samples. Note that for a nonperiodic signal, Period is equal to N .

If the period $P$ is less than $N$, the software computes instead a maximum length PRBS of order $n 2$ $=n+1$. The software then stretches this PRBS such that the period is now $P_{2}=B^{-1}\left(2^{n 2}-1\right)$. The software then returns the first N samples of this signal as u .

- To generate a periodic signal, the software generates u as NumPeriod repetitions of the stretched signal of period P . Thus, the length of u is $\mathrm{P} *$ NumPeriod.

For an example, see "Generate Pseudorandom Binary Input Signal with Specified Clock Period" on page 1-541.

## References

[1] Söderström, T. and P. Stoica., Chapter C5.3 in System Identification, Prentice Hall, 1989.
[2] Ljung, L., Section 13.3 in System Identification: Theory for the User, Prentice Hall PTR, 1999.

## See Also

iddata|sim
Topics
"Ways to Obtain Identification Data"
"Generate Data Using Simulation"
"Simulate and Predict Identified Model Output"
Introduced before R2006a

## idnlarx

Nonlinear ARX model

## Description

An idnlarx model represents a nonlinear ARX model, which is an extension of the linear ARX structure and contains linear and nonlinear functions.

A nonlinear ARX model consists of model regressors and an output function. The output function includes linear and nonlinear functions that act on the model regressors to give the model output and a fixed offset for that output. This block diagram represents the structure of a nonlinear ARX model in a simulation scenario.


The software computes the nonlinear ARX model output $y$ in two stages:
1 It computes regressor values from the current and past input values and the past output data.
In the simplest case, regressors are delayed inputs and outputs, such as $u(t-1)$ and $y(t-3)$. These kind of regressors are called linear regressors. You specify linear regressors using the linearRegressor object. You can also specify linear regressors by using linear ARX model orders as an input argument. For more information, see "Nonlinear ARX Model Orders and Delay". However, this second approach constrains your regressor set to linear regressors with consecutive delays. To create polynomial regressors, use the polynomialRegressor object. You can also specify custom regressors, which are nonlinear functions of delayed inputs and outputs. For example, $u(t-1) y(t-3)$ is a custom regressor that multiplies instances of input and output together. Specify custom regressors using the customRegressor object.

You can assign any of the regressors as inputs to the linear function block of the output function, the nonlinear function block, or both.
2 It maps the regressors to the model output using an output function block. The output function block can include linear and nonlinear blocks in parallel. For example, consider the following equation:

$$
F(x)=L^{T}(x-r)+g(Q(x-r))+d
$$

Here, $x$ is a vector of the regressors, and $r$ is the mean of $x . F(x)=L^{T}(x-r)+y_{0}$ is the output of the linear function block. $g(Q(x-r))+y_{0}$ represents the output of the nonlinear function block. $Q$ is a projection matrix that makes the calculations well-conditioned. $d$ is a scalar offset that is added to the combined outputs of the linear and nonlinear blocks. The exact form of $F(x)$ depends on your choice of output function. You can select from the available mapping objects, such as tree-partition networks, wavelet networks, and multilayer neural networks. You can also exclude either the linear or the nonlinear function block from the output function.

When estimating a nonlinear ARX model, the software computes the model parameter values, such as $L, r, d, Q$, and other parameters specifying $g$.

The resulting nonlinear ARX models are idnlarx objects that store all model data, including model regressors and parameters of the output function. For more information about these objects, see "Nonlinear Model Structures".

For more information on the idnlarx model structure, see "What are Nonlinear ARX Models?".
For idnlarx object properties, see "Properties" on page 1-555.

## Creation

You can obtain an idnlarx object in one of two ways.

- Use the nlarx command to both construct an idnlarx object and estimate the model parameters.
sys = nlarx(data, reg)
- Use the idnlarx constructor to create the nonlinear ARX model and then estimate the model parameters using nlarx or pem.
sys = idnlarx(output_name,input_name,reg)


## Syntax

sys = idnlarx(output_name,input_name,orders)
sys = idnlarx(output_name,input_name,Regressors)
sys = idnlarx( $\qquad$ , OūtputFen)
sys = idnlarx(linmodel)
sys = idnlarx(linmodel,OutputFcn)
sys = idnlarx( $\qquad$ ,Name, Value)

Description

## Specify Model Directly

sys = idnlarx(output_name,input_name,orders) specifies a set of linear regressors using ARX model orders. Use this syntax when you extend an ARX linear model, or when you plan to use only regressors that are linear with consecutive lags.
sys = idnlarx(output_name,input_name,Regressors) creates a nonlinear ARX model with the output and input names of output_name and input_name, respectively, and a regressor set in

Regressors that contains any combination of linear, polynomial, and custom regressors. The software constructs sys using the default wavelet network ('wavenet') mapping object for the output function.
sys = idnlarx( $\qquad$ ,OutputFcn) specifies the output function OutputFcn that maps the regressors to the model output. You can use this syntax with any of the previous input argument combinations.

## Initialize Model Values Using Linear Model

sys = idnlarx(linmodel) uses a linear model linmodel to extract certain properties such as names, units, and sample time, and to initialize the values of the linear coefficients of the model. Use this syntax when you want to create a nonlinear ARX model as an extension of, or an improvement upon, an existing linear model.
sys = idnlarx(linmodel,OutputFcn) specifies the output function OutputFen that maps the regressors to the model output.

## Specify Model Properties

sys = idnlarx(__ ,Name, Value) specifies additional properties on page 1-555 of the idnlarx model structure using one or more name-value arguments.

## Input Arguments

## orders - ARX model orders

nlarx orders [na nb nk]
ARX model orders, specified as the matrix [na nb nk]. na denotes the number of delayed outputs, nb denotes the number of delayed inputs, and nk denotes the minimum input delay. The minimum output delay is fixed to 1 . For more information on how to construct the orders matrix, see arx.

When you specify orders, the software converts the order information into a linear regressor form in the idnlarx Regressors property. For an example, see "Create Nonlinear ARX Model Using ARX Model Orders" on page 1-561.

## linmodel - Discrete-time linear model

idpoly object | idss object | idtf object | idproc object
Discrete-time identified input/output linear model, specified as any linear model created using estimators, that is, an idpoly object, an idss object, an idtf object, or an idproc object with Ts > 0 . Create this model using the constructor function for the object or estimate the model using the associated estimation command. For example, to create an ARX model, use arx, and specify the resulting idpoly object as linmodel.

## Properties

## Regressors - Regressor specification

linearRegressor object | polynomialRegressor object | customRegressor object | column array of regressor specification objects

Regressor specification, specified as a column vector containing one or more regressor specification objects, which are the linearRegressor objects, polynomialRegressor objects, and customRegressor customRegressor objects. Each object specifies a formula for generating regressors from lagged variables. For example:

- L = linearRegressor(\{'y1','u1'\},\{1,[2 5]\}) generates the regressors $y_{1}(t-1), u_{1}(t-2)$, and $u_{2}(t-5)$.
- $P=$ polynomialRegressor('y2', $4: 7,2)$ generates the regressors $y_{2}(t-4)^{2}, y_{2}(t-5)^{2}, y_{2}(t-6)^{2}$, and $y_{2}(t-7)^{2}$.
- C = customRegressor(\{'y1','u1','u2'\},\{1 22$\}, @(x, y, z) \sin (x . * y+z))$ generates the single regressor $\sin \left(y_{1}(t-1) u_{1}(t-2)+u_{2}(t-2)\right.$

For an example that implements these regressors, see "Create and Combine Regressor Types" on page 1-566.

To add regressors to an existing model, create a vector of specification objects and use dot notation to set Regressors to this vector. For example, the following code first creates the idnlarx model sys and then adds the regressor objects $L, P$, and $C$ to the regressors of sys.
sys = idnlarx(\{'y1','y2'\},\{'u1','u2'\});
R = [L;P;C];
sys.Regressors = R;
For an example of creating and using a linear regressor set, see "Create Nonlinear ARX Model Using Linear Regressors" on page 1-563.

## OutputFcn - Output function

'wavenet' (default)|'linear'|[]|''|'sigmoidnet'|'treepartition'|'neuralnet'| 'customnet ' | mapping object | array of mapping objects

Output function that maps the regressors of the idnlarx model into the model output, specified as a column array containing zero or more of the following strings or mapping objects:
'wavenet ' or wavenet object
'linear' or ' ' or [] or linear object
'sigmoidnet' or sigmoidnet object
'treepartition' or treepartition object
neuralnet object
customnet object

Wavelet network
Linear function
Sigmoid network
Binary tree partition regression model
Neural network - Feedforward network of Deep Learning Toolbox ${ }^{\mathrm{TM}}$.
Custom network - Similar to sigmoidnet, but with a user-defined replacement for the sigmoid function.

The wavenet, sigmoidnet, treepartion, and customnet objects contain both linear and nonlinear components. You can remove (not use) the linear components of wavenet, sigmoidnet, and customnet by setting the LinearFcn. Use value to false.

The neuralnet function has only a nonlinear component that is the network object of Deep Learning Toolbox. The linear function, as the name implies, has only a linear component.

Specifying a character vector, for example 'sigmoidnet', creates a mapping object with default settings. Alternatively, you can specify mapping object properties in two other ways:

- Create the mapping object using arguments to modify default properties.

```
MO = sigmoidnet(15)
```

- Create a default mapping object first and then use dot notation to modify properties.

```
MO = sigmoidnet;
MO.NumberOfUnits = 15
```

For $n_{y}$ output channels, you can specify mapping objects individually for each channel by setting OutputFcn to an array of $n_{y}$ mapping objects. For example, the following code specifies OutputFcn using dot notation for a system with two input channels and two output channels.

```
sys = idnlarx({'y1','y2'},{'u1','u2'});
sys.OutputFcn = [wavenet; sigmoidnet]
```

To specify the same mapping for all outputs, specify OutputFcn as a character vector or a single mapping object.

OutputFcn represents a static mapping function that transforms the regressors of the nonlinear ARX model into the model output. OutputFcn is static because it does not depend on the time. For example, if $y(t)=y_{0}+a_{1} y(t-1)+a_{2} y(t-2)+\ldots+b_{1} u(t-1)+b_{2} u(t-2)+\ldots$, then OutputFcn is a linear function represented by the linear object.

For an example of specifying the output function, see "Specify Output Function for Nonlinear ARX Model" on page 1-565.

## RegressorUsage - Regressor assignments

## table with logical entries

Regressor assignments to the linear and nonlinear components of the nonlinear ARX model, specified as an $n_{r}$-by- $n_{c}$ table with logical entries that specify which regressors to use for which component. Here, $n_{r}$ is the number of regressors. $n_{c}$ is the total number of linear and nonlinear components in OutputFcn. The rows of the table correspond to individual regressors. The row names are set to regressor names. If the table value for row $i$ and component index $j$ is true, then the $i$ th regressor is an input to the linear or nonlinear component $j$.

For multi-output systems, OutputFcn contains one mapping object for each output. Each mapping object can use both linear and nonlinear components or only one of the two components.

For an example of viewing and modifying the RegressorUsage property, see "Modify Regressor Assignments to Output Function Components" on page 1-568.

## Report - Summary report

report field values
This property is read-only.
Summary report that contains information about the estimation options and results for a nonlinear ARX model obtained using the nlarx command. Use Report to find estimation information for the identified model, including:

- Estimation method
- Estimation options
- Search termination conditions
- Estimation data fit

The contents of Report are irrelevant if the model was constructed using idnlarx.

```
sys = idnlarx('y1','u1',reg);
sys.Report.OptionsUsed
ans =
    []
```

If you use nlarx to estimate the model, the fields of Report contain information on the estimation data, options, and results.

```
load iddatal;
sys = nlarx(z1,reg);
m.Report.OptionsUsed
Option set for the nlarx command:
    IterativeWavenet: 'auto'
            Focus: 'prediction'
            Display: 'off'
        Regularization: [1x1 struct]
        SearchMethod: 'auto'
        SearchOptions: [1x1 idoptions.search.identsolver]
        OutputWeight: 'noise'
            Advanced: [1x1 struct]
```

For more information on this property and how to use it, see "Output Arguments" on page 1-878 in the nlarx reference page and "Estimation Report".

## TimeVariable - Independent variable

't' (default)| character vector
Independent variable for the inputs, outputs, and-when available-internal states, specified as a character vector.

## NoiseVariance - Noise variance

matrix
Noise variance (covariance matrix) of the model innovations $e$. The estimation algorithm typically sets this property. However, you can also assign the covariance values by specifying an ny-by-ny matrix.

## Ts - Sample time

1 (default) | positive scalar
Sample time, specified as a positive scalar representing the sampling period. This value is expressed in the unit specified by the TimeUnit property of the model.

## TimeUnit - Units for time variable

```
'seconds' (default)|'nanoseconds'|'microseconds' | 'milliseconds' |'minutes'|
'hours'| 'days'| 'weeks'| 'months' | 'years'
```

Units for the time variable, the sample time Ts, and any time delays in the model, specified as one of the following values:

- 'nanoseconds'
- 'microseconds'
- 'milliseconds'
- 'seconds'
- 'minutes'
- 'hours'
- 'days'
- 'weeks'
- 'months'
- 'years'

Changing this property has no effect on other properties, and therefore changes the overall system behavior. Use chgTimeUnit to convert between time units without modifying system behavior.

## InputName - Input channel names

' ' for all input channels (default) | character vector | cell array of character vectors
Input channel names, specified as one of the following:

- Character vector - For single-input models, for example, ' controls'.
- Cell array of character vectors - For multi-input models.

Input names in Nonlinear ARX models must be valid MATLAB variable names after you remove any spaces.

Alternatively, use automatic vector expansion to assign input names for multi-input models. For example, if sys is a two-input model, enter:
sys.InputName = 'controls';
The input names automatically expand to \{'controls(1)' ;'controls(2)'\}.
When you estimate a model using an iddata object, data, the software automatically sets InputName to data. InputName.

You can use the shorthand notation $u$ to refer to the InputName property. For example, sys. $u$ is equivalent to sys. InputName.

Input channel names have several uses, including:

- Identifying channels on model display and plots
- Extracting subsystems of MIMO systems
- Specifying connection points when interconnecting models


## InputUnit - Input channel units

' ' for all input channels (default) | character vector | cell array of character vectors
Input channel units, specified as one of the following:

- Character vector - For single-input models, for example, ' seconds '.
- Cell array of character vectors - For multi-input models.

Use InputUnit to keep track of input signal units. InputUnit has no effect on system behavior.

## InputGroup - Input channel groups

structure with no fields (default) | structure

Input channel groups. The InputGroup property lets you assign the input channels of MIMO systems into groups and refer to each group by name. Specify input groups as a structure. In this structure, field names are the group names, and field values are the input channels belonging to each group. For example:
sys.InputGroup.controls = [1 2];
sys.InputGroup.noise = [3 5];
creates input groups named controls and noise that include input channels 1,2 and 3, 5, respectively. You can then extract the subsystem from the controls inputs to all outputs using:
sys(:,'controls')

## OutputName - Output channel names

' ' for all output channels (default) | character vector | cell array of character vectors
Output channel names, specified as one of the following:

- Character vector - For single-output models. For example, 'measurements ' .
- Cell array of character vectors - For multi-output models.

Output names in Nonlinear ARX models must be valid MATLAB variable names after you remove any spaces.

Alternatively, use automatic vector expansion to assign output names for multi-output models. For example, if sys is a two-output model, enter:
sys.OutputName = 'measurements';
The output names automatically expand to \{'measurements(1)';'measurements(2)'\}.
When you estimate a model using an iddata object, data, the software automatically sets OutputName to data.OutputName.

You can use the shorthand notation $y$ to refer to the OutputName property. For example, sys.y is equivalent to sys.OutputName.

Output channel names have several uses, including:

- Identifying channels on model display and plots
- Extracting subsystems of MIMO systems
- Specifying connection points when interconnecting models


## OutputUnit - Output channel units

' ' for all output channels (default) | character vector | cell array of character vectors
Output channel units, specified as one of the following:

- Character vector - For single-output models. For example, ' seconds '.
- Cell array of character vectors - For multi-output models.

Use OutputUnit to keep track of output signal units. OutputUnit has no effect on system behavior.

## OutputGroup - Output channel groups

structure with no fields (default) | structure

Output channel groups. The OutputGroup property lets you assign the output channels of MIMO systems into groups and refer to each group by name. Specify output groups as a structure. In this structure, field names are the group names, and field values are the output channels belonging to each group. For example:
sys.OutputGroup.temperature = [1];
sys.InputGroup.measurement = [3 5];
creates output groups named temperature and measurement that include output channels 1, and 3,5 , respectively. You can then extract the subsystem from all inputs to the measurement outputs using:
sys('measurement',:)

## Name - System Name

' ' (default) | character vector
System name, specified as a character vector. For example, 'system 1'.

## Notes - Notes on system

0 -by-1 string (default) | string | character vector
Any text that you want to associate with the system, specified as a string or a cell array of character vectors. The property stores whichever data type you provide. For instance, if sys1 and sys2 are dynamic system models, you can set their Notes properties as follows.

```
sys1.Notes = "sys1 has a string.";
sys2.Notes = 'sys2 has a character vector.';
sysl.Notes
sys2.Notes
ans =
    "sys1 has a string."
ans =
    'sys2 has a character vector.'
```


## UserData - Data to associate with system

[ ] (default) | any MATLAB data type
Any data you want to associate with the system, specified as any MATLAB data type.

## Object Functions

For information about object functions for idnlarx, see "Nonlinear ARX Models".

## Examples

## Create Nonlinear ARX Model Using ARX Model Orders

Create an idnlarx model by specifying an ARX model order vector.
Create an order vector of the form [na nb nk], where na and nb are the orders of the $A$ and $B$ ARX model polynomials and $n k$ is the number of input/output delays.

```
na = 2;
nb = 3;
nk = 5;
orders = [na nb nk];
```

Construct a nonlinear ARX model sys.

```
output_name = 'y1';
input_name = 'u1';
sys = idnlarx(output_name,input_name,[2 3 5]);
```

View the output function.

```
disp(sys.OutputFcn)
```

Wavelet Network
Nonlinear Function: Wavelet network with number of units chosen automatically.
Linear Function: uninitialized
Output Offset: uninitialized
Input: [1×1 idpack.Channel]
Output: [1×1 idpack.Channel]
LinearFcn: [1×1 nlident.internal.UseProjectedLinearFcn]
NonlinearFcn: [1×1 nlident.internal. WavenetFcn]
Offset: [1×1 nlident.internal. ChooseableOffset]

By default, the model uses a wavelet network, represented by a wavenet object, for the output function. The wavenet object includes linear and nonlinear components.

View the Regressors property.

```
disp(sys.Regressors)
Linear regressors in variables yl, ul
        Variables: {'y1' 'ul'}
            Lags: {[1 2] [5 6 7]}
        UseAbsolute: [0 0]
    TimeVariable: 't'
    Regressors described by this set
```

The idnlarx constructor transforms the model orders into the Regressors form.

- The Lags array for y1, [12], is equivalent to the na value of 2 . Both forms specify two consecutive output regressors, $\mathrm{y} 1(\mathrm{t}-1)$ and $\mathrm{y} 1(\mathrm{t}-2)$.
- The Lags array for u1, [ $\left.\begin{array}{ll}5 & 6\end{array}\right]$, incorporates the three delays specified by the nb value of 3, and shifts them by the $n k$ value of 5 . The input regressors are therefore $u 1(t-5), u 1(t-6)$, and $u 1(t-7)$.

View the regressors.

```
getreg(sys)
ans = 5x1 cell
    {'y1(t-1)'}
    {'y1(t-2)'}
    {'u1(t-5)'}
    {'u1(t-6)'}
    {'u1(t-7)'}
```

You can use the orders syntax to specify simple linear regressors. However, to create more complex regressors, use the regressor commands linearRegressor, polynomialRegressor, and customRegressor to create a combined regressor for the Regressors syntax.

## Create Nonlinear ARX Model Using Linear Regressors

Construct an idnlarx model by specifying linear regressors.
Create a linear regressor that contains two output lags and two input lags.

```
output_name = 'y1';
input_name = 'ul';
var_names = {output_name,input_name};
output_lag = [1 2];
input_lag = [1 2];
lags = {output_lag,input_lag};
reg = linearRegressor(var_names,lags)
reg =
Linear regressors in variables y1, ul
            Variables: {'yl' 'ul'}
            Lags: {[1 2] [1 2]}
        UseAbsolute: [0 0]
        TimeVariable: 't'
    Regressors described by this set
```

The model contains the regressors $y(t-1), y(t-2), u(t-1)$, and $u(t-2)$.
Construct the idnlarx model and view the regressors.

```
sys = idnlarx(output_name,input_name,reg);
getreg(sys)
ans = 4\times1 cell
    {'y1(t-1)'}
    {'y1(t-2)'}
    {'u1(t-1)'}
    {'u1(t-2)'}
```

View the output function.

```
disp(sys.OutputFen)
Wavelet Network
Nonlinear Function: Wavelet network with number of units chosen automatically.
Linear Function: uninitialized
Output Offset: uninitialized
            Input: [1×1 idpack.Channel]
            Output: [1×1 idpack.Channel]
            LinearFcn: [1\times1 nlident.internal.UseProjectedLinearFcn]
NonlinearFcn: [1×1 nlident.internal.WavenetFcn]
            Offset: [1\times1 nlident.internal.ChooseableOffset]
```

View the regressor usage table.

```
disp(sys.RegressorUsage)
```

    y1:LinearFcn y1:NonlinearFcn
    | y1 (t-1) | true | true |
| :--- | :--- | :--- |
| y1(t-2) | true | true |
| u1(t-1) | true | true |
| u1( $t-2)$ | true | true |

All the regressors are inputs to both the linear and nonlinear components of the wavenet object.

## Create and Configure Nonlinear ARX Model

Create a nonlinear ARX model with a linear regressor set.
Create a linear regressor that contains three output lags and two input lags.

```
output_name = 'y1';
input name = 'u1';
var_names = {output_name,input_name};
output_lag = [ll 2 3];
input_\
lags = {output_lag,input_lag};
reg = linearRegressor(var_names,lags)
reg =
Linear regressors in variables y1, ul
            Variables: {'y1' 'ul'}
            Lags: {[1 2 3] [1 2]}
        UseAbsolute: [0 0]
        TimeVariable: 't'
    Regressors described by this set
```

Construct the nonlinear ARX model.

```
sys = idnlarx(output_name,input_name,reg);
```

View the Regressors property.

```
disp(sys.Regressors)
```

Linear regressors in variables y1, u1
Variables: \{'y1' 'ul'\}

UseAbsolute: [0 0]
TimeVariable: 't'
Regressors described by this set
sys uses wavenet as the default output function. Reconfigure the output function to sigmoidnet.

```
sys.OutputFcn = 'sigmoidnet';
disp(sys.OutputFcn)
Sigmoid Network
    Nonlinear Function: Sigmoid network with 10 units.
    Linear Function: uninitialized
    Output Offset: uninitialized
            Input: [1×1 idpack.Channel]
            Output: [1×1 idpack.Channel]
            LinearFcn: [1\times1 nlident.internal.UseProjectedLinearFcn]
        NonlinearFcn: [1×1 nlident.internal.RidgenetFcn]
            Offset: [1×1 nlident.internal.ChooseableOffset]
```


## Specify Output Function for Nonlinear ARX Model

Specify the sigmoid network output function when you construct a nonlinear ARX model.
Assign variable names and specify a regressor set.

```
output name = 'y1';
input name = 'ul';
r = linearRegressor({output_name,input_name},{1 1});
```

Construct a nonlinear ARX model that specifies the sigmoidnet output function. Set the number of terms in the sigmoid expansion to 15 .

```
sys = idnlarx(output_name,input_name,r,sigmoidnet(15));
```

View the output function specification.

```
disp(sys.OutputFcn)
Sigmoid Network
    Nonlinear Function: Sigmoid network with 15 units.
    Linear Function: uninitialized
    Output Offset: uninitialized
    Input: [1×1 idpack.Channel]
        Output: [1\times1 idpack.Channel]
```


## Create Nonlinear ARX Model Without Nonlinear Mapping Function

Construct an idnlarx model that uses only linear mapping in the output function. An argument value of [] is equivalent to an argument value of 'linear'.

```
sys = idnlarx([2 2 1],[])
sys =
Nonlinear ARX model with 1 output and 1 input
    Inputs: u1
    Outputs: y1
Regressors:
    Linear regressors in variables y1, ul
    List of all regressors
Model output is linear in regressors.
Sample time: 1 seconds
Status:
Created by direct construction or transformation. Not estimated.
```


## Create and Combine Regressor Types

Create a regressor set that includes linear, polynomial, and custom regressors.
Specify L as the set of linear regressors $y_{1}(t-1), u_{1}(t-2)$, and $u_{1}(t-5)$.
L = linearRegressor(\{'y1','u1'\},\{1, [2 5]\});
Specify $P$ as the set of polynomial regressors $y_{2}(t-4)^{2}, y_{2}(t-5)^{2}, y_{2}(t-6)^{2}$, and $y_{2}(t-7)^{2}$.
P = polynomialRegressor('y2',4:7,2);
Specify C as the custom regressor $\sin \left(y_{1}(t-1) u_{1}(t-2)+u_{2}(t-2)\right)$, using the @ symbol to create an anonymous function handle.

```
C = customRegressor({'y1','u1','u2'},{1 2 2},@(x,y,z)sin(x.*y+z));
```

Combine the regressors into one regressor set R.
$R=[L ; P ; C]$
$\mathrm{R}=3 \times 1$ object
[3 1] array of linearRegressor, polynomialRegressor, customRegressor objects.

1. Linear regressors in variables y1, u1 Variables: \{'y1' 'ul'\}
```
        Lags: {[1] [2 5]}
        UseAbsolute: [0 0]
    TimeVariable: 't'
2. Order 2 regressors in variables y2
                    Order: 2
            Variables: {'y2'}
                Lags: {[4 5 6 7]}
            UseAbsolute: 0
    AllowVariableMix: 0
        AllowLagMix: 0
    TimeVariable: 't'
3. Custom regressor: sin(y1(t-1).*u1(t-2)+u2(t-2))
        VariablesToRegressorFcn: @(x,y,z)sin(x.*y+z)
                        Variables: {'y1''u1' 'u2'}
                        Lags: {[1] [2] [2]}
            Vectorized: 1
            TimeVariable: 't'
    Regressors described by this set
Create a nonlinear ARX model.
sys = idnlarx({'y1','y2'},{'u1','u2'},R)
sys =
Nonlinear ARX model with 2 outputs and 2 inputs
    Inputs: u1, u2
    Outputs: y1, y2
Regressors:
    1. Linear regressors in variables y1, ul
    2. Order 2 regressors in variables y2
    3. Custom regressor: sin(y1(t-1).*u1(t-2)+u2(t-2))
    List of all regressors
Output functions:
    Output 1: Wavelet Network
    Output 2: Wavelet Network
Sample time: 1 seconds
Status:
Created by direct construction or transformation. Not estimated.
```


## Create Nonlinear ARX Model Using Linear Model

Use a linear ARX model instead of a regressor set to construct a nonlinear ARX model.
Construct a linear ARX model using idpoly.

```
A = [1 -1.2 0.5];
B = [0.8 1];
LinearModel = idpoly(A, B, 'Ts', 0.1);
```

Specify input and output names for the model using dot notation.

```
LinearModel.OutputName = 'yl';
```

LinearModel.InputName = 'u1';

Construct a nonlinear ARX model using the linear ARX model.

```
m1 = idnlarx(LinearModel)
```

$\mathrm{ml}=$
Nonlinear ARX model with 1 output and 1 input
Inputs: ul
Outputs: y1
Regressors:
Linear regressors in variables y1, ul
List of all regressors
Output function: Wavelet Network
Sample time: 0.1 seconds
Status:
Created by direct construction or transformation. Not estimated.

You can create a linear ARX model from any identified discrete-time linear model.
Estimate a second-order state-space model from estimation data z1.

```
load iddatal z1
ssModel = ssest(z1,2,'Ts',0.1);
```

Construct a nonlinear ARX model from ssModel. The software uses the input and output names that ssModel extracts from zl.

```
m2 = idnlarx(ssModel)
m2 =
Nonlinear ARX model with 1 output and 1 input
    Inputs: ul
    Outputs: y1
Regressors:
    Linear regressors in variables y1, ul
    List of all regressors
Output function: Wavelet Network
Sample time: 0.1 seconds
Status:
Created by direct construction or transformation. Not estimated.
```


## Modify Regressor Assignments to Output Function Components

Modify regressor assignments by modifying the RegressorUsage table.
Construct a nonlinear ARX model that has two inputs and two outputs.
Create the variable names and the regressors.

```
varnames = {'y1','y2','u1','u2'};
lags = {[1 2 3],[1 2],[1 2],[1 3]};
reg = linearRegressor(varnames,lags);
```

Create an output function specification fon that uses wavenet for mapping regressors to output y1 and sigmoidnet for mapping regressors to output y2. Both mapping objects contain linear and nonlinear components.
fcn = [wavenet;sigmoidnet];
Construct the nonlinear ARX model.

```
output_name = {'y1' 'y2'};
input_name = {'u1' 'u2'};
sys =-idnlarx(output_name,input_name,reg,fcn)
sys =
Nonlinear ARX model with 2 outputs and 2 inputs
    Inputs: u1, u2
    Outputs: y1, y2
Regressors:
    Linear regressors in variables y1, y2, u1, u2
    List of all regressors
Output functions:
    Output 1: Wavelet Network
    Output 2: Sigmoid Network with 10 units
Sample time: 1 seconds
Status:
Created by direct construction or transformation. Not estimated.
```

Display the RegressorUsage table.

```
disp(sys.RegressorUsage)
```

    y1:LinearFcn y1:NonlinearFcn y2:LinearFcn y2:NonlinearFcn
    | y1 (t-1) | true | true | true | true |
| :--- | :--- | :--- | :--- | :--- |
| y1(t-2) | true | true | true | true |
| y1(t-3) | true | true | true | true |
| y2(t-1) | true | true | true | true |
| y2(t-2) | true | true | true | true |
| u1(t-1) | true | true | true | true |
| u1(t-2) | true | true | true | true |
| u2(t-1) | true | true | true | true |

The rows of the table represent the regressors. The first two columns of the table represent the linear and nonlinear components of the mapping to output yl (wavenet). The last two columns represent the two components of the mapping to output y 2 (sigmoidnet).

In this table, all the input and output regressors are inputs to all components.
Remove the $\mathrm{y} 2(\mathrm{t}-2)$ regressor from the y 2 nonlinear component.
sys.RegressorUsage\{4,4\}=false;
disp(sys.RegressorUsage)

di:LinearFcn
$y y n n n n$
$y 1(t-1)$

The table displays false for this regressor-component pair.
Remove all output regressors from the y1 nonlinear component.
Store regressor names in Names.
Names = sys.RegressorUsage.Properties.RowNames;
Determine the indices of the rows that contain y1 or y2 and set the corresponding values of y1:NonlinearFcn to False.

```
idx = contains(Names,'y1')|contains(Names,'y2');
```

sys.RegressorUsage\{idx,2\} = false;
disp(sys.RegressorUsage)

|  | y1:LinearFcn | y1:NonlinearFcn | y2:LinearFcn | y2:NonlinearFcn |
| :---: | :---: | :---: | :---: | :---: |
| y1(t-1) | true | false | true | true |
| y1(t-2) | true | false | true | true |
| y1(t-3) | true | false | true | true |
| y2(t-1) | true | false | true | false |
| y2(t-2) | true | false | true | true |
| u1(t-1) | true | true | true | true |
| u1(t-2) | true | true | true | true |
| u2(t-1) | true | true | true | true |
| u2 (t-3) | true | true | true | true |

The table values reflect the new assignments.
The RegressorUsage table provides complete flexibility for individually controlling regressor assignments.

## More About

## Definition of idnlarx States

The states of an idnlarx object are an ordered list of delayed input and output variables that define the structure of the model. The toolbox uses this definition of states for creating the initial state vector that sim, predict, and compare use for simulation and prediction. linearize also uses this definition for linearization of nonlinear ARX models.

This toolbox provides several options to facilitate how you specify the initial states. For example, you can use findstates and data2state to search for state values in simulation and prediction applications. For linearization, use findop. You can also specify the states manually.

The states of an idnlarx model depend on the maximum delay in each input and output variable used by the regressors. If a variable $p$ has a maximum delay of $D$ samples, then it contributes $D$ elements to the state vector at time $t: p(t-1), p(t-2), \ldots, p(t-D)$.

For example, if you have a single-input, single-output idnlarx model.

```
m = idnlarx([2 3 0],'wavenet','CustomRegressors',{'y1(t-10)*u1(t-1)'});
```

This model has these regressors.

```
getreg(m)
ans = 6x1 cell
    {'y1(t-1)'
    {'y1(t-2)'
    {'u1(t)'
    {'u1(t-1)'
    {'ul(t-2)' }
    {'y1(t-10)*u1(t-1)'}
```

The regressors show that the maximum delay in the output variable y1 is 10 samples and the maximum delay in the input u1 is two samples. Thus, this model has a total of 12 states:
$X(t)=[y 1(t-1), y 2(t-2), \ldots, y 1(t-10), u 1(t-1), u 1(t-2)]$

Note The state vector includes the output variables first, followed by input variables.

As another example, consider the 2-output and 3-input model.

```
m = idnlarx([2 0 2 2 1 1 0 0; 1 0 1 5 0 1 1 0],[wavenet; linear]);
```

This model has these regressors.

```
getreg(m)
ans = 11x1 cell
    {'y1(t-1)'}
    {'y1(t-2)'}
    {'u1(t-1)'}
    {'u1(t-2)'}
    {'u2(t)' }
```

```
{'u2(t-1)'}
{'u2(t-2)'}
{'u2(t-3)'}
{'u2(t-4)'}
{'u2(t-5)'}
{'u3(t)' }
```

The maximum delay in output variable y1 is two samples. This delay occurs in the regressor set for output 1. The maximum delays in the three input variables are 2,5 , and 0 , respectively. Thus, the state vector is:

```
X(t) = [y1(t-1), y1(t-2), u1(t-1), u1(t-2), u2(t-1),
    u2(t-2), u2(t-3), u2(t-4), u2(t-5)]
```

Variables y2 and u3 do not contribute to the state vector because the maximum delay in these variables is zero.

A simpler way to determine states by inspecting regressors is to use getDelayInfo, which returns the maximum delays in all I/O variables across all model outputs. For the multi-input multi-output model m, getDelayInfo returns:

```
maxDel = getDelayInfo(m)
maxDel = 1×5
```

    \(\begin{array}{lllll}2 & 0 & 2 & 5 & 0\end{array}\)
    maxDel contains the maximum delays for all input and output variables in the order ( $y 1, y 2, u 1, u 2$, u3). The total number of model states is sum(maxDel) $=9$.

The set of states for an idnlarx model is not required to be minimal.

## Compatibility Considerations

## Use of previous idnlarx properties is not recommended.

Not recommended starting in R2021a
Starting in R2021a, several properties of idnlarx have been modified or replaced.
These changes affect the syntaxes in both idnlarx and nlarx. The use of the pre-R2021a properties in the following table is discouraged. However, the software still accepts calling syntaxes that include these properties. There are no plans to exclude these syntaxes at this time. The command syntax that uses ARX model orders continues be a recommended syntax.

| Pre-R2021a Property | R2021a Property | Usage |
| :--- | :--- | :--- |
| ARX model orders na, nb, nk | Regressors, which can include <br> linearRegressor, <br> polynomialRegressor, and <br> customRegressor objects. | na, nb, nk remains a valid <br> idnlarx and nlarx input <br> argument that the software <br> converts to a <br> linearRegressor object. <br> You can no longer change order <br> values in an existing idnlarx <br> model by dot assignment or by <br> using the set function. Create a <br> new model object instead. |
| customRegressors | Regressors | Use polynomialRegressor or <br> customRegressor to create <br> regressor objects and add the <br> objects to the Regressors <br> array. |
| NonlinearRegressors | RegressorUsage | RegressorUsage is a table that <br> contains regressor assignments <br> to linear and nonlinear output <br> components. Change <br> assignments by modifying the <br> corresponding <br> RegressorUsagetable entries. |
| Nonlinearity |  | Change is in name only. <br> Property remains an object or <br> an array or objects that map <br> regressor inputs to an output. |

## See Also

customRegressor| getreg | idnlarx/findop|linear|linearRegressor|linearize| nlarx|pem | polynomialRegressor| sigmoidnet | wavenet

## Topics

"Use nlarx to Estimate Nonlinear ARX Models"
"Estimate Nonlinear ARX Models Initialized Using Linear ARX Models"
"Initialize Nonlinear ARX Estimation Using Linear Model"
"Identifying Nonlinear ARX Models"
"Nonlinear Model Structures"

## Introduced in R2007a

## idnlgrey

Nonlinear grey-box model

## Syntax

```
sys = idnlgrey(FileName,Order,Parameters)
sys = idnlgrey(FileName,Order,Parameters,InitialStates)
sys = idnlgrey(FileName,Order,Parameters,InitialStates,Ts)
sys = idnlgrey(FileName,Order,Parameters,InitialStates,Ts,Name,Value)
```


## Description

sys = idnlgrey (FileName,Order, Parameters) creates a nonlinear grey-box model using the specified model structure in FileName, number of outputs, inputs, and states in Order, and the model parameters.
sys = idnlgrey(FileName,Order,Parameters,InitialStates) specifies the initial states of the model.
sys = idnlgrey(FileName,Order,Parameters,InitialStates,Ts) specifies the sample time of a discrete-time model.
sys = idnlgrey(FileName,Order,Parameters,InitialStates,Ts,Name,Value) specifies additional attributes of the idnlgrey model structure using one or more Name, Value pair arguments.

## Object Description

idnlgrey represents a nonlinear grey-box model. For information about the nonlinear grey-box models, see "Estimate Nonlinear Grey-Box Models".

Use the idnlgrey constructor to create the nonlinear grey-box model and then estimate the model parameters using nlgreyest.

For idnlgrey object properties, see "Properties" on page 1-579.

## Examples

## Create a Nonlinear Grey-Box Model

Load data.

```
load(fullfile(matlabroot,'toolbox','ident','iddemos','data','dcmotordata'));
```

z = iddata(y,u,0.1,'Name','DC-motor');

The data is from a linear DC motor with one input (voltage), and two outputs (angular position and angular velocity). The structure of the model is specified by dcmotor_m.m file.

Create a nonlinear grey-box model.

```
file_name = 'dcmotor_m';
Order = [2 1 2];
Parameters = [1;0.28];
InitialStates = [0;0];
sys = idnlgrey(file_name,Order,Parameters,InitialStates,0, ...
    'Name','DC-motor');
```


## Selectively Estimate Parameters of Nonlinear Grey-Box Model

Load data.

```
load(fullfile(matlabroot,'toolbox','ident','iddemos','data','twotankdata'));
z = iddata(y,u,0.2,'Name','Two tanks');
```

The data contains 3000 input-output data samples of a two tank system. The input is the voltage applied to a pump, and the output is the liquid level of the lower tank.

Specify file describing the model structure for a two-tank system. The file specifies the state derivatives and model outputs as a function of time, states, inputs, and model parameters.

```
FileName = 'twotanks_c';
```

Specify model orders [ny nu nx].

```
Order = [1 1 2];
```

Specify initial parameters $(\mathrm{Np}=6)$.

```
Parameters = {0.5;0.0035;0.019; ...
    9.81;0.25;0.016};
```

Specify initial initial states.
InitialStates = [0;0.1];
Specify as continuous system.
Ts = 0;
Create idnlgrey model object.

```
nlgr = idnlgrey(FileName,Order,Parameters,InitialStates,Ts, ...
    'Name','Two tanks');
```

Set some parameters as constant.

```
nlgr.Parameters(1).Fixed = true;
nlgr.Parameters(4).Fixed = true;
nlgr.Parameters(5).Fixed = true;
```

Estimate the model parameters.

```
nlgr = nlgreyest(z,nlgr);
```


## Input Arguments

## FileName - Name of the function or MEX-file that stores the model structure <br> character vector | function handle

Name of the function or MEX-file storing the model structure, specified as a character vector (without the file extension) or a function handle for computing the states and the outputs. If FileName is a character vector, for example 'twotanks c', then it must point to a MATLAB file, P-code file, or MEX-file. For more information about the file variables, see "Specifying the Nonlinear Grey-Box Model Structure".

## Order - Number of outputs, inputs, and states of the model <br> vector | structure

Number of outputs, inputs, and states of the model, specified as one of the following:

- Vector [ $\mathrm{Ny} \mathrm{Nu} N x$ ], specifying the number of model outputs $N y$, inputs $N u$, and states $N x$.
- Structure with fields' Ny ', ' Nu ', and ' Nx '.

For time series, Nu is set to 0 , and for static model structures, NX is set to 0 .

## Parameters - Parameters of the model

structure | vector | cell array
Parameters of the model, specified as one of the following:

- Np-by-1 structure array, where Np is the number of parameters. The structure contains the following fields:

| Field | Description | Default |
| :--- | :--- | :--- |
| Name | Name of the parameter, <br> specified as a character <br> vector. For example, <br> 'pressure '. | 'pi', where i is an integer in <br> $[1, \mathrm{~Np}]$ |
| Unit | Unit of the parameter, <br> specified as a character <br> vector. | '' |
| Value | Initial value of the parameter, <br> specified as: |  |
|  | - Finite real scalar <br> - Finite real column vector <br> - Two-dimensional real <br> matrix |  |


| Field | Description | Default |
| :--- | :--- | :--- |
| Minimum | Minimum value of the <br> parameter, specified as a real <br> scalar, column vector, or <br> matrix of the same size as <br> Value. | -Inf(size(Value)) |
|  | Minimum >= Value for all <br> components. |  |
| Maximum | Maximum value of the <br> parameter, specified as a real <br> scalar, column vector, or <br> matrix of the same size as <br> Value. | Inf(size(Value)) |
| Value <= Maximum for all <br> components. |  |  |
| Fixed | Specifies whether parameter <br> is fixed to their initial values, <br> specified as a boolean scalar, <br> column vector, or matrix of <br> the same size as Value. | Implies, estimate all <br> parameters |

Use dot notation to access the subfields of the ith parameter. For example, for idnlgrey model M, the ith parameter is accessed through M. Parameters(i) and its subfield Fixed by M. Parameters(i). Fixed.

- Np-by-1 vector of real finite initial values, InParameters.

The data is converted into a structure with default values for the fields Name, Unit, Minimum, Maximum, and Fixed.

Value is assigned the value InParameters(i), where $i$ is an integer in [ $1, N p$ ]

- Np-by-1 cell array containing finite real scalars, finite real vectors, or finite real two-dimensional matrices of initial values.

Default values are used for the fields Name, Unit, Minimum, Maximum, and Fixed.

## InitialStates - Initial states of the model

structure | [] | cell array | \{\}
Initial states of the model parameters specified as one of the following:

- Nx -by-1 structure array, where Nx is the number of states. The structure contains the following fields:

| Field | Description | Default |
| :--- | :--- | :--- |
| Name | Name of the states, specified <br> as a character vector. | 'xi', where $i$ is an integer in <br> $[1, N x]$ |
| Unit | Unit of the states, specified as <br> a character vector. |  |


| Field | Description | Default |
| :---: | :---: | :---: |
| Value | Initial value of the initial states, specified as: <br> - A finite real scalar <br> - A finite real 1-by-Ne vector, where Ne is the number of experiments in the data set to be used for estimation |  |
| Minimum | Minimum value of the initial states, specified as a real scalar or 1-by-Ne vector of the same size as Value. <br> Minimum >= Value for all components. | -Inf(size(Value)) |
| Maximum | Maximum value of the parameters, specified as a real scalar or 1-by-Ne vector of the same size as Value. <br> Value <= Maximum for all components. | Inf(size(Value)) |
| Fixed | Specifies whether initial states are fixed to their initial values, specified as boolean scalar or 1-by-Ne vector of the same size as Value | true(size(Value)) <br> Implies, do not estimate the initial states. |

Use dot notation to access the subfields of the ith initial state. For example, for idnlgrey model $M$, the ith initial state is accessed through M.InitialStates(i) and its subfield Fixed by M.InitialStates(i). Fixed.

- [].

A structure is created with default values for the fields Name, Unit, Minimum, Maximum, and Fixed.

Value is assigned the value 0 .

- A real finite Nx -by-Ne matrix (InitStates).

Value of the ith structure array element is InitStates(i,Ne), a row vector with Ne elements. Minimum, Maximum, and Fixed will be - Inf, Inf and true row vectors of the same size as InitStates(i,Ne).

- Cell array with finite real vectors of size 1-by-Ne or \{\} (same as [ ]).


## Ts - Sample time

## 0 (default) | scalar

Sample time, specified as a positive scalar representing the sampling period. The value is expressed in the unit specified by the TimeUnit property of the model. For a continuous time model Ts is equal to 0 (default).

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Use Name, Value arguments to specify additional properties on page 1-579 of idnlgrey models during model creation.

## Properties

idnlgrey object properties include:

## FileName

Name of the function or MEX-file storing the model structure, specified as a character vector (without extension) or a function handle for computing the states and the outputs. If FileName is a character vector, for example ' twotanks_c' , then it must point to a MATLAB file, P-code file, or MEX-file. For more information about the file variables, see "Specifying the Nonlinear Grey-Box Model Structure".

## Order

Number of outputs, inputs, and states of the model, specified as one of the following:

- Vector [Ny Nu Nx], specifying the number of model outputs $N y$, inputs $N u$, and states $N x$.
- Structure with fields ' Ny ', ' Nu ', and ' $N \mathrm{x}$ '.

For time series, Nu is set to 0 , and for static model structures, Nx is set to 0 .

## Parameters

Parameters of the model, specified as one of the following:

- Np-by-1 structure array, where Np is the number of parameters. The structure contains the following fields:

| Field | Description | Default |
| :--- | :--- | :--- |
| Name | Name of the parameter, <br> specified as a character <br> vector. For example, <br> 'pressure '. | 'pi' where i is an integer in <br> [1,Np] |
| Unit | Unit of the parameter, <br> specified as a character <br> vector. | '' |
| Value | Initial value of the parameter, <br> specified as: |  |
| - Finite real scalar <br> - Finite real column vector <br> - Two-dimensional real <br> matrix |  |  |


| Field | Description | Default |
| :--- | :--- | :--- |
| Minimum | Minimum value of the <br> parameter, specified as a real <br> scalar, column vector, or <br> matrix of the same size as <br> Value. | -Inf(size(Value)) |
| Minimum >= Value for all <br> components. |  |  |
| Maximum | Maximum value of the <br> parameter, specified as a real <br> scalar, column vector, or <br> matrix of the same size as <br> Value. | Inf(size(Value)) |
| Value <= Maximum for all <br> components. |  |  |
| Fixed | Specifies whether parameter <br> is fixed to their initial values, <br> specified as a boolean scalar, <br> column vector, or matrix of <br> the same size as Value. | Implies, estimate all <br> parameters |

- Np-by-1 vector of real finite initial values, InPa rameters.

The data is converted into a structure with default values for the fields Name, Unit, Minimum, Maximum, and Fixed.

Value is assigned the value InParameters(i), where i is an integer in [1,Np]

- Np-by-1 cell array containing finite real scalars, finite real vectors, or finite real two-dimensional matrices of initial values.

A structure is created with default values for the fields Name, Unit, Minimum, Maximum, and Fixed.

Use dot notation to access the subfields of the ith parameter. For example, for idnlgrey model M, the ith parameter is accessed through M. Parameters(i) and its subfield Fixed by M. Parameters(i). Fixed.

## InitialStates

Initial states of the model parameters specified as one of the following:

- Nx -by-1 structure array, where Nx is the number of states. The structure contains the following fields:

| Field | Description | Default |
| :--- | :--- | :--- |
| Name | Name of the states, specified <br> as a character vector. | 'xi', where $i$ is an integer in <br> $[1, \mathrm{Nx}]$ |
| Unit | Unit of the states, specified as <br> a character vector. |  |


| Field | Description | Default |
| :--- | :--- | :--- |
| Value | Initial value of the initial <br> states, specified as: <br> - A finite real scalar <br> - A finite real 1-by-Ne vector, <br> where Ne is the number of <br> experiments in the data set <br> to be used for estimation |  |
| Minimum | Minimum value of the initial <br> states, specified as a real <br> scalar or 1-by-Ne vector of the <br> same size as Value. | -Inf (size(Value)) |
| Maximum | Minimum >= Value for all <br> components. |  |
| Fixed | Maximum value of the <br> parameters, specified as a <br> real scalar or 1-by-Ne vector <br> of the same size as Value. <br> Value <= Maximum for all <br> components. | Inf(size(Value)) |
|  | Specifies whether initial <br> states are fixed to their initial <br> values, specified as boolean <br> scalar or 1-by-Ne vector of the <br> same size as Value | true(size(Value)) |
| Implies, do not estimate the states. |  |  |

- [].

A structure is created with default values for the fields Name, Unit, Minimum, Maximum, and Fixed.

Value is assigned the value 0.

- A real finite Nx-by-Ne matrix (InitStates).

Value of the ith structure array element is InitStates (i, Ne), a row vector with Ne elements. Minimum, Maximum, and Fixed will be - Inf, Inf and true row vectors of the same size as InitStates(i,Ne).

- Cell array with finite real vectors of size 1-by-Ne or \{\} (same as []).

A structure is created with default values for the fields Name, Unit, Minimum, Maximum, and Fixed.

Use dot notation to access the subfields of the ith initial state. For example, for idnlgrey model M, the ith initial state is accessed through M. InitialStates (i) and its subfield Fixed by M.InitialStates(i). Fixed.

## FileArgument

Contains auxiliary variables passed to the ODE file (function or MEX-file) specified in FileName, specified as a cell array. These variables are used as extra inputs for specifying the state and/or output equations.
Default: \{\}.

## SimulationOptions

A structure that specifies the simulation method and related options, containing the following fields:

| Field | Description | Default |
| :---: | :---: | :---: |
| AbsTol | Absolute error tolerance. This scalar applies to all components of the state vector. <br> Applicable to: Variable step solvers. <br> Assignable value: A positive real value. | 1e-6 |
| FixedStep | Step size used by the solver. <br> Applicable to: Fixed-step time-continuous solvers. <br> Assignable values: <br> - 'Auto ' - Automatically chooses the initial step. <br> - A real value such that $0<$ FixedStep $<=1$. | 'Auto' <br> Automatically chooses the initial step. |
| InitialStep | Specifies the initial step at which the ODE solver starts. <br> Applicable to: Variable-step, time-continuous solvers. <br> Assignable values: <br> - 'Auto ' - Automatically chooses the initial step. <br> - A positive real value such that MinStep<=InitialStep<=MaxStep. | 'Auto ' <br> Automatically chooses the initial step. |
| Max0rder | Specifies the order of the Numerical Differentiation Formulas (NDF). <br> Applicable to: ode15s. <br> Assignable values: $1,2,3,4$ or 5 . | 5 |
| MaxStep | Specifies the largest time step of the ODE solver. <br> Applicable to: Variable-step, time-continuous solvers. <br> Assignable values: <br> - 'Auto ' - Automatically chooses the time step. <br> - A positive real value > MinStep. | 'Auto ' <br> Automatically chooses the time step. |


| Field | Description | Default |
| :--- | :--- | :--- |
| MinStep | Specifies the smallest time step of the ODE solver. <br> Applicable to: Variable-step, time-continuous solvers. <br> Assignable values: <br> - 'Auto' - Automatically chooses the time step. <br> - A positive real value < MaxStep. | Auto ' <br> Automatically chooses the time <br> step. |
| RelTol | Relative error tolerance that applies to all <br> components of the state vector. The estimated error <br> in each integration step satisfies \|e(i)| < <br> max(RelTol*abs (x(i)), AbsTol (i)). <br> Applicable to: Variable-step, time-continuous solvers. | le-3 |
| Assignable value: A positive real value.. | $(0.1 \%$ accuracy). |  |


| Field | Description | Default |
| :---: | :---: | :---: |
| Solver | ODE (Ordinary Differential/Difference Equation) solver for solving state space equations. <br> - Variable-step solvers for time-continuous idnlgrey models: <br> - 'ode45' - Runge-Kutta $(4,5)$ solver for nonstiff problems. <br> - 'ode23 ' - Runge-Kutta $(2,3)$ solver for nonstiff problems. <br> - 'ode113' - Adams-Bashforth-Moulton solver for nonstiff problems. <br> - 'ode15s' - Numerical Differential Formula solver for stiff problems. <br> - 'ode23s ' - Modified Rosenbrock solver for stiff problems. <br> - ' ode23t' - Trapezoidal solver for moderately stiff problems. <br> - 'ode23tb' - Implicit Runge-Kutta solver for stiff problems. <br> - Fixed-step solvers for time-continuous idnlgrey models: <br> - 'ode5' - Dormand-Prince solver. <br> - 'ode4 ' - Fourth-order Runge-Kutta solver. <br> - 'ode3' - Bogacki-Shampine solver. <br> - 'ode2 ' - Heun or improved Euler solver. <br> - 'ode1' - Euler solver. <br> - Fixed-step solvers for time-discrete idnlgrey models: 'FixedStepDiscrete' <br> - General: 'Auto' - Automatically chooses one of the previous solvers. | 'Auto ' <br> Automatically chooses one of the solvers. |

## Report

Summary report that contains information about the estimation options and results when the model is estimated using the nlgreyest command. Use Report to query a model for how it was estimated, including:

- Estimation method
- Estimation options
- Search termination conditions
- Estimation data fit

The contents of Report are irrelevant if the model was created by construction.

```
nlgr = idnlgrey('dcmotor_m',[2,1,2],[1;0.28],[0;0],0,'Name','DC-motor');
nlgr.Report.OptionsUsed
ans =
    []
```

If you use nlgreyest to estimate the model, the fields of Report contain information on the estimation data, options, and results.

```
load(fullfile(matlabroot,'toolbox','ident','iddemos','data','dcmotordata'));
z = iddata(y,u,0.1,'Name','DC-motor');
nlgr = idnlgrey('dcmotor_m',[2,1,2],[1;0.28],[0;0],0,'Name','DC-motor');
nlgr = nlgreyest(z,nlgr);
nlgr.Report.OptionsUsed
Option set for the nlgreyest command:
    GradientOptions: [1x1 struct]
EstimateCovariance: 1
            Display: 'off'
    Regularization: [1x1 struct]
        SearchMethod: 'auto'
    SearchOptions: [1x1 idoptions.search.lsqnonlin]
        OutputWeight: []
            Advanced: [1x1 struct]
```

Report is a read-only property.
For more information on this property and how to use it, see "Output Arguments" on page 1-896 in the $n l$ greyest reference page and "Estimation Report".

## TimeVariable

Independent variable for the inputs, outputs, and-when available-internal states, specified as a character vector.

Default: 't'

## NoiseVariance

Noise variance (covariance matrix) of the model innovations $e$.
Assignable value is an ny-by-ny matrix.
Typically set automatically by the estimation algorithm.

## Ts

Sample time. Ts is a positive scalar representing the sampling period. This value is expressed in the unit specified by the TimeUnit property of the model. For a continuous time model, Ts is equal to 0 (default).

Changing this property does not discretize or resample the model.

## Default: 0

## TimeUnit

Units for the time variable, the sample time Ts, and any time delays in the model, specified as one of the following values:

- 'nanoseconds'
- 'microseconds'
- 'milliseconds'
- 'seconds'
- 'minutes'
- 'hours'
- 'days'
- 'weeks'
- 'months'
- 'years'

Changing this property has no effect on other properties, and therefore changes the overall system behavior. Use chgTimeUnit to convert between time units without modifying system behavior.

Default: 'seconds'

## InputName

Input channel names, specified as one of the following:

- Character vector - For single-input models, for example, ' controls '.
- Cell array of character vectors - For multi-input models.

Alternatively, use automatic vector expansion to assign input names for multi-input models. For example, if sys is a two-input model, enter:
sys.InputName = 'controls';
The input names automatically expand to \{'controls(1)';'controls(2)'\}.
When you estimate a model using an iddata object, data, the software automatically sets InputName to data. InputName.

You can use the shorthand notation $u$ to refer to the InputName property. For example, sys. $u$ is equivalent to sys.InputName.

Input channel names have several uses, including:

- Identifying channels on model display and plots
- Extracting subsystems of MIMO systems
- Specifying connection points when interconnecting models

Default: ' ' for all input channels

## InputUnit

Input channel units, specified as one of the following:

- Character vector - For single-input models, for example, ' seconds ' .
- Cell array of character vectors - For multi-input models.

Use InputUnit to keep track of input signal units. InputUnit has no effect on system behavior.
Default: ' ' for all input channels

## InputGroup

Input channel groups. The InputGroup property lets you assign the input channels of MIMO systems into groups and refer to each group by name. Specify input groups as a structure. In this structure, field names are the group names, and field values are the input channels belonging to each group. For example:
sys.InputGroup.controls = [1 2];
sys.InputGroup.noise = [3 5];
creates input groups named controls and noise that include input channels 1,2 and 3,5 , respectively. You can then extract the subsystem from the controls inputs to all outputs using:
sys(:,'controls')
Default: Struct with no fields

## OutputName

Output channel names, specified as one of the following:

- Character vector - For single-output models. For example, 'measurements '.
- Cell array of character vectors - For multi-output models.

Alternatively, use automatic vector expansion to assign output names for multi-output models. For example, if sys is a two-output model, enter:
sys.OutputName = 'measurements';
The output names automatically expand to \{'measurements (1)' ; 'measurements (2)' \}.
When you estimate a model using an iddata object, data, the software automatically sets OutputName to data. OutputName.

You can use the shorthand notation y to refer to the OutputName property. For example, sys.y is equivalent to sys.OutputName.

Output channel names have several uses, including:

- Identifying channels on model display and plots
- Extracting subsystems of MIMO systems
- Specifying connection points when interconnecting models

Default: ' ' for all output channels

## OutputUnit

Output channel units, specified as one of the following:

- Character vector - For single-output models. For example, ' seconds ' .
- Cell array of character vectors - For multi-output models.

Use OutputUnit to keep track of output signal units. OutputUnit has no effect on system behavior.
Default: ' ' for all output channels

## OutputGroup

Output channel groups. The OutputGroup property lets you assign the output channels of MIMO systems into groups and refer to each group by name. Specify output groups as a structure. In this structure, field names are the group names, and field values are the output channels belonging to each group. For example:

```
sys.OutputGroup.temperature = [1];
```

sys.InputGroup.measurement = [3 5];
creates output groups named temperature and measurement that include output channels 1, and 3,5 , respectively. You can then extract the subsystem from all inputs to the measurement outputs using:

```
sys('measurement',:)
```

Default: Struct with no fields

## Name

System name, specified as a character vector. For example, ' system_1'.

## Default: ' '

## Notes

Any text that you want to associate with the system, stored as a string or a cell array of character vectors. The property stores whichever data type you provide. For instance, if sys1 and sys2 are dynamic system models, you can set their Notes properties as follows:

```
sys1.Notes = "sys1 has a string.";
sys2.Notes = 'sys2 has a character vector.';
sys1.Notes
sys2.Notes
ans =
    "sys1 has a string."
ans =
    'sys2 has a character vector.'
```

Default: [ $0 \times 1$ string]

## UserData

Any type of data you want to associate with system, specified as any MATLAB data type.
Default: []

## Output Arguments

sys - Nonlinear grey-box model
idnlgrey object
Nonlinear grey-box model, returned as an idnlgrey object.

## More About

## Definition of idnlgrey States

The states of an idnlgrey model are defined explicitly in the function or MEX-file storing the model structure. States are required for simulation and prediction of nonlinear grey-box models. Use findstates to search for state values for simulation and prediction with sim, predict, and compare.

Note The initial values of the states are configured by the InitialStates property of the idnlgrey model.

See Also<br>get | getinit| getpar|nlgreyest|pem| set| setinit| setpar<br>Topics<br>"Represent Nonlinear Dynamics Using MATLAB File for Grey-Box Estimation"<br>"Creating IDNLGREY Model Files"<br>"Estimate Nonlinear Grey-Box Models"

Introduced in R2007a

## idnlhw

Hammerstein-Wiener model

## Syntax

sys = idnlhw(Orders)
sys = idnlhw(Orders,InputNL,OutputNL)
sys $=$ idnlhw(Orders,InputNL,OutputNL,Name,Value)
sys = idnlhw(LinModel)
sys = idnlhw(LinModel,InputNL,OutputNL)
sys $=$ idnlhw(LinModel,InputNL,OutputNL, Name, Value)

## Description

sys = idnlhw(Orders) creates a Hammerstein-Wiener model with the specified orders, and using piecewise linear functions as input and output nonlinearity estimators.
sys = idnlhw(Orders,InputNL,OutputNL) uses InputNL and OutputNL as the input and output nonlinearity estimators, respectively.
sys = idnlhw(Orders,InputNL,OutputNL,Name,Value) specifies additional attributes of the idnlhw model structure using one or more Name, Value pair arguments.
sys = idnlhw(LinModel) uses a linear model LinModel to specify the model orders and default piecewise linear functions for the input and output nonlinearity estimators.
sys = idnlhw(LinModel,InputNL,OutputNL) specifies input and output nonlinearity estimators for the model.
sys = idnlhw(LinModel,InputNL,OutputNL,Name,Value) specifies additional attributes of the idnlhw model structure using one or more Name, Value pair arguments.

## Object Description

idnlhw represents a Hammerstein-Wiener model. The Hammerstein-Wiener structure on page 1600 represents a linear model with input-output nonlinearities.

Use the nlhw command to both construct an idnlhw object and estimate the model parameters.
You can also use the idnlhw constructor to create the Hammerstein-Wiener model and then estimate the model parameters using $n l h w$.

For idnlhw object properties, see "Properties" on page 1-594.

## Examples

## Create a Hammerstein-Wiener Model Structure with Default Nonlinearities

Create a Hammersein-Wiener model with nb and $\mathrm{nf}=2$ and $\mathrm{nk}=1$.
m = idnlhw([2 2 1]);
m has piecewise linear input and output nonlinearity.

## Create Hammerstein-Wiener Model with Specific Input-Output Nonlinearities

m = idnlhw([2 2 1],'sigmoidnet','deadzone');
The above is equivalent to:

```
m = idnlhw([2 2 1],'sig','dead');
```

The specified nonlinearities have a default configuration.

## Create Hammerstein-Wiener Model and Configure the Nonlinearities

```
m = idnlhw([2 2 1],sigmoidnet('num',5),deadzone([-1,2]),'InputName','Volts','OutputName','Time')
```


## Create a Wiener Model and Estimate Model Parameters

Create a Wiener model (no input nonlinearity).
m = idnlhw([2 2 1],[],'saturation');
Estimate the model.
load iddata1;
m = nlhw(zl,m);

## Create Hammerstein-Wiener Model Using Input-Output Polynomial Model of Output-Error Structure

Construct an input-output polynomial model of OE structure.

```
B = [0.8 1];
F = [1 -1.2 0.5];
LinearModel = idpoly(1,B,1,1,F,'Ts',0.1);
```

Construct Hammerstein-Wiener model using OE model as its linear component.
m1 = idnlhw(LinearModel,'saturation',[],'InputName','Control');

## Input Arguments

Orders - Order and delays of the linear subsystem transfer function
[ $n b \mathrm{nf} n \mathrm{n}$ ] vector of positive integers | [ nb nf nk ] vector of matrices

Order and delays of the linear subsystem transfer function, specified as a [ nb nf nk ] vector.
Dimensions of Orders:

- For a SISO transfer function, Orders is a vector of positive integers.
nb is the number of zeros plus $1, \mathrm{nf}$ is the number of poles, and nk is the input delay.
- For a MIMO transfer function with $\mathrm{n}_{\mathrm{u}}$ inputs and $\mathrm{n}_{\mathrm{y}}$ outputs, Orders is a vector of matrices.
$n b, n f$, and $n k$ are $n_{y}-$-by $-n_{u}$ matrices whose $i-j t h$ entry specifies the orders and delay of the transfer function from the $j$ th input to the $i$ th output.


## InputNL - Input static nonlinearity

'pwlinear' (default)|'sigmoidnet'|'wavenet'|'saturation'|'deadzone'|'polyld'|
'unitgain ' | nonlinearity estimator object | array of nonlinearity estimators
Input static nonlinearity estimator, specified as one of the following.
'pwlinear' or pwlinear object (default)
'sigmoidnet' or sigmoidnet object
'wavenet' or wavenet object
'saturation' or saturation object
'deadzone' or deadzone object
' poly1d' or polyld object
'unitgain' or [] or unitgain object
customnet object

Piecewise linear function
Sigmoid network
Wavelet network
Saturation
Dead zone
One-dimensional polynomial
Unit gain
Custom network - Similar to sigmoidnet, but with a user-defined replacement for the sigmoid function.

Specifying a character vector, for example ' sigmoidnet ', creates a nonlinearity estimator object with default settings. Use object representation instead to configure the properties of a nonlinearity estimator.

```
InputNL = wavenet;
InputNL.NumberOfUnits = 10;
```

Alternatively, use the associated input nonlinearity estimator function with Name-Value pair arguments.

InputNL = wavenet('NumberOfUnits',10);
For $n_{u}$ input channels, you can specify nonlinear estimators individually for each input channel by setting InputNL to an $n_{u}$-by-1 array of nonlinearity estimators.

```
InputNL = [sigmoidnet('NumberofUnits',5); deadzone([-1,2])]
```

To specify the same nonlinearity for all inputs, specify a single input nonlinearity estimator.

## OutputNL - Output static nonlinearity

'pwlinear' (default)|'sigmoidnet'|'wavenet'|'saturation'|'deadzone'|'polyld'|
'unitgain' | nonlinearity estimator object | array of nonlinearity estimators

Output static nonlinearity estimator, specified as one of the following:
'pwlinear' or pwlinear object (default)
'sigmoidnet' or sigmoidnet object
'wavenet' or wavenet object
'saturation' or saturation object
' deadzone' or deadzone object
'polyld' or poly1d object
'unitgain' or [] or unitgain object
customnet object

Piecewise linear function

Sigmoid network
Wavelet network
Saturation
Dead zone
One-dimensional polynomial
Unit gain
Custom network - Similar to sigmoidnet, but with a user-defined replacement for the sigmoid function.

Specifying a character vector creates a nonlinearity estimator object with default settings. Use object representation instead to configure the properties of a nonlinearity estimator.

```
OutputNL = sigmoidnet;
OutputNL.NumberOfUnits = 10;
```

Alternatively, use the associated input nonlinearity estimator function with Name-Value pair arguments.

```
OutputNL = sigmoidnet('NumberOfUnits',10);
```

For $\mathrm{n}_{\mathrm{y}}$ output channels, you can specify nonlinear estimators individually for each output channel by setting OutputNL to an $n_{y}$-by-1 array of nonlinearity estimators. To specify the same nonlinearity for all outputs, specify a single output nonlinearity estimator.

## LinModel - Discrete time linear model

idpoly|idss with $\mathrm{K}=0 \mid$ idtf
Discrete-time linear model used to specify the linear subsystem, specified as one of the following:

- Input-output polynomial model of Output-Error (OE) structure (idpoly)
- State-space model with no disturbance component (idss with $K=0$ )
- Transfer function model (idtf)

Typically, you estimate the model using oe, n4sid, or tfest.

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Use Name, Value arguments to specify additional properties on page 1-594 of idnlhw models during model creation. For example, $m=i d n l h w([23$
1], 'pwlinear', 'wavenet','InputName', 'Volts', 'Ts', 0.1) creates an idnlhw model object with input nonlinearity estimator 'pwlinear', output nonlinearity estimator 'wavenet', input name Volts, and a sample time of 0.1 seconds.

## Properties

idnlhw object properties include:
nb, nf, nk
Model orders and delays of the linear subsystem transfer function, where nb is the number of zeros plus $1, \mathrm{nf}$ is the number of poles, and nk is the input delay.

For a MIMO transfer function with $n_{u}$ inputs and $n_{y}$ outputs, $n b, n f$, and $n k$ are $n_{y}-$ by $-n_{u}$ matrices whose $i$-jth entry specifies the orders and delay of the transfer function from the $j$ th input to the $i$ th output.

B
$B$ polynomial of the linear block in the model structure on page 1-600, specified as a cell array of $\mathrm{n}_{\mathrm{y}}$ -by- $n_{u}$ elements, where $n_{y}$ is the number of outputs and $n_{u}$ is the number of inputs. An element $\mathrm{B}\{\mathrm{i}, \mathrm{j}\}$ is a row vector representing the numerator polynomial for the $j$ th input to $i$ th output transfer function. The element contains nk leading zeros, where $n k$ is the number of input delays.

## F

$F$ polynomial of the linear block in the model structure on page 1-600, specified as a cell array of $\mathrm{n}_{\mathrm{y}}$ -by- $n_{u}$ elements, where $n_{y}$ is the number of outputs and $n_{u}$ is the number of inputs. An element $\mathrm{F}\{\mathrm{i}, \mathrm{j}\}$ is a row vector representing the denominator polynomial for the $j$ th input to $i$ th output transfer function.

## InputNonlinearity

Input nonlinearity estimator, specified as one of the following:

| 'pwlinear' or pwlinear object | Piecewise linear function |
| :--- | :--- |
| (default) | Sigmoid network |
| 'sigmoidnet ' or sigmoidnet object | Wavelet network |
| 'wavenet' or wavenet object | Saturation |
| 'saturation ' or saturation object | Dead zone |
| 'deadzone' or deadzone object | One- <br> dimensional polynomial <br> 'poly1d' or poly1d object |
| Unitgain' or [] or unitgain object | Unit gain |
| customnet object | Custom network |

Specifying a character vector creates a nonlinearity estimator object with default settings. Use object representation instead to configure the properties of a nonlinearity estimator.

```
InputNonlinearity = wavenet;
InputNonlinearity.NumberOfUnits = 10;
```

Alternatively, use the associated input nonlinearity estimator function with Name-Value pair arguments.

```
InputNonlinearity = wavenet('NumberOfUnits',10);
```

For $n_{u}$ input channels, you can specify nonlinear estimators individually for each input channel by setting InputNL to an $n_{u}$-by-1 array of nonlinearity estimators. To specify the same nonlinearity for all inputs, specify a single input nonlinearity estimator.

## Default: 'pwlinear'

## OutputNonlinearity

Output nonlinearity estimator, specified as one of the following:

| 'pwlinear' or pwlinear object | Piecewise linear function |
| :--- | :--- |
| (default) | Sigmoid network |
| 'sigmoidnet ' or sigmoidnet object | Wavelet network |
| 'wavenet' or wavenet object | Saturation |
| 'saturation' or saturation object | Dead zone |
| 'deadzone' or deadzone object | One- <br> dimensional polynomial <br> 'polyld' or polyld object |
|  | Unit gain |
| 'unitgain' or [] or unitgain object | Custom network |
| customnet object |  |

Specifying a character vector creates a nonlinearity estimator object with default settings. Use object representation instead to configure the properties of a nonlinearity estimator.

OutputNonlinearity = sigmoidnet;
OutputNonlinearity.NumberOfUnits = 10;
Alternatively, use the associated input nonlinearity estimator function with Name-Value pair arguments.

OutputNonlinearity = sigmoidnet('NumberOfUnits',10);
For $n_{y}$ output channels, you can specify nonlinear estimators individually for each output channel by setting OutputNL to an $n_{y}$-by-1 array of nonlinearity estimators. To specify the same nonlinearity for all outputs, specify a single output nonlinearity estimator.

Default: 'pwlinear'
LinearModel
The linear model in the linear block of the model structure, represented as an idpoly object. This property is read only.

## Report

Summary report that contains information about the estimation options and results when the model is estimated using the nlhw command. Use Report to query a model for how it was estimated, including:

- Estimation method
- Estimation options
- Search termination conditions
- Estimation data fit

The contents of Report are irrelevant if the model was created by construction.

```
m = idnlhw([2 2 1]);
m.Report.OptionsUsed
ans =
    []
```

If you use nlhw to estimate the model, the fields of Report contain information on the estimation data, options, and results.

```
load iddata1;
m = nlhw(z1,[2 2 1],[],'pwlinear');
m.Report.OptionsUsed
Option set for the nlhw command:
    InitialCondition: 'zero'
            Display: 'off'
        Regularization: [1x1 struct]
        SearchMethod: 'auto'
        SearchOption: [1x1 idoptions.search.identsolver]
        OutputWeight: 'noise'
            Advanced: [1x1 struct]
```

Report is a read-only property.
For more information on this property and how to use it, see "Output Arguments" on page 1-917 in the nlhw reference page and "Estimation Report".

## TimeVariable

Independent variable for the inputs, outputs, and—when available-internal states, specified as a character vector.

Default: 't'

## NoiseVariance

Noise variance (covariance matrix) of the model innovations $e$.
Assignable value is an ny-by-ny matrix.
Typically set automatically by the estimation algorithm.

## Ts

Sample time. Ts is a positive scalar representing the sampling period. This value is expressed in the unit specified by the TimeUnit property of the model.

Changing this property does not discretize or resample the model.

## Default: 1

## TimeUnit

Units for the time variable, the sample time Ts, and any time delays in the model, specified as one of the following values:

- 'nanoseconds'
- 'microseconds'
- 'milliseconds'
- 'seconds'
- 'minutes'
- 'hours'
- 'days'
- 'weeks'
- 'months'
- 'years'

Changing this property has no effect on other properties, and therefore changes the overall system behavior. Use chgTimeUnit to convert between time units without modifying system behavior.

Default: 'seconds'

## InputName

Input channel names, specified as one of the following:

- Character vector - For single-input models, for example, ' controls '.
- Cell array of character vectors - For multi-input models.

Alternatively, use automatic vector expansion to assign input names for multi-input models. For example, if sys is a two-input model, enter:
sys.InputName = 'controls';
The input names automatically expand to \{'controls(1)'; 'controls(2)'\}.
When you estimate a model using an iddata object, data, the software automatically sets InputName to data. InputName.

You can use the shorthand notation $u$ to refer to the InputName property. For example, sys. $u$ is equivalent to sys. InputName.

Input channel names have several uses, including:

- Identifying channels on model display and plots
- Extracting subsystems of MIMO systems
- Specifying connection points when interconnecting models

Default: ' ' for all input channels

## InputUnit

Input channel units, specified as one of the following:

- Character vector - For single-input models, for example, ' seconds ' .
- Cell array of character vectors - For multi-input models.

Use InputUnit to keep track of input signal units. InputUnit has no effect on system behavior.
Default: ' ' for all input channels

## InputGroup

Input channel groups. The InputGroup property lets you assign the input channels of MIMO systems into groups and refer to each group by name. Specify input groups as a structure. In this structure, field names are the group names, and field values are the input channels belonging to each group. For example:
sys.InputGroup.controls = [1 2];
sys.InputGroup.noise = [3 5];
creates input groups named controls and noise that include input channels 1,2 and 3,5 , respectively. You can then extract the subsystem from the controls inputs to all outputs using:
sys(:,'controls')
Default: Struct with no fields

## OutputName

Output channel names, specified as one of the following:

- Character vector - For single-output models. For example, 'measurements '.
- Cell array of character vectors - For multi-output models.

Alternatively, use automatic vector expansion to assign output names for multi-output models. For example, if sys is a two-output model, enter:
sys.OutputName = 'measurements';
The output names automatically expand to \{'measurements (1)'; 'measurements (2)' \}.
When you estimate a model using an iddata object, data, the software automatically sets OutputName to data. OutputName.

You can use the shorthand notation y to refer to the OutputName property. For example, sys.y is equivalent to sys.OutputName.

Output channel names have several uses, including:

- Identifying channels on model display and plots
- Extracting subsystems of MIMO systems
- Specifying connection points when interconnecting models

Default: ' ' for all output channels

## OutputUnit

Output channel units, specified as one of the following:

- Character vector - For single-output models. For example, ' seconds '.
- Cell array of character vectors - For multi-output models.

Use OutputUnit to keep track of output signal units. OutputUnit has no effect on system behavior.
Default: ' ' for all output channels

## OutputGroup

Output channel groups. The OutputGroup property lets you assign the output channels of MIMO systems into groups and refer to each group by name. Specify output groups as a structure. In this structure, field names are the group names, and field values are the output channels belonging to each group. For example:
sys.OutputGroup.temperature = [1];
sys.InputGroup.measurement = [35];
creates output groups named temperature and measurement that include output channels 1, and 3,5 , respectively. You can then extract the subsystem from all inputs to the measurement outputs using:
sys('measurement',:)
Default: Struct with no fields

## Name

System name, specified as a character vector. For example, ' system_1'.

## Default: ' '

## Notes

Any text that you want to associate with the system, stored as a string or a cell array of character vectors. The property stores whichever data type you provide. For instance, if sys1 and sys2 are dynamic system models, you can set their Notes properties as follows:

```
sys1.Notes = "sys1 has a string.";
sys2.Notes = 'sys2 has a character vector.';
sys1.Notes
sys2.Notes
ans =
    "sys1 has a string."
ans =
    'sys2 has a character vector.'
```

Default: [ $0 \times 1$ string]

## UserData

Any type of data you want to associate with system, specified as any MATLAB data type.
Default: []

## Output Arguments

## sys - Hammerstein-Wiener model

idnlhw object
Hammerstein-Wiener model, returned as an idnlhw object. This model is created using the specified model orders and delays, input and output nonlinearity estimators, and properties.

## More About

## Hammerstein-Wiener Model Structure

This block diagram represents the structure of a Hammerstein-Wiener model:

Where,

- $f$ is a nonlinear function that transforms input data $u(t)$ as $w(t)=f(u(t))$.
$w(t)$, an internal variable, is the output of the Input Nonlinearity block and has the same dimension as $u(t)$.
- $B / F$ is a linear transfer function that transforms $w(t)$ as $\chi(t)=(B / F) w(t)$.
$x(t)$, an internal variable, is the output of the Linear block and has the same dimension as $y(t)$.
$B$ and $F$ are similar to polynomials in a linear Output-Error model. For more information about Output-Error models, see "What Are Polynomial Models?".

For ny outputs and $n u$ inputs, the linear block is a transfer function matrix containing entries:

$$
\frac{B_{j, i}(q)}{F_{i j}(q)}
$$

where $j=1,2, \ldots$, ny and $i=1,2, \ldots$, nu.

- $h$ is a nonlinear function that maps the output of the linear block $x(t)$ to the system output $y(t)$ as $y(t)=h(x(t))$.

Because $f$ acts on the input port of the linear block, this function is called the input nonlinearity. Similarly, because $h$ acts on the output port of the linear block, this function is called the output nonlinearity. If your system contains several inputs and outputs, you must define the functions $f$ and $h$ for each input and output signal. You do not have to include both the input and the output nonlinearity in the model structure. When a model contains only the input nonlinearity $f$, it is called a Hammerstein model. Similarly, when the model contains only the output nonlinearity $h$, it is called a Wiener model.

The software computes the Hammerstein-Wiener model output $y$ in three stages:

1 Compute $w(t)=f(u(t))$ from the input data.
$w(t)$ is an input to the linear transfer function $B / F$.
The input nonlinearity is a static (memoryless) function, where the value of the output a given time $t$ depends only on the input value at time $t$.

You can configure the input nonlinearity as a sigmoid network, wavelet network, saturation, dead zone, piecewise linear function, one-dimensional polynomial, or a custom network. You can also remove the input nonlinearity.
2 Compute the output of the linear block using $w(t)$ and initial conditions: $x(t)=(B / F) w(t)$.
You can configure the linear block by specifying the orders of numerator $B$ and denominator $F$.
3 Compute the model output by transforming the output of the linear block $x(t)$ using the nonlinear function $h$ as $y(t)=h(x(t))$.

Similar to the input nonlinearity, the output nonlinearity is a static function. You can configure the output nonlinearity in the same way as the input nonlinearity. You can also remove the output nonlinearity, such that $y(t)=x(t)$.

Resulting models are idnlhw objects that store all model data, including model parameters and nonlinearity estimators. For more information about these objects, see "Nonlinear Model Structures".

## Definition of idnlhw States

The states of a Hammerstein-Wiener model correspond to the states of the linear block in the model structure. The linear block contains all the dynamic elements of the model. If the linear block is not a state-space structure, the states are defined as those of model Mss, where Mss = idss(Model.LinearModel) and Model is the idnlhw object.

States are required for simulation, prediction, and linearization of Hammerstein-Wiener models. To specify the initial states:

- Use findstates to search for state values for simulation and prediction with sim, predict, and compare.
- Use findop when linearizing the model with linearize.
- Alternatively, specify the states manually.


## See Also

customnet | findop|linear|linearize | $n l$ hw | pem | polyld | saturation | saturation | sigmoidnet|wavenet

## Topics

"Estimate Multiple Hammerstein-Wiener Models"
"Estimate Hammerstein-Wiener Models Initialized Using Linear OE Models"
"Identifying Hammerstein-Wiener Models"
"Initialize Hammerstein-Wiener Estimation Using Linear Model"

## Introduced in R2007a

## idpar

Create parameter for initial states and input level estimation

## Syntax

p = idpar(paramvalue)
p = idpar(paramname, paramvalue)

## Description

$\mathrm{p}=$ idpar(paramvalue) creates an estimable parameter with initial value paramvalue. The parameter, p , is either scalar or array-valued, with the same dimensions as paramvalue. You can configure attributes of the parameter, such as which elements are fixed and which are estimated, and lower and upper bounds.
$\mathrm{p}=$ idpar(paramname, paramvalue) sets the Name property of p to paramname.

## Input Arguments

## paramvalue

Initial parameter value.
paramvalue is a numeric scalar or array that determines both the dimensions and initial values of the estimable parameter $p$. For example, p = idpar(eye(3)) creates a 3-by-3 parameter whose initial value is the identity matrix.
paramvalue should be:

- A column vector of length $N_{x}$, the number of states to estimate, if you are using $p$ for initial state estimation.
- An $N_{x}$-by- $N_{e}$ array, if you are using p for initial state estimation with multi-experiment data. $N_{e}$ is the number of experiments.
- A column vector of length $N_{u}$, the number of inputs to estimate, if you are using p for input level estimation.
- An $N_{u}$-by- $N_{e}$ array, if you are using p for input level estimation with multi-experiment data.

If the initial value of a parameter is unknown, use NaN .

## paramname

Name property of $p$, specified as a character vector. For example, you can assign ' $x 0$ ' as the name of a parameter created for initial state estimation.

The Name property is not used in state estimation or input level estimation. You can optionally assign a name for convenience.

Default: 'par'

## Output Arguments

## p

Estimable parameter, specified as a param. Continuous object.
$p$ can be either scalar- or array-valued. $p$ takes its dimensions and initial value from paramvalue.
p contains the following fields:

- Value - Scalar or array value of the parameter.

The dimension and initial value of $p$. Value are taken from paramvalue when $p$ is created.

- Minimum - Lower bound for the parameter value. When you use p in state estimation or input value estimation, the estimated value of the parameter does not drop below p.Minimum.

The dimensions of $p$.Minimum must match the dimensions of $p$.Value.
For array-valued parameters, you can:

- Specify lower bounds on individual array elements. For example, p.Minimum([14]) =-5.
- Use scalar expansion to set the lower bound for all array elements. For example, p.Minimum = -5

Default: - Inf

- Maximum - Upper bound for the parameter value. When you use p in state estimation or input value estimation, the estimated value of the parameter does not exceed p.Maximum.

The dimensions of $p$. Maximum must match the dimensions of $p$.Value.
For array-valued parameters, you can:

- Specify upper bounds on individual array elements. For example, p.Maximum ([14]) $=5$.
- Use scalar expansion to set the upper bound for all array elements. For example, p. Maximum $=5$

Default: Inf

- Free - Boolean specifying whether the parameter is a free estimation variable.

The dimensions of $p$. Free must match the dimensions of $p$.Value. By default, all values are free (p.Free = true).

If you want to estimate p . Value(k), set p.Free(k) = true. To fix p.Value(k), set p.Free (k) = false. Doing so allows you to control which states or input values are estimated and which are not.

For array-valued parameters, you can:

- Fix individual array elements. For example, p.Free([ 14 4]) = false;
p.Free = [1 0; 0 1].
- Use scalar expansion to fix all array elements. For example, p.Free $=$ false.

Default: true (1)

- Scale - Scaling factor for normalizing the parameter value.
p. Scale is not used in initial state estimation or input value estimation.


## Default: 1

- Info - Structure array for storing parameter units and labels. The structure has Label and Unit fields.

Use these fields for your convenience, to store parameter units and labels. For example, p.Info(1,1).Unit = 'rad/m'; p.Info(1,1).Label = 'engine speed'.

The dimensions of $p$. Info must match the dimensions of $p$. Value.
Default: ' ' for both Label and Unit fields

- Name - Parameter name.

This property is read-only. It is set to the paramname input argument when you create the parameter.

Default: ' '

## Examples

## Create and Configure Parameter for State Estimation

Create and configure a parameter for estimating the initial state values of a 4 -state system. Fix the first state value to 1 . Limit the second and third states to values between 0 and 1.

```
paramvalue = [1; nan(3,1)];
p = idpar('x0',paramvalue);
p.Free(1) = 0;
p.Minimum([2 3]) = 0;
p.Maximum([2 3]) = 1;
```

The column vector paramvalue specifies an initial value of 1 for the first state. paramvalue further specifies unknown values for the remaining 3 states.

Setting p. Free(1) to false fixes p.Value(1) to 1 . Estimation using $p$ does not alter that value.
Setting p.Minimum and p.Maximum for the second and third entries in plimits the range that those values can take when $p$ is used in estimation.

You can now use $p$ in initial state estimation, such as with the findstates command. For example, use opt $=$ findstatesOptions('InitialState', p) to create a findstates options set that uses $p$. Then, call findstates with that options set.

## Tips

Use idpar to create estimable parameters for:

- Initial state estimation for state-space model estimation (ssest), prediction (predict), and forecasting (forecast)
- Explicit initial state estimation with findstates
- Input level estimation for process model estimation with pem

Specifying estimable state values or input levels gives you explicit control over the behavior of individual state values during estimation.

## See Also

findstates|findstatesOptions|forecast|pem|predict|ssest

## Introduced in R2012a

## idpoly

Polynomial model with identifiable parameters

## Description

An idpoly model represents a system as a continuous-time or discrete-time polynomial model with identifiable (estimable) coefficients. Use idpoly to create a polynomial model or to convert "Dynamic System Models" to polynomial form.

A polynomial model of a system with input vector $u$, output vector $y$, and disturbance $e$ takes the following form in discrete time:

$$
A(q) y(t)=\frac{B(q)}{F(q)} u(t)+\frac{C(q)}{D(q)} e(t)
$$

The variables $A, B, C, D$, and $F$ are polynomials expressed with the time-shift operator $q^{-1}$. For instance, the $A$ polynomial takes this form:

$$
A(q)=1+a_{1} q^{-1}+a_{2} q^{-2}+\cdots+a_{n a} q^{-n a}
$$

Here, na is the order of the $A$ polynomial. $q^{-1} y(t)$ is equivalent to $y(t-1)$.
For example, if $A(q)=1+a_{1} q^{-1}+a_{2} q^{-2}$, then $A(y(t))=1+a_{1}(t-1)+a_{2}(t-2)$.
The $C, D$, and $F$ polynomials take the same form as the $A$ polynomial, starting with 1 . The $B$ polynomial does not start with 1 .

In continuous time, a polynomial model takes the following form:

$$
A(s) Y(s)=\frac{B(s)}{F(s)} U(s)+\frac{C(s)}{D(s)} E(s)
$$

$U(s)$ contains the Laplace transformed inputs to sys. $Y(s)$ contains the Laplace transformed outputs. $E(s)$ contains the Laplace transform of the disturbances for each output.

For idpoly models, the coefficients of the polynomials $A, B, C, D$, and $F$ can be estimable parameters. The idpoly model stores the values of these matrix elements in the A, B, C, D, and F properties of the model.

Time-series models are special cases of polynomial models for systems without measured inputs. For AR models, B and F are empty, and C and D are 1 for all outputs. For ARMA models, B and F are empty, while D is 1 .

Although idpoly supports continuous-time models, idtf and idproc enable more choices for estimation of continuous-time models. Therefore, for most continuous-time applications, these other model types are preferable.

For more information about polynomial models, see "What Are Polynomial Models?"

## Creation

You can obtain an idpoly model in one of three ways.

- Estimate the idpoly model based on output or input-output measurements of a system by using commands such as polyest, arx, armax, oe, bj, iv4, or ivar. These commands estimate the values of the free polynomial coefficients. The estimated values are stored in the A, B, C, D, and F properties of the resulting idpoly model. The Report property of the resulting model stores information about the estimation, such as information on the handling of initial conditions and options used in estimation.

When you obtain an idpoly model by estimation, you can extract estimated coefficients and their uncertainties from the model using commands such as polydata, getpar, or getcov.

- Create an idpoly model using the idpoly command. You can create an idpoly model to configure an initial parameterization for estimation of a polynomial model to fit measured response data. When you do so, you can specify constraints on the polynomial coefficients. For example, you can fix the values of some coefficients, or specify minimum or maximum values for the free coefficients. You can then use the configured model as an input argument to polyest to estimate parameter values with those constraints.
- Convert an existing dynamic system model to an idpoly model using the idpoly command.


## Syntax

```
sys = idpoly(A,B,C,D,F,NoiseVariance,Ts)
sys = idpoly(A,B,C,D,F,NoiseVariance,Ts,Name,Value)
sys = idpoly(A)
sys = idpoly(A,[],C,D,[],NoiseVariance,Ts)
sys = idpoly(A,[],C,D,[],NoiseVariance,Ts,Name,Value)
sys = idpoly(sys0)
sys = idpoly(sys0,'split')
```


## Description

## Create Input-Output Polynomial Model

sys = idpoly (A, B, C, D, F,NoiseVariance,Ts) creates a polynomial model with identifiable coefficients. A, B, C, D, and F specify the initial values of the coefficients. NoiseVariance specifies the initial value of the variance of the white noise source. Ts is the model sample time.
sys $=$ idpoly(A,B,C,D,F,NoiseVariance,Ts,Name, Value) creates a polynomial model using additional options specified by one or more name-value pair arguments.

## Create Time-Series Model

sys = idpoly (A) creates a time-series model with only an autoregressive term. In this case, sys represents the AR model given by $A(q) y(t)=e(t)$. The noise $e(t)$ has variance 1 . A specifies the initial values of the estimable coefficients.
sys = idpoly(A,[], C, D, [],NoiseVariance,Ts) creates a time-series model with an autoregressive and a moving average term. The inputs $A, C$, and $D$, specify the initial values of the estimable coefficients. NoiseVariance specifies the initial value of the noise $e(t)$. Ts is the model sample time. (Omit NoiseVariance and Ts to use their default values.)

If $D$ is set to [ ], then sys represents the ARMA model given by

$$
A(q) y(t)=C(q) e(t)
$$

sys = idpoly(A,[],C,D,[],NoiseVariance,Ts,Name, Value) creates a time-series model using additional options specified by one or more name-value pair arguments.

## Convert Dynamic System Model to Polynomial Model

sys = idpoly(sys0) converts the dynamic system model sys0 to idpoly model form. sys0 can be any dynamic system model.
sys = idpoly(sys0,'split') converts sys0 to idpoly model form, and treats the last $N_{y}$ input channels of sys0 as noise channels in the returned model. sys0 must be a numeric tf , zpk , or ss model object. Also, sys0 must have at least as many inputs as outputs.

## Input Arguments

## sys0 - Dynamic system

dynamic system model
Dynamic system, specified as a dynamic system model to convert to an idpoly model.
When sys0 is an identified model, its estimated parameter covariance is lost during conversion. If you want to translate the estimated parameter covariance during the conversion, use translatecov.

For the syntax sys = idpoly(sys0,'split'), sys0 must meet the following requirements.

- sys0 is a numeric $t f, z p k$, or ss model object.
- sys0 has at least as many inputs as outputs.
- The subsystem sys0(:, Ny+1:Nu) must be biproper.


## Properties

## A, B, C, D, F - Values of polynomial coefficients

[ ] | 1 | row vectors | array of row vectors
Values of the polynomial coefficients, specified as row vectors for SISO models or, for MIMO models, cell arrays of row vectors that correspond to each of the A, B, C, D, and F polynomials. For each polynomial, the coefficients are stored in the following order:

- Ascending powers of $z^{-1}$ or $q^{-1}$ (for discrete-time polynomial models).
- Descending powers of $s$ or $p$ (for continuous-time polynomial models).

The leading coefficients of A, C, D, and F are always 1 .
For MIMO models with $N_{y}$ outputs and $N_{u}$ inputs, A, B, C, D, and F are cell arrays of row vectors. Each entry in the cell array contains the coefficients of a particular polynomial that relates input, output, and noise values.

| Polynomial | Dimension and Constraints | Relation Described |
| :---: | :---: | :---: |
| A | $N_{y}$-by- $N_{y}$ cell array of row vectors <br> Leading coefficients: <br> - Diagonal entries - Fixed to 1 <br> - Off-diagonal entries - Fixed to 0 | A\{i,j\} contains coefficients that relate the output $y_{i}$ to the output $y_{j}$. <br> For example, for a two-output system, A is a 2-by-2 cell array, such as: $\left.\begin{array}{l} \mathrm{A}\{1,1\}=\left[\begin{array}{ll} 1 & .1 \end{array}\right] \\ \mathrm{A}\{1,2\}=[0.4 \quad-0.6 \end{array}\right]$ |
| B, F | $N_{y}$-by- $N_{u}$ array of row vectors <br> Leading coefficients: <br> - B - Not constrained <br> - F - Fixed to 1 | $B\{i, j\}$ and $F\{i, j\}$ contain coefficients that relate the output $y_{i}$ to the input $u_{j}$. <br> For example, for a two-output system, B and $F$ are 2-by-1 cell arrays, such as: $\begin{aligned} & \mathrm{B}\{1,1\}=\left[\begin{array}{ll} 0.1 & 0.2 \end{array}\right] \\ & \mathrm{B}\{2,1\}=\left[\begin{array}{ll} 0.5 & 0.3 \end{array}\right] \\ & \mathrm{F}\{1,1\}=\left[\begin{array}{ll} 1 & 0.8 \end{array}\right] \\ & \mathrm{F}\{2,1\}=\left[\begin{array}{ll} 1 & 0.4 \end{array}\right] \end{aligned}$ |
| C, D | $N_{y}$-by-1 array of row vectors <br> Leading coefficients: <br> - C - Fixed to 1 <br> - D - Fixed to 1 | $C\{i\}$ and $D\{i\}$ contain coefficients that relate the output $y_{i}$ to the noise $e_{i}$. <br> For example, for a two-output system, C and $D$ are 2-by- 1 cell arrays, such as: $\begin{aligned} & C\{1,1\}=\left[\begin{array}{ll} 1 & 0.3 \end{array}\right] \\ & C\{2,1\}=\left[\begin{array}{lll} 1 & 0.5 & 0.3 \end{array}\right] \\ & D\{1,1\}=\left[\begin{array}{lll} 1 & 0.7 \end{array}\right] \\ & D\{2,1\}=\left[\begin{array}{lll} 1 & 0.1 & 0.2 \end{array}\right] \end{aligned}$ |

For a time series model (a model with no measured inputs), $\mathrm{B}=[\mathrm{]}$ and $\mathrm{F}=[\mathrm{]}$.
If you obtain an idpoly model sys by identification using a function such as polyest or arx, then sys.A, sys.B, sys.C, sys.D, and sys.F contain the estimated values of the polynomial coefficients.

If you create an idpoly model sys using the idpoly command, sys.A, sys.B, sys.C, sys.D, and sys. $F$ contain the initial coefficient values that you specify with the $A, B, C, D, F$ input arguments. Use NaN for any coefficient whose initial value is not known. Use [] for any polynomial that is not present in the model structure that you want to create. For example, to create an ARX model, use [] for C, D, and F. For an ARMA time series model, use [ ] for B and F. Default initial values when you create an idpoly model are:

- B-[]
- C -1 for all outputs
- D - 1 for all outputs
- $\mathrm{F}-[]$

For an idpoly model sys, each property sys.A, sys.B, sys.C, sys.D, and sys.F is an alias of the corresponding Value entry in the Structure property of sys. For example, sys.A is an alias of the value of the property sys.Structure.A.Value.

## Variable - Polynomial model display variable

's' (default) |' $\left.\mathrm{p}^{\prime}\right|^{\prime} z^{\wedge}-1$ ' | ' $q^{\wedge}$ - 1'
Polynomial model display variable, specified as one of the following values:

- ' $z^{\wedge}-1$ ' - Default for discrete-time models
- ' $q^{\wedge}-1$ ' - Equivalent to ${ }^{\prime} z^{\wedge}-1$ '
- 's' - Default for continuous-time models
- 'p' - Equivalent to 's'

The value of Variable is reflected in the display, and also affects the interpretation of the $A, B, C, D$, and $F$ coefficient vectors for discrete-time models. When Variable is set to ' $z^{\wedge}-1$ ' or ' $q^{\wedge}-1$ ', the coefficient vectors are ordered as ascending powers of the variable.

## IODelay - Transport delays

0 (default) | scalar | numeric array
Transport delays, specified as a numeric array containing a separate transport delay for each inputoutput pair or as a scalar that applies the same delay to each input-output pair.

For continuous-time systems, transport delays are expressed in the time unit stored in the TimeUnit property. For discrete-time systems, transport delays are expressed as integers denoting a delay of a multiple of the sample time Ts.

For a MIMO system with $N_{y}$ outputs and $N_{u}$ inputs, IODelay is an $N_{y}$-by- $N_{u}$ array. Each entry of this array is a numerical value representing the transport delay for the corresponding input-output pair. You can set IODelay to a scalar value to apply the same delay to all input-output pairs.

If you create an idpoly model sys using the idpoly command, sys.IODelay contains the initial values of the transport delay that you specify with a name-value pair argument.

If you obtain an idpoly model sys by identification using a function such as polyest or arx, then sys.IODelay contains the estimated values of the transport delay.

For an idpoly model sys, the property sys.IODelay is an alias for the value of the property sys.Structure.IODelay.Value.

## IntegrateNoise - Presence of integration on noise channels

logical vector of zeros (default) | logical vector
Logical vector denoting the presence or absence of integration on noise channels, specified as a logical vector with length equal to the number of outputs.

IntegrateNoise(i) = true indicates that the noise channel for the ith output contains an integrator. In this case, the corresponding $D$ polynomial contains an additional term that is not represented in the property sys.D. This integrator term is equal to $1 / s$ for continuous-time systems and $1 /\left(1-z^{-1}\right)$ for discrete-time systems.

## Structure - Information about the estimable parameters

structure property values
Property-specific information about the estimable parameters of the idpoly model, specified as a structure.

For a system with $N_{y}$ outputs and $N_{u}$ inputs, the dimensions of the Structure elements are as follows:

- sys.Structure. $\mathrm{A}-N_{y}$-by- $N_{y}$
- sys.Structure.B $-N_{y}$-by- $N_{u}$
- sys.Structure.C $-N_{y}$-by-1
- sys.Structure.D $-N_{y}$-by-1
- sys.Structure.F $-N_{y}$-by- $N_{u}$
sys.Structure.A, sys.Structure.B, sys.Structure.C, sys.Structure.D, and
sys. Structure. F contain information about the polynomial coefficients.
sys. Structure.IODelay contains information about the transport delay.
sys. Structure. IntegrateNoise contains information about the integration terms on the noise. Each parameter in Structure contains the following fields.

| Field | Description | Examples |
| :---: | :---: | :---: |
| Value | Parameter values. Each property is an alias of the corresponding Value entry in the Structure property of sys. NaN represents unknown parameter values. | sys.Structure.A.Value contains the initial or estimated values of the SISO A polynomial. sys. $A$ is an alias of the value of this property. sys. $A\{i, j\}$ is the alias of the MIMO property sys.Structure.A(i, j).Val ue. |
| Minimum | Minimum value that the parameter can assume during estimation | sys.Structure.IODelay.Mi nimum = 0.1 constrains the transport delay to values greater than or equal to 0.1. sys.Structure.IODelay.Mi nimum must be greater than or equal to zero. |
| Maximum | Maximum value that the parameter can assume during estimation |  |
| Free | Boolean specifying whether the parameter is a free estimation variable. If you want to fix the value of a parameter during estimation, set the corresponding Free to false. For fixed values, such as the leading coefficients of the values of $A$ polynomial, which are always equal to 1 , the corresponding value of Free is always false. | If $B$ is a 3 -by- 3 matrix,sys.Structure.B.Fre $e=$ eye(3) fixes all of the offdiagonal entries in $B$ to the values specified in sys.Structure.B.Value. In this case, only the diagonal entries in $B$ are estimable. |
| Scale | Scale of the value of the parameter. The estimation algorithm does not use Scale. |  |


| Field | Description | Examples |
| :--- | :--- | :--- |
| Info | Structure array that contains <br> the fields Label and Unit for <br> storing parameter labels and <br> units. Specify parameter labels <br> and units as character vectors. | Example: 'Time' |

An inactive polynomial, such as the B polynomial in a time series model, is not available as a parameter in the Structure property. For example, sys $=$ idpoly ([1-0.2 0.5]) creates an AR model. sys.Structure contains the fields sys.Structure.A and
sys.Structure. IntegrateNoise. However, there is no field in Structure corresponding to B, C, D, F, or IODelay.

## NoiseVariance - Variance of model innovations

positive scalar | matrix
Variance (covariance matrix) of the model innovations $e$, specified as a scalar or a positive semidefinite matrix.

- SISO model - Scalar
- MIMO model with $N_{y}$ outputs $-N_{y}$-by- $N_{y}$ positive semidefinite matrix

An identified model includes a white Gaussian noise component $e(t)$. NoiseVariance is the variance of this noise component. Typically, the model estimation function (such as polyest) determines this variance.

## Report - Summary report

report field values
This property is read-only.
Summary report that contains information about the estimation options and results for a state-space model obtained using estimation commands, such as polyest, armax, oe, and bj. Use Report to find estimation information for the identified model, including:

- Estimation method
- Estimation options
- Search termination conditions
- Estimation data fit and other quality metrics

If you create the model by construction, the contents of Report are irrelevant.

```
m = idpoly({[1 0.5]},{[1 5]},{[1 0.01]});
m.Report.OptionsUsed
ans =
    []
```

If you obtain the model using estimation commands, the fields of Report contain information on the estimation data, options, and results.

```
load iddata2 z2;
m = polyest(z2,[2 2 3 3 2 1]);
m.Report.OptionsUsed
Option set for the polyest command:
    InitialCondition: 'auto'
            Focus: 'prediction'
    EstimateCovariance: 1
            Display: 'off'
            InputOffset: []
        OutputOffset: []
        Regularization: [1x1 struct]
            SearchMethod: 'auto'
        SearchOptions: [1x1 idoptions.search.identsolver]
            Advanced: [1x1 struct]
```

For more information on this property and how to use it, see the Output Arguments section of the corresponding estimation command reference page and "Estimation Report".

## InputDelay - Input delay for each input channel

## 0 (default) | scalar | vector

Input delay for each input channel, specified as a scalar value or numeric vector. For continuous-time systems, specify input delays in the time unit stored in the TimeUnit property. For discrete-time systems, specify input delays in integer multiples of the sample time Ts. For example, setting InputDelay to 3 specifies a delay of three sample times.

For a system with $N_{u}$ inputs, set InputDelay to an $N_{u}$-by-1 vector. Each entry of this vector is a numerical value that represents the input delay for the corresponding input channel.

You can also set InputDelay to a scalar value to apply the same delay to all channels.
In estimation, InputDelay is a fixed constant of the model. The software uses the IODelay property for estimating time delays. To specify initial values and constraints for estimation of time delays, use sys.Structure.IODelay.

## OutputDelay - Output delay for each output channel <br> 0 (default)

This property is read-only.
Output delay for each output channel, specified as 0 . This value is fixed for identified systems such as idpoly.

## Ts - Sample Time

- 1 (default) | 0 | positive scalar

Sample time, specified as one of the following.

- Discrete-time model with an unspecified sample time - -1
- Continuous-time model - 0
- Discrete-time model with a specified sampling time - Positive scalar representing the sampling period expressed in the unit specified by the TimeUnit property of the model

Changing this property does not discretize or resample the model. Use c2d and d2c to convert between continuous- and discrete-time representations. Use d2d to change the sample time of a discrete-time system.

## TimeUnit - Units for time variable

'seconds' (default)|'nanoseconds'|'microseconds' |'milliseconds'|'minutes'|
Units for the time variable, the sample time Ts, and any time delays in the model, specified as a scalar.

Changing this property does not resample or convert the data. Modifying the property changes only the interpretation of the existing data. Use chgTimeUnit to convert data to different time units

## InputName - Input channel names

' ' (default) | character vector | cell array
Input channel names, specified as a character vector or cell array.

- Single-input model - Character vector, for example, ' controls '
- Multi-input model - Cell array of character vectors

Alternatively, use automatic vector expansion to assign input names for multi-input models. For example, if sys is a two-input model, enter the following:
sys.InputName = 'controls';
The input names automatically expand to \{'controls(1)';'controls(2)'\}.
When you estimate a model using an iddata object data, the software automatically sets InputName to data. InputName.

You can use the shorthand notation $u$ to refer to the InputName property. For example, sys. $u$ is equivalent to sys.InputName.

You can use input channel names in several ways, including:

- To identify channels on model display and plots.
- To extract subsystems of MIMO systems.
- To specify connection points when interconnecting models.


## InputUnit - Input channel units

' ' (default) | character vector | cell array
Input channel units, specified as a character vector or cell array:

- Single-input model - Character vector
- Multi-input Model - Cell array of character vectors

Use InputUnit to keep track of input signal units. InputUnit has no effect on system behavior.

## InputGroup - Input channel groups

structure with no fields (default) | structure

Input channel groups, specified as a structure. The InputGroup property lets you divide the input channels of MIMO systems into groups so that you can refer to each group by name. In the InputGroup structure, set field names to the group names, and field values to the input channels belonging to each group.

For example, create input groups named controls and noise that include input channels 1,2 and 3,5 , respectively.
sys.InputGroup.controls = [1 2];
sys.InputGroup.noise = [3 5];
You can then extract the subsystem from the controls inputs to all outputs using the following syntax:
sys(:,'controls')

## OutputName - Output channel names

' ' (default) | character vector | cell array
Output channel names, specified as a character vector or cell array.

- Single-input model - Character vector, for example, ' measurements '
- Multi-input model - Cell array of character vectors

Alternatively, use automatic vector expansion to assign output names for multi-output models. For example, if sys is a two-output model, enter the following:
sys.OutputName = 'measurements';
The output names automatically expand to \{'measurements(1)';'measurements(2)'\}.
When you estimate a model using an iddata object data, the software automatically sets OutputName to data.OutputName.

You can use the shorthand notation $y$ to refer to the OutputName property. For example, sys.y is equivalent to sys.OutputName.

You can use output channel names in several ways, including:

- To identify channels on model display and plots.
- To extract subsystems of MIMO systems.
- To specify connection points when interconnecting models.


## OutputUnit - Output channel units

' ' (default) | character vector | cell array
Output channel units, specified as a character vector or cell array.

- Single-input model - Character vector, for example, ' seconds '
- Multi-input Model - Cell array of character vectors

Use OutputUnit to keep track of output signal units. OutputUnit has no effect on system behavior.

## OutputGroup - Output channel groups

struct with no fields (default) | struct

Output channel groups, specified as a structure. The OutputGroup property lets you divide the output channels of MIMO systems into groups and refer to each group by name. In the 0utputGroup structure, set field names to the group names, and field values to the output channels belonging to each group.

For example, create output groups named temperature and measurement that include output channels 1 , and 3,5 , respectively.

```
sys.OutputGroup.temperature = [1];
sys.OutputGroup.measurement = [3 5];
```

You can then extract the subsystem from all inputs to the measurement outputs using the following syntax:
sys('measurement',:)

## Name - System name

' ' (default)| character vector
System name, specified as a character vector, for example, 'system_1'.

## Notes - Text to associate with system

0 -by-1 string (default) | string | string array | character vector
Any text that you want to associate with the system, specified as a string.

- For a single note, specify Notes as a string or a character vector
- For multiple notes, specify Notes as a string array.

The property preserves the string or character data type that you specify. When you specify a character vector, the software packages the character vector in a 1-by-1 cell array.

For example, if sys1, sys2, and sys3 are dynamic system models, you can set their Notes properties as follows.

```
sys1.Notes = "sys1 has a string.";
sys2.Notes = ["sys2 has a first string";"sys3 has a second string"];
sys3.Notes = 'sys3 has a character vector.';
sys1.Notes
sys2.Notes
sys3.Notes
ans =
    "sys1 has a string."
ans =
    2x1 string array
        "sys2 has a first string"
        "sys2 has a second string"
ans =
    1\times1 cell array
```

\{'sys3 has a character vector'\}

## UserData - Data to associate with system

[ ] (default) | any MATLAB data type
Data to associate with the system, specified as any MATLAB data type.

## SamplingGrid - Sampling grid

[] (default) | structure
Sampling grid for model arrays, specified as a structure.
For arrays of identified linear (IDLTI) models that you derive by sampling one or more independent variables, this property tracks the variable values associated with each model. This information appears when you display or plot the model array. Use this information to trace results back to the independent variables.

Set the field names of the data structure to the names of the sampling variables. Set the field values to the sampled variable values associated with each model in the array. All sampling variables must be numeric and scalar valued, and all arrays of sampled values must match the dimensions of the model array.

For example, suppose that you collect data at various operating points of a system. You can identify a model for each operating point separately and then stack the results together into a single system array. You can tag the individual models in the array with information regarding the operating point.

```
nominal_engine_rpm = [1000 5000 10000];
sys.SamplingGr\overline{id}= struct('rpm', nominal_engine_rpm)
```

Here, sys is an array containing three identified models obtained at 1000, 5000, and 10000 rpm , respectively.

For model arrays that you generate by linearizing a Simulink model at multiple parameter values or operating points, the software populates SamplingGrid automatically with the variable values that correspond to each entry in the array.

## Object Functions

In general, any function applicable to "Dynamic System Models" is applicable to an idpoly model object. These functions are of four general types.

- Functions that operate and return idpoly model objects enable you to transform and manipulate idpoly models. For instance:
- Use merge to merge estimated idpoly models.
- Use c2d to convert an idpoly model from continuous to discrete time. Use d2c to convert an idpoly model from discrete to continuous time.
- Functions that perform analytical and simulation functions on idpoly models, such as bode and sim
- Functions that retrieve or interpret model information, such as advice and getpar
- Functions that convert idpoly models into a different model type, such as idtf for time domain or idfrd for frequency domain

The following lists contain a representative subset of the functions that you can use with idpoly models.

## Transformation and Manipulation

| translatecov | Translate parameter covariance across model transformation operations |
| :--- | :--- |
| setpar | Set attributes such as values and bounds of linear model parameters |
| chgTimeUnit | Change time units of dynamic system |
| d2d | Resample discrete-time model |
| d2c | Convert model from discrete to continuous time |
| c2d | Convert model from continuous to discrete time |
| merge | Merge estimated models |

## Analysis and Simulation

sim Simulate response of identified model
predict Predict state and state estimation error covariance at next time step using extended or unscented Kalman filter, or particle filter
compare Compare identified model output and measured output
impulse Impulse response plot of dynamic system; impulse response data
step Step response plot of dynamic system; step response data
bode Bode plot of frequency response, or magnitude and phase data

## Information Extraction and Interpretation

tfdata Access transfer function data
get Access model property values
getpar Obtain attributes such as values and bounds of linear model parameters
getcov Parameter covariance of identified model
advice Analysis and recommendations for data or estimated linear models

## Conversion to Other Model Structures

idtf Transfer function model with identifiable parameters
idss State-space model with identifiable parameters
idfrd Frequency response data or model

## Examples

## Create Polynomial Model

Create an idpoly model representing the single-input, single-output ARMAX model described by the following equation:

$$
y(t)+0.5 y=u(t)+5 u(t-1)+2 u(t-2)+e(t)+0.01 e(t-1)
$$

$y$ is the output, $u$ is the input, and $e$ is the white-noise disturbance on $y$.
To create the idpoly model, define the $A, B$, and $C$ polynomials that describe the relationships between the output, input, and noise values, respectively. Because there are no denominator terms in the system equation, D and F are 1.

```
A = [1 0.5];
B = [1 5 2];
C = [1 0.01];
```

Create an idpoly model with the specified coefficients.

```
sys = idpoly(A,B,C)
sys =
Discrete-time ARMAX model: A(z)y(t) = B(z)u(t) + C(z)e(t)
    A(z) = 1 + 0.5 z^-1
    B(z) = 1 + 5 z^-1 + 2 z^-2
    C(z) = 1 + 0.01 z^-1
Sample time: unspecified
Parameterization:
    Polynomial orders: na=1 nb=3 nc=1 nk=0
    Number of free coefficients: 5
    Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.
Status:
Created by direct construction or transformation. Not estimated.
```

The display shows all the polynomials and allows you to verify them. The display also states that there are five free coefficients.

Create an idpoly model with specified noise variance nv and sample time Ts. To do so, you must also include values of 1 for $D$ and $F$.

```
Ts = 0.1;
nv = 0.01;
sys = idpoly(A,B,C,1,1,nv,Ts)
sys =
Discrete-time ARMAX model: A(z)y(t) = B(z)u(t) + C(z)e(t)
    A(z) = 1 + 0.5 z^-1
    B(z) = 1 + 5 z^-1 + 2 z^-2
    C(z) = 1 + 0.01 z^-1
Sample time: 0.1 seconds
Parameterization:
        Polynomial orders: na=1 nb=3 nc=1 nk=0
        Number of free coefficients: 5
        Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.
Status:
Created by direct construction or transformation. Not estimated.
```

The display shows a sample time of 0.1 seconds.
Specify an input-output delay iod of one sample when you create an idpoly model.

```
iod = 1;
sys = idpoly(A,B,C,1,1,nv,Ts,'IODelay',1)
sys =
Discrete-time ARMAX model: A(z)y(t) = B(z)u(t) + C(z)e(t)
    A(z) = 1 + 0.5 z^-1
    B(z) = 1 + 5 z^-1 + 2 z^-2
    C(z) = 1 + 0.01 z^-1
Input delays (listed by channel): 1
Sample time: 0.1 seconds
Parameterization:
    Polynomial orders: na=1 nb=3 nc=1 nk=0
    Number of free coefficients: 5
    Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.
Status:
Created by direct construction or transformation. Not estimated.
```

The display shows an input delay of one sample.
You can use sys to specify an initial parameterization for estimation with commands such as polyest or armax.

## Create Polynomial Time-Series Model

Create an idpoly model representing the single-output ARMA model described by the following equation:

$$
y(t)+0.5 y=e(t)+0.01 e(t-1)
$$

Because a time series has no measured inputs, this model contains only A and C polynomials.

```
A = [1 0.5];
C = [1 0.01];
```

Create a discrete-time time-series model without specifying a sample time.

```
sys = idpoly(A,[],C)
sys =
Discrete-time ARMA model: A(z)y(t) = C(z)e(t)
    A(z) = 1 + 0.5 z^-1
    C(z) = 1 + 0.01 z^-1
Sample time: unspecified
Parameterization:
    Polynomial orders: na=1 nc=1
    Number of free coefficients: 2
    Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.
```

Status:
Created by direct construction or transformation. Not estimated.
The display reflects your specifications.
Create a continuous-time time-series by specifying a sample time of 0 for the name-value pair argument 'Ts'.

```
sys = idpoly(A,[],C,'Ts',0)
sys =
Continuous-time ARMA model: A(s)y(t) = C(s)e(t)
    A(s) = s + 0.5
    C(s) = s + 0.01
Parameterization:
    Polynomial orders: na=1 nc=1
    Number of free coefficients: 2
    Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.
Status:
Created by direct construction or transformation. Not estimated.
```

You can also set the sample time using the Ts input argument rather than the name-value pair argument, but the syntax is more complex. You must specify the $D$ value as 1 or empty, and set both the $F$ position and the noise variance position (if you are not specifying noise variance) to empty.

```
Ts = 0;
sys = idpoly(A,[],C,1,[],[],Ts)
sys =
Continuous-time ARMA model: A(s)y(t) = C(s)e(t)
    A(s) = s + 0.5
    C(s) = s + 0.01
Parameterization:
    Polynomial orders: na=1 nc=1
    Number of free coefficients: 2
    Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.
Status:
Created by direct construction or transformation. Not estimated.
```


## Multi-Output ARMAX Model

Create an idpoly model representing the one-input, two-output ARMAX model described by the following equations:

$$
\begin{aligned}
& y_{1}(t)+0.5 y_{1}(t-1)+0.9 y_{2}(t-1)+0.1 y_{2}(t-2)= \\
& u(t)+5 u(t-1)+2 u(t-2)+e_{1}(t)+0.01 e_{1}(t-1) \\
& y_{2}(t)+0.05 y_{2}(t-1)+0.3 y_{2}(t-2)= \\
& 10 u(t-2)+e_{2}(t)+0.1 e_{2}(t-1)+0.02 e_{2}(t-2) .
\end{aligned}
$$

$y_{1}$ and $y_{2}$ are the two outputs, and $u$ is the input. $e_{1}$ and $e_{2}$ are the white noise disturbances on the outputs $y_{1}$ and $y_{2}$, respectively.

To create the idpoly model, define the $A, B$, and $C$ polynomials that describe the relationships between the outputs, inputs, and noise values. (Because there are no denominator terms in the system equations, D and F are 1.)

Define the cell array containing the coefficients of the A polynomials.
A $=\operatorname{cell}(2,2)$;
$\mathrm{A}\{1,1\}=\left[\begin{array}{ll}10.5] ;\end{array}\right.$
$\mathrm{A}\{1,2\}=\left[\begin{array}{lll}0 & 0.9 & 0.1\end{array}\right] ;$
$\mathrm{A}\{2,1\}=[0] ;$
$A\{2,2\}=\left[\begin{array}{lll}1 & 0.05 & 0.3\end{array}\right] ;$
You can read the values of each entry in the A cell array from the left side of the equations describing the system. For example, $A\{1,1\}$ describes the polynomial that gives the dependence of $y_{1}$ on itself. This polynomial is $A_{11}=1+0.5 q^{-1}$, because each factor of $q^{-1}$ corresponds to a unit time decrement. Therefore, $\mathrm{A}\{1,1\}=[10.5]$, giving the coefficients of $A_{11}$ in increasing exponents of $q^{-1}$.

Similarly, $\mathrm{A}\{1,2\}$ describes the polynomial that gives the dependence of $y_{1}$ on $y_{2}$. From the equations, $A_{12}=0+0.9 q^{-1}+0.1 q^{-2}$. Thus, $\mathrm{A}\{1,2\}=\left[\begin{array}{lll}0 & 0.9 & 0.1\end{array}\right]$.

The remaining entries in A are similarly constructed.
Define the cell array containing the coefficients of the B polynomials.

```
B = cell(2,1);
B{1,1} = [1 5 2];
B{2,1} = [0 0 10];
```

B describes the polynomials that give the dependence of the outputs $y_{1}$ and $y_{2}$ on the input $u$. From the equations, $B_{11}=1+5 q^{-1}+2 q^{-2}$. Therefore, $\mathrm{B}\{1,1\}=\left[\begin{array}{lll}1 & 5 & 2\end{array}\right]$.

Similarly, from the equations, $B_{21}=0+0 q^{-1}+10 q^{-2}$. Therefore, $B\left\{\begin{array}{l}2,1\end{array}\right\}=\left[\begin{array}{lll}0 & 0 & 10\end{array}\right]$.
Define the cell array containing the coefficients of the C polynomials.

```
C = cell(2,1);
C{1,1} = [1 0.01];
C{2,1} = [1 0.1 0.02];
```

$C$ describes the polynomials that give the dependence of the outputs $y_{1}$ and $y_{2}$ on the noise terms $e_{1}$ and $e_{2}$. The entries of $C$ can be read from the equations similarly to those of $A$ and $B$.

Create an idpoly model with the specified coefficients.

```
sys = idpoly(A,B,C)
sys =
Discrete-time ARMAX model:
    Model for output number 1: A(z)y_1(t) = - A_i(z)y_i(t) + B(z)u(t) + C(z)e_1(t)
        A(z) = 1 + 0.5 z^-1
```

```
    A_2(z) = 0.9 z^-1 + 0.1 z^-2
    B(z) = 1 + 5 z^-1 + 2 z^-2
    C(z) = 1 + 0.01 z^-1
Model for output number 2: A(z)y_2(t) = B(z)u(t) + C(z)e_2(t)
    A(z) = 1 + 0.05 z^-1 + 0.3 z^-2
    B(z) = 10 z^-2
    C(z) = 1 + 0.1 z^-1 + 0.02 z^-2
Sample time: unspecified
Parameterization:
    Polynomial orders: na=[1 2;0 2] nb=[3;1] nc=[1;2]
    nk=[0;2]
    Number of free coefficients: 12
    Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.
Status:
Created by direct construction or transformation. Not estimated.
```

The display shows all the polynomials and allows you to verify them. The display also states that there are 12 free coefficients. Leading terms of diagonal entries in $A$ are always fixed to 1 . Leading terms of all other entries in A are always fixed to 0 .

You can use sys to specify an initial parameterization for estimation with commands such as polyest or armax.

## Convert Transfer Function Model into Polynomial Model

Model a dynamic system using a transfer function. Then use idpoly to convert the transfer-function model into polynomial form.

Using idtf, construct the continuous-time, single-input, single-output (SISO) transfer function model described by the following equation:

$$
G(s)=\frac{s+4}{s^{2}+20 s+5}
$$

num $=$ [14];
den = [1 20 5];
G = idtf(num,den)
G =

```
        s + 4
    * - - - - - 
    s^2 + 20 s + 5
Continuous-time identified transfer function.
```

```
Parameterization:
    Number of poles: 2 Number of zeros: 1
    Number of free coefficients: 4
    Use "tfdata", "getpvec", "getcov" for parameters and their uncertainties.
Status:
Created by direct construction or transformation. Not estimated.
Convert the transfer function into polynomial form.
```

```
sys = idpoly(G)
```

sys = idpoly(G)
sys =
sys =
Continuous-time OE model: y(t) = [B(s)/F(s)]u(t) + e(t)
Continuous-time OE model: y(t) = [B(s)/F(s)]u(t) + e(t)
B(s) = s + 4
B(s) = s + 4
F(s) = s^2 + 20 s + 5
F(s) = s^2 + 20 s + 5
Parameterization:
Parameterization:
Polynomial orders: nb=2 nf=2 nk=0
Polynomial orders: nb=2 nf=2 nk=0
Number of free coefficients: 4
Number of free coefficients: 4
Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.
Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.
Status:
Status:
Created by direct construction or transformation. Not estimated.

```
Created by direct construction or transformation. Not estimated.
```

The display shows the polynomial form and the polynomial coefficients.

## See Also

ar|armax|arx|bj|idproc|idss|idtf|iv4|ivar|oe|polydata|polyest| setPolyFormat|translatecov

## Topics

"Dynamic System Models"
"What Are Polynomial Models?"
"Estimate Polynomial Models in the App"
"Estimate Polynomial Models at the Command Line"
"Polynomial Sizes and Orders of Multi-Output Polynomial Models"

Introduced before R2006a

## idproc

Continuous-time process model with identifiable parameters

## Syntax

sys = idproc(type)
sys $=$ idproc(type,Name, Value)

## Description

sys = idproc(type) creates a continuous-time process model with identifiable parameters. type specifies aspects of the model structures, such as the number of poles in the model, whether the model includes an integrator, and whether the model includes a time delay.
sys = idproc(type,Name, Value) creates a process model with additional attributes specified by one or more Name, Value pair arguments.

## Object Description

An idproc model represents a system as a continuous-time process model with identifiable (estimable) coefficients.

A simple SISO process model has a gain, a time constant, and a delay:

$$
\text { sys }=\frac{K_{p}}{1+T_{p 1} s} e^{-T_{d} s} .
$$

$K_{p}$ is a proportional gain. $T_{p 1}$ is the time constant of the real pole, and $T_{d}$ is the transport delay (dead time).

More generally, idproc can represent process models with up to three poles and a zero:

$$
\text { sys }=K_{p} \frac{1+T_{z} s}{\left(1+T_{p 1} s\right)\left(1+T_{p 2} s\right)\left(1+T_{p 3} s\right)} e^{-T_{d} s} .
$$

Two of the poles can be a complex conjugate (underdamped) pair. In that case, the general form of the process model is:

$$
\text { sys }=K_{p} \frac{1+T_{z} s}{\left(1+2 \zeta T_{\omega} s+\left(T_{\omega} s\right)^{2}\right)\left(1+T_{p 3} s\right)} e^{-T_{d} s} .
$$

$T_{\omega}$ is the time constant of the complex pair of poles, and $\zeta$ is the associated damping constant.
In addition, any idproc model can have an integrator. For example, the following is a process model that you can represent with idproc:

$$
\text { sys }=K_{p} \frac{1}{s\left(1+2 \zeta T_{\omega} s+\left(T_{\omega} s\right)^{2}\right)} e^{-T_{d} s} .
$$

This model has no zero ( $T_{z}=0$ ). The model has a complex pair of poles. The model also has an integrator, represented by the $1 / s$ term.

For idproc models, all the time constants, the delay, the proportional gain, and the damping coefficient can be estimable parameters. The idproc model stores the values of these parameters in properties of the model such as Kp, Tp1, and Zeta. (See "Properties" on page 1-630 for more information.)

A MIMO process model contains a SISO process model corresponding to each input-output pair in the system. For idproc models, the form of each input-output pair can be independently specified. For example, a two-input, one-output process can have one channel with two poles and no zero, and another channel with a zero, a pole, and an integrator. All the coefficients are independently estimable parameters.

There are two ways to obtain an idproc model:

- Estimate the idproc model based on output or input-output measurements of a system, using the procest command. procest estimates the values of the free parameters such as gain, time constants, and time delay. The estimated values are stored as properties of the resulting idproc model. For example, the properties sys.Tz and sys. Kp of an idproc model sys store the zero time constant and the proportional gain, respectively. (See "Properties" on page 1-630 for more information.) The Report property of the resulting model stores information about the estimation, such as handling of initial conditions and options used in estimation.

When you obtain an idproc model by estimation, you can extract estimated coefficients and their uncertainties from the model using commands such as getpar and getcov.

- Create an idproc model using the idproc command.

You can create an idproc model to configure an initial parameterization for estimation of a process model. When you do so, you can specify constraints on the parameters. For example, you can fix the values of some coefficients, or specify minimum or maximum values for the free coefficients. You can then use the configured model as an input argument to procest to estimate parameter values with those constraints.

## Examples

## Create SISO Process Model with Complex Poles and Time Delay

Create a process model with a pair of complex poles and a time delay. Set the initial value of the model to the following:

$$
\text { sys }=\frac{0.01}{1+2(0.1)(10) s+(10 s)^{2}} e^{-5 s}
$$

Create a process model with the specified structure.

```
sys = idproc('P2DU')
sys =
Process model with transfer function:
            Kp
    G(s) = -----------------** exp(-Td*s)
        1+2*Zeta*Tw*s+(Tw*s)^2
```

$$
\begin{aligned}
\mathrm{Kp} & =\mathrm{NaN} \\
\mathrm{Tw} & =\mathrm{NaN} \\
\mathrm{Zeta} & =\mathrm{NaN} \\
\mathrm{Td} & =\mathrm{NaN}
\end{aligned}
$$

```
Parameterization:
    {'P2DU'}
    Number of free coefficients: 4
    Use "getpvec", "getcov" for parameters and their uncertainties.
Status:
Created by direct construction or transformation. Not estimated.
```

The input ' P2DU ' specifies an underdamped pair of poles and a time delay. The display shows that sys has the desired structure. The display also shows that the four free parameters, Kp, Tw, Zeta, and Td are all initialized to NaN .

Set the initial values of all parameters to the desired values.
sys.Kp = 0.01;
sys.Tw = 10;
sys.Zeta = 0.1;
sys.Td = 5;
You can use sys to specify this parameterization and these initial guesses for process model estimation with procest.

## Create a MIMO Process Model

Create a one-input, three-output process model, where each channel has two real poles and a zero, but only the first channel has a time delay, and only the first and third channels have an integrator.

```
type = {'P2ZDI';'P2Z';'P2ZI'};
sys = idproc(type)
sys =
Process model with 3 outputs: y_k = Gk(s)u
    From input 1 to output 1:
                1+Tz*s
    G1(s) = Kp * ----------------- * exp(-Td*s)
        s(1+Tp1*s)(1+Tp2*s)
            Kp = NaN
            Tp1 = NaN
            Tp2 = NaN
            Td = NaN
            Tz = NaN
    From input 1 to output 2:
                1+Tz*s
    G1(s) = Kp * ----------------
                (1+Tp1*s)(1+Tp2*s)
            Kp = NaN
```

```
        Tp1 = NaN
        Tp2 = NaN
        Tz = NaN
From input 1 to output 3:
        1+Tz*s
G1(s) = Kp * -----------------
    s(1+Tp1*s)(1+Tp2*s)
        Kp = NaN
        Tp1 = NaN
        Tp2 = NaN
            Tz = NaN
```

Parameterization:
\{'P2DIZ'\}
\{'P2Z' \}
\{'P2IZ' \}
Number of free coefficients: 13
Use "getpvec", "getcov" for parameters and their uncertainties.
Status:
Created by direct construction or transformation. Not estimated.
idproc creates a MIMO model where each character vector in the type array defines the structure of the corresponding I/O pair. Since type is a column vector of character vectors, sys is a one-input, three-output model having the specified parameterization structure. type $\{\mathrm{k}, 1\}$ specifies the structure of the subsystem sys $(\mathrm{k}, 1)$. All identifiable parameters are initialized to NaN .

## Create Array of Process Models

Create a 3-by-1 array of process models, each containing one output and two input channels.
Specify the structure for each model in the array of process models.

```
type1 = {'P1D','P2DZ'};
type2 = {'P0','P3UI'};
type3 = {'P2D','P2DI'};
type = cat(3,type1,type2,type3);
size(type)
ans = 1\times3
    1 2 3
```

Use type to create the array.
sysarr = idproc(type);
The first two dimensions of the cell array type set the output and input dimensions of each model in the array of process models. The remaining dimensions of the cell array set the array dimensions. Thus, sysarr is a 3-model array of 2-input, one-output process models.

Select a model from the array.

```
sysarr(:,:,2)
ans =
Process model with 2 inputs: y = G11(s)u1 + G12(s)u2
    From input 1 to output 1:
    G11(s) = Kp
        Kp = NaN
    From input 2 to output 1:
    Kp
                s(1+2*Zeta*Tw*s+(Tw*s)^2)(1+Tp3*s)
            Kp = NaN
            Tw = NaN
            Zeta = NaN
            Tp3 = NaN
Parameterization:
            {'P0'} {'P3IU'}
    Number of free coefficients: 5
    Use "getpvec", "getcov" for parameters and their uncertainties.
Status:
Created by direct construction or transformation. Not estimated.
```

This two-input, one-output model corresponds to the type 2 entry in the type cell array.

## Input Arguments

## type

Model structure, specified as a character vector or cell array of character vectors.
For SISO models, type is a character vector made up of one or more of the following characters that specify aspects of the model structure:

| Characters | Meaning |
| :--- | :--- |
| Pk | A process model with $k$ poles (not including an integrator). $k$ must be $0,1,2$, <br> or 3. |
| Z | The process model includes a zero $\left(T_{z} \neq 0\right)$. A type with P0 cannot include $Z$ <br> (a process model with no poles cannot include a zero). |
| D | The process model includes a time delay (deadtime) $\left(T_{d} \neq 0\right)$. |
| I | The process model includes an integrator (1/s). |
| U | The process model is underdamped. In this case, the process model includes <br> a complex pair of poles |

Every type character vector must begin with one of P0, P1, P2, or P3. All other components are optional. For example:

- 'P1D ' specifies a process model with one pole and a time delay (deadtime) term:

$$
\text { sys }=\frac{K_{p}}{1+T_{p 1} S} e^{-T_{d} S} .
$$

$\mathrm{Kp}, \mathrm{Tp1}$, and Td are the identifiable parameters of this model.

- 'P2U' creates a process model with a pair of complex poles:

$$
\text { sys }=\frac{K_{p}}{\left(1+2 \zeta T_{\omega} s+\left(T_{\omega} s\right)^{2}\right)} .
$$

$\mathrm{Kp}, \mathrm{Tw}$, and Zeta are the identifiable parameters of this model.

- 'P3ZDI ' creates a process model with three poles. All poles are real, because U is not included. The model also includes a zero, a time delay, and an integrator:

$$
\text { sys }=K_{p} \frac{1+T_{\chi} s}{s\left(1+T_{p 1} s\right)\left(1+T_{p 2} s\right)\left(1+T_{p 3} s\right)} e^{-T_{d} s} .
$$

The identifiable parameters of this model are $\mathrm{Kp}, \mathrm{Tz}, \mathrm{Tp} 1, \mathrm{Tp} 2, \mathrm{Tp} 3$, and Td.
The values of all parameters in a particular model structure are initialized to NaN. You can change them to finite values by setting the values of the corresponding idproc model properties after you create the model. For example, sys.Td $=5$ sets the initial value of the time delay of sys to 5 .

For a MIMO process model with Ny outputs and Nu inputs, type is an Ny-by-Nu cell array of character vectors specifying the structure of each input/output pair in the model. For example, type $\{i, j\}$ specifies the type of the subsystem sys $(i, j)$ from the $j$ th input to the yth output.

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Use Name, Value arguments to specify parameter initial values and additional properties on page 1630 of idproc models during model creation. For example, sys $=$ idproc('p2z','InputName','Voltage','Kp',10,'Tz',0); creates an idproc model with the InputName property set to Voltage. The command also initializes the parameter Kp to a value of 10, and Tz to 0 .

## Properties

idproc object properties include:

## Type

Model structure, specified as a character vector or cell array of character vectors.
For a SISO model sys, the property sys. Type contains a character vector specifying the structure of the system. For example, ' P1D '.

For a MIMO model with Ny outputs and Nu inputs, sys. Type is an Ny-by-Nu cell array of character vectors specifying the structure of each input/output pair in the model. For example, type\{i, j\} specifies the structure of the subsystem sys $(i, j)$ from the $j$ th input to the $i$ th output.

The character vectors are made up of one or more of the following characters that specify aspects of the model structure:

| Characters | Meaning |
| :--- | :--- |
| Pk | A process model with $k$ poles (not including an integrator). $k$ is $0,1,2$, or 3. |
| $Z$ | The process model includes a zero $\left(T_{z} \neq 0\right)$. |
| D | The process model includes a time delay (deadtime) $\left(T_{d} \neq 0\right)$. |
| I | The process model includes an integrator $(1 / s)$. |
| U | The process model is underdamped. In this case, the process model includes <br> a complex pair of poles |

If you create an idproc model sys using the idproc command, sys. Type contains the model structure that you specify with the type input argument.

If you obtain an idproc model by identification using procest, then sys. Type contains the model structures that you specified for that identification.

In general, you cannot change the type of an existing model. However, you can change whether the model contains an integrator using the property sys. Integration.

## Kp,Tp1,Tp2,Tp3,Tz,Tw,Zeta,Td

Values of process model parameters.
If you create an idproc model using the idproc command, the values of all parameters present in the model structure initialize by default to NaN. The values of parameters not present in the model structure are fixed to 0 . For example, if you create a model, sys, of type ' P1D' , then Kp, Tp1, and Td are initialized to NaN and are identifiable (free) parameters. All remaining parameters, such as Tp2 and $T z$, are inactive in the model. The values of inactive parameters are fixed to zero and cannot be changed.

For a MIMO model with Ny outputs and Nu inputs, each parameter value is an Ny-by-Nu cell array of character vectors specifying the corresponding parameter value for each input/output pair in the model. For example, sys. $\mathrm{Kp}(\mathrm{i}, \mathrm{j})$ specifies the Kp value of the subsystem sys $(i, j)$ from the $j$ th input to the ith output.

For an idproc model sys, each parameter value property such as sys.Kp, sys.Tp1, sys.Tz, and the others is an alias to the corresponding Value entry in the Structure property of sys. For example, sys. Tp3 is an alias to the value of the property sys. Structure.Tp3.Value.

Default: For each parameter value, NaN if the process model structure includes the particular parameter; 0 if the structure does not include the parameter.

## Integration

Logical value or matrix denoting the presence or absence of an integrator in the transfer function of the process model.

For a SISO model sys, sys. Integration $=$ true if the model contains an integrator.
For a MIMO model, sys. Integration ( $i, j$ ) $=$ true if the transfer function from the $j$ th input to the ith output contains an integrator.

When you create a process model using the idproc command, the value of sys. Integration is determined by whether the corresponding type contains I.

## NoiseTF

Coefficients of the noise transfer function.
sys.NoiseTF stores the coefficients of the numerator and the denominator polynomials for the noise transfer function $H(s)=N(s) / D(s)$.
sys. NoiseTF is a structure with fields num and den. Each field is a cell array of $N_{y}$ row vectors, where $N_{y}$ is the number of outputs of sys. These row vectors specify the coefficients of the noise transfer function numerator and denominator in order of decreasing powers of $s$.

Typically, the noise transfer function is automatically computed by the estimation function procest. You can specify a noise transfer function that procest uses as an initial value. For example:

```
NoiseNum = {[ll 2.2]; [1 0.54]};
NoiseDen = {[1 1.3]; [1 2]};
NoiseTF = struct('num', {NoiseNum}, 'den', {NoiseDen});
sys = idproc({'p2'; 'p1di'}); % 2-output, 1-input process model
sys.NoiseTF = NoiseTF;
```

Each vector in sys.NoiseTF. num and sys. NoiseTF. den must be of length 3 or less (second-order in $s$ or less). Each vector must start with 1. The length of a numerator vector must be equal to that of the corresponding denominator vector, so that $H(s)$ is always biproper.

Default: struct('num', \{num2cell(ones(Ny,1))\},'den',\{num2cell(ones(Ny,1))\})

## Structure

Information about the estimable parameters of the idproc model.
sys.Structure includes one entry for each parameter in the model structure of sys. For example, if sys is of type 'P1D', then sys includes identifiable parameters Kp, Tp1, and Td. Correspondingly, sys.Structure.Kp, sys.Structure.Tp1, and sys.Structure.Td contain information about each of these parameters, respectively.

Each of these parameter entries in sys.Structure contains the following fields:

- Value - Parameter values. For example, sys.Structure.Kp.Value contains the initial or estimated values of the $K_{p}$ parameter.

NaN represents unknown parameter values.
For SISO models, each parameter value property such as sys.Kp, sys.Tp1, sys.Tz, and the others is an alias to the corresponding Value entry in the Structure property of sys. For example, sys.Tp3 is an alias to the value of the property sys. Structure.Tp3.Value.

For MIMO models, sys.Kp\{i,j\} is an alias to sys.Structure(i,j).Kp.Value, and similarly for the other identifiable coefficient values.

- Minimum - Minimum value that the parameter can assume during estimation. For example, sys.Structure.Kp.Minimum = 1 constrains the proportional gain to values greater than or equal to 1.
- Maximum - Maximum value that the parameter can assume during estimation.
- Free - Logical value specifying whether the parameter is a free estimation variable. If you want to fix the value of a parameter during estimation, set the corresponding Free $=$ false. For example, to fix the dead time to 5 :

```
sys.Td = 5;
```

sys.Structure.Td.Free $=$ false;

- Scale - Scale of the parameter's value. Scale is not used in estimation.
- Info - Structure array for storing parameter units and labels. The structure has Label and Unit fields.

Specify parameter units and labels as character vectors. For example, 'Time'.
Structure also includes a field Integration that stores a logical array indicating whether each corresponding process model has an integrator. sys. Structure. Integration is an alias to sys.Integration.

For a MIMO model with Ny outputs and Nu input, Structure is an Ny-by-Nu array. The element Structure ( $i, j$ ) contains information corresponding to the process model for the ( $i, j$ ) inputoutput pair.

## NoiseVariance

The variance (covariance matrix) of the model innovations $e$.
An identified model includes a white, Gaussian noise component $e(t)$. NoiseVariance is the variance of this noise component. Typically, the model estimation function (such as procest) determines this variance.

For SISO models, NoiseVariance is a scalar. For MIMO models, NoiseVariance is a $N_{y}$-by- $N_{y}$ matrix, where $N_{y}$ is the number of outputs in the system.

## Report

Summary report that contains information about the estimation options and results when the process model is obtained using the procest estimation command. Use Report to query a model for how it was estimated, including its:

- Estimation method
- Estimation options
- Search termination conditions
- Estimation data fit and other quality metrics

The contents of Report are irrelevant if the model was created by construction.

```
m = idproc('P2DU');
m.Report.OptionsUsed
ans =
    []
```

If you obtain the process model using estimation commands, the fields of Report contain information on the estimation data, options, and results.

```
load iddata2 z2;
m = procest(z2,'P2DU');
m.Report.OptionsUsed
DisturbanceModel: 'estimate'
    InitialCondition: 'auto'
            Focus: 'prediction'
    EstimateCovariance: 1
                Display: 'off'
            InputOffset: [1x1 param.Continuous]
            OutputOffset: []
        Regularization: [1x1 struct]
            SearchMethod: 'auto'
        SearchOptions: [1x1 idoptions.search.identsolver]
            OutputWeight: []
                Advanced: [1x1 struct]
```

Report is a read-only property.
For more information on this property and how to use it, see the Output Arguments section of the corresponding estimation command reference page and "Estimation Report".

## InputDelay

Input delays. InputDelay is a numeric vector specifying a time delay for each input channel. Specify input delays in the time unit stored in the TimeUnit property.

For a system with Nu inputs, set InputDelay to an Nu-by-1 vector, where each entry is a numerical value representing the input delay for the corresponding input channel. You can also set InputDelay to a scalar value to apply the same delay to all channels.

Default: 0 for all input channels

## OutputDelay

Output delays.
For identified systems, like idproc, OutputDelay is fixed to zero.

## Ts

Sample time. For idproc, Ts is fixed to zero because all idproc models are continuous time.

## TimeUnit

Units for the time variable, the sample time Ts, and any time delays in the model, specified as one of the following values:

- 'nanoseconds'
- 'microseconds'
- 'milliseconds'
- 'seconds'
- 'minutes'
- 'hours'
- 'days'
- 'weeks'
- 'months'
- 'years'

Changing this property has no effect on other properties, and therefore changes the overall system behavior. Use chgTimeUnit to convert between time units without modifying system behavior.

Default: 'seconds'

## InputName

Input channel names, specified as one of the following:

- Character vector - For single-input models, for example, ' controls '.
- Cell array of character vectors - For multi-input models.

Alternatively, use automatic vector expansion to assign input names for multi-input models. For example, if sys is a two-input model, enter:

```
sys.InputName = 'controls';
```

The input names automatically expand to \{'controls(1)';'controls(2)'\}.
When you estimate a model using an iddata object, data, the software automatically sets InputName to data. InputName.

You can use the shorthand notation $u$ to refer to the InputName property. For example, sys. $u$ is equivalent to sys.InputName.

Input channel names have several uses, including:

- Identifying channels on model display and plots
- Extracting subsystems of MIMO systems
- Specifying connection points when interconnecting models

Default: ' ' for all input channels

## InputUnit

Input channel units, specified as one of the following:

- Character vector - For single-input models, for example, ' seconds ' .
- Cell array of character vectors - For multi-input models.

Use InputUnit to keep track of input signal units. InputUnit has no effect on system behavior.
Default: ' ' for all input channels

## InputGroup

Input channel groups. The InputGroup property lets you assign the input channels of MIMO systems into groups and refer to each group by name. Specify input groups as a structure. In this structure,
field names are the group names, and field values are the input channels belonging to each group. For example:

```
sys.InputGroup.controls = [1 2];
sys.InputGroup.noise = [3 5];
```

creates input groups named controls and noise that include input channels 1,2 and 3,5 , respectively. You can then extract the subsystem from the controls inputs to all outputs using:
sys(:,'controls')
Default: Struct with no fields

## OutputName

Output channel names, specified as one of the following:

- Character vector - For single-output models. For example, ' measurements ' .
- Cell array of character vectors - For multi-output models.

Alternatively, use automatic vector expansion to assign output names for multi-output models. For example, if sys is a two-output model, enter:
sys.OutputName = 'measurements';
The output names automatically expand to \{'measurements(1)';'measurements(2)'\}.
When you estimate a model using an iddata object, data, the software automatically sets OutputName to data.OutputName.

You can use the shorthand notation $y$ to refer to the OutputName property. For example, sys.y is equivalent to sys.OutputName.

Output channel names have several uses, including:

- Identifying channels on model display and plots
- Extracting subsystems of MIMO systems
- Specifying connection points when interconnecting models

Default: ' ' for all output channels

## OutputUnit

Output channel units, specified as one of the following:

- Character vector - For single-output models. For example, ' seconds '.
- Cell array of character vectors - For multi-output models.

Use OutputUnit to keep track of output signal units. OutputUnit has no effect on system behavior.
Default: ' ' for all output channels

## OutputGroup

Output channel groups. The OutputGroup property lets you assign the output channels of MIMO systems into groups and refer to each group by name. Specify output groups as a structure. In this
structure, field names are the group names, and field values are the output channels belonging to each group. For example:

```
sys.OutputGroup.temperature = [1];
sys.InputGroup.measurement = [3 5];
```

creates output groups named temperature and measurement that include output channels 1 , and 3,5 , respectively. You can then extract the subsystem from all inputs to the measurement outputs using:

```
sys('measurement',:)
```

Default: Struct with no fields

## Name

System name, specified as a character vector. For example, 'system_1'.

## Default: ' '

## Notes

Any text that you want to associate with the system, stored as a string or a cell array of character vectors. The property stores whichever data type you provide. For instance, if sys1 and sys2 are dynamic system models, you can set their Notes properties as follows:

```
sys1.Notes = "sys1 has a string.";
sys2.Notes = 'sys2 has a character vector.';
sys1.Notes
sys2.Notes
ans =
    "sys1 has a string."
ans =
    'sys2 has a character vector.'
```

Default: [0×1 string]

## UserData

Any type of data you want to associate with system, specified as any MATLAB data type.

## Default: []

## SamplingGrid

Sampling grid for model arrays, specified as a data structure.
For arrays of identified linear (IDLTI) models that are derived by sampling one or more independent variables, this property tracks the variable values associated with each model. This information appears when you display or plot the model array. Use this information to trace results back to the independent variables.

Set the field names of the data structure to the names of the sampling variables. Set the field values to the sampled variable values associated with each model in the array. All sampling variables should be numeric and scalar valued, and all arrays of sampled values should match the dimensions of the model array.

For example, if you collect data at various operating points of a system, you can identify a model for each operating point separately and then stack the results together into a single system array. You can tag the individual models in the array with information regarding the operating point:
nominal_engine_rpm = [1000 5000 10000];
sys.SamplingGrid = struct('rpm', nominal_engine_rpm)
where sys is an array containing three identified models obtained at rpms 1000, 5000 and 10000, respectively.

For model arrays generated by linearizing a Simulink model at multiple parameter values or operating points, the software populates SamplingGrid automatically with the variable values that correspond to each entry in the array. For example, the Simulink Control Design commands linearize and slLinearizer populate SamplingGrid in this way.

Default: []

## See Also

idss|idtf|pem|procest|ssest|tfest
Introduced before R2006a

## idresamp

Resample time-domain data by decimation or interpolation

## Syntax

```
datar = idresamp(data,R)
datar = idresamp(data,R,order,tol)
[datar,res_fact] = idresamp(data,R,order,tol)
```


## Description

datar = idresamp(data, R ) resamples data on a new sample interval $R$ and stores the resampled data as datar.
datar = idresamp(data,R,order, tol) filters the data by applying a filter of specified order before interpolation and decimation. Replaces R by a rational approximation that is accurate to a tolerance tol.
[datar,res_fact] = idresamp(data,R,order,tol) returns res_fact, which corresponds to the value of $R$ approximated by a rational expression.

## Input Arguments

data
Name of time-domain iddata object or a matrix of data. Can be input-output or time-series data.
Data must be sampled at equal time intervals.
R
Resampling factor, such that $\mathrm{R}>1$ results in decimation and $\mathrm{R}<1$ results in interpolation.
Any positive number you specify is replaced by the rational approximation, Q/P.
order
Order of the filters applied before interpolation and decimation.
Default: 8
tol
Tolerance of the rational approximation for the resampling factor R .
Smaller tolerance might result in larger P and Q values, which produces more accurate answers at the expense of slower computation.

Default: 0.1

## Output Arguments

datar
Name of the resampled data variable. datar class matches the data class, as specified.

Rational approximation for the specified resampling factor R and tolerance tol.
Any positive number you specify is replaced by the rational approximation, $\mathrm{Q} / \mathrm{P}$, where the data is interpolated by a factor P and then decimated by a factor Q .

## See Also

resample

## Introduced in R2007a

## idss

State-space model with identifiable parameters

## Description

Use idss to create a continuous-time or discrete-time state-space model with identifiable (estimable) coefficients, or to convert "Dynamic System Models" to state-space form.

A state-space model of a system with input vector $u$, output vector $y$, and disturbance $e$ takes the following form in continuous time:

$$
\begin{aligned}
\frac{d x(t)}{d t} & =A x(t)+B u(t)+K e(t) \\
y(t) & =C x(t)+D u(t)+e(t)
\end{aligned}
$$

In discrete time, the state-space model takes the following form:

$$
\begin{gathered}
x[k+1]=A x[k]+B u[k]+K e[k] \\
y[k]=C x[k]+D u[k]+e[k]
\end{gathered}
$$

For idss models, the elements of the state-space matrices $A, B, C$, and $D$ can be estimable parameters. The elements of the state disturbance $K$ can also be estimable parameters. The idss model stores the values of these matrix elements in the $A, B, C, D$, and $K$ properties of the model.

## Creation

You can obtain an idss model object in one of three ways.

- Estimate the idss model based on the input-output measurements of a system by using n4sid or ssest. These estimation commands estimate the values of the estimable elements of the statespace matrices. The estimated values are stored in the A, B, C, D, and K properties of the resulting idss model. The Report property of the resulting model stores information about the estimation, such as on the handling of initial state values and the options used in estimation. For example:
sys = ssest(data, nx);
A = sys.A;
$B=$ sys. $B$;
sys.Report
For more examples of estimating an idss model, see ssest or n4sid.
- Create an idss model using the idss command. For example:
sys = idss(A,B,C,D)
You can create an idss model to configure an initial parameterization for estimation of a statespace model to fit measured response data. When you do so, you can specify constraints on one or more of the state-space matrix elements. For instance, you can fix the values of some elements, or specify minimum or maximum values for the free elements. You can then use the configured model as an input argument to an estimation command (ssest or n4sid) to estimate parameter values
with those constraints. For examples, see "Create State-Space Model with Identifiable Parameters" on page 1-651 and "Configure Identifiable Parameters of State-Space Model" on page 1-652.
- Convert an existing dynamic system model to an idss model using the idss command. For example:

```
sys_ss = idss(sys_tf);
```

For information on functions you can use to extract information from or transform idss model objects, see "Object Functions" on page 1-650.

## Syntax

```
sys = idss(A,B,C,D)
sys = idss(A,B,C,D,K)
sys = idss(A,B,C,D,K,x0)
sys = idss(A,B,C,D,K,x0,Ts)
sys = idss(
```

$\qquad$

``` , Name, Value)
sys = idss(sys0)
sys = idss(sys0,'split')
```


## Description

## Create State-Space Model

sys $=$ idss $(A, B, C, D)$ creates a state-space model with specified state-space matrices $A, B, C, D$. By default, sys is a discrete-time model with an unspecified sample time and no state disturbance element. Use this syntax especially when you want to configure an initial parameterization as an input to a state-space estimation function such as n4sid or ssest.
sys $=$ idss ( $A, B, C, D, K)$ specifies a disturbance matrix $K$.
sys $=\operatorname{idss}(A, B, C, D, K, x 0)$ initializes the state values with the vector $x 0$.
sys $=\operatorname{idss}(A, B, C, D, K, x 0, T s)$ specifies the sample time property Ts. Use Ts $=0$ to create a continuous-time model.
sys $=$ idss( $\qquad$ , Name, Value) sets additional properties using one or more name-value pair arguments. Specify name-value pair arguments after any of the input argument combinations in the previous syntaxes.

## Convert Dynamic System Model to State-Space Model

sys $=$ idss(sys0) converts any dynamic system model sys0 to idss model form.
sys $=$ idss(sys0,'split') converts sys0 to idss model form, and treats the last Ny input channels of sys0 as noise channels in the returned model. sys 0 must be a numeric (nonidentified) tf, zpk, or ss model object. Also, sys0 must have at least as many inputs as outputs.

## Input Arguments

## x0 - Initial state values

column vector of zeros (default) | vector

Initial state values, specified as a column vector of $N_{x}$ values.

## sys0 - Dynamic system

dynamic system model
Dynamic system, specified as a dynamic system model to convert to an idss model.

- When sys 0 is an identified model, its estimated parameter covariance is lost during conversion. If you want to translate the estimated parameter covariance during the conversion, use translatecov.
- When sys0 is a numeric (nonidentified) model, the state-space data of sys0 defines the $\mathrm{A}, \mathrm{B}, \mathrm{C}$, and $D$ matrices of the converted model. The disturbance matrix $K$ is fixed to zero. The NoiseVariance value defaults to eye ( Ny ), where Ny is the number of outputs of sys.

For the syntax sys = idss(sys0,'split'), sys0 must be a numeric (nonidentified) tf, zpk, or ss model object. Also, sys0 must have at least as many inputs as outputs. Finally, the subsystem sys0( : , Ny+1:Ny+Nu) must contain a nonzero feedthrough term (the subsystem must be biproper).

## Properties

## A, B, C, D - Values of state-space matrices

matrices
Values of the state-space matrices, specified as matrices that correspond to each of the A, B, C, and D matrices.

For a system with $N_{y}$ outputs, $N_{u}$ inputs, and $N_{\chi}$ states, the state-space matrices have the following dimensions:

- $\mathrm{A}-N_{x}$-by- $N_{x}$ matrix
- B $-N_{x}$-by- $N_{u}$ matrix
- $\mathrm{C}-N_{y}$-by- $N_{x}$ matrix
- D $-N_{y}$-by- $N_{u}$ matrix

If you obtain an idss model sys by identification using ssest or n4sid, then sys.A, sys.B, sys.C, and sys.D contain the estimated values of the matrix elements.

If you create an idss model sys using the idss command, sys.A, sys.B, sys.C, and sys.D contain the initial values of the state-space matrices that you specify with the $A, B, C, D$ input arguments.

For an idss model sys, each property sys.A, sys.B, sys.C, and sys.D is an alias of the corresponding Value entry in the Structure property of sys. For example, sys.A is an alias of the value of the property sys.Structure.A.Value.

## K - Value of state disturbance matrix

zero matrix (default) | matrix
Value of the state disturbance matrix $K$, specified as an $N_{x}$-by- $N_{y}$ matrix, where $N_{x}$ is the number of states and $N_{y}$ is the number of outputs.

If you obtain an idss model sys by identification using ssest or n4sid, then sys.K contains the estimated values of the matrix elements.

If you create an idss model sys using the idss command, sys.K contains the initial values of the state-space matrices that you specify with the K input argument.

For an idss model sys, sys. K is an alias to the value of the property sys.Structure.K.Value.

## StateName - State names

' ' (default) | character vector | cell array
State names, specified as a character vector or cell array.

- First-order model - Character vector
- Model with two or more states - Cell array of character vectors
- Unnamed states - ' '

Example: 'velocity ' names the only state in a first-order model

## StateUnit - State units

' ' (default) | character vector | cell array
State units, specified as a character vector or cell array.

- First-order model - Character vector
- Model with two or more states - Cell array of character vectors
- States without specified units - '

Use StateUnit to keep track of the units each state is expressed in. StateUnit has no effect on system behavior.

Example: ' rad ' corresponds to the units of the only state in a first-order model

## Structure - Information about estimable parameters

structure property values
Information about the estimable parameters of the idss model, specified as property-specific values. Structure.A, Structure.B, Structure.C, Structure.D, and Structure.K contain information about the $A, B, C, D$, and $K$ matrices, respectively. Each parameter in Structure contains the following fields.

| Field | Description | Examples |
| :--- | :--- | :--- |
| Value | Parameter Values - Each <br> property sys.A, sys.B, sys.C, <br> and sys.D is an alias of the <br> corresponding Value entry in <br> the Structure property of <br> sys.NaN represents unknown <br> parameter values. | sys.Structure.A.Value <br> contains the initial or estimated <br> values of the $A$ matrix.sys.A is <br> an alias of the value of the <br> property <br> sys.Structure.A. Value. |
| Minimum | Minimum value that the <br> parameter can assume during <br> estimation | sys. Structure.K.Minimum <br> = 0 constrains all entries in the <br> K matrix to be greater than or <br> equal to zero. |

$\left.\begin{array}{|l|l|l|}\hline \text { Field } & \text { Description } & \text { Examples } \\ \hline \text { Maximum } & \begin{array}{l}\text { Maximum value that the } \\ \text { parameter can assume during } \\ \text { estimation }\end{array} & \begin{array}{l}\text { Boolean specifying whether the } \\ \text { parameter is a free estimation } \\ \text { variable. If you want to fix the } \\ \text { value of a parameter during } \\ \text { estimation, set the } \\ \text { corresponding Free }=\text { false. }\end{array}\end{array} \begin{array}{l}\text { If } A \text { is a 3-by-3 matrix, } \\ \text { sys. Structure. A. Free }= \\ \text { eyes (3) fixes all of the off- } \\ \text { diagonal entries in } A \text { to the } \\ \text { values specified in } \\ \text { sys. Structure. A. Value. In } \\ \text { this case, only the diagonal } \\ \text { entries in } A \text { are estimable. }\end{array}\right]$.

For an example of configuring model parameters using the Structure property, see "Configure Identifiable Parameters of State-Space Model" on page 1-652.

## NoiseVariance - Variance of model innovations

scalar | matrix
Variance (covariance matrix) of the model innovations $e$, specified as a scalar or matrix.

- SISO model - Scalar
- MIMO model with $N_{y}$ outputs $-N_{y}$-by- $N_{y}$ matrix

An identified model includes a white Gaussian noise component $e(t)$. NoiseVariance is the variance of this noise component. Typically, the model estimation function (such as ssest) determines this variance.

## Report - Summary report

report field values
This property is read-only.
Summary report that contains information about the estimation options and results for a state-space model obtained using estimation commands, such as ssest, ss regest, and n4sid. Use Report to find estimation information for the identified model, including the:

- Estimation method
- Estimation options
- Search termination conditions
- Estimation data fit and other quality metrics

If you create the model by construction, the contents of Report are irrelevant.

```
\(\mathrm{A}=[-0.10 .4 ;-0.4-0.1]\);
\(B=[1 ; 0]\);
\(C=[10] ;\)
D = 0;
m = idss(A,B,C,D);
sys.Report.OptionsUsed
ans =
    []
```

If you obtain the using estimation commands, the fields of Report contain information on the estimation data, options, and results.

```
load iddata2 z2;
m = ssest(z2,3);
m.Report.OptionsUsed
InitialState: 'auto'
            N4Weight: 'auto'
            N4Horizon: 'auto'
            Focus: 'prediction'
EstimateCovariance: 1
            Display: 'off'
        InputOffset: []
        OutputOffset: []
        OutputWeight: []
        SearchMethod: 'auto'
        SearchOptions: [1x1 idoptions.search.identsolver]
        Regularization: [1x1 struct]
            Advanced: [1x1 struct]
```

For more information on this property and how to use it, see the Output Arguments section of the corresponding estimation command reference page and "Estimation Report".

## InputDelay - Input delay for each input channel

0 (default) | scalar | vector
Input delay for each input channel, specified as a scalar value or numeric vector. For continuous-time systems, specify input delays in the time unit stored in the TimeUnit property. For discrete-time systems, specify input delays in integer multiples of the sample time Ts. For example, setting InputDelay to 3 specifies a delay of three sample times.

For a system with $N_{u}$ inputs, set InputDelay to an $N_{u}$-by-1 vector. Each entry of this vector is a numerical value that represents the input delay for the corresponding input channel.

You can also set InputDelay to a scalar value to apply the same delay to all channels.

## OutputDelay - Output delay for each output channel

0 (default)
For identified systems such as idss, OutputDelay is fixed to zero.

## Ts - Sample Time

- 1 (default) | 0 | positive scalar

Sample time, specified as one of the following.

- Continuous-time model - 0
- Discrete-time model with a specified sampling time - a positive scalar representing the sampling period expressed in the unit specified by the TimeUnit property of the model
- Discrete-time model with unspecified sample time - - 1

Changing this property does not discretize or resample the model. Use c2d and d2c to convert between continuous- and discrete-time representations. Use $d 2 d$ to change the sample time of a discrete-time system.

## TimeUnit - Units for time variable

'seconds' (default)|'nanoseconds'|'microseconds' | 'milliseconds'|'minutes'| 'hours' |'days' | 'weeks' |'months'|'years'

Units for the time variable, the sample time Ts, and any time delays in the model, specified as a scalar.

Changing this property does not resample or convert the data. Modifying the property changes only the interpretation of the existing data. Use chgTimeUnit to convert data to different time units

## InputName - Input channel names

' ' (default) | character vector | cell array
Input channel names, specified as a character vector or cell array.

- Single-input model - Character vector. For example, ' controls'.
- Multi-input model - Cell array of character vectors.

Alternatively, use automatic vector expansion to assign input names for multi-input models. For example, if sys is a two-input model, enter:

```
sys.InputName = 'controls';
```

The input names automatically expand to \{'controls(1)' ;'controls(2)'\}.
When you estimate a model using an iddata object data, the software automatically sets InputName to data. InputName.

You can use the shorthand notation $u$ to refer to the InputName property. For example, sys. $u$ is equivalent to sys. InputName.

You can use input channel names in several ways, including:

- To identify channels on model display and plots
- To extract subsystems of MIMO systems
- To specify connection points when interconnecting models


## InputUnit - Input channel units

' ' (default) | character vector | cell array
Input channel units, specified as a character vector or cell array:

- Single-input model - Character vector
- Multi-input Model - Cell array of character vectors

Use InputUnit to keep track of input signal units. InputUnit has no effect on system behavior.

## InputGroup - Input channel groups

struct with no fields (default) | struct
Input channel groups, specified as a structure. The InputGroup property lets you divide the input channels of MIMO systems into groups so that you can refer to each group by name. In the InputGroup structure, set field names to the group names, and field values to the input channels belonging to each group.

For example, create input groups named controls and noise that include input channels 1, 2 and 3,5 , respectively.
sys.InputGroup.controls = [1 2];
sys.InputGroup.noise = [3 5];
You can then extract the subsystem from the controls inputs to all outputs using the following syntax:
sys(:,'controls')

## OutputName - Output channel names

' ' (default) | character vector | cell array
Output channel names, specified as a character vector or cell array.

- Single-input model - Character vector. For example, 'measurements '.
- Multi-input model - Cell array of character vectors.

Alternatively, use automatic vector expansion to assign output names for multi-output models. For example, if sys is a two-output model, enter:
sys.OutputName = 'measurements';
The output names automatically expand to \{'measurements(1)';'measurements(2)'\}.
When you estimate a model using an iddata object, data, the software automatically sets OutputName to data.OutputName.

You can use the shorthand notation $y$ to refer to the OutputName property. For example, sys.y is equivalent to sys.OutputName.

You can use output channel names in several ways, including:

- To identify channels on model display and plots
- To extract subsystems of MIMO systems
- To specify connection points when interconnecting models


## OutputUnit - Output channel units

' ' (default) | character vector | cell array
Output channel units, specified as a character vector or cell array.

- Single-input model - Character vector. For example, ' seconds ' .
- Multi-input Model - Cell array of character vectors.

Use OutputUnit to keep track of output signal units. OutputUnit has no effect on system behavior.

## OutputGroup - Output channel groups

struct with no fields (default) | struct
Output channel groups, specified as a structure. The OutputGroup property lets you divide the output channels of MIMO systems into groups and refer to each group by name. In the OutputGroup structure, set field names to the group names, and field values to the output channels belonging to each group.

For example, create output groups named temperature and measurement that include output channels 1 , and 3,5 , respectively.

```
sys.OutputGroup.temperature = [1];
sys.OutputGroup.measurement = [3 5];
```

You can then extract the subsystem from all inputs to the measurement outputs using the following syntax:

```
sys('measurement',:)
```


## Name - System Name

' ' (default) | character vector
System name, specified as a character vector. For example, 'system_1'.

## Notes - Notes on system

0 -by-1 string (default) | string | character vector
Any text that you want to associate with the system, specified as a string or a cell array of character vectors. The property stores whichever data type you provide. For instance, if sys1 and sys2 are dynamic system models, you can set their Notes properties as follows.

```
sys1.Notes = "sys1 has a string.";
sys2.Notes = 'sys2 has a character vector.';
sys1.Notes
sys2.Notes
ans =
    "sys1 has a string."
ans =
    'sys2 has a character vector.'
```


## UserData - Data to associate with system

[] (default) | any MATLAB data type
Data to associate with the system, specified as any MATLAB data type.

## SamplingGrid - Sampling grid

[] (default) | struct
Sampling grid for model arrays, specified as a structure.
For arrays of identified linear (IDLTI) models that you derive by sampling one or more independent variables, this property tracks the variable values associated with each model. This information appears when you display or plot the model array. Use this information to trace results back to the independent variables.

Set the field names of the data structure to the names of the sampling variables. Set the field values to the sampled variable values associated with each model in the array. All sampling variables must be numeric and scalar valued, and all arrays of sampled values must match the dimensions of the model array.

For example, suppose that you collect data at various operating points of a system. You can identify a model for each operating point separately and then stack the results together into a single system array. You can tag the individual models in the array with information regarding the operating point.
nominal_engine_rpm = [1000 5000 10000];
sys.SamplingGrid = struct('rpm', nominal_engine_rpm)
Here, sys is an array containing three identified models obtained at 1000, 5000, and 10000 rpm , respectively.

For model arrays that you generate by linearizing a Simulink model at multiple parameter values or operating points, the software populates SamplingGrid automatically with the variable values that correspond to each entry in the array.

## Object Functions

In general, any function applicable to "Dynamic System Models" is applicable to an idss model object. These functions are of four general types.

- Functions that operate and return idss model objects enable you to transform and manipulate idss models. For instance:
- Use canon to transform an idss model into canonical form
- Use merge to merge estimated idss models.
- Use c2d to convert an idss from continuous to discrete time. Use d2c to convert an idss from discrete to continuous time.
- Functions that perform analytical and simulation functions on idss objects, such as bode and sim
- Functions that retrieve or interpret model information, such as advice and getpar
- Functions that convert idss objects into a different model type, such as idpoly or idtf for time domain or idfrd for continuous domain

The following lists contain a representative subset of the functions that you can use with idss models.

## Transformation and Manipulation

canon Canonical state-space realization

```
ss2ss State coordinate transformation for state-space model
balred Model order reduction
translatecov Translate parameter covariance across model transformation operations
setpar
chgTimeUn
d2d
d2c Convert model from discrete to continuous time
c2d Convert model from continuous to discrete time
merge Merge estimated models
```


## Analysis and Simulation

sim Simulate response of identified model
predict Predict state and state estimation error covariance at next time step using extended or unscented Kalman filter, or particle filter
compare Compare identified model output and measured output
impulse Impulse response plot of dynamic system; impulse response data
step $\quad$ Step response plot of dynamic system; step response data
bode Bode plot of frequency response, or magnitude and phase data
data2state Map past data to states of state-space and nonlinear ARX models
findstates Estimate initial states of model

## Information Extraction and Interpretation

idssdata State-space data of identified system
get Access model property values
getpar Obtain attributes such as values and bounds of linear model parameters
getcov Parameter covariance of identified model
advice Analysis and recommendations for data or estimated linear models

## Conversion to Other Model Structures

idpoly Polynomial model with identifiable parameters
idtf Transfer function model with identifiable parameters
idfrd Frequency response data or model

## Examples

## Create State-Space Model with Identifiable Parameters

Create a 4th-order SISO state-space model with identifiable parameters. Initialize the initial state values to 0.1 for all entries. Set the sample time to 0.1 s .

```
A = blkdiag([-0.1 0.4; -0.4 -0.1],[-1 5; -5 -1]);
B = [1; zeros(3,1)];
C = [1 0 1 0];
D = 0;
K = zeros(4,1);
x0 = [0.1,0.1,0.1,0.1];
Ts = 0.1;
sys = idss(A,B,C,D,K,x0,Ts);
```

sys is a 4th-order SISO idss model. The number of states and input-output dimensions are determined by the dimensions of the state-space matrices. By default, all entries in the matrices A, B, $\mathrm{C}, \mathrm{D}$, and K are identifiable parameters.

You can use sys to specify an initial parameterization for state-space model estimation with ssest or n4sid.

## Specify Additional Attributes of State-Space Model

Create a 4th-order SISO state-space model with identifiable parameters. Name the input and output channels of the model, and specify minutes as the model time unit.

You can use name-value pair arguments to specify additional model properties during model creation.

```
A = blkdiag([-0.1 0.4; -0.4 -0.1],[-1 5; -5 -1]);
B = [1; zeros(3,1)];
C = [1 0 1 0];
D = 0;
sys = idss(A,B,C,D,'InputName','Drive','TimeUnit','minutes');
```

To change or specify most attributes of an existing model, you can use dot notation. For example, change the output name.
sys.OutputName = 'Torque';

## Configure Identifiable Parameters of State-Space Model

Configure an idss model so that it has no state disturbance element and only the nonzero entries of the $A$ matrix are estimable. Additionally, fix the values of the $B$ matrix.

You can configure individual parameters of an idss model to specify constraints for state-space model estimation with ssest or n4sid.

Create an idss model.
$\mathrm{A}=\mathrm{blkdiag}([-0.1$ 0.4; -0.4-0.1],[-1 5; -5 -1]);
$B=[1 ;$ zeros $(3,1)]$;
$C=\left[\begin{array}{llll}1 & 0 & 1 & 0\end{array}\right] ;$
D = 0;
$K=z e r o s(4,1) ;$
$x 0=[0.1,0.1,0.1,0.1] ;$
sys $=$ idss(A, $B, C, D, K, x 0,0) ;$
Setting all entries of $K$ to 0 creates an idss model with no state disturbance element.
Use the Structure property of the model to fix the values of some of the parameters.

```
sys.Structure.A.Free = (A~=0);
sys.Structure.B.Free = false;
sys.Structure.K.Free = false;
```

The entries in sys.Structure.A.Free determine whether the corresponding entries in sys.A are free (identifiable) or fixed. The first line sets sys.Structure.A. Free to a logical matrix that is true wherever A is nonzero, and false everywhere else. This setting fixes the values of the zero entries in sys.A.

The remaining lines fix all the values in sys.B and sys. $K$ to the values that you specified during model creation.

## Convert Transfer Function into State-Space Model

Model a dynamic system using a transfer function. Then use idss to convert the transfer-function model into state-space form.

Using idtf, construct a continuous-time, single-input, single-output (SISO) transfer function described by:

```
    G(s)}=\frac{s+4}{\mp@subsup{s}{}{2}+20s+5
num = [1 4];
den = [1 20 5];
G = idtf(num,den)
G =
    s + 4
    s^2 + 20 s + 5
Continuous-time identified transfer function.
Parameterization:
    Number of poles: 2 Number of zeros: 1
    Number of free coefficients: 4
    Use "tfdata", "getpvec", "getcov" for parameters and their uncertainties.
Status:
Created by direct construction or transformation. Not estimated.
```

Convert the transfer function into state-space form.

```
sys0 = idss(G)
sys0 =
    Continuous-time identified state-space model:
                dx/dt = A x(t) + B u(t) + K e(t)
                y(t) = C x(t) + D u(t) + e(t)
    A =
        x1 x2
        x1 -20 -2.5
        x2 2 0
    B =
        ul
```

```
    x1 2
    x2 0
    C =
        x1 x2
    y1 0.5 1
    D =
        u1
    y1 0
    K =
        y1
    x1 0
    x2 0
Parameterization:
    FREE form (all coefficients in A, B, C free).
    Feedthrough: none
    Disturbance component: none
    Number of free coefficients: 8
    Use "idssdata", "getpvec", "getcov" for parameters and their uncertainties.
Status:
Created by direct construction or transformation. Not estimated.
```


## Array of State-Space Models

Create an array of state-space models.
You can create an array of state-space models in one of several ways:

- Direct array construction using $n$-dimensional state-space arrays
- Array-building by indexed assignment
- Array-building using the stack command
- Sampling an identified model using the rsample command

Create an array by providing $n$-dimensional arrays as an input argument to idss, instead of 2dimensional matrices.

```
A = rand(2,2,3,4);
sysarr = idss(A,[2;1],[1 1],0);
```

When you provide a multi-dimensional array to idss in place of one of the state-space matrices, the first two dimensions specify the numbers of states, inputs, or outputs of each model in the array. The remaining dimensions specify the dimensions of the array itself. A is a 2 -by-2-by-3-by-4 array. Therefore, sysarr is a 3-by-4 array of idss models. Each model in sysarr has two states, specified by the first two dimensions of A. Further, each model in sysarr has the same B, C, and D values.

Create an array by indexed assignment.

```
sysarr = idss(zeros(1,1,2));
sysarr(:,:,1) = idss([4 -3; -2 0],[2;1],[1 1],0);
sysarr(:,:,2) = idss(rand(2),rand(2,1),rand(1,2),1);
```

The first command preallocates the array. The first two dimensions of the array are the I/O dimensions of each model in the array. Therefore, sysarr is a 2 -element vector of SISO models.

The remaining commands assign an idss model to each position in sysarr. Each model in an array must have the same I/O dimensions.

Add another model to sysarr using stack.
stack is an alternative to building an array by indexing.
sysarr $=\operatorname{stack}(1$, sysarr,idss([1 -2; -4 9],[0;-1],[1 1],0));
This command adds another idss model along the first array dimension of sysarr. sysarr is now a 3-by-1 array of SISO idss models.

## See Also

idgrey|idpoly|idproc|idssdata|idtf|n4sid|pem|ssest|ssestOptions|
translatecov

## Topics

"Dynamic System Models"
"What Are State-Space Models?"
"Canonical State-Space Realizations"
"Estimate State-Space Models with Structured Parameterization"
Introduced in R2006a

## idssdata

State-space data of identified system

## Syntax

[A,B,C,D,K] = idssdata(sys)
[A,B,C,D,K, x0] = idssdata(sys)
[A,B,C,D,K,x0,dA,dB,dC,dD,dK,dx0] = idssdata(sys)
[A,B,C,D,K,__] = idssdata(sys, $11, \ldots, j N$ )
[A,B,C,D,K,__] = idssdata(sys,'cell')

## Description

$[A, B, C, D, K]=$ idssdata(sys) returns the $A, B, C, D$ and $K$ matrices of the identified state-space model sys.
$[A, B, C, D, K, x 0]=$ idssdata(sys) returns the initial state values, $x 0$.
$[A, B, C, D, K, x 0, d A, d B, d C, d D, d K, d x 0]=i d s s d a t a(s y s)$ returns the uncertainties in the system matrices for sys.
$[A, B, C, D, K, \ldots]=i d s s d a t a(s y s, j 1, \ldots, j N)$ returns data for the $j 1, \ldots, j n$ entries in the model array sys.
[A, B, C,D,K,__] = idssdata(sys,'cell') returns data for all the entries in the model array sys as separate cells in cell arrays.

## Input Arguments

## sys

Identified model.
If sys is not an identified state-space model (idss or idgrey), then it is first converted to an idss model. This conversion results in a loss of the model uncertainty information.
sys can be an array of identified models.
j1,..., jN
Integer indices of N entries in the array sys of identified systems.

## Output Arguments

## A,B,C,D,K

State-space matrices that represent sys as:

$$
\begin{aligned}
& x[k+1]=A x[k]+B u[k]+K e[k] ; x[0]=x 0 ; \\
& y[k]=C x[k]+D u[k]+e[k] ;
\end{aligned}
$$

If sys is an array of identified models, then $A, B, C, D, K$ are multi-dimension arrays. To access the state-space matrix, say A, for the $k$-th entry of sys, use $A(:,:, k)$.
x0
Initial state.
If sys is an idss or idgrey model, then $x 0$ is the value obtained during estimation. It is also stored using the Report. Parameters property of sys.

For other model types, x 0 is zero.
If sys is an array of identified models, then $x 0$ contains a column for each entry in sys.
$d A, d B, d C, d D, d K$
Uncertainties associated with the state-space matrices A, B, C , D, K.
The uncertainty matrices represents 1 standard deviation of uncertainty.
If sys is an array of identified models, then $d A, d B, d C, d D, d K$ are multi-dimension arrays. To access the state-space matrix, say A, for the $k$-th entry of sys, use $A(:, i, k)$.
dx0
Uncertainty associated with the initial state.
$d x 0$ represents 1 standard deviation of uncertainty.
If sys is an array of identified models, then $d x 0$ contains a column for each entry in sys.

## Examples

## Obtain Identified State-Space Matrices

Obtain the identified state-space matrices for a model estimated from data.
Identify a model using data.

```
load icEngine.mat
data = iddata(y,u,0.04);
sys = n4sid(data,4,'InputDelay',2);
```

data is an iddata object representing data sampled at a sampling rate of 0.04 seconds.
sys is an idss model representing the identified system.
Obtain identified state-space matrices of sys.

```
[A,B,C,D,K] = idssdata(sys);
```


## Obtain Initial State of Identified Model

Obtain the initial state associated with an identified model.
Identify a model using data.

```
load icEngine.mat
data = iddata(y,u,0.04);
sys = n4sid(data,4,'InputDelay',2);
```

data is an iddata object representing data sampled at a sampling rate of 0.04 seconds.
sys is an idss model representing the identified system.
Obtain the initial state associated with sys.
[A,B,C,D,K,x0] = idssdata(sys);
$A, B, C, D$ and $K$ represent the state-space matrices of the identified model sys. $x 0$ is the initial state identified for sys.

## Obtain Uncertainty Data of State-Space Matrices of Identified Model

Obtain the uncertainty matrices of the state-space matrices of an identified model.
Identify a model using data.

```
load icEngine.mat
data = iddata(y,u,0.04);
sys = n4sid(data,4,'InputDelay',2);
```

data is an iddata object representing data sampled at a sampling rate of 0.04 seconds.
sys is an idss model representing the identified system.
Obtain the uncertainty matrices associated with the state-space matrices of sys.
[A,B,C,D,K,x0,dA,dB,dC,dD,dx0] = idssdata(sys);
$d A, d B, d C, d D$ and $d K$ represent the uncertainty associated with the state-space matrices of the identified model sys. dx 0 represents the uncertainty associated with the estimated initial state.

## Obtain State-Space Matrices for Multiple Identified Models

Obtain the state-space matrices for multiple models from an array of identified models.
Identify multiple models using data.

```
load icEngine.mat
data = iddata(y,u,0.04);
sys2 = n4sid(data,2,'InputDelay',2);
sys3 = n4sid(data,3,'InputDelay',2);
```

sys4 = n4sid(data,4,'InputDelay',2);
sys = stack(1,sys2,sys3,sys4);
data is an iddata object representing data sampled at a sampling rate of 0.04 seconds.
sys is an array of idss models. The first entry of sys is a second-order identified system. The second and third entries of sys are third- and fourth-order identified systems, respectively.

Obtain the state-space matrices for the first and third entries of sys.
$[A, B, C, D, K, x 0]=$ idssdata(sys,1);
[A,B,C,D,K,x0] = idssdata(sys,3);

## Obtain State-Space Matrices for Identified Model as Cell Array

Obtain the state-space matrices of an array of identified models in cell arrays.
Identify multiple models using data.

```
load icEngine.mat
data = iddata(y,u,0.04);
sys3 = n4sid(data,3,'InputDelay',2);
sys4 = n4sid(data,4,'InputDelay',2);
sys = stack(1,sys3,sys4);
```

data is an iddata object representing data sampled at a sampling rate of 0.04 seconds.
sys is an array of idss models. The first entry of sys is a third-order identified system and the second entry is a fourth-order identified system.

Obtain the state-space matrices of sys in cell arrays.
[A,B,C,D,K,x0] = idssdata(sys,'cell');
$A, B, C, D$ and $K$ are cell arrays containing the state-space matrices of the individual entries of the identified model array sys. x 0 is a cell array containing the estimated initial state of the individual entries of the identified model array sys.

## See Also

idss|polydata|ssdata|tfdata|zpkdata

## Introduced in R2012a

## idtf

Transfer function model with identifiable parameters

## Description

An idtf model represents a system as a continuous-time or discrete-time transfer function with identifiable (estimable) coefficients. Use idtf to create a transfer function model, or to convert "Dynamic System Models" to transfer function form.

A SISO transfer function is a ratio of polynomials with an exponential term. In continuous time,

$$
G(s)=e^{-\tau s} \frac{b_{n} s^{n}+b_{n-1} s^{n-1}+\ldots+b_{0}}{s^{m}+a_{m-1} s^{m-1}+\ldots+a_{0}} .
$$

In discrete time,

$$
G\left(z^{-1}\right)=z^{-k} \frac{b_{n} z^{-n}+b_{n-1} z^{-n+1}+\ldots+b_{0}}{z^{-m}+a_{m-1} z^{-m+1}+\ldots+a_{0}} .
$$

In discrete time, $z^{-k}$ represents a time delay of $k T_{s}$, where $T_{s}$ is the sample time.
For idtf models, the denominator coefficients $a_{0}, \ldots, a_{m-1}$ and the numerator coefficients $b_{0}, \ldots, b_{n}$ can be estimable parameters. (The leading denominator coefficient is always fixed to 1.) The time delay $\tau$ (or $k$ in discrete time) can also be an estimable parameter. The idtf model stores the polynomial coefficients $a_{0}, \ldots, a_{m-1}$ and $b_{0}, \ldots, b_{n}$ in the Denominator and Numerator properties of the model, respectively. The time delay $\tau$ or $k$ is stored in the IODelay property of the model.

Unlike idss and idpoly, idtf fixes the noise parameter to 1 rather than parameterizing it. So, in $y=G u+H e, H=1$.

A MIMO transfer function contains a SISO transfer function corresponding to each input-output pair in the system. For idtf models, the polynomial coefficients and transport delays of each input-output pair are independently estimable parameters.

## Creation

You can obtain an idtf model object in one of three ways.

- Estimate the idtf model based on input-output measurements of a system using tfest. The tfest command estimates the values of the transfer function coefficients and transport delays. The estimated values are stored in the Numerator, Denominator, and IODelay properties of the resulting idtf model. When you reference numerator and denominator properties, you can use the shortcuts num and den. The Report property of the resulting model stores information about the estimation, such as handling of initial conditions and options used in estimation. For example, you can use the following commands to estimate and get information about a transfer function.

```
sys = tfest(data,nx);
num = sys.Numerator;
```

```
den = sys.den;
```

sys.Report

For more examples of estimating an idtf model, see tfest.
When you obtain an idtf model by estimation, you can extract estimated coefficients and their uncertainties from the model. To do so, use commands such as tfdata, getpar, or getcov.

- Create an idtf model using the idtf command. For example, create an idtf model with the numerator and denominator that you specify.

```
sys = idtf(num,den)
```

You can create an idtf model to configure an initial parameterization for estimation of a transfer function to fit measured response data. When you do so, you can specify constraints on such values as the numerator and denominator coefficients and transport delays. For example, you can fix the values of some parameters, or specify minimum or maximum values for the free parameters. You can then use the configured model as an input argument to tfest to estimate parameter values with those constraints. For examples, see "Create Continuous-Time Transfer Function Model" on page 1-671 and "Create Discrete-Time Transfer Function" on page 1-672.

- Convert an existing dynamic system model to an idtf model using the idtf command. For example, convert the state-space model sys_ss to a transfer function.

```
sys_tf = idtf(sys_ss);
```

For a more detailed example, see "Convert Identifiable State-Space Model to Identifiable Transfer Function" on page 1-674

For information on functions you can use to extract information from or transform idtf model objects, see "Object Functions" on page 1-670.

## Syntax

```
sys = idtf(numerator,denominator)
sys = idtf(numerator,denominator,Ts)
sys = idtf(
```

$\qquad$

``` , Name, Value)
sys = idtf(sys0)
```


## Description

## Create Transfer Function Model

sys = idtf(numerator, denominator) creates a continuous-time transfer function model with identifiable parameters. numerator specifies the current values of the transfer function numerator coefficients. denominator specifies the current values of the transfer function denominator coefficients.
sys = idtf(numerator, denominator,Ts) creates a discrete-time transfer function model with sample time Ts.
sys = idtf(__, , Name, Value) creates a transfer function with the properties on page 1-662 specified by one or more Name, Value pair arguments. Specify name-value pair arguments after any of the input argument combinations in the previous syntaxes.

## Convert Dynamic System Model to Transfer Function Model

sys $=$ idtf(sys0) converts any dynamic system model sys0 to idtf model form.
Input Arguments
sys0 - Dynamic system
dynamic system model
Any dynamic system to convert to an idtf model.
When sys0 is an identified model, its estimated parameter covariance is lost during conversion. If you want to translate the estimated parameter covariance during the conversion, use translatecov.

## Properties

## Numerator - Values of transfer function numerator coefficients <br> vector | cell array

Values of transfer function numerator coefficients, specified as a row vector or a cell array.
For SISO transfer functions, the values of the numerator coefficients are stored as a row vector in the following order:

- Descending powers of $s$ or $p$ (for continuous-time transfer functions)
- Ascending powers of $z^{-1}$ or $q^{-1}$ (for discrete-time transfer functions)

Any coefficient whose initial value is not known is stored as NaN .
For MIMO transfer functions with Ny outputs and Nu inputs, Numerator is a Ny-by-Nu cell array of numerator coefficients for each input/output pair. For an example of a MIMO transfer function, see "Create MIMO Discrete-Time Transfer Function" on page 1-673.

If you create an idtf model sys using the idtf command, sys. Numerator contains the initial values of numerator coefficients that you specify with the numerator input argument.

If you obtain an idtf model by identification using tfest, then sys. Numerator contains the estimated values of the numerator coefficients.

For an idtf model sys, the property sys.Numerator is an alias for the value of the property sys.Structure.Numerator.Value.

## Denominator - Values of transfer function denominator coefficients

vector | cell array
Values of transfer function denominator coefficients, specified as a row vector or a cell array.
For SISO transfer functions, the values of the denominator coefficients are stored as a row vector in the following order:

- Descending powers of $s$ or $p$ (for continuous-time transfer functions)
- Ascending powers of $z^{-1}$ or $q^{-1}$ (for discrete-time transfer functions)

The leading coefficient in Denominator is fixed to 1. Any coefficient whose initial value is not known is stored as NaN .

For MIMO transfer functions with $N y$ outputs and $N u$ inputs, Denominator is an $N y$-by- $N u$ cell array of denominator coefficients for each input-output pair. For an example of a MIMO transfer function, see "Create MIMO Discrete-Time Transfer Function" on page 1-673.

If you create an idtf model sys using theidtf command, sys. Denominator contains the initial values of denominator coefficients that you specify with the denominator input argument.

If you obtain an idtf model sys by identification using tfest, then sys. Denominator contains the estimated values of the denominator coefficients.

For an idtf model sys, the property sys. Denominator is an alias for the value of the property sys.Structure. Denominator. Value.

## Variable - Transfer function display variable

's' (default) | ' $\left.\mathrm{p}^{\prime}\right|^{\prime} z^{\wedge}-1$ ' | ' $q^{\wedge}$ - 1'
Transfer function display variable, specified as one of the following values:

- 's' - Default for continuous-time models
- ' p ' - Equivalent to ' s '
- ' $z^{\wedge}-1^{\prime}$ - Default for discrete-time models
- ' $q^{\wedge}-1^{\prime}$ - Equivalent to ${ }^{\prime} z^{\wedge}-1$ '

The value of Variable is reflected in the display, and also affects the interpretation of the Numerator and Denominator coefficient vectors for discrete-time models. When Variable is set to ' $z^{\wedge}-1$ ' or ' $q^{\wedge}-1$ ', the coefficient vectors are ordered as ascending powers of the variable.

For an example of using the Variable property, see "Specify Transfer Function Display Variable" on page 1-673.

## IODelay - Transport delays

0 (default) | numeric array
Transport delays, specified as a numeric array containing a separate transport delay for each inputoutput pair.

For continuous-time systems, transport delays are expressed in the time unit stored in the TimeUnit property. For discrete-time systems, transport delays are expressed as integers denoting delay of a multiple of the sample time Ts.

For a MIMO system with $N y$ outputs and $N u$ inputs, set IODelay as an $N y$-by- $N u$ array. Each entry of this array is a numerical value representing the transport delay for the corresponding input-output pair. You can set IODelay to a scalar value to apply the same delay to all input-output pairs.

If you create an idtf model sys using the idtf command, then sys. IODelay contains the initial values of the transport delay that you specify with a name-value pair argument.

If you obtain an idtf model sys by identification using tfest, then sys. IODelay contains the estimated values of the transport delay.

For an idtf model sys, the property sys.IODelay is an alias for the value of the property sys.Structure.IODelay.Value.

## Structure - Information about estimable parameters

structure property values | array of structure property values
Property-specific information about the estimable parameters of the idtf model, specified as a single structure or an array of structures.

- SISO system - Single structure.
- MIMO system with $N y$ outputs and $N u$ inputs $-N y$-by- $N u$ array. The element Structure (i, j) contains information corresponding to the transfer function for the ( $i, j$ ) input-output pair.

Structure.Numerator, Structure.Denominator, and Structure.IODelay contain information about the numerator coefficients, denominator coefficients, and transport delay, respectively. Each parameter in Structure contains the following fields.

| Field | Description | Examples |
| :---: | :---: | :---: |
| Value | Parameter values. Each property is an alias of the corresponding Value entry in the Structure property of sys.NaN represents unknown parameter values. | sys.Structure.Numerator. Value contains the initial or estimated values of SISO numerator coefficients. sys. Numerator is an alias of the value of this property. sys.Numerator $\{\mathrm{i}, \mathrm{j}\}$ is the alias of the MIMO property sys.Structure(i,j).Numer ator.Value. |
| Minimum | Minimum value that the parameter can assume during estimation. | sys.Structure.IODelay.Mi nimum $=0.1$ constrains the transport delay to values greater than or equal to 0.1. sys.Structure.IODelay.Mi nimum must be greater than or equal to zero. |
| Maximum | Maximum value that the parameter can assume during estimation. | sys.Structure.IODelay.Ma ximum $=0.5$ constrains the transport delay to values less than or equal to 0.5 . <br> sys.Structure.IODelay.Ma ximum must be greater than or equal to zero. |


| Field | Description | Examples |
| :--- | :--- | :--- |
| Free | Boolean specifying whether the <br> parameter is a free estimation <br> variable. If you want to fix the <br> value of a parameter during <br> estimation, set the <br> corresponding Free value to <br> false. For denominators, the <br> value of Free for the leading <br> coefficient, specified by <br> sys.Structure.Denominato <br> r.Free (1), is always false <br> (the leading denominator <br> coefficient is always fixed to 1). | sys.Structure.Denominato <br> r. Free = false fixes all of <br> the denominator coefficients in <br> sys to the values specified in <br> sys.Structure.Denominato <br> r.Value. |
| Scale | Scale of the value of the <br> parameter. The estimation <br> algorithm does not use Scale. |  |
| Info | Structure array that contains <br> the fields Label and Unit for <br> storing parameter labels and <br> units. Specify parameter labels <br> and units as character vectors. | 'Time ' |

## NoiseVariance - Variance of model innovations

## scalar | matrix

Variance (covariance matrix) of the model innovations $e$, specified as a scalar or matrix.

- SISO model - Scalar
- MIMO model with $N_{y}$ outputs $-N_{y}$-by- $N_{y}$ matrix

An identified model includes a white Gaussian noise component $e(t)$. NoiseVariance is the variance of this noise component. Typically, the model estimation function (such as tfest) determines this variance.

## Report - Summary report

report field values
This property is read-only.
Summary report that contains information about the estimation options and results for a transfer function model obtained using estimation commands, such as tfest and impulseest. Use Report to find estimation information for the identified model, including:

- Estimation method
- Estimation options
- Search termination conditions
- Estimation data fit and other quality metrics

If you create the model by construction, the contents of Report are irrelevant.

```
sys = idtf([1 4],[1 20 5]);
sys.Report.OptionsUsed
ans =

If you obtain the model using estimation commands, the fields of Report contain information on the estimation data, options, and results.
```

load iddata2 z2;
sys = tfest(z2,3);
sys.Report.OptionsUsed
InitializeMethod: 'iv'
InitializeOptions: [1×1 struct]
InitialCondition: 'auto'
Display: 'off'
InputOffset: []
OutputOffset: []
EstimateCovariance: 1
Regularization: [1×1 struct]
SearchMethod: 'auto'
SearchOptions: [1\times1 idoptions.search.identsolver]
WeightingFilter: []
EnforceStability: 0
OutputWeight: []
Advanced: [1\times1 struct]

```

For more information on this property and how to use it, see the Output Arguments section of the corresponding estimation command reference page and "Estimation Report".

\section*{InputDelay - Input delay for each input channel}

0 (default) | scalar | vector
Input delay for each input channel, specified as a scalar value or numeric vector. For continuous-time systems, specify input delays in the time unit stored in the TimeUnit property. For discrete-time systems, specify input delays in integer multiples of the sample time Ts. For example, setting InputDelay to 3 specifies a delay of three sample times.

For a system with \(N_{u}\) inputs, set InputDelay to an \(N_{u}\)-by-1 vector. Each entry of this vector is a numerical value that represents the input delay for the corresponding input channel.

You can also set InputDelay to a scalar value to apply the same delay to all channels.
Estimation treats InputDelay as a fixed constant of the model. Estimation uses the IODelay property for estimating time delays. To specify initial values and constrains for estimation of time delays, use sys.Structure.IODelay.

\section*{OutputDelay - Output delay for each output channel \\ 0 (default)}

For identified systems such as idtf, OutputDelay is fixed to zero.

\section*{Ts - Sample Time}

0 (default) | - 1 | positive scalar

Sample time, specified as one of the following.
- Continuous-time model - 0
- Discrete-time model with a specified sampling time - Positive scalar representing the sampling period expressed in the unit specified by the TimeUnit property of the model
- Discrete-time model with unspecified sample time - - 1

Changing this property does not discretize or resample the model. Use c2d and d2c to convert between continuous- and discrete-time representations. Use \(d 2 d\) to change the sample time of a discrete-time system.

\section*{TimeUnit - Units for time variable}
```

'seconds' (default)|'nanoseconds'| 'microseconds' | 'milliseconds'|'minutes'|
'hours'|'days'| 'weeks'| 'months' | 'years'

```

Units for the time variable, the sample time Ts, and any time delays in the model, specified as a scalar.

Changing this property does not resample or convert the data. Modifying the property changes only the interpretation of the existing data. Use chgTimeUnit to convert data to different time units.

\section*{InputName - Input channel names}
' ' (default) | character vector | cell array
Input channel names, specified as a character vector or cell array.
- Single-input model - Character vector, for example, ' controls'
- Multi-input model - Cell array of character vectors

Alternatively, use automatic vector expansion to assign input names for multi-input models. For example, if sys is a two-input model, enter:
```

sys.InputName = 'controls';

```

The input names automatically expand to \{'controls(1)';'controls(2)'\}.
When you estimate a model using an iddata object data, the software automatically sets InputName to data. InputName.

You can use the shorthand notation \(u\) to refer to the InputName property. For example, sys.u is equivalent to sys. InputName.

You can use input channel names in several ways, including:
- To identify channels on model display and plots
- To extract subsystems of MIMO systems
- To specify connection points when interconnecting models

\section*{InputUnit - Input channel units}
' ' (default) | character vector | cell array
Input channel units, specified as a character vector or cell array.
- Single-input model - Character vector
- Multi-input Model - Cell array of character vectors

Use InputUnit to keep track of input signal units. InputUnit has no effect on system behavior.

\section*{InputGroup - Input channel groups}
struct with no fields (default) | struct
Input channel groups, specified as a structure. The InputGroup property lets you divide the input channels of MIMO systems into groups so that you can refer to each group by name. In the InputGroup structure, set field names to the group names, and field values to the input channels belonging to each group.

For example, create input groups named controls and noise that include input channels 1,2 and 3,5 , respectively.
sys.InputGroup.controls = [1 2];
sys.InputGroup.noise = [3 5];
You can then extract the subsystem from the controls inputs to all outputs using the following syntax:
```

sys(:,'controls')

```

\section*{OutputName - Output channel names}
' ' (default) | character vector | cell array
Output channel names, specified as a character vector or cell array.
- Single-input model - Character vector, for example, ' measurements '
- Multi-input model - Cell array of character vectors

Alternatively, use automatic vector expansion to assign output names for multi-output models. For example, if sys is a two-output model, enter:
sys.OutputName = 'measurements';
The output names automatically expand to \{'measurements(1)';'measurements(2)'\}.
When you estimate a model using an iddata object data, the software automatically sets OutputName to data.OutputName.

You can use the shorthand notation \(y\) to refer to the OutputName property. For example, sys.y is equivalent to sys.OutputName.

You can use output channel names in several ways, including:
- To identify channels on model display and plots
- To extract subsystems of MIMO systems
- To specify connection points when interconnecting models

\section*{OutputUnit - Output channel units}
' ' (default) | character vector | cell array
Output channel units, specified as a character vector or cell array.
- Single-input model - Character vector, for example, ' seconds '
- Multi-input Model - Cell array of character vectors

Use OutputUnit to keep track of output signal units. OutputUnit has no effect on system behavior.

\section*{OutputGroup - Output channel groups}
struct with no fields (default) | struct
Output channel groups, specified as a structure. The OutputGroup property lets you divide the output channels of MIMO systems into groups and refer to each group by name. In the OutputGroup structure, set field names to the group names, and field values to the output channels belonging to each group.

For example, create output groups named temperature and measurement that include output channels 1 and 3,5 , respectively.
```

sys.OutputGroup.temperature = [1];
sys.OutputGroup.measurement = [3 5];

```

You can then extract the subsystem from all inputs to the measurement outputs using the following syntax:
sys('measurement', :)

\section*{Name - System Name}
' ' (default) | character vector
System name, specified as a character vector, for example, 'system_1'.

\section*{Notes - Notes on system}

0 -by-1 string (default) | string | character vector
Any text that you want to associate with the system, specified as a string or a cell array of character vectors. The property stores whichever data type you provide. For instance, if sys1 and sys2 are dynamic system models, you can set their Notes properties as follows.
```

sys1.Notes = "sys1 has a string.";
sys2.Notes = 'sys2 has a character vector.';
sys1.Notes
sys2.Notes
ans =
"sys1 has a string."
ans =
'sys2 has a character vector.'

```
UserData - Data to associate with system
[ ] (default) | any MATLAB data type

Data to associate with the system, specified as any MATLAB data type.

\section*{SamplingGrid - Sampling grid}
[] (default) | struct
Sampling grid for model arrays, specified as a structure.
For arrays of identified linear (IDLTI) models that you derive by sampling one or more independent variables, this property tracks the variable values associated with each model. This information appears when you display or plot the model array. Use this information to trace results back to the independent variables.

Set the field names of the data structure to the names of the sampling variables. Set the field values to the sampled variable values associated with each model in the array. All sampling variables must be numeric and scalar valued, and all arrays of sampled values must match the dimensions of the model array.

For example, suppose that you collect data at various operating points of a system. You can identify a model for each operating point separately and then stack the results together into a single system array. You can tag the individual models in the array with information regarding the operating point.
```

nominal_engine_rpm = [1000 5000 10000];
sys.SamplingGrid = struct('rpm', nominal_engine_rpm)

```

Here, sys is an array containing three identified models obtained at 1000, 5000, and 10000 rpm , respectively.

For model arrays that you generate by linearizing a Simulink model at multiple parameter values or operating points, the software populates SamplingGrid automatically with the variable values that correspond to each entry in the array.

\section*{Object Functions}

In general, any function applicable to "Dynamic System Models" is applicable to an idtf model object. These functions are of four general types.
- Functions that operate and return idtf model objects enable you to transform and manipulate idtf models. For instance:
- Use merge to merge estimated idtf models.
- Use c2d to convert an idtf from continuous to discrete time. Use d2c to convert an idtf from discrete to continuous time.
- Functions that perform analytical and simulation functions on idtf objects, such as bode and sim
- Functions that retrieve or interpret model information, such as advice and getpar
- Functions that convert idtf objects into a different model type, such as idpoly for time domain or idfrd for frequency domain

The following lists contain a representative subset of the functions that you can use with idtf models.

\section*{Transformation and Manipulation}
translatecov Translate parameter covariance across model transformation operations
setpar \(\quad\) Set attributes such as values and bounds of linear model parameters
```

chgTimeUnit Change time units of dynamic system
d2d Resample discrete-time model
d2c Convert model from discrete to continuous time
c2d Convert model from continuous to discrete time
merge Merge estimated models

```

\section*{Analysis and Simulation}
sim Simulate response of identified model
predict Predict state and state estimation error covariance at next time step using extended or unscented Kalman filter, or particle filter
compare Compare identified model output and measured output
impulse Impulse response plot of dynamic system; impulse response data
step Step response plot of dynamic system; step response data
bode Bode plot of frequency response, or magnitude and phase data

\section*{Information Extraction and Interpretation}
tfdata Access transfer function data
get Access model property values
getpar Obtain attributes such as values and bounds of linear model parameters
getcov Parameter covariance of identified model
advice Analysis and recommendations for data or estimated linear models

\section*{Conversion to Other Model Structures}
idpoly Polynomial model with identifiable parameters
idss State-space model with identifiable parameters
idfrd Frequency response data or model

\section*{Examples}

\section*{Create Continuous-Time Transfer Function Model}

Specify a continuous-time, single-input, single-output (SISO) transfer function with estimable parameters. The initial values of the transfer function are given by the following equation:
\(G(s)=\frac{s+4}{s^{2}+20 s+5}\)
num \(=\left[\begin{array}{ll}14] ;\end{array}\right.\)
den = [1 20 5];
G = idtf(num,den)
G \(=\)
s +4
\(s^{\wedge} 2+20 s+5\)
Continuous-time identified transfer function.
Parameterization:
Number of poles: 2 Number of zeros: 1

Number of free coefficients: 4
Use "tfdata", "getpvec", "getcov" for parameters and their uncertainties.
Status:
Created by direct construction or transformation. Not estimated.
G is an idtf model. num and den specify the initial values of the numerator and denominator polynomial coefficients in descending powers of \(s\). The numerator coefficients with initial values 1 and 4 are estimable parameters. The denominator coefficients with initial values 20 and 5 are also estimable parameters. The leading denominator coefficient is always fixed to 1.

You can use \(G\) to specify an initial parameterization for estimation with \(t\) fest.

\section*{Create Transfer Function with Known Input Delay and Specified Attributes}

Specify a continuous-time, SISO transfer function with known input delay. The transfer function initial values are given by the following equation:
\[
G(s)=e^{-5.8 s} \frac{5}{s+5}
\]

Label the input of the transfer function with the name 'Voltage' and specify the input units as volt.

Use name-value pair arguments to specify the delay, input name, and input unit.
```

num = 5;
den = [1 5];
input_delay = 5.8;
input_name = 'Voltage';
input_unit = 'volt';
G = idtf(num,den,'InputDelay',input_delay,...
'InputName',input_name,'InputUnit',input_unit);

```
\(G\) is an idtf model. You can use \(G\) to specify an initial parameterization for estimation with tfest. If you do so, model properties such as InputDelay, InputName, and InputUnit are applied to the estimated model. The estimation process treats InputDelay as a fixed value. If you want to estimate the delay and specify an initial value of 5.8 s , use the IODelay property instead.

\section*{Create Discrete-Time Transfer Function}

Specify a discrete-time SISO transfer function with estimable parameters. The initial values of the transfer function are given by the following equation:
\[
H(z)=\frac{z-0.1}{z+0.8}
\]

Specify the sample time as 0.2 seconds.
```

num = [1 -0.1];
den = [1 0.8];

```

Ts = 0.2;
H = idtf(num,den,Ts);
num and den are the initial values of the numerator and denominator polynomial coefficients. For discrete-time systems, specify the coefficients in ascending powers of \(z^{-1}\).

Ts specifies the sample time for the transfer function as 0.2 seconds.
\(H\) is an idtf model. The numerator and denominator coefficients are estimable parameters (except for the leading denominator coefficient, which is fixed to 1 ).

\section*{Create MIMO Discrete-Time Transfer Function}

Specify a discrete-time, two-input, two-output transfer function. The initial values of the MIMO transfer function are given by the following equation:
\[
H(z)=\left[\begin{array}{cc}
\frac{1}{z+0.2} & \frac{z}{z+0.7} \\
\frac{-z+2}{z-0.3} & \frac{3}{z+0.3}
\end{array}\right]
\]

Specify the sample time as 0.2 seconds.
```

nums = {1,[1,0];[-1,2],3};
dens = {[1,0.2],[1,0.7];[1,-0.3],[1,0.3]};
Ts = 0.2;
H = idtf(nums,dens,Ts);

```
nums and dens specify the initial values of the coefficients in cell arrays. Each entry in the cell array corresponds to the numerator or denominator of the transfer function of one input-output pair. For example, the first row of nums is \(\{1,[1,0]\}\). This cell array specifies the numerators across the first row of transfer functions in H . Likewise, the first row of dens, \(\{[1,0.2],[1,0.7]\}\), specifies the denominators across the first row of H .

Ts specifies the sample time for the transfer function as 0.2 seconds.
\(H\) is an idtf model. All of the polynomial coefficients are estimable parameters, except for the leading coefficient of each denominator polynomial. These coefficients are always fixed to 1.

\section*{Specify Transfer Function Display Variable}

Specify the following discrete-time transfer function in terms of \(q^{\wedge}-1\) :
\[
H\left(q^{-1}\right)=\frac{1+0.4 q^{-1}}{1+0.1 q^{-1}-0.3 q^{-2}}
\]

Specify the sample time as 0.1 seconds.
```

num = [1 0.4];
den = [ll 0.1 -0.3];

```

Ts = 0.1;
convention_variable = 'q^-1';
H = idtf(nūm,den,Ts,'Variable', convention_variable);
Use a name-value pair argument to specify the variable \(q^{\wedge}-1\).
num and den are the numerator and denominator polynomial coefficients in ascending powers of \(q^{-1}\).
Ts specifies the sample time for the transfer function as 0.1 seconds.
\(H\) is an idtf model.

\section*{Gain Matrix Transfer Function}

Specify a transfer function with estimable coefficients whose initial value is given by the following static gain matrix:
\[
H(s)=\left[\begin{array}{lll}
1 & 0 & 1 \\
1 & 1 & 0 \\
3 & 0 & 2
\end{array}\right]
\]

M = [1 0 1; 1 1 0; 302\(]\);
H = idtf(M);
H is an idtf model that describes a three input \((\mathrm{Nu}=3)\), three output \((\mathrm{Ny}=3)\) transfer function. Each input-output channel is an estimable static gain. The initial values of the gains are given by the values in the matrix \(M\).

\section*{Convert Identifiable State-Space Model to Identifiable Transfer Function}

Convert a state-space model with identifiable parameters to a transfer function with identifiable parameters.

Convert the following identifiable state-space model to an identifiable transfer function.
```

    \(\widetilde{x}(t)=\left[\begin{array}{cc}-0.2 & 0 \\ 0 & -0.3\end{array}\right] x(t)+\left[\begin{array}{c}-2 \\ 4\end{array}\right] u(t)+\left[\begin{array}{l}0.1 \\ 0.2\end{array}\right] e(t)\)
    \(y(t)=\left[\begin{array}{ll}1 & 1\end{array}\right] x(t)\)
    $A=[-0.2,0 ; 0,-0.3] ;$
$B=[2 ; 4]$;
C $=[1,1]$;
D = 0;
K = [0.1; 0.2];
sys0 = idss(A,B,C,D,K,'NoiseVariance',0.1);
sys = idtf(sys0);

```
\(A, B, C, D\), and \(K\) are matrices that specify sys 0 , an identifiable state-space model with a noise variance of 0.1.
sys \(=\) idtf(sys0) creates an idtf model sys.

\section*{Estimate Transfer Function Model by Specifying Number of Poles}

Load the time-domain system-response data \(z 1\).
```

load iddatal z1;

```

Set the number of poles \(n \mathrm{p}\) to 2 and estimate a transfer function.
```

np = 2;

```
sys = tfest(z1,np);
sys is an idtf model containing the estimated two-pole transfer function.
View the numerator and denominator coefficients of the resulting estimated model sys.
```

sys.Numerator

```
ans \(=1 \times 2\)
    \(2.4554 \quad 176.9856\)
sys.Denominator
ans \(=1 \times 3\)
    \(1.0000 \quad 3.1625 \quad 23.1631\)

To view the uncertainty in the estimates of the numerator and denominator and other information, use tfdata.

\section*{Create Array of Transfer Function Models}

Create an array of transfer function models with identifiable coefficients. Each transfer function in the array is of the form:
\[
H(s)=\frac{a}{s+a}
\]

The initial value of the coefficient \(a\) varies across the array, from 0.1 to 1.0 , in increments of 0.1 .
```

H = idtf(zeros(1,1,10));
for k = 1:10
num = k/10;
den = [1 k/10];
H(:,:,k) = idtf(num,den);
end

```

The first command preallocates a one-dimensional, 10-element array, \(H\), and fills it with empty idtf models.

The first two dimensions of a model array are the output and input dimensions. The remaining dimensions are the array dimensions. \(\mathrm{H}(:,:, k)\) represents the \(k^{\text {th }}\) model in the array. Thus, the for loop replaces the \(k^{\text {th }}\) entry in the array with a transfer function whose coefficients are initialized with \(a=k / 10\).

\section*{See Also}
getcov|getpar|idfrd|idpoly|idproc|idss|oe|tfdata|tfest|translatecov

\section*{Topics}
"Dynamic System Models"
"What are Transfer Function Models?"
"Estimate Transfer Function Models With Prior Knowledge of Model Structure and Constraints"
Introduced in R2012a

\section*{ifft}

Transform iddata objects from frequency to time domain

\section*{Syntax}
```

dat = ifft(Datf)

```

\section*{Description}
ifft transforms a frequency-domain iddata object to the time domain. It requires the frequencies on Datf to be equally spaced from frequency 0 to the Nyquist frequency. This means that if there are N frequencies in Datf and the sample time is Ts, then

Datf.Frequency = [0:df:F], where \(F\) is pi/Ts if \(N\) is odd and \(F=p i / T s *(1-1 / N)\) if \(N\) is even.

\section*{See Also}
fft|iddata

\section*{Introduced in R2007a}

\section*{impulse}

Impulse response plot of dynamic system; impulse response data

\section*{Syntax}
```

impulse(sys)
impulse(sys,Tfinal)
impulse(sys,t)
impulse(sys1,sys2,...,sysN)
impulse(sys1,sys2,...,sysN,Tfinal)
impulse(sys1,sys2,...,sysN,t)
[y,t] = impulse(sys)
[y,t] = impulse(sys,Tfinal)
y = impulse(sys,t)
[y,t,x] = impulse(sys)
[y,t,x,ysd] = impulse(sys)

```

\section*{Description}
impulse calculates the unit impulse response of a dynamic system model. For continuous-time dynamic systems, the impulse response is the response to a Dirac input \(\delta(t)\). For discrete-time systems, the impulse response is the response to a unit area pulse of length Ts and height 1/Ts, where Ts is the sample time of the system. (This pulse approaches \(\delta(t)\) as Ts approaches zero.) For state-space models, impulse assumes initial state values are zero.
impulse(sys) plots the impulse response of the dynamic system model sys. This model can be continuous or discrete, and SISO or MIMO. The impulse response of multi-input systems is the collection of impulse responses for each input channel. The duration of simulation is determined automatically to display the transient behavior of the response.
impulse(sys,Tfinal) simulates the impulse response from \(t=0\) to the final time \(t=T f i n a l\). Express Tfinal in the system time units, specified in the TimeUnit property of sys. For discretetime systems with unspecified sample time (Ts = -1), impulse interprets Tfinal as the number of sampling periods to simulate.
impulse(sys,t) uses the user-supplied time vector \(t\) for simulation. Express \(t\) in the system time units, specified in the TimeUnit property of sys. For discrete-time models, \(t\) should be of the form Ti:Ts:Tf, where Ts is the sample time. For continuous-time models, \(t\) should be of the form Ti:dt:Tf, where \(d t\) becomes the sample time of a discrete approximation to the continuous system (see "Algorithms" on page 1-681). The impulse command always applies the impulse at \(\mathrm{t}=0\), regardless of Ti.

To plot the impulse responses of several models sys1,..., sysN on a single figure, use:
```

impulse(sys1,sys2,...,sysN)
impulse(sys1,sys2,...,sysN,Tfinal)
impulse(sys1,sys2,...,sysN,t)

```

As with bode or plot, you can specify a particular color, linestyle, and/or marker for each system, for example,
impulse(sys1,'y:',sys2,'g--')
See "Plotting and Comparing Multiple Systems" and the bode entry in this section for more details.
When invoked with output arguments:
[y,t] = impulse(sys)
[y,t] = impulse(sys,Tfinal)
y = impulse(sys,t)
impulse returns the output response \(y\) and the time vector \(t\) used for simulation (if not supplied as an argument to impulse). No plot is drawn on the screen. For single-input systems, y has as many rows as time samples (length of t ), and as many columns as outputs. In the multi-input case, the impulse responses of each input channel are stacked up along the third dimension of \(y\). The dimensions of \(y\) are then

For state-space models only:
[y,t,x] = impulse(sys)
(length of \(t) \times\) (number of outputs) \(\times\) (number of inputs)
and \(y(:,:, j)\) gives the response to an impulse disturbance entering the \(j\) th input channel. Similarly, the dimensions of \(x\) are
(length of \(t) \times\) (number of states) \(\times\) (number of inputs)
[ \(y, t, x, y s d]=\) impulse(sys) returns the standard deviation YSD of the response \(Y\) of an identified system SYS. YSD is empty if SYS does not contain parameter covariance information.

\section*{Examples}

\section*{Impulse Response Plot of Second-Order State-Space Model}

Plot the impulse response of the second-order state-space model
\[
\left[\begin{array}{l}
\dot{x}_{1} \\
\dot{x}_{2}
\end{array}\right]=\left[\begin{array}{cc}
-0.5572 & -0.7814 \\
0.7814 & 0
\end{array}\right]\left[\begin{array}{l}
x_{1} \\
x_{2}
\end{array}\right]+\left[\begin{array}{cc}
1 & -1 \\
0 & 2
\end{array}\right]\left[\begin{array}{l}
u_{1} \\
u_{2}
\end{array}\right]
\]
\[
y=\left[\begin{array}{ll}
1.9691 & 6.4493
\end{array}\right]\left[\begin{array}{l}
x_{1} \\
x_{2}
\end{array}\right]
\]
\(a=[-0.5572-0.7814 ; 0.78140] ;\)
\(\mathrm{b}=[1-1 ; 02]\);
c = [1.9691 6.4493];
sys = ss(a,b,c,0);
impulse(sys)


The left plot shows the impulse response of the first input channel, and the right plot shows the impulse response of the second input channel.

You can store the impulse response data in MATLAB arrays by
[y,t] = impulse(sys);
Because this system has two inputs, y is a 3-D array with dimensions
size(y)
ans \(=1 \times 3\)
\(139 \quad 1 \quad 2\)
(the first dimension is the length of t ). The impulse response of the first input channel is then accessed by
```

ch1 = y(:,:,1);

```
size(ch1)
ans \(=1 \times 2\)
\(139 \quad 1\)

\section*{Impulse Data from Identified System}

Fetch the impulse response and the corresponding 1 std uncertainty of an identified linear system .
```

load(fullfile(matlabroot, 'toolbox', 'ident', 'iddemos', 'data', 'dcmotordata'));
z = iddata(y, u, 0.1, 'Name', 'DC-motor');
set(z, 'InputName', 'Voltage', 'InputUnit', 'V');
set(z, 'OutputName', {'Angular position', 'Angular velocity'});
set(z, 'OutputUnit', {'rad','rad/s'});
set(z, 'Tstart', 0, 'TimeUnit', 's');
model = tfest(z,2);
[y,t,~,ysd] = impulse(model,2);
% Plot 3 std uncertainty
subplot(211)
plot(t,y(:,1), t,y(:,1)+3*ysd(:,1),'k:', t,y(:,1)-3*ysd(:,1),'k:')
subplot(212)
plot(t,y(:,2), t,y(:,2)+3*ysd(:,2),'k:', t,y(:,2)-3*ysd(:,2),'k:')

```

\section*{Limitations}

The impulse response of a continuous system with nonzero \(D\) matrix is infinite at \(t=0\). impulse ignores this discontinuity and returns the lower continuity value \(C b\) at \(t=0\).

\section*{Tips}

You can change the properties of your plot, for example the units. For information on the ways to change properties of your plots, see "Ways to Customize Plots" (Control System Toolbox).

\section*{Algorithms}

Continuous-time models are first converted to state space. The impulse response of a single-input state-space model
\[
\begin{aligned}
& \dot{x}=A x+b u \\
& y=C x
\end{aligned}
\]
is equivalent to the following unforced response with initial state \(b\).
\[
\begin{aligned}
& \dot{x}=A x, x(0)=b \\
& y=C x
\end{aligned}
\]

To simulate this response, the system is discretized using zero-order hold on the inputs. The sample time is chosen automatically based on the system dynamics, except when a time vector \(\mathrm{t}=0: \mathrm{dt}: \mathrm{Tf}\) is supplied ( dt is then used as sample time).

\author{
See Also \\ Linear System Analyzer | impulseest | lsim | step \\ Introduced before R2006a
}

\section*{impulseest}

Nonparametric impulse response estimation

\section*{Syntax}
sys = impulseest(data)
sys = impulseest(data,n)
sys = impulseest(data,n,nk)
sys = impulseest( \(\qquad\) , opt)

\section*{Description}
sys = impulseest(data) estimates an impulse response model sys, also known as a finite impulse response (FIR) model, using time-domain or frequency-domain data data. The function uses persistence-of-excitation analysis on the input data to select the model order (number of nonzero impulse response coefficients.)

Use nonparametric impulse response estimation to analyze input/output data for feedback effects, delays and significant time constants.
sys = impulseest(data, \(n\) ) estimates an nth order impulse response model that corresponding to the time range \(0: T \mathrm{Ts}:(\mathrm{n}-1) * \mathrm{Ts}\), where Ts is the data sample time.
sys = impulseest(data, \(\mathrm{n}, \mathrm{nk}\) ) specifies a transport delay of nk samples in the estimated impulse response.
sys = impulseest( __ , opt) specifies estimation options using the options set opt.

\section*{Examples}

\section*{Identify Nonparametric Impulse Response Model from Data}

Estimate a nonparametric impulse response model using data from a hair dryer. The input is the voltage applied to the heater and the output is the heater temperature. Use the first 500 samples for estimation.

Load the data and use the first 500 samples to estimate the model.
```

load dry2
ze = dry2(1:500);
sys = impulseest(ze);

```
ze is an iddata object that contains time-domain data. sys, the identified nonparametric impulse response model, is an idtf model.

Analyze the impulse response of the identified model from time 0 to 1 .
```

h = impulseplot(sys,1);

```


Right-click the plot and select Characteristics > Confidence Region to view the region of statistically zero response. Alternatively, you can use the showConfidence command.
showConfidence(h);


The first significantly nonzero response value occurs at 0.24 seconds, or the third lag. This implies that the transport delay is 3 samples. To generate a model that imposes the 3-lag delay, set the transport delay, which is the third argument, to 3 . You must also set the second argument, order \(n\), to its default value of [] as a placeholder.
```

sys1 = impulseest(ze,[],3);
h1 = impulseplot(sys1,1);
showConfidence(h1);

```


The response is identically zero until 0.24 s .

\section*{Specify Order of FIR Model}

Load the estimation data
load iddata3 z3;
Estimate a 35th order FIR model.
n = 35;
sys = impulseest(z3,n);
You can confirm the model order of sys by displaying the number of terms.
nsys = size(sys.num)
nsys \(=1 \times 2\)
135

Set n to [ ] so that the function automatically determines n . Display the model order.
```

n = [];
sys1 = impulseest(z3,n);
nsys1 = size(sys1.Numerator)
nsys1 = 1\times2
170

```

The model order is 70. The default value for the order is [ ], so setting the order to [] is equivalent to omitting the specification.

\section*{Specify Transport Delay in FIR Model}

Estimate an impulse response model with transport delay of 3 samples.
If you know about the presence of delay in the input/output data in advance, use the delay value as a transport delay for impulse response estimation.

Generate data that contains a 3 -sample input-to-output lag.
Create a random input signal. Construct an idpoly model that includes three sample delays that you implement by using three leading zeros in the B polynomial.
```

u = rand(100,1);
A = [1 . 1 .4];
B = [0 0 0 4 -2];
C = [lllll
sys = idpoly(A,B,C);

```

Simulate the model response \(y\) to the noise signal, using the AddNoise option and a sample time of 1s. Encapsulate y in an iddata object.
```

opt = simOptions('AddNoise',true);
y = sim(sys,u,opt);
data = iddata(y,u,1);

```

Estimate and plot a 20th order model with no transport delay.
```

n = 20;
model1 = impulseest(data,n);
impulseplot(model1);

```


The plot shows that the impulse response includes nonzero samples during the 3-second delay period. Estimate a model with a 3-sample transport delay.
nk \(=3\);
model2 = impulseest(data,n,nk);
impulseplot(model2)


The first three samples are identically 0 .

\section*{Obtain Regularized Estimate of Impulse Response Model}

Obtain regularized estimates of impulse response model using the regularizing kernel estimation option.

Estimate a model using regularization. impulseest performs regularized estimates by default, using the tuned and correlated kernel ('TC ').
```

load iddata3 z3;
sys1 = impulseest(z3);

```

Estimate a model with no regularization.
```

opt = impulseestOptions('RegularizationKernel','none');
sys2 = impulseest(z3,opt);

```

Compare the impulse response of both models.
h = impulseplot(sys2,sys1,70);
legend('sys2','sys1')


As the plot shows, using regularization makes the response smoother.
Plot the confidence interval.
showConfidence(h);


The uncertainty in the computed response is reduced at larger lags for the model using regularization. Regularization decreases variance at the price of some bias. The tuning of the 'TC' regularization is such that the variance error dominates the overall error.

\section*{Use Regularized Impulse Response Model to Estimate State-Space Model}

Load the estimation data.
load regularizationExampleData eData;
Recreate the transfer function model that was used for generating the estimation data (true system).
```

num = [0.02008 0.04017 0.02008];
den = [1 -1.561 0.6414];
Ts = 1;
trueSys = idtf(num,den,Ts);

```

Obtain a regularized impulse response (FIR) model with an order of 70.
```

opt = impulseestOptions('RegularizationKernel','DC');

```
m0 = impulseest(eData,70,opt);

Convert the model into a state-space model and reduce the model order.
m1 = idss(m0);
m1 = balred(m1,15);
Estimate a second state-space model directly from eData by using regularized reduction of an ARX model.
m2 = ssregest(eData, 15);
Compare the impulse responses of the true system and the estimated models.
```

impulse(trueSys,m1,m2,50);

```
legend('trueSys','m1','m2');


The three model responses are similar.

\section*{Test Measured Data for Feedback Effects}

Use the empirical impulse response to measured data to determine whether the data includes feedback effects. Feedback effects may be present when the impulse response includes statistically significant response values for negative time values.

Compute the noncausal impulse response using a fourth-order prewhitening filter and no regularization, automatic order selection, and negative lag.
```

load iddata3 z3;
opt = impulseestOptions('pw',4,'RegularizationKernel','none');
sys = impulseest(z3,[],'negative',opt);

```
sys is a noncausal model containing response values for negative time.
Analyze the impulse response of the identified model.
```

h = impulseplot(sys);

```


View the statistically zero-response region by right-clicking on the plot and selecting Characteristics > Confidence Region. Alternatively, you can use the showConfidence command.
showConfidence(h);


The large response value at \(\mathrm{t}=0\) (zero lag) suggests that the data comes from a process containing feedthrough. That is, the input affects the output instantaneously. The large response value could also indicate direct feedback, such as proportional control without some delay so that \(y(t)\) partly determines \(u(t)\).

Other indications of feedback in the data are the significant response values such as those at -7 s and -9 s .

\section*{Compute Impulse Response on Frequency Response Data}

Compute an impulse response model for frequency response data.
Load the frequency response data, which contains measured amplitude AMP and phase PHA for the frequency vector \(W\).
load demofr;
Create the complex frequency response \(z f r\) and encapsulate it in an idfrd object that has a sample time of 0.1 s . Plot the data.
```

zfr = AMP.*exp(1i*PHA*pi/180);
Ts = 0.1;
data = idfrd(zfr,W,Ts);

```

Estimate an impulse response model from data and plot the response.
sys = impulseest(data);
impulseplot(sys)


\section*{Compare Identified Nonparametric and Parametric Models}

Identify parametric and nonparametric models for a data set, and compare their step response.
Estimate the impulse response model sys 1 (nonparametric) and state-space model sys 2 (parametric) using the estimation data set \(z 1\).
load iddatal z1;
sys1 = impulseest(z1);
sys2 = ssest(z1,4);
sys1 is a discrete-time identified transfer function model. sys2 is a continuous-time identified statespace model.

Compare the step responses for sys1 and sys2.
step(sys1,'b',sys2,'r');
legend('impulse response model','state-space model');


\section*{Input Arguments}

\section*{data - Estimation data}
iddata object|idfrd object| frd object
Estimation data, specified as an iddata object, an idfrd, or an frd object, with at least one input signal and a nonzero sample time.

For time-domain estimation, specify data as an iddata object containing the input and output signal values.

For frequency-domain estimation, specify data as one of the following:
- Frequency response data (idfrd object or frd object)
- iddata object with properties specified as follows:
- InputData - Fourier transform of the input signal
- OutputData - Fourier transform of the output signal
- Domain - 'Frequency'

\section*{n - Order of FIR model}
[ ] (default) | positive integer | matrix
Order of the FIR model, specified as a positive integer, [ ], or a matrix.
- If data contains a single input channel and output channel, or if you want to apply the same model order to all input/output pairs, specify n as a positive integer.
- If data contains \(N_{u}\) input channels and \(N_{y}\) output channels, and you want to specify individual model orders for the input/output pairs, specify n as an \(N_{y}\)-by- \(N_{u}\) matrix of positive integers, such that \(N(i, j)\) represents the length of impulse response from input \(j\) to output \(i\).
- If you want the function to determine the order automatically, specify n as []. The software uses persistence-of-excitation analysis on the input data to select the order.

Example: sys \(=\) impulseest(data,70) estimates an impulse response model of order 70.

\section*{nk - Transport delay}
zero matrix (default)|'negative' | \(0|1|\) scalar integer | matrix
Transport delay in the estimated impulse response, specified as a scalar integer, 'negative ', or an \(N_{y}\)-by- \(N_{u}\) matrix, where \(N_{y}\) is the number of outputs and \(N_{u}\) is the number of inputs. The impulse response (input \(j\) to output i) coefficients correspond to the time span nk(i,j)*Ts : Ts : ( \(n(i j)+n k(i, j)-1) * T s\).
- If you know the value of the transport delay, specify nk as a scalar integer or a matrix of scalar integers.
- If you do not know the delay value, specify nk as 0 . Once you have estimated the impulse response, you can determine the true delay from the insignificant impulse response values in the beginning portion of the response. For an example of finding true delay, see "Identify Nonparametric Impulse Response Model from Data" on page 1-682.
- To generate the impulse response coefficients for negative time values, which is useful for feedback analysis, use a negative integer. If you specify a negative value, the value must be the same across all output channels. You can also specify nk as 'negative' to automatically pick negative lags for all input/output channels of the model. For an example of using negative time values, see "Test Measured Data for Feedback Effects" on page 1-691.
- To create a system whose leading numerator coefficient is zero, specify nk as 1.

The function stores positive values of nk greater than 1 in the IODelay property of sys (sys.IODelay \(=\max (n k-1,0)\) ), and negative values in the InputDelay property.

\section*{opt - Estimation options}
impulseestOptions option set
Estimation options, specified as an impulseestOptions option set, that specify the following:
- Prefilter order
- Regularization algorithm
- Input and output data offsets
- Advanced options such as structure

Use impulseest0ptions to create the options set.

\section*{Output Arguments}

\section*{sys - Estimated impulse response model}
idtf object

Estimated impulse response model, returned as an idtf model that encapsulates an FIR model.
Information about the estimation results and options used is stored in the Report property of the model. Report has the following fields:
\begin{tabular}{|c|c|}
\hline Report Field & Description \\
\hline Status & Summary of the model status, which indicates whether the model was created by construction or obtained by estimation. \\
\hline Method & Estimation command used. \\
\hline \multirow[t]{10}{*}{Fit} & Quantitative assessment of the estimation, returned as a structure. See "Loss Function and Model Quality Metrics" for more information on these quality metrics. The structure has the following fields: \\
\hline & Field Description \\
\hline & \begin{tabular}{|l|l|}
\hline FitP & \begin{tabular}{l} 
Normalized root mean squared error \\
erce \\
nt
\end{tabular} \\
\begin{tabular}{ll} 
(NRMSE) measure of how well the \\
response of the model fits the estimation \\
data, expressed as the percentage fit \(=\) \\
\(100(1-N R M S E)\).
\end{tabular}
\end{tabular} \\
\hline & Loss Value of the loss function when the Fcn estimation completes. \\
\hline & \begin{tabular}{|l|l|}
\hline MSE & \begin{tabular}{l} 
Mean squared error (MSE) measure of \\
how well the response of the model fits \\
the estimation data.
\end{tabular} \\
\hline
\end{tabular} \\
\hline & FPE Final prediction error for the model. \\
\hline & \begin{tabular}{|l|l}
\hline AIC & \begin{tabular}{l} 
Raw Akaike Information Criteria (AIC) \\
measure of model quality.
\end{tabular} \\
\hline
\end{tabular} \\
\hline & AICc Small sample-size corrected AIC. \\
\hline & nAIC Normalized AIC. \\
\hline & BIC Bayesian Information Criteria (BIC). \\
\hline Parameters & Estimated values of model parameters. \\
\hline OptionsUsed & Option set used for estimation. If no custom options were configured, this is a set of default options. See impulseestOptions for more information. \\
\hline RandState & State of the random number stream at the start of estimation. Empty, [ ], if randomization was not used during estimation. For more information, see rng. \\
\hline
\end{tabular}
\begin{tabular}{|c|c|c|}
\hline Report Field & \multicolumn{2}{|l|}{Description} \\
\hline \multirow[t]{9}{*}{DataUsed} & \multicolumn{2}{|l|}{Attributes of the data used for estimation, returned as a structure with the following fields:} \\
\hline & Field & Description \\
\hline & Name & Name of the data set. \\
\hline & Type & Data type. \\
\hline & Leng th & Number of data samples. \\
\hline & Ts & Sample time. \\
\hline & Inte rSam ple & \begin{tabular}{l}
Input intersample behavior, returned as one of the following values: \\
- 'zoh' - Zero-order hold maintains a piecewise-constant input signal between samples. \\
- 'foh ' - First-order hold maintains a piecewise-linear input signal between samples. \\
- 'bl' - Band-limited behavior specifies that the continuous-time input signal has zero power above the Nyquist frequency.
\end{tabular} \\
\hline & Inpu t0ff set & Offset removed from time-domain input data during estimation. For nonlinear models, it is []. \\
\hline & Outp ut0f fset & Offset removed from time-domain output data during estimation. For nonlinear models, it is []. \\
\hline
\end{tabular}

For more information on using Report, see "Estimation Report".

\section*{Tips}
- To view the impulse or step response of sys, use either impulseplot or stepplot, respectively.
- A significant value of the impulse response of sys for negative time values indicates the presence of feedback in the data.
- To view the region of insignificant impulse response (statistically zero) in a plot, right-click on the plot and select Characteristics > Confidence Region. A patch depicting the zero-response region appears on the plot. The impulse response at any time value is significant only if it lies outside the zero response region. The level of significance depends on the number of standard deviations specified in showConfidence or options in the property editor. A common choice is 3 standard deviations, which gives \(99.7 \%\) significance.

\section*{Algorithms}

Correlation analysis refers to methods that estimate the impulse response of a linear model, without specific assumptions about model orders.

The impulse response, \(g\), is the system output when the input is an impulse signal. The output response to a general input, \(u(t)\), is the convolution with the impulse response. In continuous time:
\[
y(t)=\int_{-\infty}^{t} g(\tau) u(t-\tau) d \tau
\]

In discrete-time:
\[
y(t)=\sum_{k=1}^{\infty} g(k) u(t-k)
\]

The values of \(g(k)\) are the discrete-time impulse response coefficients.
You can estimate the values from observed input/output data in several different ways. impulseest estimates the first \(n\) coefficients using the least-squares method to obtain a finite impulse response (FIR) model of order \(n\).
impulseest provides several important options for the estimation:
- Regularization - Regularize the least-squares estimate. With regularization, the algorithm forms an estimate of the prior decay and mutual correlation among \(g(k)\), and then merges this prior estimate with the current information about \(g\) from the observed data. This approach results in an estimate that has less variance but also some bias. You can choose one of several kernels to encode the prior estimate.

This option is essential because the model order \(n\) can often be quite large. In cases where there is no regularization, \(n\) can be automatically decreased to secure a reasonable variance.

Specify the regularizing kernel using the RegularizationKernel Name-Value pair argument of impulseest0ptions.
- Prewhitening - Prewhiten the input by applying an input-whitening filter of order PW to the data. Use prewhitening when you are performing unregularized estimation. Using a prewhitening filter minimizes the effect of the neglected tail ( \(k>n\) ) of the impulse response. To achieve prewhitening, the algorithm:

1 Defines a filter A of order PW that whitens the input signal \(u\) :
\(1 / A=A(u) e\), where \(A\) is a polynomial and \(e\) is white noise.
2 Filters the inputs and outputs with A:
\[
u f=A u, y f=A y
\]

3 Uses the filtered signals uf and yf for estimation.
Specify prewhitening using the PW name-value pair argument of impulseest0ptions.
- Autoregressive Parameters - Complement the basic underlying FIR model by NA autoregressive parameters, making it an ARX model.
\[
y(t)=\sum_{k=1}^{n} g(k) u(t-k)-\sum_{k=1}^{N A} a_{k} y(t-k)
\]

This option gives both better results for small \(n\) values and allows unbiased estimates when data are generated in closed loop. impulseest uses \(\mathrm{NA}=5\) for \(\mathrm{t}>0\) and \(\mathrm{NA}=0\) (no autoregressive component) for \(t<0\).
- Noncausal effects - Include response to negative lags. Use this option if the estimation data includes output feedback:
\[
u(t)=\sum_{k=0}^{\infty} h(k) y(t-k)+r(t)
\]
where \(h(k)\) is the impulse response of the regulator and \(r\) is a setpoint or disturbance term. The algorithm handles the existence and character of such feedback \(h\), and estimates \(h\) in the same way as \(g\) by simply trading places between \(y\) and \(u\) in the estimation call. Using impulseest with an indication of negative delays, \(\mathrm{mi}=\operatorname{impulseest(data,~} n k, n b), n k<0\) returns a model mi with an impulse response
\[
[h(-n k), h(-n k-1), \ldots, h(0), g(1), g(2), \ldots, g(n b+n k)]
\]
that has an alignment that corresponds to lags \([n k, n k+1, . ., 0,1,2, \ldots, n b+n k]\). The algorithm achieves this alignment because the input delay (InputDelay) of model mi is nk.

For a multi-input multi-output system, the impulse response \(g(k)\) is an \(n y\)-by-nu matrix, where \(n y\) is the number of outputs and \(n u\) is the number of inputs. The \(i-j\) element of the matrix \(g(k)\) describes the behavior of the \(i\) th output after an impulse in the \(j\) th input.

\section*{See Also}
cra|idtf|impulse|impulseestOptions|impulseplot|spa|step

\section*{Topics}
"What Is Time-Domain Correlation Analysis?"

\section*{Introduced in R2012a}

\section*{impulseestOptions}

Options set for impulseest

\section*{Syntax}
options = impulseestOptions
options = impulseestOptions(Name,Value)

\section*{Description}
options = impulseestOptions creates a default options set for impulseest.
options = impulseestOptions(Name,Value) creates an options set with the options specified by one or more Name, Value pair arguments.

\section*{Input Arguments}

\section*{Name-Value Pair Arguments}

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

\section*{RegularizationKernel}

Regularizing kernel, used for regularized estimates of impulse response for all input-output channels. Regularization reduces variance of estimated model coefficients and produces a smoother response by trading variance for bias. For more information, see [1].

Regularization is specified as one of the following values:
- 'TC' - Tuned and correlated kernel
- 'none' - No regularization is used
- 'CS' - Cubic spline kernel
- 'SE' - Squared exponential kernel
- 'SS' - Stable spline kernel
- 'HF ' - High frequency stable spline kernel
- 'DI ' - Diagonal kernel
- 'DC' - Diagonal and correlated kernel

Default: 'TC'

\section*{PW}

Order of the input prewhitening filter. Must be one of the following:
- 'auto' - Uses a filter of order 10 when RegularizationKernel is ' none'; otherwise, 0.
- Nonnegative integer

Use a nonzero value of prewhitening only for unregularized estimation (RegularizationKernel is 'none').

Default: 'auto'

\section*{InputOffset}

Input signal offset level of time-domain estimation data. Must be one of the following:
- An Nu-element column vector, where Nu is the number of inputs. For multi-experiment data, specify a Nu-by-Ne matrix, where Ne is the number of experiments. The offset value InputOffset \((\mathrm{i}, \mathrm{j})\) is subtracted from the \(\mathrm{i}^{\text {th }}\) input signal of the \(\mathrm{j}^{\text {th }}\) experiment.
- [] - No offsets.

\section*{Default: []}

\section*{OutputOffset}

Output signal offset level of time-domain estimation data. Must be one of the following:
- An Ny-element column vector, where Ny is the number of outputs. For multi-experiment data, specify a Ny-by-Ne matrix, where Ne is the number of experiments. The offset value OputOffset \((i, j)\) is subtracted from the \(i^{\text {th }}\) output signal of the \(j^{\text {th }}\) experiment.
- [] - No offsets.

Default: []

\section*{Advanced}

Structure, used during regularized estimation, with the following fields:
- MaxSize - Maximum allowable size of Jacobian matrices formed during estimation. Specify a large positive number.

Default: 250e3
- SearchMethod - Search method for estimating regularization parameters, specified as one of the following values:
- 'fmincon': Trust-region-reflective constrained minimizer. In general, 'fmincon' is better than 'gn' for handling bounds on regularization parameters that are imposed automatically during estimation.
- 'gn': Quasi-Newton line search.

SearchMethod is used only when RegularizationKernel is not 'none'.
Default: 'fmincon'
- AROrder - Order of the AR-part in the model from input to output. Specify as a positive integer.

An order>0 allows more accurate models of the impulse response in case of feedback and nonwhite output disturbances.

Default: 5
- FeedthroughInSys - Specify whether the impulse response value at zero lag must be attributed to feedthrough in the system (true) or to feedback effects (false). Applies only when you compute the response values for negative lags.

Default: false

\section*{Output Arguments}

\section*{options}

Option set containing the specified options for impulseest.

\section*{Examples}

\section*{Create Default Options Set for Impulse Response Estimation}

Create a default options set for impulseest.
```

options = impulseestOptions;

```

\section*{Specify Regularizing Kernel and Prewhitening Options for Impulse Response Estimation}

Specify 'HF' regularizing kernel and order of prewhitening filter for impulseest.
```

options = impulseestOptions('RegularizationKernel','HF','PW',5);

```

Alternatively, use dot notation to specify these options.
```

options = impulseestOptions;
options.RegularizationKernel = 'HF';
options.PW = 5;

```

\section*{Tips}
- A linear model cannot describe arbitrary input-output offsets. Therefore, before using the data, you must either detrend it or remove the levels using Input0ffset and Output0ffset. You can reintroduce the removed data during simulations by using the Input0ffset and Output0ffset simulation options. For more information, see sim0ptions.
- Estimating the impulse response by specifying either Input0ffset, Output0ffset or both is equivalent to detrending the data using getTrend and detrend. For example:
```

opt = impulseestOptions('InputOffset',in off,'OutputOffset',out off);

```
impulseest(data,opt);
is the same as:
Tr = getTrend(data),
Tr.InputOffset = in_off
TR.OutputOffset = out_off
```

dataT = detrend(data,Tr)
impulseest(dataT)

```

\section*{Compatibility Considerations}

\section*{Renaming of Estimation and Analysis Options}

The names of some estimation and analysis options were changed in R2018a. Prior names still work. For details, see the R2018a release note "Renaming of Estimation and Analysis Options".

\section*{References}
[1] T. Chen, H. Ohlsson, and L. Ljung. "On the Estimation of Transfer Functions, Regularizations and Gaussian Processes - Revisited", Automatica, Volume 48, August 2012.

\section*{See Also}
impulseest

Introduced in R2012b

\section*{impulseplot}

Plot impulse response and return plot handle

\section*{Syntax}
```

impulseplot(sys)
impulseplot(sys,Tfinal)
impulseplot(sys,t)
impulseplot(sys1,sys2,...,sysN)
impulseplot(sys1,sys2,...,sysN,Tfinal)
impulseplot(sys1,sys2,...,sysN,t)
impulseplot(AX,...)
impulseplot(..., plotoptions)
h = impulseplot(...)

```

\section*{Description}
impulseplot plots the impulse response of the dynamic system model sys. For multi-input models, independent impulse commands are applied to each input channel. The time range and number of points are chosen automatically. For continuous systems with direct feedthrough, the infinite pulse at \(\mathrm{t}=0\) is disregarded. impulseplot can also return the plot handle, h . You can use this handle to customize the plot with the getoptions and setoptions commands. Type
help timeoptions
for a list of available plot options.
impulseplot (sys) plots the impulse response of the LTI model without returning the plot handle.
impulseplot(sys,Tfinal) simulates the impulse response from \(t=0\) to the final time \(t=\) Tfinal. Express Tfinal in the system time units, specified in the TimeUnit property of sys. For discrete-time systems with unspecified sample time ( \(\mathrm{Ts}=-1\) ), impulseplot interprets Tfinal as the number of sampling intervals to simulate.
impulseplot (sys,t) uses the user-supplied time vector \(t\) for simulation. Express \(t\) in the system time units, specified in the TimeUnit property of sys. For discrete-time models, \(t\) should be of the form Ti:Ts:Tf, where Ts is the sample time. For continuous-time models, \(t\) should be of the form Ti:dt:Tf, where dt becomes the sample time of a discrete approximation to the continuous system (see impulse). The impulseplot command always applies the impulse at \(\mathrm{t}=0\), regardless of Ti .

To plot the impulse response of multiple LTI models sys 1, sys \(2, \ldots\) on a single plot, use:
```

impulseplot(sys1,sys2,...,sysN)
impulseplot(sys1,sys2,...,sysN,Tfinal)
impulseplot(sys1,sys2,...,sysN,t)

```

You can also specify a color, line style, and marker for each system, as in
```

impulseplot(sys1,'r',sys2,'y--',sys3,'gx')

```
```

impulseplot(AX,...) plots into the axes with handle AX.
impulseplot(..., plotoptions) plots the impulse response with the options specified in
plotoptions. Type
help timeoptions
for more detail.
h = impulseplot(...) plots the impulse response and returns the plot handle h.

```

\section*{Examples}

\section*{Example 1}

Normalize the impulse response of a third-order system.
```

sys = rss(3);
h = impulseplot(sys);
% Normalize responses
setoptions(h,'Normalize','on');

```

\section*{Example 2}

Plot the impulse response and the corresponding 1 std "zero interval" of an identified linear system.
```

load(fullfile(matlabroot, 'toolbox', 'ident', 'iddemos', 'data', 'dcmotordata'));
z = iddata(y, u, 0.1, 'Name', 'DC-motor');
set(z, 'InputName', 'Voltage', 'InputUnit', 'V');
set(z, 'OutputName', {'Angular position', 'Angular velocity'});
set(z, 'OutputUnit', {'rad', 'rad/s'});
set(z, 'Tstart', 0, 'TimeUnit', 's');
model= n4sid(z,4,n4sidOptions('Focus', 'simulation'));
h = impulseplot(model,2);
showConfidence(h);

```

\section*{Tips}

You can change the properties of your plot, for example the units. For information on the ways to change properties of your plots, see "Ways to Customize Plots" (Control System Toolbox).

\author{
See Also \\ getoptions |impulse | setoptions | showConfidence
}

Introduced in R2012a

\section*{init}

Set or randomize initial parameter values

\section*{Syntax}
\(\mathrm{m}=\) init(m0)
\(m=\operatorname{init}(m 0, R, p a r s, s p)\)

\section*{Description}
\(\mathrm{m}=\) init( m 0 ) randomizes initial parameter estimates for model structures m 0 for any linear or nonlinear identified model. It does not support idnlgrey models. \(m\) is the same model structure as \(m 0\), but with a different nominal parameter vector. This vector is used as the initial estimate by pem.
\(m=\operatorname{init}(m 0, R, p a r s, s p)\) randomizes parameters around pars with variances given by the row vector R. Parameter number \(k\) is randomized as pars \((k)+e^{*} \operatorname{sqrt}(R(k))\), where e is a normal random variable with zero mean and a variance of 1 . The default value of \(R\) is all ones, and the default value of pars is the nominal parameter vector in m 0 .

Only models that give stable predictors are accepted. If \(s p=\) ' \(b\) ', only models that are both stable and have stable predictors are accepted.
\(s p=\) ' \(s\) ' requires stability only of the model, and \(s p=\) ' \(p\) ' requires stability only of the predictor. \(s p=\) ' \(p\) ' is the default.

Sufficiently free parameterizations can be stabilized by direct means without any random search. To just stabilize such an initial model, set \(R=0\). With \(R>0\), randomization is also done.

For model structures where a random search is necessary to find a stable model/predictor, a maximum of 100 trials is made by init. It can be difficult to find a stable predictor for high-order systems by trial and error.

\section*{See Also}
idnlarx|idnlhw|rsample|simsd

\section*{Introduced before R2006a}

\section*{initialCondition}

Initial condition representation for linear time-invariant systems

\section*{Description}

An initialCondition object encapsulates the initial-condition information for a linear timeinvariant (LTI) model. The object generalizes the numeric vector representation of the initial states of a state-space model so that the information applies to linear models of any form-transfer functions, polynomial models, or state-space models.

You can estimate and retrieve initial conditions when you identify a linear model using commands such as tfest or compare model response to measured input/output data using compare. The software estimates the initial condition value by minimizing the simulation or prediction error against the measured output data. You can then apply those initial conditions in a subsequent simulation, using commands such as sim or predict, to confirm model performance with respect to the same measurement data. Use the initialCondition command to create an initialCondition object from a state-space model specification or from any LTI model of a free response.

The initialCondition object can also be seen as a representation of the free response of a linear model. The simulation functions use this information to compute the model response in the following manner:

1 Compute the forced response of the model to the input signal. The forced response is the standard simulation output when there are no specified initial conditions.
2 Compute the impulse response of the model and scale the result to generate the free response of the model to the specified initial conditions.
3 Add the forced response and the free response together to form the total system response.
The figure illustrates this process.


For continuous systems ( \(T s=0\) ), the free response \(G(s)\) for the initial state vector \(x_{0}\) is
\[
G(s)=C(s I-A)^{-1} x_{0}
\]

Here, \(C\) is equivalent to the state-space measurement matrix \(C\) and \(A\) is equivalent to the state-space state matrix \(A\).

For discrete systems (Ts \(>0\) ), the free response \(G(z)\) is
\[
G(z)=z C(z I-A)^{-1} x_{0}
\]

The initialCondition object represents the free response in state-space form. The object is a structure with properties containing the state-space \(A\) and \(C\) matrices and the initial state vector \(x_{0}\). For idtf and idpoly models, using an initialCondition object is the only way to represent and use initial conditions for simulation. For idss models, you can use either an initialCondition object or a numeric initial state vector. When you obtain initial conditions ic for multiexperiment data, ic is an object array, with one initialCondition object for each experiment.

\section*{Creation}

You can obtain an initialCondition object in one of four ways.
- Model estimation - Specify that the estimation function return the estimated initial condition that corresponds to the estimation input/output data. For example, you can use the following command to obtain the estimated initial condition ic for a transfer function model sys that is estimated with input/output data \(z\).
```

[sys,ic] = tfest(z,2,1)

```

For an example, see "Obtain Estimated Initial Conditions" on page 1-713.
- Model-to-data comparison using any input/output data - Specify that compare return the estimated initial condition that the function estimates internally to support the fit assessment. For example, you can use the following command to obtain the initial condition ic for the linear model sys when determining the fit against input/output data \(z\). yp is the simulated or predicted model output.
[yp,fit,ic] = compare(z,sys)
For an example, see "Obtain Initial Conditions for New Data" on page 1-717.
- Direct construction - Use the initialCondition command to encapsulate the state-space form of a free-response model in an initialCondition object.
ic \(=\) initialCondition (A, X0, C,Ts)
For an example, see "Construct initialCondition Object from State-Space Model" on page 1-723.
- Free-response model conversion - Use the initialCondition command to convert an LTI freeresponse model into an initialCondition object.
```

ic = initialCondition(G)

```

For an example, see "Convert Free-Response Model to initialCondition Object" on page 1-726.
For information on functions you can use to extract information from or transform initialCondition objects, see "Object Functions" on page 1-711.

\section*{Syntax}
```

ic = initialCondition(A,X0,C,Ts)
ic = initialCondition(G)

```

\section*{Description}
ic \(=\) initialCondition \((A, X 0, C, T s)\) creates an initialCondition object that represents the free response to an initial condition, expressed in state-space form, of an LTI model.
\[
\begin{aligned}
& d x=A x \\
& y=C x \\
& x(0)=x_{0}
\end{aligned}
\]
ic stores this model in the form of properties on page 1-711. A and C correspond to a state-space realization of the model, \(X 0\) to the initial state vector \(x_{0}\), and Ts to the sample time. You can use ic to specify initial conditions when simulating any type of LTI system.
ic = initialCondition(G) creates an initialCondition object corresponding to a linear model \(G\) of the free response.

\section*{Input Arguments}

G - Free-response model
LTI model
Free-response model, specified as an LTI model with no inputs. In the continuous-time case, G must be strictly proper. In the discrete-time case, \(G\) must be biproper. For an example of using a free-
response model to obtain an initialCondition object, see "Convert Free-Response Model to initialCondition Object" on page 1-726.

\section*{Properties}

\section*{A - A matrix of state-space realization of LTI free response}
numeric matrix
A matrix of the state-space realization of the LTI free response, specified as an \(N_{x}\)-by- \(N_{x}\) numeric matrix, where \(N_{x}\) is the number of states. For an example of using this property, see "Obtain Estimated Initial Conditions" on page 1-713.

\section*{X0 - Initial states of state-space realization of LTI free response}
numeric vector
Initial states of the state-space realization of the LTI free response, specified as a numeric vector of length \(N_{x}\). For an example of using this property, see "Obtain Estimated Initial Conditions" on page 1713.

\section*{C - C matrix of state-space realization of LTI free response \\ numeric matrix}
\(C\) matrix of the state-space realization of the LTI free response, specified as an \(N_{y}\)-by- \(N_{x}\) numeric matrix, where \(N_{y}\) is the number of outputs. For an example of using this property, see "Obtain Estimated Initial Conditions" on page 1-713.

\section*{Ts - Sample time}

0|-1 | positive scalar
Sample time of the LTI free response, specified as one of the following:
- Continuous-time model - 0
- Discrete-time model with a specified sampling time - Positive scalar representing the sampling period expressed in the unit specified by the TimeUnit property of the model
- Discrete-time model with unspecified sample time --1

The sample time of an initialCondition object is the same as for the dynamic system model that the object corresponds to.

\section*{Object Functions}

Functions applicable to initialCondition objects are those that can return, use, or convert the objects.
\begin{tabular}{|c|c|c|c|}
\hline Initial Condition (IC) Role & LTI Function Type & Syntax Example & Example Links \\
\hline Return estimated IC objects & All estimation functions, compare & \[
\begin{aligned}
& \text { [sys,ic] }= \\
& \text { tfest(data, } 2,1 \text { ) }
\end{aligned}
\] & \begin{tabular}{l}
"Obtain Estimated Initial Conditions" on page 1-713 \\
"Obtain Initial Conditions for Multiexperiment Data" on page 1-720 \\
"Obtain Initial Conditions for New Data" on page 1-717
\end{tabular} \\
\hline Use IC objects for model response & Option sets for model response functions & ```
opt =
simOptions('Initia
lCondition',ic)
``` & "Apply Initial Conditions in Simulation" on page 1-715 \\
\hline Convert IC objects into Dynamic System Models (DSMs) & DSM object functions & \(\mathrm{g}=\mathrm{idtf}(\mathrm{ic})\) & "Visualize Free Response to Initial Condition" on page 1714 \\
\hline Analyze models converted from IC objects & DSM analysis functions & y_g = impulse(g) & "Visualize Free Response to Initial Condition" on page 1714 \\
\hline
\end{tabular}

\section*{Estimate and Return Initial Conditions}
\begin{tabular}{ll} 
tfest & Estimate transfer function \\
procest & Estimate process model using time or frequency data \\
arx & Estimate parameters of ARX, ARIX, AR, or ARI model \\
armax & Estimate parameters of ARMAX, ARIMAX, ARMA, or ARIMA model using time-domain \\
& data \\
bj & Estimate Box-Jenkins polynomial model using time domain data \\
oe & Estimate output-error polynomial model using time-domain or frequency-domain data \\
polyest & Estimate polynomial model using time- or frequency-domain data \\
compare & Compare identified model output and measured output
\end{tabular}

\section*{Use Initial Conditions}
sim Simulate response of identified model
simOptions Option set for sim
predict Predict state and state estimation error covariance at next time step using extended or unscented Kalman filter, or particle filter
predictOptions Option set for predict
pe Prediction error for identified model
peOptions Option set for pe
resid Compute and test residuals
residOptions
compare
compareOptions
Option set for resid
Compare identified model output and measured output
Option set for compare

\section*{Convert Initial Conditions to Dynamic System Model}
idss State-space model with identifiable parameters
idpoly Polynomial model with identifiable parameters
idtf Transfer function model with identifiable parameters

\section*{Analyze Free Model Response using idss Form}
impulse Impulse response plot of dynamic system; impulse response data freqresp Frequency response over grid

\section*{Examples}

\section*{Obtain Estimated Initial Conditions}

Estimate a transfer function model and obtain estimated initial conditions.
Load and plot the data.
```

load iddatalic.mat zli
plot(z1i)

```


The output data does not start at 0 .
Estimate a second-order transfer function sys_tf. Specify that the function return the initial conditions ic.
[sys_tf,ic] = tfest(z1i,2,1);
Examine the contents of ic. ic includes, in state-space form, the free response model defined by matrices A and C, the initial state vector X0, and the sample time Ts.
```

A = ic.A
A = 2\times2
-2.9841 -5.5848
4.0000 0
C = ic.C
C = 1\times2
0.2957 5.2441
x0 = ic.X0
x0 = 2x1
-0.9019
-0.6161
Ts = ic.Ts
Ts = 0

```
ic is specific to the estimation data zli. You can use ic to establish initial conditions when you simulate any LTI model using the input signal from z1i and compare the response with the z1i output signal.

\section*{Visualize Free Response to Initial Condition}

Visualize the free response encapsulated in an initialCondition object by generating an impulse response.

Estimate a transfer function and return the initial condition ic_tf.
```

load iddatalic zli

```
[sys_tf,ic_tf] = tfest(z1i,2,1);
ic_tf
ic_tf =
    initialCondition with properties:
        A: [2×2 double]
        X0: [2x1 double]
        C: [0.2957 5.2441]
        Ts: 0
ic_tf contains the information necessary to compute the free response to an initial condition.

Convert ic_tf into an idss object that can be passed to the impulse function.
ic_tfss = idss(ic_tf);
Create a time vector \(t\) that spans the data set. Compute the impulse response.
\(\mathrm{t}=0: 0.1: 9.9\);
\(\mathrm{t}=\mathrm{t}^{\prime}\);
yimp = impulse(ic_tfss,t);
plot(t,yimp)
title('Free Response to Initial Condition')


The free response is a transient that lasts for about four seconds.

\section*{Apply Initial Conditions in Simulation}

Load the data and estimate a second-order transfer function sys. Return initial conditions in ic.
load iddatalic zli
[sys,ic] = tfest(z1i,2,1);
Simulate sys using the estimation data, but without incorporating the initial condition. Plot the simulated output with the measured output.
y_no_ic = sim(sys,zli);
plot(y_no_ic,zli)
legend('Módel Response','Measured Output')
Input-Output Data


The measured and simulated outputs do not agree at the beginning of the simulation.
Incorporate ic into the simOptions option set opt. Simulate and plot the model response using opt.
```

opt = simOptions('InitialCondition',ic);

```
y_ic = sim(sys,zli,opt);
plot(y_ic,zli);
legend('Model Response','Measured Output')

\section*{Input-Output Data}


The simulation combines the model response to the input signal with the free response to the initial condition. The measured and simulated outputs now have better agreement at the beginning of the simulation.

\section*{Obtain Initial Conditions for New Data}

An estimated initialCondition object is specific to the data from which you estimated it. If you want to simulate your model with new data, such as validation data, you need to estimate a new initial condition for that data. To do so, use the compare command.

Load data and split it into estimation and validation data sets.
```

load iddatal z1
z1_est = z1(1:150);
z1 val = z1(151:300);
plōt(z1_est,z1_val);
legend('Estimation Data','Validation Data')

```

\section*{Input-Output Data}


Examine the start points of each output data set.
```

e0 = z1_est.y(1)
e0 = -0.5872
v0 = z1_val.y(1)
v0 = -7.4390

```

The two data sets have different starting conditions.
Estimate a second-order transfer function model using z1_est. Return the estimated initial conditions in ic_est. Display the X0 property of ic_est. This property represents the estimated initial state vector that the free-response model defined by ic_est.A and ic_est.C responds to.
[sys,ic_est] = tfest(z1_est,2,1);
ic_est.X0
ans \(=2 \times 1\)
-0. 4082
0.0095

You can use ic_est if you want to simulate sys using z1_est. Alternatively, you can use compare, which estimates the initial condition independently. Use compare twice, once to plot the data and once to return the results. Display the initial state vector ic_estc. X0 that compare estimates.
compare(z1_est,sys)

```

[yce,fit,ic_estc] = compare(z1_est,sys);
ic estc.X0
ans = 2×1
-0.4082
0.0095

```

The initial state vector ic_estc. X 0 is identical to ic_est.
Now evaluate the model with the validation data set. Estimate the initial conditions with the validation data.
```

compare(zl_val,sys)

```


You can use ic_val when you simulate sys with the z1_val input signal and compare the model response to the z1_val output signal.

\section*{Obtain Initial Conditions for Multiexperiment Data}

Estimate an initialCondition object array using multiexperiment data.
Load data from two experiments. Merge the two data sets into one multiexperiment data set.
load iddatal z1
load iddata2 z2
z12 = merge(z1,z2)
z12 =
Time domain data set containing 2 experiments.
\begin{tabular}{ccc} 
Experiment & Samples & Sample Time \\
Exp1 & 300 & 0.1 \\
Exp2 & 400 & 0.1 \\
Outputs & Unit (if specified) \\
yl & & \\
\begin{tabular}{c} 
Inputs \\
ul
\end{tabular} & Unit (if specified)
\end{tabular}
plot(z12)

\section*{Input-Output Data}


Estimate the second-order transfer function sys and return the initial conditions in ic.
\(n p=2 ;\)
nz = 1;
[sys,ic] \(=\) tfest(z12,np,nz);
ic
ic=1×2 object
1x2 initialCondition array with properties:
A
X0
C
Ts
ic is an object array. Display the contents of each object.
```

ic(1,1)
ans =
initialCondition with properties:
A: [2x2 double]
X0: [2x1 double]
C: [-0.7814 5.2530]
Ts: 0
ic(1,2)
ans =
initialCondition with properties:
A: [2x2 double]
X0: [2x1 double]
C: [-0.7814 5.2530]
Ts: 0

```

Compare the \(\mathrm{A}, \mathrm{X} 0\), and C properties for each object.
```

A1 = ic(1,1).A
A1 = 2\times2
-3.4824 -5.5785
4.0000 0
A2 = ic(1,2).A
A2 = 2\times2
-3.4824 -5.5785
4.0000 0
C1 = ic(1,1).C
C1 = 1\times2
-0.7814 5.2530
C2 = ic(1,2).C
C2 = 1\times2
-0.7814 5.2530

```
\(X 01=i c(1,1) . X 0\)
X01 = 2×1
```

X02 =ic(1,2).X0
X02 = 2×1
0.3076
-0.0715

```

The A and C matrices are identical. These matrices represent the state-space form of sys. The X0 vectors are different. This difference results from the different initial conditions for the two experiments.

\section*{Construct initialCondition Object from State-Space Model}

Estimate a state-space model and return the initial states. From the model and the initial state vector, construct an initialCondition object that can be used with any linear model.

Load and plot the data.
load iddatalic z1i
plot(z1i)

\section*{Input-Output Data}


Estimate a state-space model and obtain the estimated initial conditions.
First, set the 'InitialState' name-value pair argument in ssest0ptions to 'estimate', which overrides the default setting of 'auto'. The 'estimate' setting always estimates the initial states. The 'auto' setting uses the 'zero' setting if the effect of the initial states on the overall model estimation error is relatively small, and can therefore result in an initial-state vector containing only zeros.
```

opt = ssestOptions;
opt = ssestOptions('InitialState','estimate');

```

Estimate a second-order state-space model sys_ss. Specify the output argument x0 to return the initial state vector. Specify the input argument opt to use your 'InitialState' setting. After estimating, examine \(\times 0\).
```

[sys_ss,x0] = ssest(zli,2,opt);
x0
x0 = 2x1
0.0631
0.0329

```
x 0 is a nonzero initial state vector.
Simulate the model using \(x 0\) and compare the output with the original output data.
To use \(x 0\) as the initial condition, specify the 'InitialCondition' name-value pair argument in sim0ptions as x0.
```

opt = simOptions;
opt = simOptions('InitialCondition',x0);

```

Simulate the model using opt and store the response in xss.
```

xss = sim(sys_ss,zli,opt);

```

Plot the model response with the original output data.
```

t = 0:0.1:19.9;
plot(t',[xss.y zli.y])
legend('ss model','output data')
title('Simulated State-Space Model Using Estimated Initial States')

```


The simulation starts at a point close to the starting point of the data.
With the A and C matrices, x 0 , and the sample time Ts from sys_ss, construct an initialCondition object ic that you can use with a transfer function model.

A = sys_ss.A;
C = sys_ss.C;
Ts = sys ss.Ts;
ic = initialCondition(A, x0,C,Ts)
ic =
initialCondition with properties:
A: [2x2 double]
X0: [2x1 double]
C: [-61.3674 13.4811]
Ts: 0

Estimate a transfer function model and simulate the model using ic as the initial condition. Store the response in xtf.
```

sys tf = tfest(zli,2,1);
opt = simOptions('InitialCondition',ic);
xtf = sim(sys_tf,zli,opt);

```

Plot the model responses xss and xtf together.
```

plot(t',[xss.y xtf.y])
legend('ss model','tf model')
title('Simulated SS and TF Models with Equivalent Initial Conditions')

```


The models track each other closely throughout the simulation.

\section*{Convert Free-Response Model to initialCondition Object}

Obtain the initial conditions when estimating a transfer function model. Convert the initialCondition into a free-response model, and the free-response model back into an initialCondition object.

Load the data and estimate a transfer function model sys. Obtain the estimated initial conditions ic.
```

load iddatalic.mat zli
[sys,ic] = tfest(z1i,2,1);

```

Convert ic into the idtf free-response model g.
g = idtf(ic);
Plot the impulse response of g .
```

impulse(g)
title('Impulse Response of g')

```


Convert g back into the initialCondition object ic1.
ic1 = initialCondition(g);
Plot the impulse response of ic1 by converting ic1 into an idss model.
```

impulse(idss(ic1))

```
title('Impulse Response of icl in idss form')


The impulse responses appear identical.
Compare ic and icl.
ic.A
ans \(=2 \times 2\)
\(\begin{array}{rr}-2.9841 & -5.5848 \\ 4.0000 & 0\end{array}\)
ic1.A
ans \(=2 \times 2\)
\(\begin{array}{rr}-2.9841 & -5.5848 \\ 4.0000 & 0\end{array}\)
ic. X0
ans \(=2 \times 1\)
-0.9019
-0.6161
ic1.X0
```

ans = 2×1
4
0
ic.C
ans = 1\times2
0.2957 5.2441
ic1.C
ans = 1\times2
-0.8745 -1.7215

```

The A matrices of ic and ic1 are identical. The C matrix and the X0 vector are different. There are infinitely many state-space representations possible for a given linear model. The two objects are equivalent, as illustrated by the impulse responses.

\section*{See Also}
compare |impulse|polyest|predict|predict0ptions|sim| simOptions|ssest|tfest

\section*{Topics}
"Apply Initial Conditions when Simulating Identified Linear Models"
"Estimate Initial Conditions for Simulating Identified Models"
Introduced in R2020b

\section*{initialize}

Initialize the state of the particle filter

\section*{Syntax}
```

initialize(pf,numParticles,mean,covariance)
initialize(pf,numParticles,stateBounds)
initialize(

```
\(\qquad\)
``` ,Name, Value)
```


## Description

initialize(pf,numParticles, mean, covariance) initializes a particle filter object with a specified number of particles. The initial states of the particles in the state space are determined by sampling from the multivariate normal distribution with the specified mean and covariance. The number of state variables (NumStateVariables) is retrieved automatically based on the length of the mean vector.
initialize(pf,numParticles,stateBounds) determines the initial location of numParticles particles by sampling from the multivariate uniform distribution with the given stateBounds.
initialize( $\qquad$ ,Name, Value) initializes the particles with additional options specified by one or more Name, Value pair arguments.

## Examples

## Initialize Particle Filter Object for Online State Estimation

To create a particle filter object for estimating the states of your system, create appropriate state transition function and measurement function for the system.

In this example, the functions vdpParticleFilterStateFcn and vdpMeasurementLikelihoodFcn describe a discrete-approximation to van der Pol oscillator with nonlinearity parameter, mu, equal to 1.

Create the particle filter object. Use function handles to provide the state transition and measurement likelihood functions to the object.

```
myPF = particleFilter(@vdpParticleFilterStateFcn,@vdpMeasurementLikelihoodFcn);
```

Initialize the particle filter at state [2; 0] with unit covariance, and use 1000 particles.

```
initialize(myPF, 1000, [2;0], eye(2));
myPF
myPF =
    particleFilter with properties:
                                    NumStateVariables: 2
                            NumParticles: 1000
            StateTransitionFcn: @vdpParticleFilterStateFcn
```

```
MeasurementLikelihoodFcn: @vdpMeasurementLikelihoodFcn
    IsStateVariableCircular: [0 0]
            ResamplingPolicy: [1x1 particleResamplingPolicy]
            ResamplingMethod: 'multinomial'
        StateEstimationMethod: 'mean'
            StateOrientation: 'column'
                    Particles: [2\times1000 double]
                        Weights: [1x1000 double]
                    State: 'Use the getStateEstimate function to see the value.
                StateCovariance: 'Use the getStateEstimate function to see the value.'
```

To estimate the states and state estimation error covariance from the constructed object, use the predict and correct commands.

## Input Arguments

## pf - Particle filter

particleFilter object
Particle filter, specified as a object. See particleFilter for more information.
numParticles - Number of particles used in the filter
scalar
Number of particles used in the filter, specified as a scalar.
Unless performance is an issue, do not use fewer than 1000 particles. A higher number of particles can improve the estimate but sacrifices performance speed, because the algorithm has to process more particles. Tuning the number of particles is the best way to improve the tracking of your particle filter.

## mean - Mean of particle distribution

vector
Mean of particle distribution, specified as a vector. The NumStateVariables property of pf is set based on the length of this vector.

## covariance - Covariance of particle distribution <br> $N$-by- $N$ matrix

Covariance of particle distribution, specified as an $N$-by- $N$ matrix, where $N$ is the value of NumStateVariables property from pf.

## stateBounds - Bounds of state variables

$n$-by-2 matrix
Bounds of state variables, specified as an $n$-by- 2 matrix. The NumStateVariables property of pf is set based on the value of $n$. Each row corresponds to the lower and upper limit of the corresponding state variable. The number of state variables (NumStateVariables) is retrieved automatically based on the number of rows of the stateBounds array.

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Example: ...'StateOrientation','row'

## CircularVariables - Circular variables <br> logical vector

Circular variables, the comma-separated pair consisting of CircularVariables and specified as a logical vector. Each state variable that uses circular or angular coordinates is indicated with a 1 . The length of the vector is equal to the NumStateVariables property of particleFilter.

## StateOrientation - Orientation of states

'column' (default) | 'row'
Orientation of states, specified as the comma-separated pair consisting of StateOrientation as one of these values: 'column' or 'row'. If it is 'column', State property and getStateEstimate method of the object pf returns the states as a column vector, and the Particles property has dimensions NumStateVariables-by-NumParticles. If it is 'row', the states have the row orientation and Particles has dimensions NumParticles-by-NumStateVariables.

## See Also

clone | correct | extendedKalmanFilter | particleFilter|predict| unscentedKalmanFilter

## Topics

"Nonlinear State Estimation Using Unscented Kalman Filter and Particle Filter"
"What Is Online Estimation?"
"Generate Code for Online State Estimation in MATLAB"
Introduced in R2017b

## interp

Interpolate FRD model

## Syntax

isys = interp(sys,freqs)

## Description

isys = interp(sys,freqs) interpolates the frequency response data contained in the FRD model sys at the frequencies freqs. interp, which is an overloaded version of the MATLAB function interp, uses linear interpolation and returns an FRD model isys containing the interpolated data at the new frequencies freqs. If sys is an IDFRD model, the noise spectrum, if non-empty, is also interpolated. The response and noise covariance data, if available, are also interpolated.

You should express the frequency values freqs in the same units as sys.frequency. The frequency values must lie between the smallest and largest frequency points in sys (extrapolation is not supported).

## See Also

frd|freqresp|idfrd

## Introduced in R2012a

## iopzmap

Plot pole-zero map for I/O pairs of model

## Syntax

iopzmap(sys)
iopzmap(sys1,sys2,...)

## Description

iopzmap(sys) computes and plots the poles and zeros of each input/output pair of the dynamic system model sys. The poles are plotted as x's and the zeros are plotted as o's.
iopzmap (sys1, sys2, ...) shows the poles and zeros of multiple models sys1,sys $2, \ldots$ on a single plot. You can specify distinctive colors for each model, as in iopzmap(sys1,'r',sys2,'y',sys3,'g').

The functions sgrid or zgrid can be used to plot lines of constant damping ratio and natural frequency in the $s$ or $z$ plane.

For model arrays, iopzmap plots the poles and zeros of each model in the array on the same diagram.

## Examples

## Pole-Zero Map for MIMO System

Create a one-input, two-output dynamic system.

```
H = [tf(-5 ,[1 -1]); tf([1 -5 6],[1 1 0])];
```

Plot a pole-zero map.
iopzmap(H)

iopzmap generates a separate map for each I/O pair in the system.

## Pole-Zero Map of Identified Model

View the poles and zeros of an over-parameterized state-space model estimated from input-output data. (Requires System Identification Toolbox ${ }^{\mathrm{TM}}$ ).
load iddatal
sys = ssest(z1,6,ssest0ptions('focus','simulation')); iopzmap(sys)


The plot shows that there are two pole-zero pairs that almost overlap, which hints are their potential redundancy.

## Tips

For additional options for customizing the appearance of the pole-zero plot, use iopzplot.

## See Also

iopzplot|pole|pzmap|sgrid|zero|zgrid
Introduced in R2012a

## iopzplot

Plot pole-zero map for I/O pairs and return plot handle

## Syntax

```
h = iopzplot(sys)
iopzplot(sys1,sys2,...)
iopzplot(AX,...)
iopzplot(..., plotoptions)
```


## Description

h = iopzplot(sys) computes and plots the poles and zeros of each input/output pair of the dynamic system model sys. The poles are plotted as x's and the zeros are plotted as o's. It also returns the plot handle h. You can use this handle to customize the plot with the getoptions and setoptions commands. Type
help pzoptions
for a list of available plot options. For more information on the ways to change properties of your plots, see "Ways to Customize Plots" (Control System Toolbox).
iopzplot (sys1, sys2,...) shows the poles and zeros of multiple dynamic system models sys1, sys2, .. . on a single plot. You can specify distinctive colors for each model, as in iopzplot(sys1,'r',sys2,'y',sys3,'g')
iopzplot (AX, ...) plots into the axes with handle AX.
iopzplot(..., plotoptions) plots the poles and zeros with the options specified in plotoptions. Type
help pzoptions
for more detail.
The function sgrid or zgrid can be used to plot lines of constant damping ratio and natural frequency in the s or z plane.

For arrays sys of LTI models, iopzplot plots the poles and zeros of each model in the array on the same diagram.

## Examples

## Change I/O Grouping on Pole/Zero Map

Create a pole/zero map of a two-input, two-output dynamic system.
sys = rss(3,2,2);
h = iopzplot(sys);

Pole-Zero Map


By default, the plot displays the poles and zeros of each I/O pair on its own axis. Use the plot handle to view all I/Os on a single axis.

```
setoptions(h,'IOGrouping','all')
```



## Use Pole-Zero Map to Examine Identified Model

View the poles and zeros of a sixth-order state-space model estimated from input-output data. Use the plot handle to display the confidence intervals of the identified model's pole and zero locations.

```
load iddata1
sys = ssest(z1,6,ssestOptions('focus','simulation'));
h = iopzplot(sys);
showConfidence(h)
```



There is at least one pair of complex-conjugate poles whose locations overlap with those of a complex zero, within the $1-\sigma$ confidence region. This suggests their redundancy. Hence, a lower (4th) order model might be more robust for the given data.

```
sys2 = ssest(z1,4,ssestOptions('focus','simulation'));
h = iopzplot(sys,sys2);
showConfidence(h)
legend('6th-order','4th-order')
axis([-20, 10 -30 30])
```



The fourth-order model sys2 shows less variability in the pole-zero locations.

See Also<br>getoptions |iopzmap|setoptions | showConfidence Introduced in R2012a

## isct

Determine if dynamic system model is in continuous time

## Syntax

bool = isct(sys)

## Description

bool = isct(sys) returns a logical value of 1 (true) if the dynamic system model sys is a continuous-time model. The function returns a logical value of 0 (false) otherwise.

## Input Arguments

## sys

Dynamic system model or array of such models.

## Output Arguments

## bool

Logical value indicating whether sys is a continuous-time model.
bool = 1 (true) if sys is a continuous-time model (sys.Ts = 0). If sys is a discrete-time model, bool = 0 (false).

For a static gain, both isct and isdt return true unless you explicitly set the sample time to a nonzero value. If you do so, isdt returns true and isct returns false.

For arrays of models, bool is true if the models in the array are continuous.

## See Also

isdt|isstable

Introduced in R2012a

## isdt

Determine if dynamic system model is in discrete time

## Syntax

```
bool = isdt(sys)
```


## Description

bool = isdt(sys) returns a logical value of 1 (true) if the dynamic system model sys is a discrete-time model. The function returns a logical value of 0 (false) otherwise.

## Input Arguments

## sys

Dynamic system model or array of such models.

## Output Arguments

## bool

Logical value indicating whether sys is a discrete-time model.
bool $=1$ (true) if sys is a discrete-time model (sys.Ts $\neq 0$ ). If sys is a continuous-time model, bool $=0$ (false).

For a static gain, both isct and isdt return true unless you explicitly set the sample time to a nonzero value. If you do so, isdt returns true and isct returns false.

For arrays of models, bool is true if the models in the array are discrete.

## See Also

isct|isstable

Introduced in R2012a

## isempty

Determine whether dynamic system model is empty

## Syntax

isempty(sys)

## Description

isempty (sys) returns a logical value of 1 (true) if the dynamic system model sys has no input or no output, and a logical value of 0 (false) otherwise. Where sys is a frd model, isempty (sys) returns 1 when the frequency vector is empty. Where sys is a model array, isempty(sys) returns 1 when the array has empty dimensions or when the LTI models in the array are empty.

## Examples

## Determine Whether Dynamic Model Is Empty

Create a continuous-time state-space model with 1 input and no outputs. In this example, specify the $A$ and $B$ matrices as 1 and 2, respectively.

```
sys1 = ss(1,2,[],[]);
```

Determine whether sys 1 is empty.

```
isempty(sys1)
ans = logical
    1
```

The isempty command returns 1 because the system does not have any outputs.
Similarly, isempty returns 1 for an empty transfer function.

```
isempty(tf)
ans = logical
    1
```

Now create a state-space model with 1 input and 1 output. In this example, specify the A, B, C, and D matrices as $1,2,3$, and 4 , respectively.
sys2 = ss(1,2,3,4);
Determine whether sys2 is empty.
isempty(sys2)

```
ans = logical
    0
```

The command returns 0 because the system has inputs and outputs.

## See Also

issiso|size

Introduced before R2006a

## isLocked

Locked status of online parameter estimation System object

## Syntax

L = isLocked(obj)

## Description

$\mathrm{L}=$ isLocked(obj) returns the locked status of online parameter estimation System object, obj.

## Examples

## Check Locked Status of Online Estimation System Object

Create a System object ${ }^{\mathrm{TM}}$ for online estimation of an ARMAX model with default properties.
obj = recursiveARMAX;
Check the locked status of the object.

```
L = isLocked(obj)
L = logical
    0
```

Estimate model parameters online using step and input-output data.

```
[A,B,C,EstimatedOutput] = step(obj,1,1);
```

Check the locked status of the object again.

```
L = isLocked(obj)
L = logical
    1
```

step puts the object in a locked state.

## Input Arguments

## obj - System object for online parameter estimation

```
recursiveAR object | recursiveARMA object | recursiveARX object | recursiveARMAX object |
recursive0E object| recursiveBJ object| recursiveLS object
```

System object for online parameter estimation, created using one of the following commands:

- recursiveAR
- recursiveARMA
- recursiveARX
- recursiveARMAX
- recursive0E
- recursiveBJ
- recursiveLS


## Output Arguments

L - Locked status of online estimation System object
logical
Locked status of online estimation System object, returned as a logical value. L is true if obj is locked.

## See Also

clone | recursiveAR | recursiveARMA | recursiveARMAX | recursiveARX| recursiveBJ| recursiveLS | recursive0E | release | reset \| step

Topics
"What Is Online Estimation?"

Introduced in R2015b

## isnlarx

Detect nonlinearity in estimation data

## Syntax

isnlarx(data,orders)
isnlarx(data,orders,Ky)
isnlarx (__, Name, Value)
NLHyp = isnlarx( ___ )
[NLHyp,NLValue,NLRegs,NoiseSigma,DetectRatio] = isnlarx( $\qquad$ )

## Description

isnlarx (data, orders) detects nonlinearity in data by testing whether a nonlinear ARX model with the indicated orders produces a better estimate of data than a linear ARX model. The nonlinear model uses a default treepartition nonlinearity estimator.

The result of the test is printed to the Command Window and indicates whether a nonlinearity is detected. Use the printed detection ratio to assess the reliability of the nonlinearity detection test:

- Larger values (>2) indicate that a significant nonlinearity was detected.
- Smaller values (<0.5) indicate that any error unexplained by the linear model is mostly noise. That is, no significant nonlinearity was detected.
- Values close to 1 indicate that the nonlinearity detection test is not reliable and that a weak nonlinearity may be present.
isnlarx(data, orders, Ky) restricts the nonlinearity test to output channel Ky for multi-output data.
isnlarx( $\qquad$ ,Name, Value) specifies additional nonlinear ARX model options using one or more Name, Value pair arguments.

NLHyp = isnlarx( $\qquad$ ) returns the result of the nonlinearity test and suppresses the command window output.
[NLHyp,NLValue,NLRegs,NoiseSigma, DetectRatio] = isnlarx( $\qquad$ ) additionally returns the test quantities behind the evaluation.

## Examples

## Detect Nonlinearity in Estimation Data

Load the signal transmission data set.

```
load(fullfile(matlabroot,'toolbox','ident','iddemos','data','frictiondata'))
```

Construct an iddata object from the estimation data.

```
z = iddata(f1,v,1);
```

Specify the model orders and delays.

```
orders = [1 1 0];
```

Run the test to detect nonlinearity.

```
isnlarx(z,orders);
Nonlinearity is detected in data set z.
Detection ratio: 525.05
Estimated discrepancy of the linear model found: 0.0064966
Estimated noise standard deviation: 0.00080938
```

The large detection ratio indicates that the test was robust and a significant nonlinearity was detected. Additionally, the estimated discrepancy of the linear model that was found, that is the data explained by the nonlinearity, is significantly greater than the noise error, which can indicate a significant nonlinearity.

## Detect Nonlinearity in Estimation Data Output Channel

Load the CSTR data set.

```
load(fullfile(matlabroot,'toolbox','ident','iddemos','data','cstrdata'))
```

Construct an iddata object from the estimation data using a sample time of 0.1 seconds.
$z=i d d a t a(y 1, u 1,0.1) ;$
Specify the model orders and delays.

```
orders = [3*ones(2,2),ones(2,3),2*ones(2,3)];
```

Run the test to detect nonlinearity on the second output channel.

```
isnlarx(z,orders,2);
ISNLARX results for dataset z
Nonlinearity is not detected in channel (2).
However, the test may be on the edge of detecting the nonlinearity.
Detection ratio: 0.36446
Searching for best nonlinear regressors may provide more reliable results.
```

A detection ratio less than 1 indicates that no nonlinearity was detected. However, since this value is near 0.5 , there may be a weak nonlinearity that was not detected by the test.

## Search for Best Regressors When Detecting Nonlinearity

Load the signal transmission data set.

```
load(fullfile(matlabroot,'toolbox','ident','iddemos','data','signaltransmissiondata'))
```

Construct an iddata object from the estimation data using a sample time of 0.1 seconds.
z = iddata(vout,vin,0.1);
Specify the model orders and delays.
orders = [3 0 2];
Display the model regressors for an idnlarx model with the given orders.

```
getreg(idnlarx(orders));
```

Detect nonlinearities in the data, and search for the best nonlinear regressor combination.

```
isnlarx(z,orders,'NonlinearRegressors','search');
Nonlinearity is detected in data set z.
Detection ratio: 1.4691
Estimated discrepancy of the linear model found: 0.74371
Estimated noise standard deviation: 0.74935
Corresponding NonlinearRegressors parameter: [1]
```

The regressor search found that using the first two regressors produces the best nonlinear estimation of the given data.

A detection ratio greater than 1 but less than 2 means that a nonlinearity was detected, but the test was not robust. This result may indicate that the detected nonlinearity is not significant. Additionally, the data explained by the nonlinearity is smaller than the noise error, which can be an indication of a weak nonlinearity.

## Return Nonlinearity Detection Result

Load the estimation data.
load(fullfile(matlabroot,'toolbox','ident','iddemos','data','cstrdata'))
Construct an iddata object using the estimation data.
z = iddata(y1,u1,0.1);
Specify the model orders and delays.
orders $=[3 *$ ones $(2,2)$, ones $(2,3), 2$ *ones $(2,3)]$;
Detect nonlinearities in the data, and determine the test quantities behind the evaluation.
NLHyp = isnlarx(z,orders);

## Return Nonlinearity Detection Test Quantities

Load the estimation data.

```
load(fullfile(matlabroot,'toolbox','ident','iddemos','data','narendralidata'))
```

Construct an iddata object using the estimation data.
z = iddata(u,y1,1);
Specify the model orders and delays.
orders = [1 1 2];
Detect nonlinearities in the data, and determine the test quantities behind the evaluation.
[NLHyp,NLValue,NLRegs,NoiseSigma,DetectRatio] = isnlarx(z,orders);

## Input Arguments

## data - Time-domain estimation data

iddata object | numeric matrix
Time-domain estimation data, specified as an iddata object or a numeric matrix.

- If data is an iddata object, then data can have one or more output channels and zero or more input channels.
- If data is a numeric matrix, then the number of columns of data must match the sum of the number of inputs $\left(n_{u}\right)$ and the number of outputs ( $n_{y}$.
data must be uniformly sampled and cannot contain missing ( NaN ) samples.


## orders - ARX model orders

nlarx orders [na nb nk]
ARX model order matrix [na nb nk]. na denotes the number of delayed outputs, nb denotes the number of delayed inputs, and nk denotes the minimum input delay. The minimum output delay is fixed to 1 . For more information on how to construct the orders matrix, see arx.

When you specify orders, the software converts the order information into linear regressor form in the idnlarx Regressors property. For an example, see "Create Nonlinear ARX Model Using ARX Model Orders" on page 1-561

## Ky - Output channel number in estimation data

positive integer in the range [0, $n_{y}$ ]
Output channel number in estimation data, specified as a positive integer in the range $\left[1, n_{y}\right]$, where $n_{y}$ is the number of output channels.

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.
Example: 'NonlinearRegressors', 'output' specifies that only the regressors containing output variables are used as inputs to the nonlinear block of the model.

## TimeVariable - Independent variable name

't' (default)| character vector

Independent variable name, specified as the comma-separated pair consisting of 'TimeVariable' and a character vector. For example, ' t ' .

CustomRegressors - Regressors constructed from combinations of inputs and outputs
\{\} (default) | cell array of character vectors | array of customreg objects
Regressors constructed from combinations of inputs and outputs, specified as the comma-separated pair consisting of 'CustomRegressors' and one of the following for single-output systems:

- Cell array of character vectors. For example:
- \{'y1(t-3)^3', 'y2(t-1)*u1(t-3)','sin(u3(t-2))'\}

Each character vector must represent a valid formula for a regressor contributing towards the prediction of the model output. The formula must be written using the input and output names and the time variable name as variables.

- Array of custom regressor objects, created using customreg or polyreg.

For a model with $n_{y}$ outputs, specify an $n_{y}$-by-1 cell array of customreg object arrays or character arrays.

These regressors are in addition to the standard regressors based on Orders.
Example: 'CustomRegressors', \{'y1(t-3)^3', 'y2(t-1)*u1(t-3)'\}
Example: 'CustomRegressors', \{'sin(u3(t-2))'\}

## NonlinearRegressors - Subset of regressors that enter as inputs to the nonlinear block of the model

'all'(default)|'output'|'input'|'standard'|'custom'|'search'| vector of positive integers | [] | cell array

Subset of regressors that enter as inputs to the nonlinear block of the model, specified as the commaseparated pair consisting of 'NonlinearRegressors' and one of the following values:

- 'all' - All regressors
- 'output' - Regressors containing output variables
- 'input' - Regressors containing input variables
- 'standard' - Standard regressors
- 'custom' - Custom regressors
- 'search' - The estimation algorithm performs a search for the best regressor subset. This is useful when you want to reduce a large number of regressors entering the nonlinear function block of the nonlinearity estimator. This option must be applied to all output models simultaneously.
- [ ] - No regressors. This creates a linear-in-regressor model.
- Vector of regressor indices. To determine the number and order of regressors, use getreg.

For a model with multiple outputs, specify a cell array of $n_{y}$ elements, where $n_{y}$ is the number of output channels. For each output, specify one of the preceding options. Alternatively, to apply the
same regressor subset to all model outputs, specify [] or any of the character vector options alone, for example 'standard'.

> Example: 'NonlinearRegressors', 'search' performs a best regressor search for the only output of a single output model, or all of the outputs of a multiple output model.

Example: 'NonlinearReg' , 'input' applies only input regressors to the inputs of the nonlinear function.
Example: 'NonlinearRegressors', \{'input', 'output'\} applies input regressors to the first output, and output regressors to the second output of a model with two outputs.

## Output Arguments

## NLHyp - Result of the nonlinearity test

0 | 1 | logical vector
Result of the nonlinearity test, returned as a logical vector with length equal to the number of output channels. The elements of NLHyp are 1 if nonlinearities were detected for the corresponding output. A value of 0 indicates that nonlinearities were not detected.

## NLValue - Estimated standard deviation of the data explained by the nonlinearity vector of nonnegative scalars

Estimated standard deviation of the data explained by the nonlinearity, returned as a vector of nonnegative scalars with length equal to the number of output channels. The elements of NLValue are 0 if nonlinearities are not detected for the corresponding output.

## NLRegs - Regressors that should enter nonlinearly in the model <br> vector of indices | [] | cell array

Regressors that should enter nonlinearly in the model, returned as a vector of indices for single output models. For multi-output models, NLRegs is returned as a cell array, with elements corresponding to each output channel. NLRegs is empty, [ ], if nonlinearities are not detected.

See the 'NonlinearRegressors ' Name, Value argument for more information.

## NoiseSigma - Estimated standard deviation of the unexplained error <br> vector of nonnegative scalars

Estimated standard deviation of the unexplained error, returned as a vector of nonnegative scalars with length equal to the number of output channels. The elements of NoiseSigma are 0 if nonlinearities are not detected for the corresponding output.

## DetectRatio - Ratio of the test statistic and the detection threshold

 vectorRatio of the test statistic and the detection threshold, returned as a vector with length equal to the number of output channels. Use the elements of DetectRatio to assess the reliability of the nonlinearity detection test for the corresponding output:

- Larger values (>2) indicate that a significant nonlinearity was detected.
- Smaller values (<0.5) indicate that any error unexplained by the linear model is mostly noise. That is, no significant nonlinearity was detected.
- Values close to 1 indicate that the nonlinearity detection test is not reliable and that a weak nonlinearity may be present.


## Algorithms

isnlarx estimates a nonlinear ARX model using the given data and a treepartition nonlinearity estimator.

The estimation data can be described as $Y(t)=L(t)+F_{n}(t)+E(t)$, where:

- $L(t)$ is the portion of the data explained by the linear function of the nonlinear ARX model.
- $F_{n}(t)$ is the portion of the data explained by the nonlinear function of the nonlinear ARX model. The output argument NLValue is an estimate of the standard deviation of $F_{n}(t)$. If the nonlinear function explains a significant portion of the data beyond the data explained by the linear function, a nonlinearity is detected.
- $E(t)$ is the remaining error that is unexplained by the nonlinear ARX model and is typically white noise. The output argument NoiseSigma is an estimate of the standard deviation of $E(t)$.


## See Also

getreg|idnlarx|nlarx|treepartition

## Topics

"Structure of Nonlinear ARX Models"

Introduced in R2007a

## isproper

Determine if dynamic system model is proper

## Syntax

B = isproper(sys)
B = isproper(sys,'elem')
[B,sysr] = isproper(sys)

## Description

$B=$ isproper(sys) returns a logical value of 1 (true) if the dynamic system model sys is proper and a logical value of 0 (false) otherwise.

A proper model has relative degree $\leq 0$ and is causal. SISO transfer functions and zero-pole-gain models are proper if the degree of their numerator is less than or equal to the degree of their denominator (in other words, if they have at least as many poles as zeroes). MIMO transfer functions are proper if all their SISO entries are proper. Regular state-space models (state-space models having no E matrix) are always proper. A descriptor state-space model that has an invertible E matrix is always proper. A descriptor state-space model having a singular (non-invertible) E matrix is proper if the model has at least as many poles as zeroes.

If sys is a model array, then $B$ is 1 if all models in the array are proper.
B = isproper(sys,'elem') checks each model in a model array sys and returns a logical array of the same size as sys. The logical array indicates which models in sys are proper.
[ $B$, sysr] = isproper(sys) also returns an equivalent model sys $r$ with fewer states (reduced order) and a non-singular E matrix, if sys is a proper descriptor state-space model with a noninvertible E matrix. If sys is not proper, sysr = sys.

## Examples

## Examine Whether Models are Proper

Create a SISO continuous-time transfer function, $H_{1}=s$
H1 = tf([10],1);
Check whether H 1 is proper.
B1 = isproper(H1)
B1 = logical
0

SISO transfer functions are proper if the degree of their numerator is less than or equal to the degree of their denominator That is, if the transfer function has at least as many poles as zeroes. Since H1 has one zero and no poles, the isproper command returns 0 .

Now create a transfer function with one pole and one zero, $H_{2}=s /(s+1)$
H2 = tf([10],[11]);
Check whether H 2 is proper.

```
B2 = isproper(H2)
B2 = logical
    1
```

Since H 2 has equal number of poles and zeros, isproper returns 1.

## Compute Equivalent Lower-Order Model

Combining state-space models sometimes yields results that include more states than necessary. Use isproper to compute an equivalent lower-order model.

```
H1 = ss(tf([1 1],[1 2 5]));
H2 = ss(tf([1 7],[1]));
H = H1*H2;
size(H)
State-space model with 1 outputs, 1 inputs, and 4 states.
```

H is proper and reducible. isproper returns the reduced model.

```
[isprop,Hr] = isproper(H);
size(Hr)
State-space model with 1 outputs, 1 inputs, and 2 states.
H and Hr are equivalent, as a Bode plot demonstrates.
bodeplot (H,Hr,'r--')
legend('original','reduced')
```



Bode Diagram

## References

[1] Varga, Andràs. "Computation of irreducible generalized state-space realizations." Kybernetika 26.2 (1990): 89-106.

## See Also

dss|ss
Introduced before R2006a

## isreal

Determine whether model parameters or data values are real

## Syntax

isreal(Data)
isreal(Model)

## Description

isreal (Data) returns 1 if all signals of the data set are real. Data is an iddata object.
isreal (Model) returns 1 if all parameters of the model are real. Model is any linear identified model.

## See Also

realdata

Introduced before R2006a

## issiso

Determine if dynamic system model is single-input/single-output (SISO)

## Syntax

issiso(sys)

## Description

issiso(sys) returns a logical value of 1 (true) if the dynamic system model sys is SISO and a logical value of 0 (false) otherwise.

## See Also

isempty|size
Introduced in R2012a

## isstable

Determine if dynamic system model is stable

## Syntax

B = isstable(sys)
B = isstable(sys,'elem')

## Description

$B=$ isstable(sys) returns a logical value of 1 (true) if the dynamic system model (Control System Toolbox) sys has stable dynamics, and a logical value of 0 (false) otherwise. If sys is a model array, then the function returns 1 only if all the models in sys are stable.
isstable returns a logical value of 1 (true) for stability of a dynamic system if:

- In continuous-time systems, all the poles lie in the open left half of the complex plane.
- In discrete-time systems, all the poles lie inside the open unit disk.
isstable is supported only for analytical models with a finite number of poles.
B = isstable(sys,'elem') returns a logical array of the same dimensions as the model array sys. The logical array indicates which models in sys are stable.


## Examples

## Determine Stability of Discrete-Time Transfer Function Model

Determine the stability of this discrete-time SISO transfer function model with a sample time of 0.1 seconds.

$$
\operatorname{sys}(z)=\frac{2 z}{4 z^{3}+3 z-1}
$$

Create the discrete-time transfer function model.
sys = tf([2,0], [4, 0, 3, -1], 0.1);
Examine the poles of the system.

```
P = abs(pole(sys))
P = 3\times1
    0.9159
    0.9159
    0.2980
```

All the poles of the transfer function model have a magnitude less than 1 , so all the poles lie within the open unit disk and the system is stable.

Confirm the stability of the model using isstable.

```
B = isstable(sys)
B = logical
    1
```

The system sys is stable.

## Determine Stability of Continuous-Time Zero-Pole-Gain Model

Determine the stability of this continuous-time zero-pole-gain model.

$$
\operatorname{sys}(s)=\frac{2}{(s+2+3 j)(s+2-3 j)(s-0.5)}
$$

Create the model as a zpk model object by specifying the zeros, poles, and gain.
sys = zpk([],[-2-3*j,-2+3*j, 0.5],2);
Because one pole of the model lies in the right half of the complex plane, the system is unstable.
Confirm the instability of the model using isstable.
$B=$ isstable(sys)
B = logical
0

The system sys is unstable.

## Determine Stability of Models in Model Array

Determine the stability of an array of SISO transfer function models with poles varying from - 2 to 2 .

$$
\left[\frac{1}{s+2}, \frac{1}{s+1}, \frac{1}{s}, \frac{1}{s-1}, \frac{1}{s-2}\right]
$$

To create the array, first initialize an array of dimension [length (a) , 1] with zero-valued SISO transfer functions.

```
a = [-2:2];
sys = tf(zeros(1,1,length(a)));
```

Populate the array with transfer functions of the form $1 /(s-a)$.

```
for j = 1:length(a)
    sys(1,1,j) = tf(1,[1 -a(j)]);
end
```

isstable can tell you whether all the models in model array are stable or each individual model is stable.

Examine the stability of the model array.

```
B_all = isstable(sys)
B_all = logical
    0
```

By default, isstable returns a single logical value that is 1 (true) only if all models in the array are stable. sys contains some models with nonnegative poles, which are not stable. Therefore, isstable returns 0 (false) for the entire array.

Examine the stability of each model in the array by using 'elem' flag.

```
B_elem = isstable(sys,'elem')
B_elem = 5x1 logical array
```

    1
    1
    0
    0
    0
    The function returns an array of logical values that indicate the stability of the corresponding entry in the model array. For example, B_elem (2) is 1, which indicates that the second model in the array, $\operatorname{sys}(1,1,2)$ is stable. This is because sys $(1,1,2)$ has a pole at -1 .

## Input Arguments

## sys - Dynamic system

dynamic system model | model array
Dynamic system, specified as a SISO or MIMO dynamic system model or an array of SISO or MIMO dynamic system models. Dynamic systems that you can use include continuous-time or discrete-time numeric LTI models such as tf, zpk, or ss models.

If sys is a generalized state-space model genss or an uncertain state-space model uss, isstable checks the stability of the current or nominal value of sys.

If sys is an array of models, isstable checks the stability of every model in the array.

- If you use $B=$ isstable(sys), the output is 1 (true) only if all the models in the array are stable.
- If you use $\mathbf{B}=$ isstable(sys,'elem'), the output is a logical array, the entries of which indicate the stability of the corresponding entry in the model array.

For more information on model arrays, see "Model Arrays" (Control System Toolbox).

## Output Arguments

## B - True or false result

1|0|logical array
True or false result, returned as 1 for a stable model or 0 for an unstable model.
The 'elem' flag causes isstable to return an array of logical values with same dimensions as the model array. The values in the array indicate the stability of the corresponding entry in the model array.

## See Also <br> isproper|issiso|pole <br> Introduced in R2012a

## ivar

AR model estimation using instrumental variable method

## Syntax

sys = ivar(data,na)
sys = ivar(data,na,nc)
sys = ivar(data,na,nc,max_size)

## Description

sys = ivar(data,na) estimates an AR polynomial model, sys, using the instrumental variable method and the time series data data. na specifies the order of the $A$ polynomial.

An AR model is represented by the equation:

$$
A(q) y(t)=e(t)
$$

In the above model, $e(t)$ is an arbitrary process, assumed to be a moving average process of order nc, possibly time varying. nc is assumed to be equal to na. Instruments are chosen as appropriately filtered outputs, delayed nc steps.
sys = ivar(data,na,nc) specifies the value of the moving average process order, nc , separately.
sys = ivar(data, na,nc,max_size) specifies the maximum size of matrices formed during estimation.

## Input Arguments

## data

Estimation time series data.
data must be an iddata object with scalar output data only.

## na

Order of the $A$ polynomial
nc
Order of the moving average process representing $e(t)$.
max_size
Maximum matrix size.
max_size specifies the maximum size of any matrix formed by the algorithm for estimation.
Specify max_size as a reasonably large positive integer.

Default: 250000

## Output Arguments

## sys

Identified polynomial model.
sys is an AR idpoly model which encapsulates the identified polynomial model.

## Examples

Compare spectra for sinusoids in noise, estimated by the IV method and by the forward-backward least squares method.

```
y = iddata(sin([1:500]'*1.2) + sin([1:500]'*1.5) + ...
    0.2*randn(500,1),[]);
miv = ivar(y,4);
mls = ar(y,4);
spectrum(miv,mls)
```


## References

[1] Stoica, P., et al. Optimal Instrumental Variable Estimates of the AR-parameters of an ARMA Process, IEEE Trans. Autom. Control, Volume AC-30, 1985, pp. 1066-1074.

## See Also

ar|arx|etfe|idpoly|polyest|spa|spectrum|step
Introduced before R2006a

## ivstruc

Compute loss functions for sets of ARX model structures using instrumental variable method

## Syntax

v = ivstruc(ze,zv,NN)
v = ivstruc(ze,zv,NN,p,maxsize)

## Description

$\mathrm{v}=$ ivstruc (ze, zv, NN) computes the loss functions for sets of single-output ARX model structures. NN is a matrix that defines a number of different structures of the ARX type. Each row of NN is of the form

```
nn = [na nb nk]
```

with the same interpretation as described for arx. See struc for easy generation of typical NN matrices.
ze and zv are iddata objects containing input-output data. Only time-domain data is supported. Models for each model structure defined in NN are estimated using the instrumental variable (IV) method on data set ze. The estimated models are simulated using the inputs from data set zv. The normalized quadratic fit between the simulated output and the measured output in zv is formed and returned in $v$. The rows below the first row in $v$ are the transpose of NN, and the last row contains the logarithms of the condition numbers of the IV matrix

$$
\sum \varsigma(t) \varphi^{T}(t)
$$

A large condition number indicates that the structure is of unnecessarily high order (see Ljung, L. System Identification: Theory for the User, Upper Saddle River, NJ, Prentice-Hall PTR, 1999, p. 498).

The information in $v$ is best analyzed using selstruc.
The routine is for single-output systems only.
$v=i v s t r u c(z e, z v, N N, p, m a x s i z e)$ specifies the computation of condition numbers and the size of largest matrix formed during computations. If $p$ is equal to zero, the computation of condition numbers is suppressed. maxsize affects the speed/memory trade-off.

Note The IV method used does not guarantee that the models obtained are stable. The output-error fit calculated in $v$ can then be misleading.

## Examples

## Generate Model-Order Combinations and Estimate ARX Model Using IV Method

Create estimation and validation data sets

```
load iddata1;
ze = z1(1:150);
zv = z1(151:300);
```

Generate model-order combinations for estimation, specifying ranges for model orders and delays.
NN = struc(1:3,1:2,2:4);
Estimate ARX models using the instrumental variable method, and compute the loss function for each model order combination.
$\mathrm{V}=$ ivstruc(ze,zv,NN);
Select the model order with the best fit to the validation data.
order $=\operatorname{sel}$ struc ( $\mathrm{V}, 0$ ) ;
Estimate an ARX model of selected order.
$M=i v 4(z e, o r d e r) ;$

## Suppress Condition Number Computation When Determining ARX Loss Functions

Create estimation and validation data sets.

```
load iddatal;
ze = zl(1:150);
zv = z1(151:300);
```

Generate model-order combinations for estimation, specifying ranges for model orders and a delay of 2 for all model configurations.

```
NN = struc(2:3,1:2,2);
```

Compute the loss function for each model order combination. Suppress the computation of condition numbers.

```
V = ivstruc(ze,zv,NN,0);
```


## Algorithms

A maximum-order ARX model is computed using the least squares method. Instruments are generated by filtering the input(s) through this model. The models are subsequently obtained by operating on submatrices in the corresponding large IV matrix.

## References

[1] Ljung, L. System Identification: Theory for the User, Upper Saddle River, NJ, Prentice-Hall PTR, 1999.

## See Also

arxstruc|iv4|selstruc|struc

Introduced before R2006a

## ivx

ARX model estimation using instrumental variable method with arbitrary instruments

## Syntax

sys = ivx(data,[na nb nk],x)
sys = ivx(data,[na nb nk],x,max_size)

## Description

sys = ivx(data,[na nb nk],x) estimates an ARX polynomial model, sys, using the instrumental variable method with arbitrary instruments. The model is estimated for the time series data data. [na nb nk] specifies the ARX structure orders of the $A$ and $B$ polynomials and the input to output delay, expressed in the number of samples.

An ARX model is represented as:

$$
A(q) y(t)=B(q) u(t-n k)+v(t)
$$

sys = ivx(data,[na nb nk],x,max_size) specifies the maximum size of matrices formed during estimation.

## Input Arguments

## data

Estimation data. The data can be:

- Time- or frequency-domain input-output data
- Time-series data
- Frequency-response data
data must be an iddata, idfrd, or frd object.
When using frequency-domain data, the number of outputs must be 1.


## [na nb nk]

ARX model orders.
For more details on the ARX model structure, see arx.

## x

Instrument variable matrix.
$x$ is a matrix containing the arbitrary instruments for use in the instrumental variable method.
$x$ must be of the same size as the output data, data. $y$. For multi-experiment data, specify $x$ as a cell array with one entry for each experiment.

The instruments used are analogous to the regression vector, with y replaced by $x$.
max_size
Maximum matrix size.
max_size specifies the maximum size of any matrix formed by the algorithm for estimation.
Specify max_size as a reasonably large positive integer.
Default: 250000

## Output Arguments

## sys

ARX model that fits the estimation data, returned as a discrete-time idpoly object. This model is created using the specified model orders, delays, and estimation options. ivx does not return any estimated covariance information for sys.

Information about the estimation results and options used is stored in the Report property of the model. Report has the following fields:

| Report <br> Field | Description |
| :--- | :--- |
| Status | Summary of the model status, which indicates whether the model was created by <br> construction or obtained by estimation. |
| Method <br> nditialCo | Estimation command used. <br> nandling of initial conditions during model estimation, returned as one of the following <br> values: |
|  | - 'zero' - The initial conditions were set to zero. <br> - 'estimate ' - The initial conditions were treated as independent estimation <br> parameters. <br> 'backcast ' <br> fit. The initial conditions were estimated using the best least squares <br> This field is especially useful to view how the initial conditions were handled when the <br> InitialCondition option in the estimation option set is 'auto '. |


| Report <br> Field | Description |  |
| :--- | :--- | :--- |
| Fit | Quantitative assessment of the estimation, returned as a structure. See "Loss Function <br> and Model Quality Metrics" for more information on these quality metrics. The <br> structure has the following fields: |  |
|  | Field | Description |
|  | FitPerce <br> nt | Normalized root mean squared error (NRMSE) measure of how well the <br> response of the model fits the estimation data, expressed as the <br> percentage fit = 100(1-NRMSE). |
|  | LossFcn | Value of the loss function when the estimation completes. |
|  | MSE | Mean squared error (MSE) measure of how well the response of the <br> model fits the estimation data. |
|  | FPE | Final prediction error for the model. |
|  | AIC | Raw Akaike Information Criteria (AIC) measure of model quality. |
|  | AICc | Small sample-size corrected AIC. |
|  | nAIC | Normalized AIC. |
|  | BIC | Bayesian Information Criteria (BIC). |
| Parameter <br> s | Estimated values of model parameters. <br> OptionsUs <br> edOption set used for estimation. If no custom options were configured, this is a set of <br> default options. See arx0ptions for more information. |  |
| RandState | State of the random number stream at the start of estimation. Empty, [ ], if <br> randomization was not used during estimation. For more information, see rng. |  |


| Report Field | Description |  |
| :---: | :---: | :---: |
| DataUsed | Attributes of the data used for estimation, returned as a structure with the following fields: |  |
|  | Field | Description |
|  | Name | Name of the data set. |
|  | Type | Data type. |
|  | Length | Number of data samples. |
|  | Ts | Sample time. |
|  | InterSam ple | Input intersample behavior, returned as one of the following values: <br> - 'zoh' - Zero-order hold maintains a piecewise-constant input signal between samples. <br> - ' foh ' - First-order hold maintains a piecewise-linear input signal between samples. <br> - 'bl' - Band-limited behavior specifies that the continuous-time input signal has zero power above the Nyquist frequency. |
|  | Input0ff set | Offset removed from time-domain input data during estimation. For nonlinear models, it is [ ]. |
|  | $\begin{aligned} & \text { Output0f } \\ & \text { fset } \end{aligned}$ | Offset removed from time-domain output data during estimation. For nonlinear models, it is []. |

For more information on using Report, see "Estimation Report".

## Tips

- Use iv4 first for IV estimation to identify ARX polynomial models where the instruments x are chosen automatically. Use ivx for nonstandard situations. For example, when there is feedback present in the data, or, when other instruments need to be tried. You can also use iv to automatically generate instruments from certain custom defined filters.


## References

[1] Ljung, L. System Identification: Theory for the User, page 222, Upper Saddle River, NJ, PrenticeHall PTR, 1999.

## See Also <br> arx|arxstruc|idpoly|iv4|ivar|polyest

Introduced before R2006a

## iv4

ARX model estimation using four-stage instrumental variable method

## Syntax

sys $=$ iv4(data, [na nb nk])
sys = iv4(data,'na',na,'nb',nb,'nk',nk)
sys = iv4( $\qquad$ , Name, Value)
sys $=$ iv4( $\qquad$ , opt)

## Description

sys = iv4(data, [na nb nk]) estimates an ARX polynomial model, sys, using the four-stage instrumental variable method, for the data object data. [na nb nk] specifies the ARX structure orders of the $A$ and $B$ polynomials and the input to output delay. The estimation algorithm is insensitive to the color of the noise term.
sys is an ARX model:

$$
A(q) y(t)=B(q) u(t-n k)+v(t)
$$

sys = iv4(data,'na',na,'nb',nb,'nk',nk) alternatively specify the ARX model orders separately.
sys = iv4(__ , Name, Value) estimates an ARX polynomial with additional options specified by one or more Name, Value pair arguments.
sys = iv4( __ , opt) uses the option set, opt, to configure the estimation behavior.

## Input Arguments

## data

Estimation data. The data can be:

- Time- or frequency-domain input-output data
- Time-series data
- Frequency-response data
data must be an iddata, idfrd, or frd object.
data must be discrete-time ( $\mathrm{Ts}>0$ ) for frequency domain.


## [na nb nk]

ARX polynomial orders.
For multi-output model, [na nb nk] contains one row for every output. In particular, specify na as an $N y$-by-Ny matrix, where each entry is the polynomial order relating the corresponding output pair.

Here, $N y$ is the number of outputs. Specify nb and nk as $N y$-by- $N u$ matrices, where $N u$ is the number of inputs. For more details on the ARX model structure, see arx.

## opt

Estimation options.
opt is an options set that configures the estimation options. These options include:

- estimation focus
- handling of initial conditions
- handling of data offsets

Use iv40ptions to create the options set.

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

## InputDelay

Input delay for each input channel, specified as a scalar value or numeric vector. For continuous-time systems, specify input delays in the time unit stored in the TimeUnit property. For discrete-time systems, specify input delays in integer multiples of the sample time Ts. For example, InputDelay = 3 means a delay of three sample times.

For a system with Nu inputs, set InputDelay to an Nu-by-1 vector. Each entry of this vector is a numerical value that represents the input delay for the corresponding input channel.

You can also set InputDelay to a scalar value to apply the same delay to all channels.
Default: 0

## IODelay

Transport delays. IODelay is a numeric array specifying a separate transport delay for each input/ output pair.

For continuous-time systems, specify transport delays in the time unit stored in the TimeUnit property. For discrete-time systems, specify transport delays in integer multiples of the sample time, Ts.

For a MIMO system with Ny outputs and Nu inputs, set IODelay to a Ny-by-Nu array. Each entry of this array is a numerical value that represents the transport delay for the corresponding input/output pair. You can also set IODelay to a scalar value to apply the same delay to all input/output pairs.

Default: 0 for all input/output pairs

## IntegrateNoise

Specify integrators in the noise channels.
Adding an integrator creates an ARIX model represented by:

$$
A(q) y(t)=B(q) u(t-n k)+\frac{1}{1-q^{-1}} e(t)
$$

where,$\frac{1}{1-q^{-1}}$ is the integrator in the noise channel, $e(t)$.
IntegrateNoise is a logical vector of length Ny , where Ny is the number of outputs.
Default: false(Ny,1), where Ny is the number of outputs

## Output Arguments

sys
ARX model that fits the estimation data, returned as a discrete-time idpoly object. This model is created using the specified model orders, delays, and estimation options.

Information about the estimation results and options used is stored in the Report property of the model. Report has the following fields:

| Report <br> Field | Description |
| :--- | :--- |
| Status | Summary of the model status, which indicates whether the model was created by <br> construction or obtained by estimation. |
| Method | Estimation command used. |
| InitialCo <br> ndition | Handling of initial conditions during model estimation, returned as one of the following <br> values: |
| - 'zero' - The initial conditions were set to zero. |  |
| - 'estimate' - The initial conditions were treated as independent estimation |  |
| parameters. |  |
| This field is especially useful to view how the initial conditions were handled when the |  |
| InitialCondition option in the estimation option set is 'auto'. |  |


| Report Field | Description |  |
| :---: | :---: | :---: |
| Fit | Quantitative assessment of the estimation, returned as a structure. See "Loss Function and Model Quality Metrics" for more information on these quality metrics. The structure has the following fields: |  |
|  | Field | Description |
|  | FitPerce nt | Normalized root mean squared error (NRMSE) measure of how well the response of the model fits the estimation data, expressed as the percentage fit $=100(1-$ NRMSE $)$. |
|  | LossFcn | Value of the loss function when the estimation completes. |
|  | MSE | Mean squared error (MSE) measure of how well the response of the model fits the estimation data. |
|  | FPE | Final prediction error for the model. |
|  | AIC | Raw Akaike Information Criteria (AIC) measure of model quality. |
|  | AICc | Small sample-size corrected AIC. |
|  | nAIC | Normalized AIC. |
|  | BIC | Bayesian Information Criteria (BIC). |
| Parameter <br> s | Estimated values of model parameters. |  |
| OptionsUs ed | Option set used for estimation. If no custom options were configured, this is a set of default options. See iv40ptions for more information. |  |
| RandState | State of the random number stream at the start of estimation. Empty, [ ], if randomization was not used during estimation. For more information, see rng. |  |


| Report Field | Description |  |
| :---: | :---: | :---: |
| DataUsed | Attributes of the data used for estimation, returned as a structure with the following fields: |  |
|  | Field | Description |
|  | Name | Name of the data set. |
|  | Type | Data type. |
|  | Length | Number of data samples. |
|  | Ts | Sample time. |
|  | InterSam ple | Input intersample behavior, returned as one of the following values: <br> - 'zoh' - Zero-order hold maintains a piecewise-constant input signal between samples. <br> - ' foh ' - First-order hold maintains a piecewise-linear input signal between samples. <br> - 'bl' - Band-limited behavior specifies that the continuous-time input signal has zero power above the Nyquist frequency. |
|  | Input0ff set | Offset removed from time-domain input data during estimation. For nonlinear models, it is [ ]. |
|  | $\begin{aligned} & \text { Output0f } \\ & \text { fset } \\ & \hline \end{aligned}$ | Offset removed from time-domain output data during estimation. For nonlinear models, it is [ ]. |

For more information on using Report, see "Estimation Report".

## Examples

## Estimate ARX Model Using Four-Stage Instrumental Variable Method

Load estimation data.
load iddata7;
This data has two inputs, $u 1$ and $u 2$, and one output, y 1 .
Specify the ARX model orders, using the same orders for both inputs.

```
na = 2;
nb = [2 2];
```

Specify a delay of 2 samples for input $u 2$ and no delay for input $u 1$.
nk = [0 2];
Estimate an ARX model using the four-stage instrumental variable method.
m = iv4(z7,[na nb nk]);

## Algorithms

Estimation is performed in 4 stages. The first stage uses the arx function. The resulting model generates the instruments for a second-stage IV estimate. The residuals obtained from this model are modeled as a high-order AR model. At the fourth stage, the input-output data is filtered through this AR model and then subjected to the IV function with the same instrument filters as in the second stage.

For the multiple-output case, optimal instruments are obtained only if the noise sources at the different outputs have the same color. The estimates obtained with the routine are reasonably accurate, however, even in other cases.

## References

[1] Ljung, L. System Identification: Theory for the User, equations (15.21) through (15.26), Upper Saddle River, NJ, Prentice-Hall PTR, 1999.

## See Also

armax|arx|bj|idpoly|iv40ptions|ivx|n4sid|oe|polyest
Introduced before R2006a

## iv40ptions

Option set for iv4

## Syntax

opt = iv40ptions
opt $=$ iv40ptions(Name,Value)

## Description

opt $=$ iv40ptions creates the default options set for iv4.
opt = iv40ptions(Name, Value) creates an option set with the options specified by one more Name, Value pair arguments.

## Input Arguments

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

## InitialCondition - Handling of initial conditions <br> 'auto' (default)|'zero'|'estimate'

Handling of initial conditions during estimation, specified as one of the following values:

- 'zero' - The initial condition is set to zero.
- 'estimate ' - The initial condition is treated as an independent estimation parameter.
- 'auto' - The software chooses the initial condition handling method based on the estimation data.


## Focus - Error to be minimized

 'prediction' (default) | 'simulation'Error to be minimized in the loss function during estimation, specified as the comma-separated pair consisting of 'Focus ' and one of the following values:

- 'prediction' - The one-step ahead prediction error between measured and predicted outputs is minimized during estimation. As a result, the estimation focuses on producing a good predictor model.
- 'simulation' - The simulation error between measured and simulated outputs is minimized during estimation. As a result, the estimation focuses on making a good fit for simulation of model response with the current inputs.

The Focus option can be interpreted as a weighting filter in the loss function. For more information, see "Loss Function and Model Quality Metrics".

## WeightingFilter - Weighting prefilter

[ ] (default) | vector | matrix | cell array | linear system
Weighting prefilter applied to the loss function to be minimized during estimation. To understand the effect of WeightingFilter on the loss function, see "Loss Function and Model Quality Metrics".

Specify WeightingFilter as one of the following values:

- [] - No weighting prefilter is used.
- Passbands - Specify a row vector or matrix containing frequency values that define desired passbands. You select a frequency band where the fit between estimated model and estimation data is optimized. For example, [ $w l, w h$ ] where $w l$ and wh represent lower and upper limits of a passband. For a matrix with several rows defining frequency passbands, [w1l,w1h;w2l,w2h;w3l,w3h; ...], the estimation algorithm uses the union of the frequency ranges to define the estimation passband.

Passbands are expressed in rad/TimeUnit for time-domain data and in FrequencyUnit for frequency-domain data, where TimeUnit and FrequencyUnit are the time and frequency units of the estimation data.

- SISO filter - Specify a single-input-single-output (SISO) linear filter in one of the following ways:
- A SISO LTI model
- $\{A, B, C, D\}$ format, which specifies the state-space matrices of a filter with the same sample time as estimation data.
- \{numerator, denominator\} format, which specifies the numerator and denominator of the filter as a transfer function with same sample time as estimation data.

This option calculates the weighting function as a product of the filter and the input spectrum to estimate the transfer function.

- Weighting vector - Applicable for frequency-domain data only. Specify a column vector of weights. This vector must have the same length as the frequency vector of the data set, Data. Frequency. Each input and output response in the data is multiplied by the corresponding weight at that frequency.


## EnforceStability - Control whether to enforce stability of model <br> false (default) | true

Control whether to enforce stability of estimated model, specified as the comma-separated pair consisting of 'EnforceStability' and either true or false.

Data Types: logical

## EstimateCovariance - Control whether to generate parameter covariance data

true (default) | false
Controls whether parameter covariance data is generated, specified as true or false.
If EstimateCovariance is true, then use getcov to fetch the covariance matrix from the estimated model.

## Display - Specify whether to display the estimation progress

'off' (default)|'on'
Specify whether to display the estimation progress, specified as one of the following values:

- 'on' - Information on model structure and estimation results are displayed in a progress-viewer window.
- 'off' - No progress or results information is displayed.


## InputOffset - Removal of offset from time-domain input data during estimation

[ ] (default) | vector of positive integers | matrix
Removal of offset from time-domain input data during estimation, specified as the comma-separated pair consisting of 'InputOffset' and one of the following:

- A column vector of positive integers of length $N u$, where $N u$ is the number of inputs.
- [] - Indicates no offset.
- Nu-by-Ne matrix - For multi-experiment data, specify InputOffset as an Nu -by-Ne matrix. Nu is the number of inputs, and $N e$ is the number of experiments.

Each entry specified by InputOffset is subtracted from the corresponding input data.

## OutputOffset - Removal of offset from time-domain output data during estimation

[ ] (default) | vector | matrix
Removal of offset from time-domain output data during estimation, specified as the comma-separated pair consisting of 'OutputOffset' and one of the following:

- A column vector of length $N y$, where $N y$ is the number of outputs.
- [ ] - Indicates no offset.
- $N y$-by-Ne matrix - For multi-experiment data, specify OutputOffset as a $N y$-by-Ne matrix. $N y$ is the number of outputs, and $N e$ is the number of experiments.

Each entry specified by OutputOffset is subtracted from the corresponding output data.

## Advanced - Additional advanced options

structure
Additional advanced options, specified as a structure with the following fields:

- MaxSize - Specifies the maximum number of elements in a segment when input-output data is split into segments.

MaxSize must be a positive integer.
Default: 250000

- StabilityThreshold - Specifies thresholds for stability tests.

StabilityThreshold is a structure with the following fields:

- $s$ - Specifies the location of the right-most pole to test the stability of continuous-time models. A model is considered stable when its right-most pole is to the left of $s$.

Default: 0

- $z$ - Specifies the maximum distance of all poles from the origin to test stability of discretetime models. A model is considered stable if all poles are within the distance $z$ from the origin.

Default: 1+sqrt(eps)

## Output Arguments

## opt - Options set for iv4

iv40ptions option set
Option set for iv4, returned as an iv40ptions option set.

## Examples

## Create Default Options Set for ARX Model Estimation Using 4-Stage Instrument Variable Method

```
opt = iv40ptions;
```


## Specify Options for ARX Model Estimation Using 4-Stage Instrument Variable Method

Create an options set for iv4 using the 'backcast' algorithm to initialize the state. Set Display to 'on'.

```
opt = iv4Options('InitialCondition','backcast','Display','on');
```

Alternatively, use dot notation to set the values of opt.

```
opt = iv40ptions;
opt.InitialCondition = 'backcast';
opt.Display = 'on';
```


## Compatibility Considerations

## Renaming of Estimation and Analysis Options

The names of some estimation and analysis options were changed in R2018a. Prior names still work. For details, see the R2018a release note "Renaming of Estimation and Analysis Options".

## See Also

iv4

## Topics

"Loss Function and Model Quality Metrics"
Introduced in R2012a

## linapp

Linear approximation of nonlinear ARX and Hammerstein-Wiener models for given input

## Syntax

lm = linapp( $\mathrm{nlmodel}, \mathrm{u}$ )
lm = linapp(nlmodel, umin, umax, nsample)

## Description

lm = linapp( $\mathrm{nlmodel}, \mathrm{u}$ ) computes a linear approximation of a nonlinear ARX or HammersteinWiener model by simulating the model output for the input signal $u$, and estimating a linear model lm from $u$ and the simulated output signal. $l m$ is an idpoly model.
lm = linapp(nlmodel, umin, umax, nsample) computes a linear approximation of a nonlinear ARX or Hammerstein-Wiener model by first generating the input signal as a uniformly distributed white noise from the magnitude range umin and umax and (optionally) the number of samples.

## Input Arguments

## nlmodel

Name of the idnlarx or idnlhw model object you want to linearize.
u
Input signal as an iddata object or a real matrix.
Dimensions of $u$ must match the number of inputs in nlmodel.
[umin, umax]
Minimum and maximum input values for generating white-noise input with a magnitude in this rectangular range. The sample length of this signal is nsample.
nsample
Optional argument when you specify [umin, umax]. Specifies the length of the white-noise input.
Default: 1024.

## See Also

idnlarx|idnlarx/findop|idnlarx/linearize|idnlhw|idnlhw/findop|idnlhw/
linearize

## Topics

"Linear Approximation of Nonlinear Black-Box Models"

## Introduced in R2007a

## linear

Linear mapping object for nonlinear ARX models

## Description

A linear object implements an affine function, and is a mapping function for estimating nonlinear ARX models. The mapping function uses a combination of linear weights and an offset. Unlike the other mapping objects for the nonlinear models, the linear object contains no accommodation for a nonlinear component.


Mathematically, linear is a linear function $y=F(x)$ that maps $m$ inputs $X(t)=\left[x\left(t_{1}\right), \chi_{2}(t), \ldots, \chi_{m}(t)\right]^{\mathrm{T}}$ to a scalar output $y(t) . . F$ is a (affine) function of $x$ :

$$
y(t)=y_{0}+(X(t)-\bar{X})^{T} P L
$$

Here:

- $X(t)$ is an $m$-by- 1 vector of inputs, or regressors, with mean $\bar{X}$.
- $y_{0}$ is the output offset, a scalar.
- $\quad P$ is an $m$-by- $p$ projection matrix, where $m$ is the number of regressors and is $p$ is the number of linear weights. $m$ must be greater than or equal to $p$.
- $L$ is a $p$-by- 1 vector of weights.

Set linear as the value of the OutputFen property of an idnlarx model. For example, specify linear when you estimate an idnlarx model with the following command.
sys = nlarx(data, regressors,linear)
When nlarx estimates the model, it also estimates the parameters of the linear function.
Use the linear mapping object when you want to create nonlinear ARX models that operate linearly on the regressors. The regressors themselves can be nonlinear functions of the inputs and outputs. The polynomialRegressor and customRegressor commands allow you to create such regressors. When the idnlarx model has no custom regressors and the output function is set to linear, the model is similar to a linear ARX model. However, for the nonlinear ARX model, the offset is an estimable parameter.

You can configure the linear object to disable components and fix parameters. Use evaluate to compute the output of the function for a given vector of inputs.

## Creation

## Syntax

Lin = linear

## Description

Lin = linear creates a linear object Lin with unknown parameters.

## Properties

## Input - Input signal information

signal property values
Input signal information for signals used for estimation, specified as vectors of $m$ property-specific values, where $m$ is the number of input signals. The Input properties for each input signal are as follows:

- Name - Names of the input signals, specified as a 1-by-m string or character array, where $m$ is the number of inputs
- Mean - Mean of the input signals, specified as a numeric scalar
- Range - Ranges of the input signals, specified as a 2-by-m numeric array that contains the minimum and maximum values


## Output - Output signal information

signal property values

Output signal information, specified as property-specific values. The Output properties are as follows:

- Name - Name of the output signal, specified as a string or a character array
- Mean - Mean of the output signal, specified as a numeric scalar
- Range - Range of the output signal, specified as 2-by-1 numeric array that contains the minimum and maximum values.


## LinearFcn - Parameters of linear function

linear function property values (default)
Parameters of the linear function, specified as follows:

- Value - Value of $L^{\prime}$, specified as a 1 -by- $m$ vector.
- Free - Option to update entries of Value during estimation. specified as a logical scalar. The software honors the Free specification only if the starting value of Value is finite. The default value is true.
- Minimum - Minimum bound on Value, specified as a 1 -by- $p$ vector. If Minimum is specified with a finite value and the starting value of Value is finite, then the software enforces that minimum bound during model estimation.
- Maximum - Maximum bound on Value, specified as a 1-by-p vector. If Maximum is specified with a finite value and the starting value of Value is finite, then the software enforces that maximum bound during model estimation.
- SelectedInputIndex - Indices of linear inputs (see Input. Name) that are used as inputs to the linear function, specified as an 1-by- $n_{r}$ integer vector, where $n_{r}$ is the number of inputs. The RegressorUsage property of the idnlarx model determines these indices.


## Offset - Parameters of offset term

offset property values
Parameters of the offset term, specified as follows:

- Value - Offset value, specified as a scalar.
- Free - Option to update Value during estimation, specified as a scalar logical. The software honors the Free specification of false only if the value of Value is finite. The default value is true.
- Minimum - Minimum bound on Value, specified as a numeric scalar or -Inf. If Minimum is specified with a finite value and the value of Value is finite, then the software enforces that minimum bound during model estimation. The default value is - Inf.
- Maximum - Maximum bound on Value, specified as a numeric scalar or Inf. If Maximum is specified with a finite value and the starting value of Value is finite, then the software enforces that maximum bound during model estimation. The default value is Inf.


## Examples

## Estimate Nonlinear ARX Model with linear as Output Function

Load the data.
load iddata7 z7

Create a linear mapping object L .
L = linear;
Create model regressors that include nonlinear polynomial regressors.

```
Reg1 = linearRegressor({'y1','u1'},{1:4, 0:4});
Reg2 = polynomialRegressor({'y1','u1'},{1:2, 0:2},2,false,true,true);
Reg3 = polynomialRegressor({'y1','u1'},{2, 1:3},3,false,true);
```

Estimate the nonlinear ARX model.

```
sys = nlarx(z7,[Reg1;Reg2;Reg3],L)
sys =
Nonlinear ARX model with 1 output and 2 inputs
    Inputs: u1, u2
    Outputs: y1
Regressors:
    1. Linear regressors in variables y1, u1
    2. Order 2 regressors in variables y1, ul
    3. Order 3 regressors in variables y1, ul
    List of all regressors
Model output is linear in regressors.
Sample time: 1 seconds
Status:
Estimated using NLARX on time domain data "z7".
Fit to estimation data: 43.22% (prediction focus)
FPE: 5.66, MSE: 4.963
```


## Compatibility Considerations

## Use of previous nonlinearity estimator properties is not recommended

Not recommended starting in R2021a
Starting in R2021a, the properties of the mapping objects, previously known as nonlinearity estimators, have been reorganized. These objects are wavenet (W), sigmoidnet (S), treepartition (T), customnet (C), and linear (L). The property changes do not apply to neuralnet. The use of the pre-R2021a properties in the following table is discouraged. However, the software still accepts commands that set these properties. There are no plans to exclude such commands at this time.

| Pre-R2021a Property | R2021a Property | Applicable Mapping Objects |
| :--- | :--- | :--- |
| NumberOfUnits | NonlinearFcn.NumberOfUni <br> ts | W,S,T,C |
| LinearTerm | LinearFcn.Use, Offset.Use | W,S,C |


| Pre-R2021a Property | R2021a Property | Applicable Mapping Objects |
| :--- | :--- | :--- |
| Parameters | Split into three pieces: | W,S,T,C,L |
|  | - LinearFcn.Value <br> -Offset.Value <br> - NonlinearFcn.Parameter <br> s | linear (L) excludes <br> NonlinearFcn.Parameters. |
| Options | NonlinearFcn.Structure | W,T |

## See Also

customnet | evaluate |idnlarx| linearRegressor|neuralnet|nlarx| polynomialRegressor|sigmoidnet|treepartition|wavenet

Introduced in R2007a

## idnlarx/linearize

Linearize nonlinear ARX model

## Syntax

SYS = linearize(NLSYS,U0,X0)

## Description

SYS = linearize(NLSYS, U0, X0) linearizes a nonlinear ARX model about the specified operating point U0 and X0. The linearization is based on tangent linearization. For more information about the definition of states for idnlarx models, see "Definition of idnlarx States" on page 1-571.

## Input Arguments

- NLSYS: idnlarx model.
- U0: Matrix containing the constant input values for the model.
- X0: Model state values. The states of a nonlinear ARX model are defined by the time-delayed samples of input and output variables. For more information about the states of nonlinear ARX models, see the getDelayInfo reference page.

Note To estimate U0 and X0 from operating point specifications, use the findop command.

## Output Arguments

- SYS is an idss model.

When the Control System Toolbox product is installed, SYS is an LTI object.

## Examples

## Linearize Nonlinear ARX Model at Simulation Snapshot

Linearize a nonlinear ARX model around an operating point corresponding to a simulation snapshot at a specific time.

Load sample data.

```
load iddata2
```

Estimate nonlinear ARX model from sample data.

```
nlsys = nlarx(z2,[4 3 10],'tree','custom',...
    {'sin(y1(t-2)*u1(t))+y1(t-2)*u1(t)+u1(t).*u1(t-13)',...
        'yl(t-5)*yl(t-5)*yl(t-1)'},'nlr',[1:5, 7 9]);
```

Plot the response of the model for a step input.

## Step Response



The step response is a steady-state value of 0.8383 at $\mathrm{T}=20$ seconds.
Compute the operating point corresponding to $T=20$.
stepinput $=$ iddata([],[zeros(10,1);ones(200,1)],nlsys.Ts);
[x,u] = findop(nlsys,'snapshot',20,stepinput);
Linearize the model about the operating point corresponding to the model snapshot at $\mathrm{T}=20$.
sys $=$ linearize(nlsys,u,x);
Validate the linear model.
Apply a small perturbation delta_u to the steady-state input of the nonlinear model nlsys. If the linear approximation is accurate, the following should match:

- The response of the nonlinear model $y \_n l$ to an input that is the sum of the equilibrium level and the perturbation delta_u.
- The sum of the response of the linear model to a perturbation input delta_u and the output equilibrium level.

Generate a 200-sample perturbation step signal with amplitude 0.1.
delta_u $=$ [zeros(10,1); 0.1*ones(190,1)];

For a nonlinear system with a steady-state input of 1 and a steady-state output of 0.8383 , compute the steady-state response $\mathrm{y} \_\mathrm{nl}$ to the perturbed input $\mathrm{u} \_\mathrm{nl}$. Use equilibrium state values x computed previously as initial conditions.
u_nl = 1 + delta_u;
$y_{-}^{-} n l=\operatorname{sim}\left(n l s y s, u \_n l, x\right)$;
Compute response of linear model to perturbation input and add it to the output equilibrium level.
$y_{-}$lin $=0.8383+\operatorname{lsim}\left(s y s, d e l t a \_u\right) ;$
Compare the response of nonlinear and linear models.

```
time = [0:0.1:19.9]';
plot(time,y_nl,time,y_lin)
legend('Nonlinear response','Linear response about op. pt.')
title(['Nonlinear and linear model response for small step input'])
```



## Algorithms

The following equations govern the dynamics of an idnlarx model:

$$
\begin{aligned}
& X(t+1)=A X(t)+B \tilde{u}(t) \\
& y(t)=f(X, u)
\end{aligned}
$$

where $X(t)$ is a state vector, $u(t)$ is the input, and $y(t)$ is the output. $A$ and $B$ are constant matrices. $\tilde{u}(t)$ is $[y(t), u(t)]^{T}$.

The output at the operating point is given by
$y^{*}=f\left(X^{*}, u^{*}\right)$
where $X^{*}$ and $u^{*}$ are the state vector and input at the operating point.
The linear approximation of the model response is as follows:

$$
\begin{aligned}
& \Delta X(t+1)=\left(A+B_{1} f_{X}\right) \Delta X(t)+\left(B_{1} f_{u}+B_{2}\right) \Delta u(t) \\
& \Delta y(t)=f_{X} \Delta X(t)+f_{u} \Delta u(t)
\end{aligned}
$$

where

- $\Delta X(t)=X(t)-X^{*}(t)$
- $\Delta u(t)=u(t)-u^{*}(t)$
- $\Delta y(t)=y(t)-y^{*}(t)$
- $B \tilde{U}=\left[B_{1}, B_{2}\right]\left[\begin{array}{l}Y \\ U\end{array}\right]=B_{1} Y+B_{2} U$
- $f_{X}=\left.\frac{\partial}{\partial X} f(X, U)\right|_{X^{*}, U^{*}}$
- $f_{U}=\left.\frac{\partial}{\partial U} f(X, U)\right|_{X^{*}, U^{*}}$

Note For linear approximations over larger input ranges, use linapp.

## See Also

getDelayInfo|idnlarx|idnlarx/findop|linapp

## Topics

"Linear Approximation of Nonlinear Black-Box Models"

## Introduced in R2014b

## idnlhw/linearize

Linearize Hammerstein-Wiener model

## Syntax

SYS = linearize(NLSYS,U0)
SYS = linearize(NLSYS,U0,X0)

## Description

SYS = linearize(NLSYS, U0) linearizes a Hammerstein-Wiener model around the equilibrium operating point. When using this syntax, equilibrium state values for the linearization are calculated automatically using U0.

SYS = linearize(NLSYS, U0, X0) linearizes the idnlhw model NLSYS around the operating point specified by the input U0 and state values X 0 . In this usage, X0 need not contain equilibrium state values. For more information about the definition of states for idnlhw models, see "Definition of idnlhw States" on page 1-601.

The output is a linear model that is the best linear approximation for inputs that vary in a small neighborhood of a constant input $u(t)=U$. The linearization is based on tangent linearization.

## Input Arguments

- NLSYS: idnlhw model.
- U0: Matrix containing the constant input values for the model.
- X0: Operating point state values for the model.

Note To estimate U0 and X0 from operating point specifications, use the findop command.

## Output Arguments

- SYS is an idss model.

When the Control System Toolbox product is installed, SYS is an LTI object.

## Algorithms

The idnlhw model structure represents a nonlinear system using a linear system connected in series with one or two static nonlinear systems. For example, you can use a static nonlinearity to simulate saturation or dead-zone behavior. The following figure shows the nonlinear system as a linear system that is modified by static input and output nonlinearities, where function $\boldsymbol{f}$ represents the input nonlinearity, $\boldsymbol{g}$ represents the output nonlinearity, and $[A, B, C, D]$ represents a state-space parameterization of the linear model.


The following equations govern the dynamics of an idnlhw model:
$v(t)=f(u(t))$
$X(t+1)=A X(t)+B v(t)$
$w(t)=C X(t)+D v(t)$
$y(t)=g(w(t))$
where

- $u$ is the input signal
- $v$ and $w$ are intermediate signals (outputs of the input nonlinearity and linear model respectively)
- $y$ is the model output

The linear approximation of the Hammerstein-Wiener model around an operating point ( $X^{*}, u^{*}$ ) is as follows:

$$
\begin{aligned}
& \Delta X(t+1)=A \Delta X(t)+B f_{u} \Delta u(t) \\
& \Delta y(t) \approx g_{w} C \Delta X(t)+g_{w} D f_{u} \Delta u(t)
\end{aligned}
$$

where

- $\Delta X(t)=X(t)-X^{*}(t)$
- $\Delta u(t)=u(t)-u^{*}(t)$
- $\Delta y(t)=y(t)-y^{*}(t)$
- $f_{u}=\left.\frac{\partial}{\partial u} f(u)\right|_{u=u^{*}}$
- $g_{w}=\left.\frac{\partial}{\partial w} g(w)\right|_{w=w^{*}}$
where $y^{*}$ is the output of the model corresponding to input $u^{*}$ and state vector $X^{*}, v^{*}=f\left(u^{*}\right)$, and $w^{*}$ is the response of the linear model for input $v^{*}$ and state $X^{*}$.

Note For linear approximations over larger input ranges, use linapp. For more information, see the linapp reference page.

## See Also

idnlhw |idnlhw/findop|linapp

## Topics

"Linear Approximation of Nonlinear Black-Box Models"

## Introduced in R2014b

## linearRegressor

Specify linear regressor for nonlinear ARX model

## Description

A linear regressor is a lagged output or input variable, such as $y(t-1)$ or $u(t-2)$. Here, the $y$ term has a lag of 1 sample and the $u$ term has a lag of 2 samples. A linearRegressor object encapsulates a set of linear regressors. Use linearRegressor objects when you create nonlinear ARX models using idnlarx or nlarx. linearRegressor generalizes the concept of orders in ARX models, or in other words, the [na nb nk] matrix, to allow absolute values and noncontiguous lags. Using linearRegressor objects also lets you combine linear regressors with polynomial and custom regressors in a single regressor set.

## Creation

## Syntax

lReg = linearRegressor(Variables,Lags)
lreg = linearRegressor(Variables,Lags,useAbsolute)

## Description

lReg = linearRegressor(Variables,Lags) creates a linearRegressor object that contains output and input names in Variables and the corresponding lags in Lags.
lreg = linearRegressor(Variables,Lags, useAbsolute) specifies in UseAbsolute whether to use the absolute values of the variables to create the regressors.

## Properties

## Variables - Output and input variable names

cell array of strings | iddata object properties
Output and input variable names, specified as a cell array of strings or a cell array that references the OutputName and InputName properties of an iddata object. Each entry must be a string with no special characters other than white space. For an example of using this property, see "Estimate Nonlinear ARX Model Using Linear Regressor Set" on page 1-796.
Example: \{'y1', 'u1'\}
Example: [z.OutputName; z.InputName]'

## Lags - Lags in each variable

cell array of non-negative integers
Lags in each variable, specified as a 1-by- $n_{\mathrm{v}}$ cell array of non-negative integer row vectors, where $n_{\mathrm{v}}$ is the total number of regressor variables. Each row vector contains $n_{r}$ integers that specify the $n_{r}$
regressor lags for the corresponding variable. For instance, suppose that you want the following regressors:

- Output variable $y_{1}: y_{1}(t-1)$ and $y_{1}(t-2)$
- Input variable $u_{1}: u_{1}(t-3)$

To obtain these lags, set Lags to $\left.\left\{\begin{array}{ll}1 & 2\end{array}\right], 3\right\}$.
If a lag corresponds to an output variable of an idnlarx model, the minimum lag must be greater than or equal to 1 .

For an example of using this property, see "Estimate Nonlinear ARX Model Using Linear Regressor Set" on page 1-796.
Example: \{1 1\}
Example: $\{[12],[1,3,4]\}$

## UseAbsolute - Absolute value indicator <br> false (default) | logical vector

Absolute value indicator that determines whether to use the absolute value of a regressor variable instead of the signed value, specified as a logical vector with a length equal to the number of variables. For an example of setting this property, see "Use Absolute Value in Linear Regressor Set" on page 1-798.
Example: [true,false]

## TimeVariable - Name of time variable <br> 't ' (default) | character array | string

Name of the time variable, specified as a valid MATLAB variable name that is distinct from values in Variables.
Example: 'ClockTime'

## Examples

## Estimate Nonlinear ARX Model Using Linear Regressor Set

Specify a linear regressor that is equivalent to an ARX model order matrix of $\left[\begin{array}{lll}4 & 4 & 1\end{array}\right]$.
An order matrix of [ $\left.\begin{array}{ll}4 & 4\end{array}\right]$ 1] specifies that both input and output regressor sets contain four regressors with lags ranging from 1 to 4 . For example, $u_{1}(t-2)$ represents the second input regressor.

Specify the output and input names.

```
output_name = 'y1';
input_name = 'u1';
names = {output_name,input_name};
```

Specify the output and input lags.

```
output_lag = [ll 2 3 4];
input_lag = [1 2 3 4];
lags = {output_lag,input_lag};
Create the linear regressor object.
```

```
lreg = linearRegressor(names,lags)
```

lreg = linearRegressor(names,lags)
lreg =
lreg =
Linear regressors in variables y1, ul
Linear regressors in variables y1, ul
Variables: {'y1' 'ul'}
Variables: {'y1' 'ul'}
Lags: {[1 2 3 4] [1 2 3 4]}
Lags: {[1 2 3 4] [1 2 3 4]}
UseAbsolute: [0 0]
UseAbsolute: [0 0]
TimeVariable: 't'

```
        TimeVariable: 't'
```

    Regressors described by this set
    Load the estimation data and create an iddata object.
load twotankdata
z = iddata(y,u,0.2);
Estimate the nonlinear ARX model.

```
sys = nlarx(z,lreg)
sys =
Nonlinear ARX model with 1 output and 1 input
    Inputs: ul
    Outputs: yl
Regressors:
    Linear regressors in variables y1, ul
    List of all regressors
Output function: Wavelet Network with 11 units
Sample time: 0.2 seconds
Status:
Estimated using NLARX on time domain data "z".
Fit to estimation data: 96.84% (prediction focus)
FPE: 3.482e-05, MSE: 3.431e-05
```

View the regressors

```
getreg(sys)
ans = 8\times1 cell
    {'y1(t-1)'}
    {'y1(t-2)'}
    {'y1(t-3)'}
    {'y1(t-4)'}
    {'u1(t-1)'}
    {'u1(t-2)'}
    {'u1(t-3)'}
    {'u1(t-4)'}
```

Compare the model output to the estimation data.


## Use Absolute Value in Linear Regressor Set

Create a linear regressor set that uses lags of 3,10 , and 100 in variable y1 and lags of 0 and 4 in variable u1.

```
vars = {'y1','u1'};
lags = {[3 10 100],[0,4]};
```

Specify that the y1 regressor use the absolute value of y 1 .
UseAbs = [true,false];
Create the linear regressor.

```
reg = linearRegressor(vars,lags,UseAbs)
reg =
Linear regressors in variables y1, u1
        Variables: {'y1' 'u1'}
                Lags: {[3 10 100] [0 4]}
        UseAbsolute: [1 0]
        TimeVariable: 't'
```

    Regressors described by this set
    
## Specify Linear, Polynomial, and Custom Regressors

Load the estimation data $z 1$, which has one input and one output, and obtain the output and input names.

```
load iddatal z1;
names = [z1.OutputName z1.InputName]
names = l×2 cell
    {'yl'} {'ul'}
```

Specify $L$ as the set of linear regressors that represents $y_{1}(t-1), u_{1}(t-2)$, and $u_{1}(t-5)$.
L = linearRegressor(names,\{1,[2 5]\});
Specify P as the polynomial regressor $y_{1}(t-1)^{2}$.

P = polynomialRegressor(names(1),1,2);
Specify C as the custom regressor $y_{1}(t-2) u_{1}(t-3)$. Use an anonymous function handle to define this function.

```
C = customRegressor(names,{2 3},@(x,y)x.*y)
C =
Custom regressor: y1(t-2).*u1(t-3)
    VariablesToRegressorFcn: @(x,y)x.*y
            Variables: {'y1' 'ul'}
                            Lags: {[2] [3]}
    Vectorized: 1
    TimeVariable: 't'
    Regressors described by this set
```

Combine the regressors in the column vector $R$.
$R=[L ; P ; C]$
R=3×1 object
[3 1] array of linearRegressor, polynomialRegressor, customRegressor objects.

1. Linear regressors in variables y1, ul Variables: \{'y1' 'ul'\}

Lags: \{[1] [2 5]\}
UseAbsolute: [0 0]
TimeVariable: 't'
2. Order 2 regressors in variables y1 Order: 2
Variables: \{'y1'\}
Lags: \{[1]\}
UseAbsolute: 0
AllowVariableMix: 0

$$
\begin{aligned}
& \text { AllowLagMix: } 0 \\
& \text { TimeVariable: 't' }
\end{aligned}
$$

```
3. Custom regressor: yl(t-2).*ul(t-3)
    VariablesToRegressorFcn: @(x,y)x.*y
            Variables: {'yl' 'ul'}
                        Lags: {[2] [3]}
                            Vectorized: 1
                    TimeVariable: 't'
    Regressors described by this set
```

Estimate a nonlinear ARX model with R .

```
sys = nlarx(z1,R)
sys =
Nonlinear ARX model with 1 output and 1 input
    Inputs: ul
    Outputs: y1
Regressors:
    1. Linear regressors in variables y1, ul
    2. Order 2 regressors in variables y1
    3. Custom regressor: y1(t-2).*ul(t-3)
    List of all regressors
Output function: Wavelet Network with 1 unit
Sample time: 0.1 seconds
Status:
Estimated using NLARX on time domain data "z1".
Fit to estimation data: 59.73% (prediction focus)
FPE: 3.356, MSE: 3.147
```

View the full regressor set.

```
getreg(sys)
```

ans $=5 \times 1 \mathrm{cell}$
\{'y1(t-1)' \}
\{'u1(t-2)' \}
\{'u1(t-5)' \}
\{'y1(t-1)^2' \}
\{'y1(t-2).*u1(t-3)'\}

## See Also

customRegressor|getreg|idnlarx|nlarx|polynomialRegressor
Introduced in R2021a

## Isim

Plot simulated time response of dynamic system to arbitrary inputs; simulated response data

## Syntax

```
lsim(sys,u,t)
lsim(sys,u,t,x0)
lsim(sys,u,t,x0,method)
lsim(sys1,sys2,\ldots.,sysN,u,t,___)
lsim(sys1,LineSpec1,...,sysN,LineSpecN,
```

$\qquad$

``` )
y = lsim(sys,u,t)
y = lsim(sys,u,t,x0)
y = lsim(sys,u,t,x0,method)
[y,tOut,x] = lsim(
```

$\qquad$

``` )
lsim(sys)
```


## Description

## Response Plots

lsim(sys,u,t) plots the simulated time response of the dynamic system model sys to the input history ( $\mathrm{t}, \mathrm{u}$ ). The vector t specifies the time samples for the simulation. For single-input systems, the input signal $u$ is a vector of the same length as $t$. For multi-input systems, $u$ is an array with as many rows as there are time samples (length( t )) and as many columns as there are inputs to sys.
$\operatorname{lsim}(s y s, u, t, x 0)$ further specifies a vector $x 0$ of initial state values, when sys is a state-space model.
$\operatorname{lsim}(s y s, u, t, x 0$, method) specifies how lsim interpolates the input values between samples, when sys is a continuous-time model.
lsim(sys1, sys2,...,sysN,u,t,__) simulates the responses of several dynamic system models to the same input history and plots these responses on a single figure. All systems must have the same number of inputs and outputs. You can also use the x 0 and method input arguments when computing the responses of multiple models.
lsim(sys1,LineSpec1, ...,sysN, LineSpecN, $\qquad$ ) specifies a color, line style, and marker for each system in the plot. When you need additional plot customization options, use lsimplot instead.

## Response Data

$y=\operatorname{lsim}(s y s, u, t)$ returns the system response $y$, sampled at the same times $t$ as the input. For single-output systems, $y$ is a vector of the same length as $t$. For multi-output systems, $y$ is an array having as many rows as there are time samples (length( $t$ )) and as many columns as there are outputs in sys. This syntax does not generate a plot.
$y=l \operatorname{sim}(s y s, u, t, x 0)$ further specifies a vector $x 0$ of initial state values, when sys is a statespace model.
$y=\operatorname{lsim}(s y s, u, t, x 0$, method) specifies how lsim interpolates the input values between samples, when sys is a continuous-time model.
[y,t0ut, $x$ ] = lsim( $\qquad$ ) returns the state trajectories $x$, when sys is a state-space model. $x$ is an array with as many rows as there are time samples and as many columns as there are states in sys. This syntax also returns the time samples used for the simulation in t0ut.

## Linear Simulation Tool

lsim (sys) opens the Linear Simulation Tool. For more information about using this tool for linear analysis, see Working with the Linear Simulation Tool (Control System Toolbox).

## Examples

## Simulated Response to Arbitrary Input Signal

Consider the following transfer function.

```
sys = tf(3,[1 2 3])
sys =
    3
    * 2-2 s+3
Continuous-time transfer function.
```

To compute the response of this system to an arbitrary input signal, provide lsim with a vector of the times $t$ at which you want to compute the response and a vector $u$ containing the corresponding signal values. For instance, plot the system response to a ramping step signal that starts at 0 at time $\mathrm{t}=0$, ramps from 0 at $\mathrm{t}=1$ to 1 at $\mathrm{t}=2$, and then holds steady at 1 . Define t and compute the values of $u$.

```
t = 0:0.04:8; % 201 points
u = max(0,min(t-1,1));
```

Use lsim without an output argument to plot the system response to the signal.

```
lsim(sys,u,t)
grid on
```



The plot shows the applied input ( $u, t$ ) in gray and the system response in blue.
Use lsim with an output argument to obtain the simulated response data.
$y=\operatorname{lsim}(s y s, u, t) ;$
size(y)
ans $=1 \times 2$
$201 \quad 1$

The vector y contains the simulated response at the corresponding times in t .

## Response to Periodic Signal

Use gensig (Control System Toolbox) to create periodic input signals such as sine waves and square waves for use with lsim. Simulate the response to a square wave of the following SISO state-space model.

```
A = [-3 -1.5; 5 0];
B = [1; 0];
C = [0.5 1.5];
D = 0;
sys = ss(A,B,C,D);
```

For this example, create a square wave with a period of 10 s and a duration of 20 s .
[u,t] = gensig("square", 10,20);
gensig returns the vector $t$ of time steps and the vector $u$ containing the corresponding values of the input signal. (If you do not specify a sample time for t , then gensig generates 64 samples per period.) Use these with lsim and plot the system response.
lsim(sys,u,t)
grid on


The plot shows the applied square wave in gray and the system response in blue. Call lsim with an output argument to obtain the response values at each point in $t$.
$[y, \sim]=\operatorname{lsim}(s y s, u, t) ;$

## Response of Discrete-Time System

When you simulate the response of a discrete-time system, the time vector $t$ must be of the form Ti:dT:Tf, where dT is the sample time of the model. Simulate the response of the following discrete-time transfer function to a ramp step input.
sys = tf([0.06 0.05],[1-1.56 0.67],0.05);

This transfer function has a sample time of 0.05 s . Use the same sample time to generate the time vector $t$ and a ramped step signal $u$.
$t=0: 0.05: 4 ;$
$\mathrm{u}=\max (0, \min (\mathrm{t}-1,1))$;
Plot the system response.
lsim(sys,u,t)

Linear Simulation Results


To simulate the response of a discrete-time system to a periodic input signal, use the same sample time with gensig to generate the input. For instance, simulate the system response to a sine wave with period of 1 s and a duration of 4 s .
[u,t] = gensig("sine",1,4,0.05);
Plot the system response.
lsim(sys,u,t)


## Plot Response of Multiple Systems to Same Input

lsim allows you to plot the simulated responses of multiple dynamic systems on the same axis. For instance, compare the closed-loop response of a system with a PI controller and a PID controller. Create a transfer function of the system and tune the controllers.

```
H = tf(4,[1 10 25]);
Cl = pidtune(H,'PI');
C2 = pidtune(H,'PID');
```

Form the closed-loop systems.

```
sys1 = feedback(H*C1,1);
sys2 = feedback(H*C2,1);
```

Plot the responses of both systems to a square wave with a period of 4 s .

```
[u,t] = gensig("square",4,12);
lsim(sys1,sys2,u,t)
grid on
legend("PI","PID")
```



By default, l sim chooses distinct colors for each system that you plot. You can specify colors and line styles using the LineSpec input argument.

```
lsim(sys1,"r--",sys2,"b",u,t)
grid on
legend("PI","PID")
```



The first LineSpec "r--" specifies a dashed red line for the response with the PI controller. The second LineSpec "b" specifies a solid blue line for the response with the PID controller. The legend reflects the specified colors and line styles. For more plot customization options, use lsimplot.

## Plot Simulated Response of MIMO System

In a MIMO system, at each time step $t$, the input $u(t)$ is a vector whose length is the number of inputs. To use lsim, you specify $u$ as a matrix with dimensions Nt -by-Nu, where Nu is the number of system inputs and Nt is the length of t . In other words, each column of u is the input signal applied to the corresponding system input. For instance, to simulate a system with four inputs for 201 time steps, provide $u$ as a matrix of four columns and 201 rows, where each row $u(i,:)$ is the vector of input values at the ith time step; each column $u(:, j)$ is the signal applied at the $j$ th input.

Similarly, the output $y(t)$ computed by lsim is a matrix whose columns represent the signal at each system output. When you use lsim to plot the simulated response, lsim provides separate axes for each output, representing the system response in each output channel to the input $u(t)$ applied at all inputs.

Consider the two-input, three-output state-space model with the following state-space matrices.

```
A = [-1.5 -0.2 1.0;
    -0.2 -1.7 0.6;
    1.0 0.6 -1.4];
```

```
B = [ 1.5 0.6;
    -1.8 1.0;
    0 0 ];
C = [ 0 -0.5 -0.1;
        0.35 -0.1 -0.15
        0.65 0 0.6];
D = [ 0.5 0;
        0.05 0.75
        0 0];
sys = ss(A,B,C,D);
```

Plot the response of sys to a square wave of period 4 s , applied to the first input sys and a pulse applied to the second input every 3 s . To do so, create column vectors representing the square wave and the pulsed signal using gensig. Then stack the columns into an input matrix. To ensure the two signals have the same number of samples, specify the same end time and sample time.

Tf = 10;
Ts = 0.1;
[uSq, t] = gensig("square", 4, Tf,Ts);
[uP, $\sim$ ] = gensig("pulse",3,Tf,Ts);
u = [uSq uP];
lsim(sys,u,t)


Each axis shows the response of one of the three system outputs to the signals $u$ applied at all inputs. Each plot also shows all input signals in gray.

## Plot System Evolution from Initial Condition

By default, lsim simulates the model assuming all states are zero at the start of the simulation. When simulating the response of a state-space model, use the optional x0 input argument to specify nonzero initial state values. Consider the following two-state SISO state-space model.

```
A = [-1.5 -3;
    3 -1];
B = [1.3; 0];
C = [1.15 2.3];
D = 0;
sys = ss(A,B,C,D);
```

Suppose that you want to allow the system to evolve from a known set of initial states with no input for 2 s , and then apply a unit step change. Specify the vector x 0 of initial state values, and create the input vector.

```
x0 = [-0.2 0.3];
t = 0:0.05:8;
u = zeros(length(t),1);
u(t>=2) = 1;
lsim(sys,u,t,x0)
grid on
```



The first half of the plot shows the free evolution of the system from the initial state values [-0.2 0.3 ]. At $t=2$ there is a step change to the input, and the plot shows the system response to this new signal beginning from the state values at that time.

## Extract Simulated Response Data

When you use lsim with output arguments, it returns the simulated response data in an array. For a SISO system, the response data is returned as a column vector of the same length as $t$. For instance, extract the response of a SISO system to a square wave. Create the square wave using gensig.

```
sys = tf([2 5 1],[1 2 3]);
[u,t] = gensig("square",4,10,0.05);
[y,t] = lsim(sys,u,t);
size(y)
ans = 1\times2
```

    \(201 \quad 1\)
    The vector y contains the simulated response at each time step in t . (l sim returns the time vector t as a convenience.)

For a MIMO system, the response data is returned in an array of dimensions $N$-by- $N y$-by- $N u$, where $N y$ and $N u$ are the number of outputs and inputs of the dynamic system. For instance, consider the following state-space model, representing a three-state system with two inputs and three outputs.

```
A = [-1.5 -0.2 1.0;
    -0.2 -1.7 0.6;
    1.0 0.6 -1.4];
B = [ 1.5 0.6;
    -1.8 1.0;
        0 0 ];
C = [ 0 -0.1 -0.2;
        0.7 -0.2 -0.3
    -0.65 0 -0.6];
D = [ 0.1 0;
        0.1 1.5
        0 0];
sys = ss(A,B,C,D);
```

Extract the responses of the three output channels to the square wave applied at both inputs.

```
uM = [u u];
[y,t] = lsim(sys,uM,t);
size(y)
ans = 1\times2
    201 3
```

$y(:, j)$ is a column vector containing response at the $j$ th output to the square wave applied to both inputs. That is, $y(i,:)$ is a vector of three values, the output values at the $i$ th time step.

Because sys is a state-space model, you can extract the time evolution of the state values in response to the input signal.

```
[y,t,x] = lsim(sys,uM,t);
size(x)
ans = 1\times2
    201 3
```

Each row of $x$ contains the state values [ $x 1, x 2, x 3$ ] at the corresponding time in $t$. In other words, $x(i,:)$ is the state vector at the $i$ th time step. Plot the state values.

```
plot(t,x)
```



## Response of Systems in Model Array

The example Plot Response of Multiple Systems to Same Input shows how to plot responses of several individual systems on a single axis. When you have multiple dynamic systems arranged in a model array, lsim plots all their responses at once.

Create a model array. For this example, use a one-dimensional array of second-order transfer functions having different natural frequencies. First, preallocate memory for the model array. The following command creates a 1-by-5 row of zero-gain SISO transfer functions. The first two dimensions represent the model outputs and inputs. The remaining dimensions are the array dimensions. (For more information about model arrays and how to create them, see "Model Arrays" (Control System Toolbox).)
sys $=\mathrm{tf}(z \operatorname{eros}(1,1,1,5))$;
Populate the array.

```
w0 = 1.5:1:5.5; % natural frequencies
zeta = 0.5; % damping constant
for i = 1:length(w0)
    sys(:,:,1,i) = tf(w0(i)^2,[1 2*zeta*w0(i) w0(i)^2]);
end
```

Plot the responses of all models in the array to a square wave input.

```
[u,t] = gensig("square",5,15);
lsim(sys,u,t)
```


lsim uses the same line style for the responses of all entries in the array. One way to distinguish among entries is to use the SamplingGrid property of dynamic system models to associate each entry in the array with the corresponding $w 0$ value.

```
sys.SamplingGrid = struct('frequency',w0);
```

Now, when you plot the responses in a MATLAB figure window, you can click a trace to see which frequency value it corresponds to.

## Simulate Response of Identified Model

Load estimation data to estimate a model.

```
load(fullfile(matlabroot,'toolbox','ident','iddemos','data','dcmotordata'));
```

z = iddata(y,u,0.1,'Name','DC-motor');
$z$ is an iddata object that stores the one-input two-output estimation data with a sample time of 0.1 s.

Estimate a state-space model of order 4 using estimation data z.
[sys,x0] = n4sid(z,4);
sys is the estimated model and $x 0$ is the estimated initial states.
Simulate the response of sys using the same input data as the one used for estimation and the initial states returned by the estimation command.
$[y, t, x]=$ lsim(sys,z.InputData, [], x0);
Here, y is the system response, t is the time vector used for simulation, and x is the state trajectory. Compare the simulated response $y$ to the measured response z.OutputData for both outputs.
subplot(211), plot(t,z.OutputData(:,1),'k',t,y(:,1),'r')
legend('Measured','Simulated')
subplot(212), plot(t,z.OutputData(:,2),'k',t,y(:,2),'r')
legend('Measured','Simulated')



## Effect of Sample Time on Simulation

The choice of sample time can drastically affect simulation results. To illustrate why, consider the following second-order model.

$$
\operatorname{sys}(s)=\frac{\omega^{2}}{s^{2}+2 s+\omega^{2}}, \omega=62.83 .
$$

Simulate the response of this model to a square wave with period 1 s , using a sample time of 0.1 s .

```
w2 = 62.83^2;
sys = tf(w2,[1 2 w2]);
tau = 1;
Tf = 5;
Ts = 0.1;
[u,t] = gensig("square",tau,Tf,Ts);
lsim(sys,u,t)
```



lsim simulates the model using the specified input signal, but it issues a warning that the input signal is undersampled. lsim recommends a sample time that generates at least 64 samples per period of the input $u$. To see why this recommendation matters, simulate sys again using a sample time smaller than the recommended maximum.

```
figure
Ts2 = 0.01;
[u2,t2] = gensig("square",tau,Tf,Ts2);
lsim(sys,u2,t2)
```



This response exhibits strong oscillatory behavior that is hidden in the undersampled version.

## Input Arguments

sys - Dynamic system
dynamic system model | model array
Dynamic system, specified as a SISO or MIMO dynamic system model or array of dynamic system models. Dynamic systems whose responses you can simulate include:

- Continuous-time or discrete-time numeric LTI models, such as tf, zpk, or ss models.
- Generalized or uncertain LTI models such as genss or uss models. (Using uncertain models requires Robust Control Toolbox software.)
- For tunable control design blocks, the function evaluates the model at its current value for both plotting and returning response data.
- For uncertain control design blocks, the function plots the nominal value and random samples of the model. When you use output arguments, the function returns response data for the nominal model only.
- Sparse state-space models such as sparss and mechss models.
- Identified LTI models, such as idtf, idss, or idproc models. For identified models, you can also use the sim command, which can compute the standard deviation of the simulated response and state trajectories. sim can also simulate all types of models with nonzero initial conditions, and can simulate nonlinear identified models.
lsim does not support frequency-response data models such as frd , genfrd, or idfrd models.
If sys is an array of models, the function plots the responses of all models in the array on the same axes. See "Response of Systems in Model Array" (Control System Toolbox).


## u - Input signal

vector | array
Input signal for simulation, specified as a vector for single-input systems, and an array for multi-input systems.

- For single-input systems, $u$ is a vector of the same length as $t$.
- For multi-input systems, $u$ is an array with as many rows as there are time samples (length( $t$ )) and as many columns as there are inputs to sys. In other words, each row $u(i,:)$ represents the values applied at the inputs of sys at time $t(i)$. Each column $u(:, j)$ is the signal applied to the $j$ th input of sys.


## t - Time samples

vector
Time samples at which to compute the response, specified as a vector of the form 0:dT:Tf. The lsim command interprets $t$ as having the units specified in the TimeUnit property of the model sys.

For continuous-time sys, the lsim command uses the time step dT to discretize the model. If dT is too large relative to the system dynamics (undersampling), lsim issues a warning recommending a faster sampling time. For further discussion of the impact of sampling time on simulation, see "Effect of Sample Time on Simulation" (Control System Toolbox).

For discrete-time sys, the time step dT must equal the sample time of sys. Alternatively, you can omit $t$ or set it to [ ]. In that case, lsim sets $t$ to a vector of the same length as $u$ that begins at 0 with a time step equal to sys.Ts.

## x0 - Initial state values

vector of zeros (default) | vector
Initial state values for simulating a state-space model, specified as a vector having one entry for each state in sys. If you omit this argument, then lsim sets all states to zero at $t=0$.

## method - Discretization method <br> 'zoh|'foh'

Discretization method for sampling continuous-time models, specified as one of the following.

- 'zoh' - Zero-order hold
- 'foh' - First-order hold

When sys is a continuous-time model, lsim computes the time response by discretizing the model using a sample time equal to the time step $d T=t(2)-t(1)$ of $t$. If you do not specify a discretization method, then lsim selects the method automatically based on the smoothness of the signal u. For more information about these two discretization methods, see "Continuous-Discrete Conversion Methods" (Control System Toolbox).

## LineSpec - Line style, marker, and color

character vector | string

Line style, marker, and color, specified as a string or vector of one, two, or three characters. The characters can appear in any order. You do not need to specify all three characteristics (line style, marker, and color). For example, if you omit the line style and specify the marker, then the plot shows only the marker and no line. For more information about configuring this argument, see the LineSpec input argument of the plot function.
Example: 'r--' specifies a red dashed line
Example: '*b' specifies blue asterisk markers
Example: 'y' specifies a yellow line

## Output Arguments

## y - Simulated response data <br> array

Simulated response data, returned as an array.

- For single-input systems, y is a column vector of the same length as t .
- For multi-output systems, y is an array with as many rows as there are time samples (length( t ) ) and as many columns as there are outputs in sys. Thus, the jth column of $y$, or $y(:, j)$, contains the response at the $j$ th output to $u$ applied at all inputs.


## tOut - Time vector

vector
Time vector used for simulation, returned as a column vector. When you specify an input time vector t of the form $0: \mathrm{dT}: \mathrm{Tf}$, then $\mathrm{tOut}=\mathrm{t}$. If t is nearly equisampled, l sim adjusts the sample times for simulation and returns the result in tOut. For discrete-time sys, you can omit $t$ or set it to [ ]. In that case, lsim sets $t$ to a vector of the same length as $u$ that begins at 0 with a time step equal to sys.Ts, and returns the result in tOut.

## x - State trajectories

array
State trajectories, returned as an array. When sys is a state-space model, x contains the evolution of the states of sys in response to the input. x is an array with as many rows as there are time samples (length( t )) and as many columns as there are states in sys.

## Tips

- When you need additional plot customization options, use lsimplot instead.


## Algorithms

For a discrete-time transfer function,

$$
\operatorname{sys}\left(z^{-1}\right)=\frac{a_{0}+a_{1} z^{-1}+\ldots+a_{n} z^{-n}}{1+b_{1} z^{-1}+\ldots+b_{n} z^{-n}}
$$

lsim filters the input based on the recursion associated with this transfer function:

$$
y[k]=a_{0} u[k]+\ldots+a_{n} u[k-n]-b_{1} y[k-1]-\ldots-b_{n}[k-n] .
$$

For discrete-time zpk models, lsim filters the input through a series of first-order or second-order sections. This approach avoids forming the numerator and denominator polynomials, which can cause numerical instability for higher-order models.

For discrete-time state-space models, lsim propagates the discrete-time state-space equations,

$$
\begin{gathered}
x[n+1]=A x[n]+B u[n], \\
y[n]=C x[n]+D u[n] .
\end{gathered}
$$

For continuous-time systems, lsim first discretizes the system using c2d, and then propagates the resulting discrete-time state-space equations. Unless you specify otherwise with the method input argument, lsim uses the first-order-hold discretization method when the input signal is smooth, and zero-order hold when the input signal is discontinuous, such as for pulses or square waves. The sample time for discretization is the spacing dT between the time samples you supply in t .

## See Also

## Functions

gensig|impulse|initial|lsiminfo|lsimplot|sim|step

## Apps

Linear System Analyzer
Introduced in R2012a

## Isiminfo

Compute linear response characteristics

## Syntax

```
S = lsiminfo(y,t,yfinal)
S = lsiminfo(y,t)
S = lsiminfo(...,'SettlingTimeThreshold',ST)
```


## Description

$\mathrm{S}=\mathrm{l}$ siminfo( $\mathrm{y}, \mathrm{t}, \mathrm{yfinal})$ takes the response data $(\mathrm{t}, \mathrm{y})$ and a steady-state value yfinal and returns a structure $S$ containing the following performance indicators:

- SettlingTime - Settling time
- Min - Minimum value of Y
- MinTime - Time at which the minimum value is reached
- Max - Maximum value of Y
- MaxTime - Time at which the maximum value is reached

For SISO responses, t and y are vectors with the same length NS. For responses with NY outputs, you can specify y as an NS-by-NY array and yfinal as a NY-by-1 array. lsiminfo then returns an NY-by-1 structure array S of performance metrics for each output channel.
$\mathrm{S}=$ lsiminfo( $\mathrm{y}, \mathrm{t})$ uses the last sample value of y as steady-state value y final. $\mathrm{s}=$ lsiminfo(y) assumes $t=1: N S$.

S = lsiminfo(...,'SettlingTimeThreshold',ST) lets you specify the threshold ST used in the settling time calculation. The response has settled when the error |y( $t$ ) - yfinal| becomes smaller than a fraction ST of its peak value. The default value is $\mathrm{ST}=0.02$ (2\%).

## Examples

## Compute Response Characteristics of a Transfer Function

Create the following continuous-time transfer function:

$$
H(s)=\frac{s-1}{s^{3}+2 s^{2}+3 s+4}
$$

sys = tf([1-1],[1 $\left.2 \begin{array}{lll}1 & 3 & 4\end{array}\right]$;
Calculate the impulse response.
[y,t] = impulse(sys);
impulse returns the output response $y$ and the time vector $t$ used for simulation.

Compute the response characteristics using a final response value of 0 .

```
s = lsiminfo(y,t,0)
s = struct with fields:
    SettlingTime: 22.8700
            Min: -0.4268
            MinTime: 2.0088
            Max: 0.2847
            MaxTime: 4.0733
```

You can plot the impulse response and verify these response characteristics. For example, the time at which the minimum response value (MinTime) is reached is approximately 2 seconds.

```
impulse(sys)
```



## See Also

impulse |initial|lsim|stepinfo

## Introduced in R2012a

## Isimplot

Simulate response of dynamic system to arbitrary inputs and return plot handle

## Syntax

```
h = lsimplot(sys)
lsimplot(sys1,sys2,...)
lsimplot(sys,u,t)
lsimplot(sys,u,t,x0)
lsimplot(sys1,sys2,...,u,t,x0)
lsimplot(AX,...)
lsimplot(..., plotoptions)
lsimplot(sys,u,t,x0,'zoh')
lsimplot(sys,u,t,x0,'foh')
```


## Description

h = lsimplot (sys) opens the Linear Simulation Tool for the dynamic system model sys, which enables interactive specification of driving input(s), the time vector, and initial state. It also returns the plot handle h. You can use this handle to customize the plot with the getoptions and setoptions commands. Type
help timeoptions
for a list of available plot options.
lsimplot (sys1, sys2, ...) opens the Linear Simulation Tool for multiple models sys1,sys2,.... Driving inputs are common to all specified systems but initial conditions can be specified separately for each.
lsimplot (sys,u,t) plots the time response of the model sys to the input signal described by $u$ and $t$. The time vector $t$ consists of regularly spaced time samples (in system time units, specified in the TimeUnit property of sys). For MIMO systems, $u$ is a matrix with as many columns as inputs and whose ith row specifies the input value at time $t(i)$. For SISO systems $u$ can be specified either as a row or column vector. For example,

```
t = 0:0.01:5;
u = sin(t);
lsimplot(sys,u,t)
```

simulates the response of a single-input model sys to the input $u(t)=\sin (t)$ during 5 seconds.
For discrete-time models, u should be sampled at the same rate as sys ( t is then redundant and can be omitted or set to the empty matrix).

For continuous-time models, choose the sampling period $t(2)-t(1)$ small enough to accurately describe the input $u$. lsim issues a warning when $u$ is undersampled, and hidden oscillations can occur.
lsimplot (sys, $u, t, x 0$ ) specifies the initial state vector $x 0$ at time $t(1)$ (for state-space models only). $x 0$ is set to zero when omitted.
lsimplot(sys1,sys2,..., u,t,x0) simulates the responses of multiple LTI models sys1,sys2,... on a single plot. The initial condition $x 0$ is optional. You can also specify a color, line style, and marker for each system, as in
lsimplot(sys1,'r',sys2,'y--',sys3,'gx',u,t)
lsimplot (AX,...) plots into the axes with handle AX.
lsimplot(..., plotoptions) plots the initial condition response with the options specified in plotoptions. Type
help timeoptions
for more detail.
For continuous-time models, lsimplot(sys,u,t,x0,'zoh') or lsimplot(sys,u,t,x0,'foh') explicitly specifies how the input values should be interpolated between samples (zero-order hold or linear interpolation). By default, lsimplot selects the interpolation method automatically based on the smoothness of the signal $u$.

## See Also

getoptions|lsim|setoptions
Introduced in R2012a

## mag2db

Convert magnitude to decibels (dB)

## Syntax

$y d b=m a g 2 d b(y)$

## Description

$y d b=\operatorname{mag2db}(y)$ expresses in decibels ( dB ) the magnitude measurements specified in y . The relationship between magnitude and decibels is $y d b=20 * \log _{10}(y)$

## Examples

## Display Gain Margins in Decibels

For this example, consider the following SISO feedback loop where the system contains multiple gain crossover or phase crossover frequencies, which leads to multiple gain or phase margin values:


Create the transfer function.
$\mathrm{G}=\mathrm{tf}(20,[17]) * \operatorname{tf}([13.27 .2],[1-1.20 .8]) * \operatorname{tf}([1-8400],[133$ 700]);
Use the allmargin command to compute all stability margins.

```
m = allmargin(G)
m = struct with fields:
    GainMargin: [0.3408 3.3920]
    GMFrequency: [1.9421 16.4807]
    PhaseMargin: 68.1178
    PMFrequency: 7.0762
    DelayMargin: 0.1680
    DMFrequency: 7.0762
        Stable: 1
```

Note that gain margins are expressed as gain ratios and not in decibels (dB). Use mag2db to convert the values to dB .

```
GainMargins_dB = mag2db(m.GainMargin)
GainMargins dB = 1\times2
    -9.3510 10.6091
```


## Input Arguments

## y - Input array

scalar | vector | matrix | array
Input array, specified as a scalar, vector, matrix, or an array. When y is nonscalar, mag2db is an element-wise operation.
Data Types: single | double
Complex Number Support: Yes

## Output Arguments

ydb - Magnitude measurements in decibels
scalar | vector | matrix | array
Magnitude measurements in decibels, returned as a scalar, vector, matrix, or an array of the same size as $y$.

## See Also

db2mag
Introduced in R2008a

## merge (iddata)

Merge data sets into iddata object

## Syntax

```
dat = merge(dat1,dat2,....,datN)
```


## Description

dat collects the data sets in dat1, ..., datN into one iddata object, with several experiments. The number of experiments in dat will be the sum of the number of experiments in datk. For the merging to be allowed, a number of conditions must be satisfied:

- All of datk must have the same number of input channels, and the InputNames must be the same.
- All of datk must have the same number of output channels, and the OutputNames must be the same. If some input or output channel is lacking in one experiment, it can be replaced by a vector of NaNs to conform with these rules.
- If the ExperimentNames of datk have been specified as something other than the default 'Exp1', 'Exp2', etc., they must all be unique. If default names overlap, they are modified so that dat will have a list of unique ExperimentNames.

The sampling intervals, the number of observations, and the input properties (Period, InterSample) might be different in the different experiments.

You can retrieve the individual experiments by using the command getexp. You can also retrieve them by subreferencing with a fourth index.

```
dat1 = dat(:,:,:,ExperimentNumber)
```

or
dat1 $=\operatorname{dat}(:,:,:$ ExperimentName)
Storing multiple experiments as one iddata object can be very useful for handling experimental data that has been collected on different occasions, or when a data set has been split up to remove "bad" portions of the data. All the toolbox routines accept multiple-experiment data.

## Examples

## Merge Multiple Data Sets

Remove bad portions of data to estimate models without the bad data destroying the estimate.
load iddemo8;
plot(dat);

## Input-Output Data



Bad portions of data are detected around sample 250 to 280 and between samples 600 to 650 . Cut out these bad portions to form a multiple-experiment data set and merge the data.

```
dat = merge(dat(1:250),dat(281:600),dat(651:1000));
```

You can use the first two experiments to estimate a model and the third experiment to validate the model.

```
dat_est = getexp(dat,[1,2]);
m = ssest(dat_est,2);
dat_val = getexp(dat,3);
```


## See Also

getexp|iddata|merge

## Topics

"Dealing with Multi-Experiment Data and Merging Models"
"Create Multiexperiment Data at the Command Line"
Introduced before R2006a

## merge

Merge estimated models

## Syntax

$\mathrm{m}=\mathrm{merge}(\mathrm{m} 1, \mathrm{~m} 2, \ldots . . \mathrm{mN})$
[m,tv] = merge(m1,m2)

## Description

$\mathrm{m}=\mathrm{merge}(\mathrm{m} 1, \mathrm{~m} 2, \ldots . \mathrm{mN})$ merges estimated models. The models $\mathrm{m} 1, \mathrm{~m} 2, \ldots, \mathrm{mN}$ must all be of the same structure, just differing in parameter values and covariance matrices. Then $m$ is the merged model, where the parameter vector is a statistically weighted mean (using the covariance matrices to determine the weights) of the parameters of mk .
[ $\mathrm{m}, \mathrm{tv}$ ] = merge ( $\mathrm{m} 1, \mathrm{~m} 2$ ) returns a test variable tv. When two models are merged,
[m, tv] = merge(m1,m2)
tv is $\chi^{2}$ distributed with n degrees of freedom, if the parameters of m 1 and m 2 have the same means. Here n is the length of the parameter vector. A large value of tv thus indicates that it might be questionable to merge the models.

For idfrd models, merge is a statistical average of two responses in the individual models, weighted using inverse variances. You can only merge two idfrd models with responses at the same frequencies and nonzero covariances.

Merging models is an alternative to merging data sets and estimating a model for the merged data.

```
load iddatal z1;
load iddata2 z2;
m1 = arx(z1,[2 3 4]);
m2 = arx(z2,[2 3 4]);
ma = merge(m1,m2);
and
mb = arx(merge(z1,z2),[2 3 4]);
```

result in models ma and mb that are related and should be close. The difference is that merging the data sets assumes that the signal-to-noise ratios are about the same in the two experiments. Merging the models allows one model to be much more uncertain, for example, due to more disturbances in that experiment. If the conditions are about the same, we recommend that you merge data rather than models, since this is more efficient and typically involves better conditioned calculations.

## See Also

append

## Introduced in R2007a

## midprefs

Specify location for file containing System Identification app startup information

## Syntax

midprefs
midprefs(path)

## Description

The System Identification app allows a large number of variables for customized choices. These include the window layout, the default choices of plot options, and names and directories of the four most recent sessions with the System Identification app. This information is stored in the file idprefs.mat, which should be placed on the user's MATLABPATH. The default, automatic location for this file is in the same folder as the user's startup.m file.
midprefs is used to select or change the folder where you store idprefs.mat. Either type midprefs and follow the instructions, or give the folder name as the argument. Include all folder delimiters, as in the PC case:

```
midprefs('c:\matlab\toolbox\local\')
```

or in the UNIX ${ }^{\circledR}$ case"
midprefs('/home/ljung/matlab/')

## See Also

## Apps

System Identification
Introduced before R2006a

## misdata

Reconstruct missing input and output data

## Syntax

```
Datae = misdata(Data)
Datae = misdata(Data,Model)
Datae = misdata(Data,MaxIterations,Tol)
```


## Description

Datae $=$ misdata $($ Data $)$ reconstructs missing input and output data. Data is time-domain inputoutput data in the iddata object format. Missing data samples (both in inputs and in outputs) are entered as NaNs. Datae is an iddata object where the missing data has been replaced by reasonable estimates.

Datae $=$ misdata (Data, Model) specifies a model used for the reconstruction of missing data. Model is any linear identified model (idtf, idproc, idgrey, idpoly, idss). If no suitable model is known, it is estimated in an iterative fashion using default order state-space models.

Datae = misdata(Data,MaxIterations,Tol) specifies maximum number of iterations and tolerance. MaxIterations is the maximum number of iterations carried out (the default is 10). The iterations are terminated when the difference between two consecutive data estimates differs by less than $\mathrm{Tol} \%$. The default value of Tol is 1 .

## Examples

## Reconstruct Missing Data Using Specified Model

Load data with missing data points.
load('missing_data.mat')
missing_data is an iddata object containing input-output data.
Plot the data.

```
plot(missing_data)
```


## Input-Output Data



The output data contains missing data between indices 10 and 100 .
To reconstruct missing data using a specified model, estimate the model using measured data that has no missing samples. In this example, estimate a transfer function model with 2 poles.

```
data2 = missing_data(101:end);
model = tfest(data2,2);
```

Reconstruct the missing data.

```
datae = misdata(missing_data,model);
```

Plot the original and reconstructed data.

```
plot(missing_data,'b',datae,'--r')
```


## Input-Output Data



If you do not specify a model for reconstructing the data, the software alternates between estimating missing data and estimating models, based on the current data reconstruction.

## Algorithms

For a given model, the missing data is estimated as parameters so as to minimize the output prediction errors obtained from the reconstructed data. See Section 14.2 in Ljung (1999). Treating missing outputs as parameters is not the best approach from a statistical point of view, but is a good approximation in many cases.

When no model is given, the algorithm alternates between estimating missing data and estimating models, based on the current reconstruction.

## See Also

advice|arx|pexcit|tfest
Introduced before R2006a

## n4sid

Estimate state-space model using subspace method with time-domain or frequency-domain data

## Syntax

```
sys = n4sid(data,nx)
sys = n4sid(data,nx,Name,Value)
sys = n4sid(
        ,opt)
[sys,x0] = n4sid( __ )
```


## Description

sys $=n 4 s i d(d a t a, n x)$ estimates a discrete-time state-space model sys of order $n x$ using data, which can be time-domain or frequency-domain data. sys is a model of the following form:

$$
\begin{aligned}
& \dot{x}(t)=A x(t)+B u(t)+K e(t) \\
& y(t)=C x(t)+D u(t)+e(t)
\end{aligned}
$$

$A, B, C, D$, and $K$ are state-space matrices. $u(t)$ is the input, $y(t)$ is the output, $e(t)$ is the disturbance, and $x(t)$ is the vector of nx states.

All entries of $A, B, C$, and $K$ are free estimable parameters by default. For dynamic systems, $D$ is fixed to zero by default, meaning that the system has no feedthrough. For static systems ( $\mathrm{nx}=0$ ), $D$ is an estimable parameter by default.
sys = n4sid(data, nx, Name, Value) incorporates additional options specified by one or more name-value pair arguments. For example, to estimate a continuous-time model, specify the sample time 'Ts' as 0. Use the 'Form', 'Feedthrough', and 'DisturbanceModel' name-value pair arguments to modify the default behavior of the $A, B, C, D$, and $K$ matrices.
sys $=n 4$ sid ( _ _ , opt) specifies the estimation options opt. These options can include the initial states, estimation objective, and subspace algorithm related choices to be used for estimation.
Specify opt after any of the previous input-argument combinations.
[sys,x0] = n4sid( ___ ) returns the value of initial states computed during estimation. You can use this syntax with any of the previous input-argument combinations.

## Examples

## State-Space Model

Estimate a state-space model and compare its response with the measured output.
Load the input-output data z 1 , which is stored in an iddata object.
load iddata1 z1
Estimate a fourth-order state-space model.

```
nx = 4;
sys = n4sid(z1,nx)
sys =
    Discrete-time identified state-space model:
        x(t+Ts) = A x(t) + Bu(t) + K e(t)
            y(t) = C x(t) + D u(t) + e(t)
    A =
        rrrrer
        x2 0.4768 0.6671 0.1428 0.003757
        x3 -0.01951 0.08374 -0.09761 1.046
        x4 -0.003885 -0.02914 -0.8796 -0.03171
    B =
        M u1
        x2 -0.03301
        x3 7.256e-05
        x4 0.0005861
    C =
```



```
    D =
        u1
        y1 0
    K =
        y1
        x1 0.003282
        x2 0.009339
        x3 -0.003232
        x4 0.003809
Sample time: 0.1 seconds
Parameterization:
    FREE form (all coefficients in A, B, C free).
    Feedthrough: none
    Disturbance component: estimate
    Number of free coefficients: 28
    Use "idssdata", "getpvec", "getcov" for parameters and their uncertainties.
Status:
Estimated using N4SID on time domain data "z1".
Fit to estimation data: 76.33% (prediction focus)
FPE: 1.21, MSE: 1.087
```

Compare the simulated model response with the measured output.

```
compare(z1,sys)
```

Simulated Response Comparison


The plot shows that the fit percentage between the simulated model and the estimation data is greater than $70 \%$.

You can view more information about the estimation by exploring the idss property sys.Report.

```
sys.Report
ans =
    Status: 'Estimated using N4SID with prediction focus'
        Method: 'N4SID'
    InitialState: 'estimate'
        N4Weight: 'CVA'
        N4Horizon: [6 10 10]
            Fit: [1x1 struct]
        Parameters: [1x1 struct]
    OptionsUsed: [1\times1 idoptions.n4sid]
        RandState: [1x1 struct]
        DataUsed: [1x1 struct]
```

For example, find out more information about the estimated initial state.

```
sys.Report.Parameters.X0
ans = 4\times1
    -0.0085
    0.0052
```


## Determine Optimal Estimated Model Order

Load the input-output data $z 1$, which is stored in an iddata object.
load iddatal z1
Determine the optimal model order by specifying argument nx as a range from 1 to 10 .
$n x=1: 10 ;$
sys $=n 4 s i d(z 1, n x)$;
An automatically generated plot shows the Hankel singular values for models of the orders specified by $n x$.


States with relatively small Hankel singular values can be safely discarded. The suggested default order choice is 2 .

Select the model order in the Chosen Order list and click Apply.

## Specify Estimation Options

Load estimation data.
load iddata2 z2
Specify estimation options. Set the weighting scheme 'N4Weight' to 'SSARX' and estimation-status display option 'Display' to 'on'.

```
opt = n4sidOptions('N4Weight','SSARX','Display','on')
Option set for the n4sid command:
    InitialState: 'estimate'
            N4Weight: 'SSARX'
            N4Horizon: 'auto'
                Display: 'on'
            InputOffset: []
            OutputOffset: []
    EstimateCovariance: 1
        OutputWeight: []
            Focus: 'prediction'
        WeightingFilter: []
    EnforceStability: 0
        Advanced: [1x1 struct]
```

Estimate a third-order state-space model using the updated option set.

```
nx = 3;
```

sys = n4sid(z2,nx,opt);

## Modify Form, Feedthrough, and Disturbance-Model Matrices

Modify the canonical form of the A, B, and C matrices, include a feedthrough term in the D matrix, and eliminate disturbance-model estimation in the K matrix.

Load input-output data and estimate a fourth-order system using the n4sid default options.
load iddatal z1
sys1 = n4sid(z1,4);
Specify the modal form and compare the A matrix with the default A matrix.

```
sys2 = n4sid(z1,4,'Form','modal');
A1 = sys1.A
A1 = 4×4
    0.8392 -0.3129 0.0211 0.0374
    0.4768 0.6671 0.1428 0.0038
    -0.0195 0.0837 -0.0976 1.0462
    -0.0039 -0.0291 -0.8796 -0.0317
A2 = sys2.A
A2 = 4×4
\begin{tabular}{rrrr}
0.7554 & 0.3779 & 0 & 0 \\
-0.3779 & 0.7554 & 0 & 0 \\
0 & 0 & -0.0669 & 0.9542 \\
0 & 0 & -0.9542 & -0.0669
\end{tabular}
```

Include a feedthrough term and compare the D matrices.

```
sys3 = n4sid(z1,4,'Feedthrough',1);
D1 = sys1.D
D1 = 0
D3 = sys3.D
D3 = 0.0487
```

Eliminate disturbance modeling and compare the K matrices.

```
sys4 = n4sid(z1,4,'DisturbanceModel','none');
```

K1 = sys1.K
$K 1=4 \times 1$
0.0033
0.0093
-0.0032
0.0038
$\mathrm{K} 4=$ sys $4 . \mathrm{K}$
$K 4=4 \times 1$
0
0
0
0

## Continuous-Time Canonical-Form Model

Estimate a continuous-time canonical-form model.
Load estimation data.
load iddatal z1
Estimate the model. Set Ts to 0 to specify a continuous model.
$\mathrm{nx}=2$;
sys $=$ n4sid(z1,nx,'Ts',0,'Form','canonical');
sys is a second-order continuous-time state-space model in the canonical form.

## Estimate State-Space Model from Closed-Loop Data

Estimate a state-space model from closed-loop data using the subspace algorithm SSARX. This algorithm is better at capturing feedback effects than other weighting algorithms.

Generate closed-loop estimation data for a second-order system corrupted by white noise.

```
N = 1000;
K = 0.5;
rng('default');
w = randn(N,1);
z = zeros(N,1);
u = zeros(N,1);
y = zeros(N,1);
e = randn(N,1);
v = filter([1 0.5],[1 1.5 0.7],e);
for k = 3:N
    u(k-1) = -K*y(k-2) + w(k);
    u(k-1) = -K*y(k-1) + w(k);
    z(k) = 1.5*z(k-1) - 0.7*z(k-2) + u(k-1) + 0.5*u(k-2);
    y(k) = z(k) + 0.8*v(k);
end
dat = iddata(y, u, 1);
```

Specify the weighting scheme 'N4weight ' used by the N4SID algorithm. Create two option sets. For one option set, set 'N4weight' to 'CVA'. For the other option set, set the 'N4weight' to 'SSARX'.
optCVA = n4sidOptions('N4weight','CVA');
optSSARX = n4sidOptions('N4weight','SSARX');

Estimate state-space models using the option sets.
sysCVA = n4sid(dat,2,optCVA);
sysSSARX $=$ n4sid(dat,2,optSSARX);
Compare the fit of the two models with the estimation data.
compare(dat, sysCVA, sysSSARX);


As the plot shows, the model estimated using the SSARX algorithm produces a better fit than the model estimated using the CVA algorithm.

## Input Arguments

## data - Estimation data

iddata object | frd object | idfrd object
Estimation data, specified as an iddata object, an frd object, or an idfrd object.
For time-domain estimation, data must be an iddata object containing the input and output signal values.

For frequency-domain estimation, data can be one of the following:

- Recorded frequency response data (frd or idfrd)
- iddata object with properties specified as follows.
- InputData - Fourier transform of the input signal
- OutputData - Fourier transform of the output signal
- Domain - 'Frequency'

Estimation data must be uniformly sampled. By default, the software sets the sample time of the model to the sample time of the estimation data.

For multiexperiment data, the sample times and intersample behavior of all the experiments must match.

The domain of your data determines the type of model you can estimate.

- Time-domain or discrete-time frequency-domain data - Continuous-time and discrete-time models
- Continuous-time frequency-domain data - Continuous-time models only


## nx - Order of estimated model

1:10 (default) | positive integer scalar | positive integer vector | best | 0
Order of the estimated model, specified as a nonnegative integer or as a vector containing a range of positive integers.

- If you already know what order you want your estimated model to have, specify nx as a scalar.
- If you want to compare a range of potential orders to choose the most effective order for your estimated model, specify that range for nx . n4sid creates a Hankel singular-value plot that shows the relative energy contributions of each state in the system. States with relatively small Hankel singular values contribute little to the accuracy of the model and can be discarded with little impact. The index of the highest state you retain is the model order. The plot window includes a suggestion for the order to use. You can accept this suggestion or enter a different order. For an example, see "Determine Optimal Estimated Model Order" on page 1-837.

If you do not specify $n x$, or if you specify $n x$ as best, the software automatically chooses $n x$ from the range 1:10.

- If you are identifying a static system, set nx to 0 .
opt - Estimation options
n4sidOptions option set
Estimation options, specified as an n4sid0ptions option set. Options specified by opt include:
- Estimation objective
- Handling of initial conditions
- Subspace algorithm-related choices

For examples showing how to specify options, see "Specify Estimation Options" on page 1-837 and "Continuous-Time Canonical-Form Model" on page 1-839.

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Example: sys = n4sid(data,nx,'Form','modal')

## Ts - Sample time of estimated model

sample time of data (data.Ts) (default) | $0 \mid$ positive scalar
Sample time of the estimated model, specified as the comma-separated pair consisting of 'Ts ' and either 0 or a positive scalar.

- For continuous-time models, specify 'Ts ' as 0 . In the frequency domain, using continuous-time frequency-domain data results in a continuous-time model.
- For discrete-time models, the software sets 'Ts' to the sample time of the data in the units stored in the TimeUnit property.


## InputDelay - Input delays

0 (default) | scalar | vector
Input delay for each input channel, specified as the comma-separated pair consisting of 'InputDelay' and a numeric vector.

- For continuous-time models, specify 'InputDelay ' in the time units stored in the TimeUnit property.
- For discrete-time models, specify ' InputDelay ' in integer multiples of the sample time Ts. For example, setting 'InputDelay' to 3 specifies a delay of three sampling periods.

For a system with $N_{u}$ inputs, set InputDelay to an $N_{u}$-by-1 vector. Each entry of this vector is a numerical value that represents the input delay for the corresponding input channel.

To apply the same delay to all channels, specify ' InputDelay ' as a scalar.

## Form - Type of canonical form

'free' (default)|'modal'|'companion'|'canonical'
Type of canonical form of sys, specified as the comma-separated pair consisting of 'Form' and one of the following values:

- ' free' - Treat all entries of the matrices $A, B, C, D$, and $K$ as free.
- 'modal' - Obtain sys in modal form.
- 'companion' - Obtain sys in companion form.
- 'canonical' - Obtain sys in the observability canonical form.

For definitions of the canonical forms, see "Canonical State-Space Realizations".
For more information about using these forms for identification, see "Estimate State-Space Models with Canonical Parameterization".

For an example, see "Modify Form, Feedthrough, and Disturbance-Model Matrices" on page 1-838.

## Feedthrough - Direct feedthrough from input to output

0 (default) | 1 | logical vector
Direct feedthrough from input to output, specified as the comma-separated pair consisting of
' Feedthrough' and a logical vector of length $N_{u}$, where $N_{u}$ is the number of inputs. If you specify
' Feedthrough ' as a logical scalar, that value is applied to all the inputs. For static systems, the software always assumes 'Feedthrough' is 1.

For an example, see "Modify Form, Feedthrough, and Disturbance-Model Matrices" on page 1-838.

## DisturbanceModel - Option to estimate time-domain noise component parameters <br> 'estimate'| 'none'

Option to estimate time-domain noise component parameters in the K matrix, specified as the commaseparated pair consisting of 'DisturbanceModel' and one of the following values:

- 'estimate ' - Estimate the noise component. The $K$ matrix is treated as a free parameter. For time-domain data, 'estimate' is the default.
- 'none' - Do not estimate the noise component. The elements of the $K$ matrix are fixed at zero. For frequency-domain data, 'none' is the default and the only acceptable value.

For an example, see "Modify Form, Feedthrough, and Disturbance-Model Matrices" on page 1-838.

## Output Arguments

## sys - Identified state-space model

idss model
Identified state-space model, returned as an idss model. This model is created using the specified model orders, delays, and estimation options.

Information about the estimation results and options used is stored in the Report property of the model. Report has the following fields.

| Report <br> Field | Description |
| :--- | :--- |
| Status | Summary of the model status, which indicates whether the model was created by <br> construction or obtained by estimation. |
| Method | Estimation command used. |
| InitialSt <br> ate | How initial states were handled during estimation, returned as one of the following <br> values: <br> $-\quad$ ' zero' - The initial state is set to zero. <br> $-\quad$ 'estimate ' - The initial state is treated as an independent estimation parameter. <br> This field is especially useful when the ' InitialState ' option in the estimation <br> option set is 'auto '. |
| N4Weight | Weighting scheme used for singular-value decomposition by the N4SID algorithm, <br> returned as one of the following values: <br> $-\quad$ 'MOESP ' - Uses the MOESP algorithm. <br> $-\quad$ 'CVA' - Uses the Canonical Variate Algorithm. <br> $-\quad$ 'SSARX' - A subspace identification method that uses an ARX estimation-based <br> algorithm to compute the weighting. |
| This option is especially useful when the N4Weight option in the estimation option set <br> is 'auto'. |  |
| N4Horizon | Forward and backward prediction horizons used by the N4SID algorithm, returned as a <br> row vector with three elements [r sy su], where $r$ is the maximum forward <br> prediction horizon, sy is the number of past outputs, and su is the number of past <br> inputs that are used for the predictions. |


| Report <br> Field | Description |  |
| :--- | :--- | :--- |
| Fit | Quantitative assessment of the estimation, returned as a structure. See "Loss Function <br> and Model Quality Metrics" for more information on these quality metrics. The <br> structure has the following fields: |  |
|  | Field | Description |
|  | FitPerce <br> nt | Normalized root mean squared error (NRMSE) measure of how well the <br> response of the model fits the estimation data, expressed as the <br> percentage fit = 100(1-NRMSE). |
|  | LossFcn | Value of the loss function when the estimation completes. |
|  | MSE <br> Mean squared error (MSE) measure of how well the response of the <br> model fits the estimation data. |  |
|  | FPE | Final prediction error for the model. |
|  | AIC | Raw Akaike Information Criteria (AIC) measure of model quality. |
|  | AICc | Small sample-size corrected AIC. |
| nAIC | Normalized AIC. |  |
|  | BIC | Bayesian Information Criteria (BIC). |
| Parameter <br> s | Estimated values of model parameters. <br> OptionsUs <br> edOption set used for estimation. If you did not configure any custom options, <br> OptionsUsed is the set of default options. See n4sidOptions for more information. |  |
| RandState | State of the random number stream at the start of estimation. Empty, [ ], if <br> randomization was not used during estimation. For more information, see rng. |  |


| Report Field | Description |  |
| :---: | :---: | :---: |
| DataUsed | Attributes of the data used for estimation, returned as a structure with the following fields: |  |
|  | Field | Description |
|  | Name | Name of the data set. |
|  | Type | Data type. |
|  | Length | Number of data samples. |
|  | Ts | Sample time. |
|  | InterSam ple | Input intersample behavior, returned as one of the following values: <br> - 'zoh ' - Zero-order hold maintains a piecewise-constant input signal between samples. <br> - ' foh ' - First-order hold maintains a piecewise-linear input signal between samples. <br> - 'bl' - Band-limited behavior specifies that the continuous-time input signal has zero power above the Nyquist frequency. |
|  | Input0ff set | Offset removed from time-domain input data during estimation. For nonlinear models, it is []. |
|  | Output0f \|fset | Offset removed from time-domain output data during estimation. For nonlinear models, it is [ ]. |

For more information on using Report, see "Estimation Report".
x0 - Initial states computed during estimation
column vector | matrix
Initial states computed during the estimation, returned as an array containing a column vector corresponding to each experiment.

This array is also stored in the Parameters field of the model Report property.

## References

[1] Ljung, L. System Identification: Theory for the User, Appendix 4A, Second Edition, pp. 132-134. Upper Saddle River, NJ: Prentice Hall PTR, 1999.
[2] van Overschee, P., and B. De Moor. Subspace Identification of Linear Systems: Theory, Implementation, Applications. Springer Publishing: 1996.
[3] Verhaegen, M. "Identification of the deterministic part of MIMO state space models." Automatica, 1994, Vol. 30, pp. 61-74.
[4] Larimore, W.E. "Canonical variate analysis in identification, filtering and adaptive control." Proceedings of the 29th IEEE Conference on Decision and Control, 1990, pp. 596-604.
[5] McKelvey, T., H. Akcay, and L. Ljung. "Subspace-based multivariable system identification from frequency response data." IEEE Transactions on Automatic Control, 1996, Vol. 41, pp. 960979.

## See Also

canon|iddata|idfrd|idgrey|idss|n4sid0ptions| pem|polyest|procest|ssest|
tfest

## Topics

"What Are State-Space Models?"
"Estimate State-Space Models at the Command Line"
"State-Space Model Estimation Methods"
"Estimate State-Space Models with Canonical Parameterization"
Introduced before R2006a

## n4sidOptions

Option set for n4sid

## Syntax

```
opt = n4sidOptions
opt = n4sidOptions(Name,Value)
```


## Description

opt $=\mathrm{n} 4$ sidOptions creates the default options set for n 4 sid .
opt $=n 4$ sidOptions(Name, Value) creates an option set with the options specified by one or more Name, Value pair arguments.

## Input Arguments

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, ... , NameN, ValueN.

## InitialState - Handling of initial states

'estimate' (default)|'zero'
Handling of initial states during estimation, specified as one of the following values:

- 'zero' - The initial state is set to zero.
- 'estimate' - The initial state is treated as an independent estimation parameter.


## N4Weight - Weighting scheme used for singular-value decomposition by the N4SID algorithm <br> 'auto' (default) | 'MOESP' | 'CVA'| 'SSARX'

Weighting scheme used for singular-value decomposition by the N4SID algorithm, specified as one of the following values:

- 'MOESP' - Uses the MOESP algorithm by Verhaegen [2].
- 'CVA' - Uses the Canonical Variate Algorithm by Larimore [1].

Estimation using frequency-domain data always uses 'CVA'.

- 'SSARX' - A subspace identification method that uses an ARX estimation based algorithm to compute the weighting.

Specifying this option allows unbiased estimates when using data that is collected in closed-loop operation. For more information about the algorithm, see [4].

- 'auto' - The estimating function chooses between the MOESP, CVA and SSARX algorithms.


## N4Horizon - Forward- and backward-prediction horizons used by the N4SID algorithm 'auto' (default)|vector [r sy su] |k-by-3 matrix

Forward- and backward-prediction horizons used by the N4SID algorithm, specified as one of the following values:

- A row vector with three elements - [r sy su], where $r$ is the maximum forward prediction horizon, using up to $r$ step-ahead predictors. sy is the number of past outputs, and su is the number of past inputs that are used for the predictions. See pages 209 and 210 in [3] for more information. These numbers can have a substantial influence on the quality of the resulting model, and there are no simple rules for choosing them. Making 'N4Horizon' a k-by-3 matrix means that each row of 'N4Horizon' is tried, and the value that gives the best (prediction) fit to data is selected. k is the number of guesses of [ r sy su] combinations. If you specify N4Horizon as a single column, $r=s y=s u$ is used.
- 'auto ' - The software uses an Akaike Information Criterion (AIC) for the selection of sy and su.


## Focus - Error to be minimized

'prediction' (default) | 'simulation'
Error to be minimized in the loss function during estimation, specified as the comma-separated pair consisting of 'Focus' and one of the following values:

- 'prediction' - The one-step ahead prediction error between measured and predicted outputs is minimized during estimation. As a result, the estimation focuses on producing a good predictor model.
- 'simulation' - The simulation error between measured and simulated outputs is minimized during estimation. As a result, the estimation focuses on making a good fit for simulation of model response with the current inputs.

The Focus option can be interpreted as a weighting filter in the loss function. For more information, see "Loss Function and Model Quality Metrics".

## WeightingFilter - Weighting prefilter

[ ] (default) | vector | matrix | cell array | linear system
Weighting prefilter applied to the loss function to be minimized during estimation. To understand the effect of WeightingFilter on the loss function, see "Loss Function and Model Quality Metrics".

Specify WeightingFilter as one of the following values:

- [] - No weighting prefilter is used.
- Passbands - Specify a row vector or matrix containing frequency values that define desired passbands. You select a frequency band where the fit between estimated model and estimation data is optimized. For example, [ $\mathrm{wl} \mathrm{l}, \mathrm{wh}$ ] where wl and wh represent lower and upper limits of a passband. For a matrix with several rows defining frequency passbands,
[w1l,w1h;w2l,w2h;w3l,w3h; ...], the estimation algorithm uses the union of the frequency ranges to define the estimation passband.

Passbands are expressed in rad/TimeUnit for time-domain data and in FrequencyUnit for frequency-domain data, where TimeUnit and FrequencyUnit are the time and frequency units of the estimation data.

- SISO filter - Specify a single-input-single-output (SISO) linear filter in one of the following ways:
- A SISO LTI model
- $\{A, B, C, D\}$ format, which specifies the state-space matrices of a filter with the same sample time as estimation data.
- \{numerator, denominator\} format, which specifies the numerator and denominator of the filter as a transfer function with same sample time as estimation data.

This option calculates the weighting function as a product of the filter and the input spectrum to estimate the transfer function.

- Weighting vector - Applicable for frequency-domain data only. Specify a column vector of weights. This vector must have the same length as the frequency vector of the data set, Data. Frequency. Each input and output response in the data is multiplied by the corresponding weight at that frequency.


## EnforceStability - Control whether to enforce stability of model <br> false (default) | true

Control whether to enforce stability of estimated model, specified as the comma-separated pair consisting of 'EnforceStability' and either true or false.
Data Types: logical

## EstimateCovariance - Control whether to generate parameter covariance data true (default) | false

Controls whether parameter covariance data is generated, specified as true or false.
If EstimateCovariance is true, then use getcov to fetch the covariance matrix from the estimated model.

## Display - Specify whether to display the estimation progress

'off' (default)|'on'
Specify whether to display the estimation progress, specified as one of the following values:

- ' on ' - Information on model structure and estimation results are displayed in a progress-viewer window.
- 'off' - No progress or results information is displayed.


## InputOffset - Removal of offset from time-domain input data during estimation

[ ] (default) | vector of positive integers | matrix
Removal of offset from time-domain input data during estimation, specified as the comma-separated pair consisting of 'InputOffset' and one of the following:

- A column vector of positive integers of length $N u$, where $N u$ is the number of inputs.
- [] - Indicates no offset.
- Nu-by-Ne matrix - For multi-experiment data, specify InputOffset as an $N u$-by-Ne matrix. $N u$ is the number of inputs, and $N e$ is the number of experiments.

Each entry specified by InputOffset is subtracted from the corresponding input data.

## OutputOffset - Removal of offset from time-domain output data during estimation [] (default) | vector | matrix

Removal of offset from time-domain output data during estimation, specified as the comma-separated pair consisting of 'OutputOffset' and one of the following:

- A column vector of length $N y$, where $N y$ is the number of outputs.
- [] - Indicates no offset.
- Ny -by-Ne matrix - For multi-experiment data, specify OutputOffset as a Ny -by-Ne matrix. Ny is the number of outputs, and $N e$ is the number of experiments.

Each entry specified by OutputOffset is subtracted from the corresponding output data.

## OutputWeight - Weighting of prediction errors in multi-output estimations

[] (default) | 'noise' | positive semidefinite symmetric matrix
Weighting of prediction errors in multi-output estimations, specified as one of the following values:

- 'noise' - Minimize $\operatorname{det}\left(E^{\prime *} E / N\right)$, where $E$ represents the prediction error and $N$ is the number of data samples. This choice is optimal in a statistical sense and leads to the maximum likelihood estimates in case no data is available about the variance of the noise. This option uses the inverse of the estimated noise variance as the weighting function.
- Positive semidefinite symmetric matrix $(\mathrm{W})$ - Minimize the trace of the weighted prediction error matrix trace (E'*E*W/N) where:
- $E$ is the matrix of prediction errors, with one column for each output. W is the positive semidefinite symmetric matrix of size equal to the number of outputs. Use $W$ to specify the relative importance of outputs in multiple-output models, or the reliability of corresponding data.
- $N$ is the number of data samples.
- [ ] - The software chooses between the 'noise' or using the identity matrix for W.

This option is relevant only for multi-output models.

## Advanced - Additional advanced options

structure
Additional advanced options, specified as a structure with the field MaxSize. MaxSize specifies the maximum number of elements in a segment when input-output data is split into segments.

MaxSize must be a positive integer.
Default: 250000

## Output Arguments

## opt - Option set for $\mathbf{n} 4$ sid

n4sidOptions option set
Option set for n4sid, returned as an n4sid0ptions option set.

## Examples

## Create Default Options Set for State-Space Estimation Using Subspace Method opt $=$ n4sidOptions;

## Specify Options for State-Space Estimation Using Subspace Method

Create an options set for n4sid using the 'zero' option to initialize the state. Set the Display to 'on'.

```
opt = n4sidOptions('InitialState','zero','Display','on');
```

Alternatively, use dot notation to set the values of opt.

```
opt = n4sidOptions;
opt.InitialState = 'zero';
opt.Display = 'on';
```


## Compatibility Considerations

## Renaming of Estimation and Analysis Options

The names of some estimation and analysis options were changed in R2018a. Prior names still work. For details, see the R2018a release note "Renaming of Estimation and Analysis Options".

## References

[1] Larimore, W.E. "Canonical variate analysis in identification, filtering and adaptive control." Proceedings of the 29th IEEE Conference on Decision and Control, pp. 596-604, 1990.
[2] Verhaegen, M. "Identification of the deterministic part of MIMO state space models." Automatica, Vol. 30, 1994, pp. 61-74.
[3] Ljung, L. System Identification: Theory for the User. Upper Saddle River, NJ: Prentice-Hall PTR, 1999.
[4] Jansson, M. "Subspace identification and ARX modeling." 13th IFAC Symposium on System Identification, Rotterdam, The Netherlands, 2003.

## See Also

idfilt|idpar|n4sid

## Topics

"Loss Function and Model Quality Metrics"
Introduced in R2012a

## ndims

Query number of dimensions of dynamic system model or model array

## Syntax

n = ndims(sys)

## Description

$\mathrm{n}=$ ndims(sys) is the number of dimensions of a dynamic system model or a model array sys. A single model has two dimensions (one for outputs, and one for inputs). A model array has $2+p$ dimensions, where $p \geq 2$ is the number of array dimensions. For example, a 2-by-3-by-4 array of models has $2+3=5$ dimensions.

```
ndims(sys) = length(size(sys))
```


## Examples

## Determine Dimensions of Model Array

Create a 3-by-1 array of random state-space models, each with 4 states, 1 input, and 1 output.
sys $=$ rss (4,1,1,3);
Compute the number of dimensions of the model array.
ndims(sys)
ans $=4$
The number of dimensions is $2+p$, where $p$ is the number of array dimensions. In this example, $p$ is 2 because sys is $3-b y-1$.

## See Also

size

Introduced in R2012a

## neuralnet

Class representing neural network nonlinearity estimator for nonlinear ARX models

## Syntax

net_estimator $=$ neuralnet (Network)

## Description

neuralnet is the class that encapsulates the neural network nonlinearity estimator. A neuralnet object lets you use networks, created using Deep Learning Toolbox software, in nonlinear ARX models.

The neural network nonlinearity estimator defines a nonlinear function $y=F(x)$, where $F$ is a multilayer feed-forward (static) neural network, as defined in the Deep Learning Toolbox software. y is a scalar and $x$ is an $m$-dimensional row vector.

You create multi-layer feed-forward neural networks using Deep Learning Toolbox commands such as feedforwardnet, cascadeforwardnet and linearlayer. When you create the network:

- Designate the input and output sizes to be unknown by leaving them at the default value of zero (recommended method). When estimating a nonlinear ARX model using the nlarx command, the software automatically determines the input-output sizes of the network.
- Initialize the sizes manually by setting input and output ranges to m-by-2 and 1-by-2 matrices, respectively, where $m$ is the number of nonlinear ARX model regressors and the range values are minimum and maximum values of regressors and output data, respectively.

See "Examples" on page 1-0 for more information.
Use evaluate (net_estimator, x ) to compute the value of the function defined by the neuralnet object net_estimator at input value $x$. When used for nonlinear ARX model estimation, $x$ represents the model regressors for the output for which the neuralnet object is assigned as the nonlinearity estimator.

You cannot use neuralnet when the Focus option in nlarx0ptions is 'simulation' because this nonlinearity estimator is considered to be nondifferentiable for estimation. Minimization of simulation error requires differentiable nonlinear functions.

## Construction

net_estimator = neuralnet(Network) creates a neural network nonlinearity estimator based on the feed-forward (static) network object Network created using Deep Learning Toolbox commands feedforwardnet, cascadeforwardnet, and linearlayer. Network must represent a static mapping between the inputs and output without I/O delays or feedback. The number of outputs of the network, if assigned, must be one. For a multiple-output nonlinear ARX models, create a separate neuralnet object for each output-that is, each estimator must represent a single-output network object.

## Properties

Network
Neural network object, typically created using the Deep Learning Toolbox commands feedforwardnet, cascadeforwardnet, and linearlayer.

After creating the object, you can use get or dot notation to access the object property values. For example:
\% List Network property value
get (n)
n. Network

You can also use the set function to set the value of particular properties. For example:

```
set(d, 'Network', net_obj)
```

The first argument to set must be the name of a MATLAB variable.

## Examples

## Create a Nonlinearity Estimator Using Feed-Forward Neural Network

Create a neural network nonlinearity estimator using a feed-forward neural network with three hidden layers; transfer functions of types logsig, radbas,and purelin; and unknown input and output sizes.

Create a neural network.

```
net = feedforwardnet([4 6 1]);
net.layers{1}.transferFcn = 'logsig';
net.layers{2}.transferFcn = 'radbas';
net.layers{3}.transferFcn = 'purelin';
```

View the network diagram.
view(net)


Create a neural network estimator.
net_estimator $=$ neuralnet(net);

## Estimate Nonlinear ARX Model Using a Neural Network Nonlinearity Estimator

Create a single-layer, cascade-forward network with unknown input and output sizes and use this network for nonlinear ARX model estimation.

Create a cascade-forward neural network with 20 neurons and unknown input-output sizes.
net $=$ cascadeforwardnet(20);
Create a neural network nonlinearity estimator.
net_estimator = neuralnet(net);
Load estimation data.
load twotankdata
Data $=$ iddata(y,u,0.2);
Estimate nonlinear ARX model.
Model = nlarx(Data,[2 2 1], net_estimator);
Compare model response to measured output signal.
compare(Data,Model)

Simulated Response Comparison


## Initialize Input-Output Sizes of Neural Network Nonlinearity Estimator

Initialize the input-output sizes of a two-layer feed-forward neural network based on estimation data, and use this network for nonlinear ARX estimation.

Load estimation data.
load iddata7 z7
z7 = z7(1:200);
Create a template nonlinear ARX model with no nonlinearity.
model = idnlarx([4 4 4 1 1],[]);
This model has six regressors and is used to define the regressors. The range of regressor values for input-output data in $z 7$ is then used to set the input ranges in the neural network object, as shown in the next steps.

Obtain the model regressor values.
R = getreg(model,'all',z7);
R = R.Variables;
Create a two-layer, feed-forward neural network and initialize the network input and output dimensions to 2 and 1, respectively. Use 5 neurons for first layer and 7 for second layer.
net = feedforwardnet([5 7]);
Determine input range.
InputRange $=[\min (R) ; \max (R)] . ' ;$
Initialize input dimensions of estimator.
net.inputs\{1\}.range = InputRange;
Determine output range.
OutputRange $=[\min (z 7.0 u t p u t D a t a), \max (z 7.0 u t p u t D a t a)] ;$
Initialize output dimensions of estimator and the choice of training function.

```
net.outputs{net.outputConnect}.range = OutputRange;
net.trainFcn = 'trainbfg';
```

Create a neural network nonlinearity estimator.

```
net_estimator = neuralnet(net);
```

Specify the nonlinearity estimator in the model.
model. Nonlinearity = net_estimator;
Estimate the parameters of the network to minimize the prediction error between data and model. Estimate model.
model $=$ nlarx(z7,model);
Compare model's predicted response to measured output signal.

```
compare(z7(1:100),model,1)
```


## 1-Step Predicted Response Comparison



## Algorithms

The nlarx command uses the train method of the network object, defined in the Deep Learning Toolbox software, to compute the network parameter values.

## See Also

cascadeforwardnet | customnet | feedforwardnet | linearlayer|nlarx|sigmoidnet | treepartition|wavenet

## Topics

"Identifying Nonlinear ARX Models"

## Introduced in R2007a

## nkshift

Shift data sequences

## Syntax

Datas $=$ nkshift (Data, nk)

## Description

Data contains input-output data in the iddata format.
nk is a row vector with the same length as the number of input channels in Data.
Datas is an iddata object where the input channels in Data have been shifted according to nk . A positive value of $\mathrm{nk}(\mathrm{ku})$ means that input channel number ku is delayed $\mathrm{nk}(\mathrm{ku})$ samples.
nkshift supports both frequency- and time-domain data. For frequency-domain data it multiplies with $e^{i n k \omega T}$ to obtain the same effect as shifting in the time domain. For continuous-time frequencydomain data ( $\mathrm{Ts}=0$ ), nk should be interpreted as the shift in seconds.
nkshift lives in symbiosis with the InputDelay property of linear identified models:
$\mathrm{m} 1=$ ssest(dat,4,'InputDelay',nk)
is related to

```
m2 = ssest(nkshift(dat,nk),4);
```

such that m 1 and m 2 are the same models, but m 1 stores the delay information and uses this information when computing the frequency response, for example. When using $m 2$, the delay value must be accounted for separately when computing time and frequency responses.

```
See Also
absorbDelay|delayest|idpoly|idss
```

Introduced before R2006a

## nlarx

Estimate parameters of nonlinear ARX model

## Syntax

```
sys = nlarx(data,orders)
sys = nlarx(data,regressors)
sys = nlarx(___,output_fcn)
sys = nlarx(data,linmodel)
sys = nlarx(data,linmodel,output_fcn)
sys = nlarx(data,sys0)
sys = nlarx(
```

$\qquad$

``` ,Options)
```


## Description

## Specify Regressors

sys = nlarx(data,orders) estimates a nonlinear ARX model to fit the given estimation data using the specified ARX model orders and the default wavelet network output function. Use this syntax when you extend an ARX linear model, or when you use only regressors that are linear with consecutive lags.
sys = nlarx(data, regressors) estimates a nonlinear ARX model using the specified regressor set regressors. Use this syntax when you have linear regressors that have non-consecutive lags, or when you also have polynomial regressors, custom regressors, or both.
sys $=n l a r x($ $\qquad$ , output_fcn) specifies the output function that maps the regressors to the model output. You can use this syntax with any of the previous input argument combinations.

## Specify Linear Model

sys = nlarx(data,linmodel) uses a linear ARX model linmodel to specify the model orders and the initial values of the linear coefficients of the model. Use this syntax when you want to create a nonlinear ARX model as an extension of, or an improvement upon, an existing linear model.
sys = nlarx(data, linmodel,output_fcn) specifies the output function to use for model estimation.

## Refine Existing Model

sys $=n l a r x(d a t a, s y s 0)$ estimates or refines the parameters of the nonlinear ARX model sys0.
Use this syntax to:

- Estimate the parameters of a model previously created using the idnlarx constructor. Prior to estimation, you can configure the model properties using dot notation.
- Update the parameters of a previously estimated model to improve the fit to the estimation data. In this case, the estimation algorithm uses the parameters of sys0 as initial guesses.


## Specify Options

sys $=$ nlarx( $\qquad$ , Options) specifies additional configuration options for the model estimation.

## Examples

## Estimate Nonlinear ARX Model

Load the estimation data.
load twotankdata;
Create an iddata object from the estimation data with a sample time of 0.2 seconds.
Ts = 0.2;
z = iddata(y,u,Ts);
Estimate the nonlinear ARX model using ARX model orders to specify the regressors.

```
sysNL = nlarx(z,[[4 4 1])
sysNL =
Nonlinear ARX model with 1 output and 1 input
    Inputs: u1
    Outputs: y1
Regressors:
    Linear regressors in variables y1, ul
    List of all regressors
Output function: Wavelet Network with }11\mathrm{ units
Sample time: 0.2 seconds
Status:
Estimated using NLARX on time domain data "z".
Fit to estimation data: 96.84% (prediction focus)
FPE: 3.482e-05, MSE: 3.431e-05
```

sys uses the default wavenet function as the output function.
For comparison, compute a linear ARX model with the same model orders.

```
sysL = arx(z,[4 4 1]);
```

Compare the model outputs with the original data.

```
compare(z,sysNL,sysL)
```

Simulated Response Comparison


The nonlinear model has a much better fit to the data than the linear model.

## Estimate Nonlinear ARX Model Using Linear Regressor Set

Specify a linear regressor that is equivalent to an ARX model order matrix of [ $\left.\begin{array}{lll}4 & 4 & 1\end{array}\right]$.
An order matrix of [ $\left.\begin{array}{lll}4 & 4 & 1\end{array}\right]$ specifies that both input and output regressor sets contain four regressors with lags ranging from 1 to 4 . For example, $u_{1}(t-2)$ represents the second input regressor.

Specify the output and input names.

```
output_name = 'y1';
input_name = 'ul';
names = {output_name,input_name};
```

Specify the output and input lags.

```
output_lag = [llllll
input_lag = [lllll}12% 3 4]
lags = {output_lag,input_lag};
```

Create the linear regressor object.
lreg $=$ linearRegressor(names,lags)

```
lreg =
Linear regressors in variables yl, ul
            Variables: {'y1' 'ul'}
                Lags: {[11 2 2 3 4] [lllllll
        UseAbsolute: [0 0]
        TimeVariable: 't'
    Regressors described by this set
```

Load the estimation data and create an iddata object.
load twotankdata
$z=$ iddata(y,u,0.2);
Estimate the nonlinear ARX model.

```
sys = nlarx(z,lreg)
sys =
Nonlinear ARX model with 1 output and 1 input
    Inputs: ul
    Outputs: y1
Regressors:
    Linear regressors in variables y1, ul
    List of all regressors
Output function: Wavelet Network with 11 units
Sample time: 0.2 seconds
Status:
Estimated using NLARX on time domain data "z".
Fit to estimation data: 96.84% (prediction focus)
FPE: 3.482e-05, MSE: 3.431e-05
```

View the regressors

```
getreg(sys)
ans = 8\times1 cell
    {'y1(t-1)'}
    {'y1(t-2)'}
    {'y1(t-3)'}
    {'y1(t-4)'}
    {'u1(t-1)'}
    {'u1(t-2)'}
    {'u1(t-3)'}
    {'u1(t-4)'}
```

Compare the model output to the estimation data.

```
compare(z,sys)
```

Simulated Response Comparison


## Estimate Nonlinear ARX Model from Time Series Data

Create time and data arrays.
dt = 0.01;
$t=0: d t: 10 ;$
y $=10 * \sin \left(2 *\right.$ pi $\left.^{*} \mathrm{t}\right)+$ rand $(\operatorname{size}(\mathrm{t}))$;
Create an iddata object with no input signal specified.
z = iddata(y',[],dt);
Estimate the nonlinear ARX model.

```
sys = nlarx(z,2)
sys =
Nonlinear time series model
    Outputs: yl
Regressors:
    Linear regressors in variables y1
    List of all regressors
Output function: Wavelet Network with 8 units
```

```
Sample time: 0.01 seconds
Status:
Estimated using NLARX on time domain data "z".
Fit to estimation data: 92.92% (prediction focus)
FPE: 0.2568, MSE: 0.2507
```


## Specify and Customize Output Function

Estimate a nonlinear ARX model that uses the mapping function sigmoidnet as its output function.
Load the data and divide it into the estimation and validation data sets ze and zv .

```
load twotankdata.mat u y
z = iddata(y,u,'Ts',0.2);
ze = z(1:1500);
zv = z(1501:end);
```

Configure the sigmoidnet mapping function. Fix the offset to 0.2 and the number of units to 15 .

```
s = sigmoidnet;
```

s.Offset. Value = 0.2;
s. NonlinearFcn.NumberOfUnits = 15;

Create a linear model regressor specification that contains four output regressors and five input regressors.

```
reg1 = linearRegressor({'y1','u1'},{1:4,0:4});
```

Create a polynomial model regressor specification that contains the squares of two input terms and three output terms.

```
reg2 = polynomialRegressor({'y1','u1'},{1:2,0:2},2);
```

Set estimation options for the search method and maximum number of iterations.

```
opt = nlarxOptions('SearchMethod','fmincon')';
opt.SearchOptions.MaxIterations = 40;
Estimate the nonlinear ARX model.
sys = nlarx(ze,[reg1; reg2],s,opt);
```

Validate sys by comparing the simulated model response to the validation data set.

```
compare(zv,sys)
```



## Add Output Function to Extend and Improve Linear Model

Estimate a linear model and improve the model by adding a treepartition output function.
Load the estimation data.
load throttledata ThrottleData

linsys = arx(ThrottleData,[2 2 1]);
Create an idnlarx template model that uses linsys and specifies sigmoidnet as the output function.

```
sys0 = idnlarx(linsys,treepartition);
```

Fix the linear component of sys0 so that during estimation, the linear portion of sys0 remains identical to linsys.
sys0.OutputFcn.LinearFcn.Free = false;
Estimate the free parameters of sys0, which are the nonlinear-function parameters and the offset.
sys = nlarx(ThrottleData,sys0);

Compare the fit accuracies for the linear and nonlinear models.
compare(ThrottleData, linsys,sys)

Simulated Response Comparison


## Estimate Nonlinear ARX Model Using Custom Network Mapping Object

Generating a custom network mapping object requires the definition of a user-defined unit function.
Define the unit function and save it as gaussunit.m.
\% Copyright 2015 The MathWorks, Inc.
function [f, g, a] = gaussunit(x)
$f=\exp (-x . * x)$;
if nargout>1
$\mathrm{g}=-2 * \mathrm{x} .{ }^{*} \mathrm{f}$;
a $=0.2$;
end

Create a custom network mapping object using a handle to the gaussunit function.
H = @gaussunit;
CNet $=$ customnet ( H );

Load the estimation data.
load iddata1;
Estimate a nonlinear ARX model using the custom network.

```
sys = nlarx(z1,[1 2 1],CNet)
sys =
<strong>Nonlinear ARX model with 1 output and 1 input</strong>
    Inputs: u1
    Outputs: yl
Regressors:
    Linear regressors in variables yl, ul
Output function: Custom Network with 10 units
Sample time: 0.1 seconds
Status:
Estimated using NLARX on time domain data "z1".
Fit to estimation data: 64.35% (prediction focus)
FPE: 3.58, MSE: 2.465
```


## Estimate MIMO Nonlinear ARX Model

Load the estimation data.
load motorizedcamera;
Create an iddata object.
z = iddata(y,u,0.02,'Name','Motorized Camera','TimeUnit','s');
z is an iddata object with six inputs and two outputs.
Specify the model orders.
Orders $=$ [ones $(2,2), 2 * \operatorname{ones}(2,6)$, ones $(2,6)]$;
Specify different mapping functions for each output channel.
NL = [wavenet('NumberOfUnits',2),linear];
Estimate the nonlinear ARX model.

```
sys = nlarx(z,Orders,NL)
sys =
Nonlinear ARX model with 2 outputs and 6 inputs
    Inputs: u1, u2, u3, u4, u5, u6
    Outputs: y1, y2
Regressors:
```

```
    Linear regressors in variables y1, y2, u1, u2, u3, u4, u5, u6
    List of all regressors
Output functions:
    Output 1: Wavelet Network with 2 units
    Output 2: Linear Function
Sample time: 0.02 seconds
Status:
Estimated using NLARX on time domain data "Motorized Camera".
Fit to estimation data: [98.72;98.77]% (prediction focus)
FPE: 0.5719, MSE: 1.061
```


## Estimate MIMO Nonlinear ARX Model with Same Mapping Function for All Outputs

Load the estimation data and create an iddata object z . z contains two output channels and six input channels.

```
load motorizedcamera;
z = iddata(y,u,0.02);
```

Specify a set of linear regressors that uses the output and input names from $z$ and contains:

- 2 output regressors with 1 lag.
- 6 input regressor pairs with 1 and 2 lags.

```
names = [z.OutputName; z.InputName];
lags = {1,1,[1,2],[1,2],[1,2],[1,2],[1,2],[1,2]};
reg = linearRegressor(names,lags);
```

Estimate a nonlinear ARX model using a sigmoidnet mapping function with four units for all output channels.

```
sys = nlarx(z,reg,sigmoidnet(4))
sys =
Nonlinear ARX model with 2 outputs and 6 inputs
    Inputs: u1, u2, u3, u4, u5, u6
    Outputs: y1, y2
Regressors:
    Linear regressors in variables y1, y2, u1, u2, u3, u4, u5, u6
    List of all regressors
Output functions:
    Output 1: Sigmoid Network with 4 units
    Output 2: Sigmoid Network with 4 units
Sample time: 0.02 seconds
Status:
Estimated using NLARX on time domain data "z".
Fit to estimation data: [98.86;98.79]% (prediction focus)
FPE: 2.641, MSE: 0.9233
```


## Specify Linear, Polynomial, and Custom Regressors

Load the estimation data $z 1$, which has one input and one output, and obtain the output and input names.

```
load iddatal z1;
names = [z1.OutputName z1.InputName]
names = l×2 cell
    {'y1'} {'u1'}
```

Specify $L$ as the set of linear regressors that represents $y_{1}(t-1), u_{1}(t-2)$, and $u_{1}(t-5)$.
L = linearRegressor(names,\{1,[2 5]\});
Specify P as the polynomial regressor $y_{1}(t-1)^{2}$.
$P=$ polynomialRegressor(names(1),1,2);
Specify C as the custom regressor $y_{1}(t-2) u_{1}(t-3)$. Use an anonymous function handle to define this function.

```
C = customRegressor(names,{2 3},@(x,y)x.*y)
C =
Custom regressor: y1(t-2).*u1(t-3)
    VariablesToRegressorFcn: @(x,y)x.*y
            Variables: {'y1' 'ul'}
                            Lags: {[2] [3]}
                            Vectorized: 1
                            TimeVariable: 't'
    Regressors described by this set
```

Combine the regressors in the column vector $R$.
$R=[L ; P ; C]$
R=3×1 object
[3 1] array of linearRegressor, polynomialRegressor, customRegressor objects.

1. Linear regressors in variables y1, u1

Variables: \{'y1' 'ul'\}
Lags: \{[1] [2 5]\}
UseAbsolute: [0 0]
TimeVariable: 't'
2. Order 2 regressors in variables y1

Order: 2
Variables: \{'y1'\}
Lags: \{[1]\}
UseAbsolute: 0
AllowVariableMix: 0

```
    AllowLagMix: 0
TimeVariable: 't'
3. Custom regressor: y1(t-2).*ul(t-3)
    VariablesToRegressorFcn: @(x,y)x.*y
            Variables: {'y1' 'ul'}
                        Lags: {[2] [3]}
            Vectorized: 1
            TimeVariable: 't'
    Regressors described by this set
```

Estimate a nonlinear ARX model with R.

```
sys = nlarx(zl,R)
sys =
Nonlinear ARX model with 1 output and 1 input
    Inputs: u1
    Outputs: y1
Regressors:
    1. Linear regressors in variables y1, ul
    2. Order 2 regressors in variables yl
    3. Custom regressor: y1(t-2).*ul(t-3)
    List of all regressors
Output function: Wavelet Network with 1 unit
Sample time: 0.1 seconds
Status:
Estimated using NLARX on time domain data "z1".
Fit to estimation data: 59.73% (prediction focus)
FPE: 3.356, MSE: 3.147
```

View the full regressor set.

```
getreg(sys)
ans = 5\times1 cell
    {'y1(t-1)' }
    {'ul(t-2)' }
    {'u1(t-5)' }
    {'y1(t-1)^2' }
    {'y1(t-2).*u1(t-3)'}
```


## Estimate Nonlinear ARX Model with No Linear Term in Output Function

Load the estimation data.
load iddata1;

Create a sigmoid network mapping object with 10 units and no linear term.
SN = sigmoidnet(10,false);
Estimate the nonlinear ARX model. Confirm that the model does not use the linear function.

```
sys = nlarx(z1,[2 2 1],SN);
sys.OutputFcn.LinearFcn.Use
ans = logical
    0
```


## Specify Nonlinear ARX Orders and Linear Parameters Using Linear ARX Model

Load the estimation data.
load throttledata;
Detrend the data.
Tr = getTrend(ThrottleData);
Tr.OutputOffset = 15;
DetrendedData = detrend(ThrottleData,Tr);
Estimate the linear ARX model.

```
LinearModel = arx(DetrendedData,[2 1 1]);
```

Estimate the nonlinear ARX model using the linear model. The model orders, delays, and linear parameters of NonlinearModel are derived from LinearModel.

```
NonlinearModel = nlarx(ThrottleData,LinearModel)
NonlinearModel =
Nonlinear ARX model with 1 output and 1 input
    Inputs: Step Command
    Outputs: Throttle Valve Position
Regressors:
    Linear regressors in variables Throttle Valve Position, Step Command
    List of all regressors
Output function: Wavelet Network with 7 units
Sample time: 0.01 seconds
Status:
Estimated using NLARX on time domain data "ThrottleData".
Fit to estimation data: 99.03% (prediction focus)
FPE: 0.1127, MSE: 0.1039
```


## Estimate Nonlinear ARX Model Using Constructed idnlarx Object

Load the estimation data.
load iddata1;
Create an idnlarx model.
sys = idnlarx([2 2 1]);
Configure the model using dot notation to:

- Use a sigmoid network mapping object.
- Assign a name.
sys.Nonlinearity = 'sigmoidnet';
sys.Name = 'Model 1';
Estimate a nonlinear ARX model with the structure and properties specified in the idnlarx object.

```
sys = nlarx(z1,sys)
sys =
Nonlinear ARX model with 1 output and 1 input
    Inputs: u1
    Outputs: y1
Regressors:
    Linear regressors in variables yl, ul
    List of all regressors
Output function: Sigmoid Network with 10 units
Name: Model 1
Sample time: 0.1 seconds
Status:
Estimated using NLARX on time domain data "z1".
Fit to estimation data: 69.03% (prediction focus)
FPE: 2.918, MSE: 1.86
```


## Estimate Nonlinear ARX Model and Avoid Local Minima

If an estimation stops at a local minimum, you can perturb the model using init and reestimate the model.

Load the estimation data.

```
load iddata1;
```

Estimate the initial nonlinear model.
sys1 = nlarx(z1,[4 2 1],'sigmoidnet');
Randomly perturb the model parameters to avoid local minima.

```
sys2 = init(sys1);
```

Estimate the new nonlinear model with the perturbed values.

```
sys2 = nlarx(z1,sys1);
```


## Estimate Nonlinear ARX Model Using Specific Options

Load the estimation data.
load twotankdata;
Create an iddata object from the estimation data.
$z=i d d a t a(y, u, 0.2) ;$
Create an nlarx0ptions option set specifying a simulation error minimization objective and a maximum of 50 estimation iterations.

```
opt = nlarxOptions;
opt.Focus = 'simulation';
opt.Search0ptions.MaxIterations = 50;
```

Estimate the nonlinear ARX model.

```
sys = nlarx(z,[4 4 1],'sigmoidnet',opt)
```

sys =
Nonlinear ARX model with 1 output and 1 input
Inputs: u1
Outputs: y1
Regressors:
Linear regressors in variables y1, ul
List of all regressors
Output function: Sigmoid Network with 10 units
Sample time: 0.2 seconds
Status:
Estimated using NLARX on time domain data "z".
Fit to estimation data: 88.79\% (simulation focus)
FPE: 3.341e-05, MSE: 0.0004307

## Estimate Regularized Nonlinear ARX Model with Large Number of Units

Load the regularization example data.
load regularizationExampleData.mat nldata;
Create a sigmoidnet mapping object with 30 units and specify the model orders.

MO = sigmoidnet(30);
Orders = [1 2 1];
Create an estimation option set and set the estimation search method to lm.
opt = nlarxOptions('SearchMethod','lm');
Estimate an unregularized model.
sys = nlarx(nldata,Orders,MO,opt);
Configure the regularization Lambda parameter.
opt. Regularization.Lambda $=1 \mathrm{e}-8$;
Estimate a regularized model.
sysR = nlarx(nldata,Orders,MO,opt);
Compare the two models.

```
compare(nldata,sys,sysR)
```

Simulated Response Comparison


The large negative fit result for the unregularized model indicates a poor fit to the data. Estimating a regularized model produces a significantly better result.

## Input Arguments

## data - Time-domain estimation data

iddata object | numeric matrix
Time-domain estimation data, specified as an iddata object or a numeric matrix.

- If data is an iddata object, then data can have one or more output channels and zero or more input channels.
- If data is a numeric matrix, then the number of columns of data must match the sum of the number of inputs $\left(n_{u}\right)$ and the number of outputs ( $n_{y}$.
data must be uniformly sampled and cannot contain missing ( NaN ) samples.


## orders - ARX model orders

nlarx orders [na nb nk]
ARX model orders, specified as the matrix [na nb nk]. na denotes the number of delayed outputs, nb denotes the number of delayed inputs, and nk denotes the minimum input delay. The minimum output delay is fixed to 1 . For more information on how to construct the orders matrix, see arx.

When you specify orders, the software converts the order information into linear regressor form in the idnlarx Regressors property. For an example, see "Create Nonlinear ARX Model Using ARX Model Orders" on page 1-561.

## regressors - Regressor specification

linearRegressor object | polynomialRegressor object | customRegressor object | column array of regressor specification objects

Regressor specification, specified as a column vector containing one or more regressor specification objects, which are the linearRegressor objects, polynomialRegressor objects, and customRegressor objects. Each object specifies a formula for generating regressors from lagged variables. For example:

- L = linearRegressor(\{'y1','u1'\},\{1,[2 5]\}) generates the regressors $y_{1}(t-1), u_{1}(t-2)$, and $u_{2}(t-5)$.
- $\mathrm{P}=$ polynomialRegressor('y2',4:7,2) generates the regressors $y_{2}(t-4)^{2}, y_{2}(t-5)^{2}, y_{2}(t-6)^{2}$, and $y_{2}(t-7)^{2}$.
- C = customRegressor(\{'y1','u1','u2'\},\{1 22$\}, @(x, y, z) \sin (x . * y+z))$ generates the single regressor $\sin \left(y_{1}(t-1) u_{1}(t-2)+u_{2}(t-2)\right)$.

When you create a regressor set to support estimation with an iddata object, you can use the input and output names of the object rather than create the names for the regressor function. For instance, suppose you create a linear regressor for a model, plan to use the iddata object $z$ to estimate the model. You can use the following command to create the linear regressor.

L = linearRegressor([z.outputName;z.inputName],\{1,[2 5]\})
For an example of creating and using a SISO linear regressor set, see "Estimate Nonlinear ARX Model Using Linear Regressor Set" on page 1-862. For an example of creating a MIMO linear regressor set that obtains variable names from the estimation data set, see "Estimate MIMO Nonlinear ARX Model with Same Mapping Function for All Outputs" on page 1-869.

## output_fen - Output function

'wavenet' (default)|'linear' | [] | ' ' | 'sigmoidnet' | 'treepartition' | 'neuralnet' |
'customnet' | mapping object | array of mapping objects
Output function that maps the regressors of the idnlarx model into the model output, specified as a column array containing zero or more of the following strings or objects:

```
'wavenet' or wavenet object
'linear' or ' ' or [] or linear object
'sigmoidnet' or sigmoidnet object
'treepartition' or treepartition object neuralnet object
```

customnet object

Wavelet network
Linear function
Sigmoid network
Binary tree partition regression model
Neural network - Feedforward network of Deep Learning Toolbox.
Custom network - Similar to sigmoidnet, but with a user-defined replacement for the sigmoid function.

Use a string, such as 'sigmoidnet ', to use the default properties of the mapping function object. Use the object itself, such as sigmoidnet, when you want to configure the properties of the mapping object.

The wavenet, sigmoidnet, treepartion, and customnet objects contain both linear and nonlinear components. You can remove (not use) the linear components of wavenet, sigmoidnet, and customnet by setting the LinearFcn. Use value to false.

The neuralnet function has only a nonlinear component, which is the network object of Deep Learning Toolbox. The linear function, as the name implies, has only a linear component.
output_fcn is static in that it depends only upon the data values at a specific time, but not directly on time itself. For example, if the output function $y(t)$ is equal to $y_{0}+a_{1} y(t-1)+a_{2} y(t-2)+\ldots b_{1} u(t-$ 1) $+b_{2} u(t-2)+\ldots$, then output_fcn is a linear function that the linear mapping object represents.

Specifying a character vector, for example 'sigmoidnet ', creates a mapping object with default settings. Alternatively, you can specify mapping object properties in two ways:

- Create the mapping object using arguments to modify default properties.

```
MO = sigmoidnet(15);
```

- Create a default mapping object first and then use dot notation to modify properties.

```
MO = sigmoidnet;
MO.NumberOfUnits = 15;
```

For $n_{y}$ output channels, you can specify mapping objects individually for each channel by setting output_fen to an array of $n_{y}$ mapping objects. For example, the following code specifies OutputFcn using dot notation for a system with two input channels and two output channels.

```
sys = idnlarx({'y1','y2'},{'u1','u2'});
sys.OutputFcn = [wavenet; sigmoidnet];
```

To specify the same mapping for all outputs, specify OutputFcn as a character vector or a single mapping object.
output_fen represents a static mapping function that transforms the regressors of the nonlinear ARX model into the model output. output_fen is static because it does not depend on time. For example, if $y(t)=y_{0}+a_{1} y(t-1)+a_{2} y(t-2)+\ldots+b_{1} u(t-1)+b_{2} u(t-2)+\ldots$, then output_fcn is a linear function represented by the linear object.

For an example of specifying the output function, see "Specify and Customize Output Function" on page 1-865.

## linmodel - Discrete-time linear model <br> idpoly object|idss object|idtf object|idproc object

Discrete-time identified input/output linear model, specified as any linear model created using an estimator such as arx, armax, tfest, or ssest. For example, to create a state-space idss model, estimate the model using ssest.

## sys0 - Nonlinear ARX model

idnlarx model
Nonlinear ARX model, specified as an idnlarx model. sys0 can be:

- A model previously estimated using nlarx. The estimation algorithm uses the parameters of sys0 as initial guesses. In this case, use init to slightly perturb the model properties to avoid trapping the model in local minima.

```
sys = init(sys);
sys = nlarx(data,sys);
```

- A model previously created using the idnlarx constructor and with properties set using dot notation. For example, use the following to create an idnlarx object, set its properties, and estimate the model.

```
sys1 = idnlarx('y1','u1',Regressors);
sysl.OutputFcn = 'treepartition';
sys1.Ts = 0.02;
sys1.TimeUnit = 'Minutes';
sys1.InputName = 'My Data';
sys2 = nlarx(data,sys1);
```

The preceding code is equivalent to the following nlarx command.

```
sys2 = nlarx(data,Regressors,'treepartition','Ts',0.02,'TimeUnit','Minutes', ...
'InputName','My Data');
```


## Options - Estimation options

nlarx0ptions option set
Estimation options for nonlinear ARX model identification, specified as an nlarx0ptions option set.

## Output Arguments

## sys - Nonlinear ARX model

idnlarx object
Nonlinear ARX model that fits the given estimation data, returned as an idnlarx object. This model is created using the specified model orders, nonlinearity estimator, and estimation options.

Information about the estimation results and options used is stored in the Report property of the model. The contents of Report depend upon the choice of nonlinearity and estimation focus you specified for nlarx. Report has the following fields:

| Report Field | Description |  |
| :---: | :---: | :---: |
| Status | Summary of the model status, which indicates whether the model was created by construction or obtained by estimation. |  |
| Method | Estimation command used. |  |
| Fit | Quantitative assessment of the estimation, returned as a structure. See "Loss Function and Model Quality Metrics" for more information on these quality metrics. The structure has the following fields: |  |
|  | Field | Description |
|  | FitPerce nt | Normalized root mean squared error (NRMSE) measure of how well the response of the model fits the estimation data, expressed as the percentage fit = 100(1-NRMSE). |
|  | LossFcn | Value of the loss function when the estimation completes. |
|  | MSE | Mean squared error (MSE) measure of how well the response of the model fits the estimation data. |
|  | FPE | Final prediction error for the model. |
|  | AIC | Raw Akaike Information Criteria (AIC) measure of model quality. |
|  | AICc | Small sample-size corrected AIC. |
|  | nAIC | Normalized AIC. |
|  | BIC | Bayesian Information Criteria (BIC). |
| Parameter <br> s | Estimated values of model parameters. |  |
| OptionsUs ed | Option set used for estimation. If no custom options were configured, this is a set of default options. See nlarxOptions for more information. |  |
| RandState | State of the random number stream at the start of estimation. Empty, [ ], if randomization was not used during estimation. For more information, see rng. |  |


| Report Field | Description |  |
| :---: | :---: | :---: |
| DataUsed | Attributes of the data used for estimation, returned as a structure with the following fields: |  |
|  | Field | Description |
|  | Name | Name of the data set. |
|  | Type | Data type. |
|  | Length | Number of data samples. |
|  | Ts | Sample time. |
|  | InterSam ple | Input intersample behavior, returned as one of the following values: <br> - 'zoh ' - Zero-order hold maintains a piecewise-constant input signal between samples. <br> - ' foh ' - First-order hold maintains a piecewise-linear input signal between samples. <br> - 'bl' - Band-limited behavior specifies that the continuous-time input signal has zero power above the Nyquist frequency. |
|  | InputOff set | Offset removed from time-domain input data during estimation. For nonlinear models, it is [ ]. |
|  | Output0f fset | Offset removed from time-domain output data during estimation. For nonlinear models, it is []. |
| Terminati on | Termination conditions for the iterative search used for prediction error minimization, returned as a structure with the following fields: |  |
|  | Field | Description |
|  | WhyStop | Reason for terminating the numerical search. |
|  | Iteratio ns | Number of search iterations performed by the estimation algorithm. |
|  | First0rd erOptima lity | $\infty$-norm of the gradient search vector when the search algorithm terminates. |
|  | FcnCount | Number of times the objective function was called. |
|  | UpdateNo rm | Norm of the gradient search vector in the last iteration. Omitted when the search method is 'lsqnonlin' or 'fmincon'. |
|  | LastImpr ovement | Criterion improvement in the last iteration, expressed as a percentage. Omitted when the search method is 'lsqnonlin' or 'fmincon'. |
|  | Algorith \|lm | Algorithm used by 'lsqnonlin' or 'fmincon' search method. Omitted when other search methods are used. |
|  | For estimation methods that do not require numerical search optimization, the Termination field is omitted. |  |

For more information on using Report, see "Estimation Report".

## Algorithms

## Nonlinear ARX Model Structure

A nonlinear ARX model consists of model regressors and an output function. The output function includes linear and nonlinear functions that act on the model regressors to give the model output and a fixed offset for that output. This block diagram represents the structure of a nonlinear ARX model in a simulation scenario.

Output Function


The software computes the nonlinear ARX model output $y$ in two stages:
1 It computes regressor values from the current and past input values and the past output data.
In the simplest case, regressors are delayed inputs and outputs, such as $u(t-1)$ and $y(t-3)$. These kind of regressors are called linear regressors. You specify linear regressors using the linearRegressor object. You can also specify linear regressors by using linear ARX model orders as an input argument. For more information, see "Nonlinear ARX Model Orders and Delay". However, this second approach constrains your regressor set to linear regressors with consecutive delays. To create polynomial regressors, use the polynomialRegressor object. You can also specify custom regressors, which are nonlinear functions of delayed inputs and outputs. For example, $u(t-1) y(t-3)$ is a custom regressor that multiplies instances of input and output together. Specify custom regressors using the customRegressor object.

You can assign any of the regressors as inputs to the linear function block of the output function, the nonlinear function block, or both.
2 It maps the regressors to the model output using an output function block. The output function block can include linear and nonlinear blocks in parallel. For example, consider the following equation:

$$
F(x)=L^{T}(x-r)+g(Q(x-r))+d
$$

Here, $x$ is a vector of the regressors, and $r$ is the mean of $x . F(x)=L^{T}(x-r)+y_{0}$ is the output of the linear function block. $g(Q(x-r))+y_{0}$ represents the output of the nonlinear function block. $Q$ is a projection matrix that makes the calculations well-conditioned. $d$ is a scalar offset that is added to the combined outputs of the linear and nonlinear blocks. The exact form of $F(x)$ depends on your choice of output function. You can select from the available mapping objects, such as
tree-partition networks, wavelet networks, and multilayer neural networks. You can also exclude either the linear or the nonlinear function block from the output function.

When estimating a nonlinear ARX model, the software computes the model parameter values, such as $L, r, d, Q$, and other parameters specifying $g$.

The resulting nonlinear ARX models are idnlarx objects that store all model data, including model regressors and parameters of the output function. For more information about these objects, see "Nonlinear Model Structures".

## Compatibility Considerations

## Use of previous idnlarx properties is not recommended.

Not recommended starting in R2021a
Starting in R2021a, several properties of idnlarx have been modified or replaced.
These changes affect the syntaxes in both idnlarx and nlarx. The use of the pre-R2021a properties in the following table is discouraged. However, the software still accepts calling syntaxes that include these properties. There are no plans to exclude these syntaxes at this time. The command syntax that uses ARX model orders continues be a recommended syntax.

| Pre-R2021a Property | R2021a Property | Usage |
| :--- | :--- | :--- |
| ARX model orders na, nb, nk | Regressors, which can include <br> linearRegressor, <br> polynomialRegressor, and <br> customRegressor objects. | na, nb, nk remains a valid <br> idnlarx and nlarx input <br> argument that the software <br> converts to a <br> linearRegressor object. <br> You can no longer change order <br> values in an existing idnlarx <br> model by dot assignment or by <br> using the set function. Create a <br> new model object instead. |
| customRegressors | Regressors | Use polynomialRegressor or <br> customRegressor to create <br> regressor objects and add the <br> objects to the Regressors <br> array. |
| NonlinearRegressors | RegressorUsage | RegressorUsage is a table that <br> contains regressor assignments <br> to linear and nonlinear output <br> components. Change <br> assignments by modifying the <br> corresponding <br> RegressorUsagetable entries. |
| Nonlinearity |  | Change is in name only. <br> Property remains an object or <br> an array or objects that map <br> regressor inputs to an output. |

## Extended Capabilities

## Automatic Parallel Support

Accelerate code by automatically running computation in parallel using Parallel Computing Toolbox ${ }^{\mathrm{TM}}$.
Parallel computing support is available for estimation using the lsqnonlin search method (requires
Optimization Toolbox). To enable parallel computing, use nlarxOptions, set SearchMethod to 'lsqnonlin', and set SearchOptions.Advanced.UseParallel to true, as in the following example.

```
opt = nlarxOptions;
opt.SearchMethod = 'lsqnonlin';
opt.SearchOptions.Advanced.UseParallel = true;
```


## See Also

aic|fpe|goodnessOfFit|idnlarx|isnlarx|linearRegressor|nlarxOptions| polynomialRegressor

## Topics

"Estimate Nonlinear ARX Models at the Command Line"
"Estimate Nonlinear ARX Models Initialized Using Linear ARX Models"
"Identifying Nonlinear ARX Models"
"Validate Nonlinear ARX Models"
"Using Nonlinear ARX Models"
"Loss Function and Model Quality Metrics"
"Regularized Estimates of Model Parameters"
"Estimation Report"
Introduced in R2007a

## nlarxOptions

Option set for nlarx

## Syntax

```
opt = nlarxOptions
opt = nlarxOptions(Name,Value)
```


## Description

opt $=$ nlarx0ptions creates the default option set for nlarx. Use dot notation to modify this option set for your specific application. Any options that you do not modify retain their default values.
opt = nlarx0ptions(Name, Value) creates an option set with options specified by one or more Name, Value pair arguments.

## Examples

## Create Default Option Set for Nonlinear ARX Estimation <br> opt = nlarxOptions;

## Create and Modify Default Nonlinear ARX Option Set

Create a default option set for nlarx, and use dot notation to modify specific options.
opt = nlarxOptions;
Turn on the estimation progress display.

```
opt.Display = 'on';
```

Minimize the norm of the simulation error.

```
opt.Focus = 'simulation';
```

Use a subspace Gauss-Newton least squares search with a maximum of 25 iterations.

```
opt.SearchMethod = 'gn';
opt.SearchOptions.MaxIterations = 25;
```


## Specify Options for Nonlinear ARX Estimation

Create an option set for nlarx specifying the following options:

- Turn off iterative estimation for the default wavelet network estimation.
- Turn on the estimation progress-viewer display.
opt = nlarxOptions('IterativeWavenet','off','Display','on');


## Input Arguments

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Example: 'Focus', 'simulation', 'SearchMethod', 'grad' specifies that the norm of the simulation error is minimized using a steepest descent least squares search.

## Focus - Minimization objective

'prediction' (default)|'simulation'
Minimization objective, specified as the comma-separated pair consisting of 'Focus ' and one of the following:

- 'prediction' - Minimize the norm of the prediction error, which is defined as the difference between the measured output and the one-step ahead predicted response of the model.
- 'simulation' - Minimize the norm of the simulation error, which is defined as the difference between the measured output and simulated response of the model.


## Display - Estimation progress display setting

'off' (default)|'on'
Estimation progress display setting, specified as the comma-separated pair consisting of 'Display' and one of the following:

- 'off ' - No progress or results information is displayed.
- 'on ' - Information on model structure and estimation results are displayed in a progress-viewer window.


## OutputWeight - Weighting of prediction error in multi-output estimations <br> 'noise' (default)| positive semidefinite matrix

Weighting of prediction error in multi-output model estimations, specified as the comma-separated pair consisting of 'OutputWeight' and one of the following:

- 'noise' - Optimal weighting is automatically computed as the inverse of the estimated noise variance. This weighting minimizes $\operatorname{det}\left(E^{\prime} * E\right)$, where $E$ is the matrix of prediction errors. This option is not available when using 'lsqnonlin' as a 'SearchMethod'.
- A positive semidefinite matrix, $W$, of size equal to the number of outputs. This weighting minimizes trace ( $E^{\prime} * E^{*} W / N$ ), where $E$ is the matrix of prediction errors and $N$ is the number of data samples.


## IterativeWavenet - Iterative wavenet estimation setting <br> 'auto' (default)|'on' | 'off'

Iterative wavenet estimation setting, specified as the comma-separated pair consisting of 'IterativeWavenet ' and one of the following:

- 'auto' - First estimation is noniterative and subsequent estimations are iterative.
- 'on' - Perform iterative estimation only.
- 'off' - Perform noniterative estimation only.

This option applies only when using a wavenet nonlinearity estimator.
Regularization - Options for regularized estimation of model parameters
structure
Options for regularized estimation of model parameters, specified as the comma-separated pair consisting of 'Regularization' and a structure with fields:

| Field <br> Name | Description | Default |
| :--- | :--- | :--- |
| Lambda | Bias versus variance trade-off constant, specified as a <br> nonnegative scalar. | 0 - Indicates no <br> regularization. |
| R | Weighting matrix, specified as a vector of nonnegative scalars <br> or a square positive semidefinite matrix. The length must be <br> equal to the number of free parameters in the model, np. Use <br> the nparams command to determine the number of model <br> parameters. | 1-Indicates a value of <br> eye (np). |
| Nominal | The nominal value towards which the free parameters are <br> pulled during estimation, specified as one of the following: <br> - 'zero' - Pull parameters towards zero. <br> - 'model ' - Pull parameters towards preexisting values in <br> the initial model. Use this option only when you have a well- <br> initialized idnlarx model with finite parameter values. | 'zero' |

To specify field values in Regularization, create a default $n l a r x 0 p t i o n s$ set and modify the fields using dot notation. Any fields that you do not modify retain their default values.

```
opt = nlarxOptions;
opt.Regularization.Lambda = 1.2;
opt.Regularization.R = 0.5*eye(np);
```

Regularization is a technique for specifying model flexibility constraints, which reduce uncertainty in the estimated parameter values. For more information, see "Regularized Estimates of Model Parameters".

## SearchMethod - Numerical search method used for iterative parameter estimation

 'auto' (default) |'gn'|'gna'|'lm'|'grad'|'lsqnonlin'|'fmincon'Numerical search method used for iterative parameter estimation, specified as the comma-separated pair consisting of 'SearchMethod' and one of the following:

- 'auto' - A combination of the line search algorithms, 'gn', 'lm', 'gna', and 'grad' methods is tried in sequence at each iteration. The first descent direction leading to a reduction in estimation cost is used.
- 'gn' - Subspace Gauss-Newton least squares search. Singular values of the Jacobian matrix less than GnPinvConstant*eps*max(size(J))*norm(J) are discarded when computing the search direction. $J$ is the Jacobian matrix. The Hessian matrix is approximated as $J^{T} J$. If there is no improvement in this direction, the function tries the gradient direction.
- 'gna' - Adaptive subspace Gauss-Newton search. Eigenvalues less than gamma*max (sv) of the Hessian are ignored, where sv contains the singular values of the Hessian. The Gauss-Newton direction is computed in the remaining subspace. gamma has the initial value
InitialGnaTolerance (see Advanced in 'SearchOptions' for more information). This value is increased by the factor LMStep each time the search fails to find a lower value of the criterion in fewer than five bisections. This value is decreased by the factor $2 *$ LMStep each time a search is successful without any bisections.
- 'lm' - Levenberg-Marquardt least squares search, where the next parameter value is - pinv (H $+\mathrm{d} * \mathrm{I}) * \mathrm{grad}$ from the previous one. $H$ is the Hessian, $I$ is the identity matrix, and grad is the gradient. $d$ is a number that is increased until a lower value of the criterion is found.
- 'grad ' - Steepest descent least squares search.
- 'lsqnonlin' - Trust-region-reflective algorithm of lsqnonlin. Requires Optimization Toolbox software.
- 'fmincon' - Constrained nonlinear solvers. You can use the sequential quadratic programming (SQP) and trust-region-reflective algorithms of the fmincon solver. If you have Optimization Toolbox software, you can also use the interior-point and active-set algorithms of the fmincon solver. Specify the algorithm in the SearchOptions. Algorithm option. The fmincon algorithms may result in improved estimation results in the following scenarios:
- Constrained minimization problems when there are bounds imposed on the model parameters.
- Model structures where the loss function is a nonlinear or non smooth function of the parameters.
- Multi-output model estimation. A determinant loss function is minimized by default for multioutput model estimation. fmincon algorithms are able to minimize such loss functions directly. The other search methods such as ' lm ' and 'gn' minimize the determinant loss function by alternately estimating the noise variance and reducing the loss value for a given noise variance value. Hence, the fmincon algorithms can offer better efficiency and accuracy for multi-output model estimations.


## SearchOptions - Options set for the search algorithm

search option set
Options set for the search algorithm, specified as the comma-separated pair consisting of
'Search0ptions ' and a search option set with fields that depend on the value of SearchMethod:

| SearchOptions Structure When SearchMethod is Specified as 'gn ', 'gna', 'lm', 'grad', or |
| :--- |
| 'auto' |
| Field <br> Name Description Default <br> Toleran <br> ce Minimum percentage difference between the current value of the loss <br> function and its expected improvement after the next iteration, specified as <br> a positive scalar. When the percentage of expected improvement is less <br> than Tolerance, the iterations stop. The estimate of the expected loss- <br> function improvement at the next iteration is based on the Gauss-Newton <br> vector computed for the current parameter value. $1 e-5$MaxIter <br> ationsMaximum number of iterations during loss-function minimization, specified <br> as a positive integer. The iterations stop when MaxIterations is reached <br> or another stopping criterion is satisfied, such as Tolerance. |
| Setting MaxIterations = 0 returns the result of the start-up procedure. <br> Use sys. Report. Termination. Iterations to get the actual number <br> of iterations during an estimation, where sys is an idtf model. |


| Field Name | Description |  | Default |
| :---: | :---: | :---: | :---: |
|  | Advanced search settings, specified as a structure with the following fields: |  |  |
|  | Field Name | Description | Default |
|  | GnPinvCons tant | Jacobian matrix singular value threshold, specified as a positive scalar. Singular values of the Jacobian matrix that are smaller than <br> GnPinvConstant*max(size(J)*norm(J)*eps) are discarded when computing the search direction. Applicable when SearchMethod is 'gn'. | 10000 |
|  | InitialGna Tolerance | Initial value of gamma, specified as a positive scalar. Applicable when SearchMethod is 'gna'. | 0.0001 |
|  | LMStartVal ue | Starting value of search-direction length $d$ in the Levenberg-Marquardt method, specified as a positive scalar. Applicable when SearchMethod is ' lm '. | 0.001 |
|  | LMStep | Size of the Levenberg-Marquardt step, specified as a positive integer. The next value of the search-direction length $d$ in the Levenberg-Marquardt method is LMStep times the previous one. Applicable when SearchMethod is ' 1 m '. | 2 |
|  | MaxBisecti ons | Maximum number of bisections used for line search along the search direction, specified as a positive integer. | 25 |
|  | MaxFunctio nEvaluatio ns | Maximum number of calls to the model file, specified as a positive integer. Iterations stop if the number of calls to the model file exceeds this value. | Inf |
|  | MinParamet erChange | Smallest parameter update allowed per iteration, specified as a nonnegative scalar. | 0 |
|  | RelativeIm provement | Relative improvement threshold, specified as a nonnegative scalar. Iterations stop if the relative improvement of the criterion function is less than this value. | 0 |
|  | StepReduct ion | Step reduction factor, specified as a positive scalar that is greater than 1. The suggested parameter update is reduced by the factor StepReduction after each try. This reduction continues until MaxBisections tries are completed or a lower value of the criterion function is obtained. <br> StepReduction is not applicable for SearchMethod ' lm' (Levenberg-Marquardt method). | 2 |

SearchOptions Structure When SearchMethod is Specified as ' Isqnonlin'

| Field Name | Description | Default |
| :---: | :---: | :---: |
| Function Toleranc e | Termination tolerance on the loss function that the software minimizes to determine the estimated parameter values, specified as a positive scalar. <br> The value of FunctionTolerance is the same as that of opt.SearchOptions.Advanced.TolFun. | 1e-5 |
| StepTole rance | Termination tolerance on the estimated parameter values, specified as a positive scalar. <br> The value of StepTolerance is the same as that of opt.SearchOptions.Advanced.TolX. | 1e-6 |
| MaxItera tions | Maximum number of iterations during loss-function minimization, specified as a positive integer. The iterations stop when MaxIterations is reached or another stopping criterion is satisfied, such as FunctionTolerance. <br> The value of MaxIterations is the same as that of opt.SearchOptions.Advanced.MaxIter. | 20 |
| Advanced | Advanced search settings, specified as an option set for lsqnonlin. <br> For more information, see the Optimization Options table in "Optimization Options" (Optimization Toolbox). | Use optimset('lsqnonl in') to create a default option set. |

SearchOptions Structure When SearchMethod is Specified as 'fmincon'

| Field Name | Description | Default |
| :---: | :---: | :---: |
| Algorithm | fmincon optimization algorithm, specified as one of the following: <br> 'sqp ' - Sequential quadratic programming algorithm. The algorithm satisfies bounds at all iterations, and it can recover from NaN or Inf results. It is not a large-scale algorithm. For more information, see "Large-Scale vs. MediumScale Algorithms" (Optimization Toolbox). <br> - 'trust-regionreflective' - Subspace trust-region method based on the interior-reflective Newton method. It is a largescale algorithm. <br> - 'interior-point' -Large-scale algorithm that requires Optimization Toolbox software. The algorithm satisfies bounds at all iterations, and it can recover from NaN or Inf results. <br> - 'active-set' - Requires Optimization Toolbox software. The algorithm can take large steps, which adds speed. It is not a large-scale algorithm. <br> For more information about the algorithms, see "Constrained Nonlinear Optimization Algorithms" (Optimization Toolbox) and "Choosing the Algorithm" (Optimization Toolbox). | 'sqp ' |


| Field Name | Description | Default |
| :--- | :--- | :--- |
| FunctionTolerance | Termination tolerance on the <br> loss function that the software <br> minimizes to determine the <br> estimated parameter values, <br> specified as a positive scalar. | $1 \mathrm{e}-6$ |
| StepTolerance | Termination tolerance on the <br> estimated parameter values, <br> specified as a positive scalar. | $1 \mathrm{e}-6$ |
| MaxIterations | Maximum number of iterations <br> during loss function <br> minimization, specified as a <br> positive integer. The iterations <br> stop when MaxIterations is <br> reached or another stopping <br> criterion is satisfied, such as <br> FunctionTolerance. | 100 |

To specify field values in Search0ptions, create a default nlarx0ptions set and modify the fields using dot notation. Any fields that you do not modify retain their default values.

```
opt = nlarxOptions;
opt.SearchOptions.MaxIter = 15;
opt.Search0ptions.Advanced.RelImprovement = 0.5;
```


## Advanced - Additional advanced options

structure
Additional advanced options, specified as the comma-separated pair consisting of 'Advanced' and a structure with fields:

| Field Name | Description | Default |
| :--- | :--- | :--- |
| ErrorThres <br> hold | Threshold for when to adjust the weight of large errors from <br> quadratic to linear, specified as a nonnegative scalar. Errors <br> larger than ErrorThreshold times the estimated standard <br> deviation have a linear weight in the loss function. The <br> standard deviation is estimated robustly as the median of the <br> absolute deviations from the median of the prediction errors, <br> divided by 0.7. If your estimation data contains outliers, try <br> setting ErrorThreshold to 1.6. | quadratic loss function. <br> MaxSizeMaximum number of elements in a segment when input- <br> output data is split into segments, specified as a positive <br> integer. |

To specify field values in Advanced, create a default nlarx0ptions set and modify the fields using dot notation. Any fields that you do not modify retain their default values.

```
opt = nlarxOptions;
opt.Advanced.ErrorThreshold = 1.2;
```


## Output Arguments

opt - Option set for nlarx command
nlarxOptions option set
Option set for nlarx command, returned as an nlarxOptions option set.

## Compatibility Considerations

## Renaming of Estimation and Analysis Options

The names of some estimation and analysis options were changed in R2018a. Prior names still work. For details, see the R2018a release note "Renaming of Estimation and Analysis Options".

## See Also

nlarx

Introduced in R2015a

## nlgreyest

Estimate nonlinear grey-box model parameters

## Syntax

sys= nlgreyest(data,init_sys)
sys= nlgreyest(data,init_sys,options)

## Description

sys= nlgreyest(data,init_sys) estimates the parameters of a nonlinear grey-box model, init_sys, using time-domain data, data.
sys= nlgreyest(data,init_sys,options) specifies additional model estimation options.

## Examples

## Selectively Estimate Parameters of Nonlinear Grey-Box Model

Load data.

```
load(fullfile(matlabroot,'toolbox','ident','iddemos','data','twotankdata'));
z = iddata(y,u,0.2,'Name','Two tanks');
```

The data contains 3000 input-output data samples of a two tank system. The input is the voltage applied to a pump, and the output is the liquid level of the lower tank.

Specify file describing the model structure for a two-tank system. The file specifies the state derivatives and model outputs as a function of time, states, inputs, and model parameters.
FileName = 'twotanks_c';
Specify model orders [ny nu nx].
Order = [llll $\left.1 \begin{array}{ll}1 & 2\end{array}\right]$;
Specify initial parameters ( $\mathrm{Np}=6$ ).
Parameters $=\{0.5 ; 0.0035 ; 0.019 ; . .$. 9.81;0.25;0.016\};

Specify initial initial states.
InitialStates = [0;0.1];
Specify as continuous system.
Ts = 0;
Create idnlgrey model object.

```
nlgr = idnlgrey(FileName,Order,Parameters,InitialStates,Ts, ...
    'Name','Two tanks');
```

Set some parameters as constant.

```
nlgr.Parameters(1).Fixed = true;
nlgr.Parameters(4).Fixed = true;
nlgr.Parameters(5).Fixed = true;
```

Estimate the model parameters.
$n l g r=n l g r e y e s t(z, n l g r) ;$

## Estimate a Nonlinear Grey-Box Model Using Specific Options

Create estimation option set for $n l$ greyest to view estimation progress, and to set the maximum iteration steps to 50.

```
opt = nlgreyestOptions;
opt.Display = 'on';
opt.Search0ptions.MaxIterations = 50;
```

Load data.

```
load(fullfile(matlabroot,'toolbox','ident','iddemos','data','dcmotordata'));
z = iddata(y,u,0.1,'Name','DC-motor');
```

The data is from a linear DC motor with one input (voltage), and two outputs (angular position and angular velocity). The structure of the model is specified by dcmotor_m.m file.

Create a nonlinear grey-box model.

```
file_name = 'dcmotor_m';
Order = [2 1 2];
Parameters = [1;0.28];
InitialStates = [0;0];
init_sys = idnlgrey(file_name,Order,Parameters,InitialStates,0, ...
    'Name','DC-motor');
```

Estimate the model parameters using the estimation options.
sys = nlgreyest(z,init_sys,opt);

## Input Arguments

## data - Time domain data

iddata object
Time-domain estimation data, specified as an iddata object. data has the same input and output dimensions as init_sys.

If you specify the InterSample property of data as 'bl' (band-limited) and the model is continuoustime, the software treats data as first-order-hold (foh) interpolated for estimation.

## init_sys - Constructed nonlinear grey-box model

## idnlgrey object

Constructed nonlinear grey-box model that configures the initial parameterization of sys, specified as an idnlgrey object. init_sys has the same input and output dimensions as data. Create init_sys using idnlgrey.

## options - Estimation options

nlgreyest0ptions option set
Estimation options for nonlinear grey-box model identification, specified as an nlgreyestOptions option set.

## Output Arguments

## sys - Estimated nonlinear grey-box model

idnlgrey object
Nonlinear grey-box model with the same structure as init_sys, returned as an idnlgrey object. The parameters of sys are estimated such that the response of sys matches the output signal in the estimation data.

Information about the estimation results and options used is stored in the Report property of the model. Report has the following fields:

| Report <br> Field | Description |  |
| :--- | :--- | :---: |
| Status | Summary of the model status, which indicates whether the model was created by <br> construction or obtained by estimation. |  |
| Method | Name of the simulation solver and the search method used during estimation. |  |
| Fit | Quantitative assessment of the estimation, returned as a structure. See "Loss Function <br> and Model Quality Metrics" for more information on these quality metrics. The <br> structure has the following fields: |  |
|  | Field |  |
| FitPerce <br> nt | Description <br> Normalized root mean squared error (NRMSE) measure of how well the <br> response of the model fits the estimation data, expressed as the <br> percentage fit = 100(1-NRMSE). |  |
|  | LossFcn |  |
| Value of the loss function when the estimation completes. |  |  |
| MSE | Mean squared error (MSE) measure of how well the response of the <br> model fits the estimation data. |  |
|  | FPE |  |
|  | Final prediction error for the model. |  |
| AIC | Raw Akaike Information Criteria (AIC) measure of model quality. |  |
|  | AICc |  |
| nAIC | Small sample-size corrected AIC. |  |
|  | Normalized AIC. |  |
| BIC | Bayesian Information Criteria (BIC). |  |


| Report Field | Description |  |  |
| :---: | :---: | :---: | :---: |
| Parameter s | Estimated values of the model parameters. Structure with the following fields: |  |  |
|  | Field |  | Description |
|  | InitialValues |  | Structure with values of parameters and initial states before estimation. |
|  | ParVector |  | Value of parameters after estimation. |
|  | Free |  | Logical vector specifying the fixed or free status of parameters during estimation |
|  | FreeParCovariance |  | Covariance of the free parameters. |
|  | X0 |  | Value of initial states after estimation. |
|  | X0Covariance |  | Covariance of the initial states. |
| OptionsUs ed | Option set used for estimation. If no custom options were configured, this is a set of default options. See nlgreyestOptions for more information. |  |  |
| RandState | State of the random number stream at the start of estimation. Empty, [ ], if randomization was not used during estimation. For more information, see rng. |  |  |
| DataUsed | Attributes of the data used for estimation - Structure with the following fields: |  |  |
|  | Field | Description |  |
|  | Name | Name of the data set. |  |
|  | Type | Data type - For idnlgrey models, this is set to 'Time domain data'. |  |
|  | Length | Number of data samples. |  |
|  | Ts | Sample time. This is equivalent to data.Ts. |  |
|  | InterSam ple | Input intersample behavior. One of the following values: <br> - 'zoh' - Zero-order hold maintains a piecewise-constant input signal between samples. <br> - 'foh ' - First-order hold maintains a piecewise-linear input signal between samples. <br> - 'bl' - Band-limited behavior specifies that the continuous-time input signal has zero power above the Nyquist frequency. <br> The value of Intersample has no effect on estimation results for discrete-time models. |  |
|  | Input0ff set | Empty, [ ], for nonlinear estimation methods. |  |
|  | $\begin{array}{\|l} \hline \text { Output0f } \\ \text { fset } \\ \hline \end{array}$ | Empty, [ ], for nonlinear estimation methods. |  |


| Report Field | Description |  |
| :---: | :---: | :---: |
| Terminati on | Termination conditions for the iterative search used for prediction error minimization, returned as a structure with the following fields: |  |
|  | Field | Description |
|  | WhyStop | Reason for terminating the numerical search. |
|  | Iteratio ns | Number of search iterations performed by the estimation algorithm. |
|  | First0rd erOptima lity | $\infty$-norm of the gradient search vector when the search algorithm terminates. |
|  | FcnCount | Number of times the objective function was called. |
|  | UpdateNo rm | Norm of the gradient search vector in the last iteration. Omitted when the search method is 'lsqnonlin' or 'fmincon'. |
|  | LastImpr ovement | Criterion improvement in the last iteration, expressed as a percentage. Omitted when the search method is 'lsqnonlin' or 'fmincon'. |
|  | $\begin{aligned} & \text { Algorith } \\ & \mathrm{m} \end{aligned}$ | Algorithm used by ' lsqnonlin' or 'fmincon' search method. Omitted when other search methods are used. |
|  | For estimation methods that do not require numerical search optimization, the Termination field is omitted. |  |

For more information, see "Estimation Report".

## Extended Capabilities

## Automatic Parallel Support

Accelerate code by automatically running computation in parallel using Parallel Computing Toolbox ${ }^{\mathrm{TM}}$.
Parallel computing support is available for estimation using the lsqnonlin search method (requires Optimization Toolbox). To enable parallel computing, use nlgreyestOptions, set SearchMethod to 'lsqnonlin', and set SearchOptions.Advanced.UseParallel to true.

For example:
opt $=$ nlgreyestOptions;
opt.SearchMethod = 'lsqnonlin';
opt.SearchOptions.Advanced.UseParallel = true;

## See Also

aic|fpe|goodnessOfFit|idnlgrey|nlgreyestOptions | pem

## Topics

"Creating IDNLGREY Model Files"
"Represent Nonlinear Dynamics Using MATLAB File for Grey-Box Estimation"
"Estimate Nonlinear Grey-Box Models"
"Loss Function and Model Quality Metrics"
"Regularized Estimates of Model Parameters"
"Estimation Report"
Introduced in R2015a

## nlgreyestOptions

Option set for nlgreyest

## Syntax

```
opt = nlgreyestOptions
opt = nlgreyestOptions(Name,Value)
```


## Description

opt $=$ nlgreyestOptions creates the default option set for nlgreyest. Use dot notation to customize the option set, if needed.
opt $=\mathrm{nl}$ greyestOptions(Name, Value) creates an option set with options specified by one or more Name, Value pair arguments. The options that you do not specify retain their default value.

## Examples

## Create Default Option Set for Nonlinear Grey-Box Model Estimation

```
opt = nlgreyestOptions;
```


## Estimate a Nonlinear Grey-Box Model Using Specific Options

Create estimation option set for nlgreyest to view estimation progress, and to set the maximum iteration steps to 50 .

```
opt = nlgreyestOptions;
opt.Display = 'on';
opt.SearchOptions.MaxIterations = 50;
```

Load data.
load(fullfile(matlabroot,'toolbox','ident','iddemos','data','dcmotordata'));
z = iddata(y,u,0.1,'Name','DC-motor');
The data is from a linear DC motor with one input (voltage), and two outputs (angular position and angular velocity). The structure of the model is specified by dcmotor_m.m file.

Create a nonlinear grey-box model.

```
file_name = 'dcmotor m';
Order = [2 1 2];
Parameters = [1;0.28];
InitialStates = [0;0];
init_sys = idnlgrey(file_name,Order,Parameters,InitialStates,0, ...
    'Name','DC-motor');
```

Estimate the model parameters using the estimation options.
sys = nlgreyest(z,init_sys,opt);

## Specify Options for Nonlinear Grey-Box Model Estimation

Create an option set for $n l$ greyest where:

- Parameter covariance data is not generated.
- Subspace Gauss-Newton least squares method is used for estimation.

```
opt = nlgreyest0ptions('EstimateCovariance',false,'SearchMethod','gn');
```


## Input Arguments

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Example: nlgreyest0ptions('Display', 'on')
GradientOptions - Options for computing Jacobians and gradients
structure
Options for computing Jacobians and gradients, specified as the comma-separated pair consisting of 'GradientOptions' and a structure with fields:

| Field Name | Description | Default |
| :--- | :--- | :--- |
| MaxDifference | Largest allowed parameter perturbation <br> when computing numerical derivatives. <br> Specified as a positive real value > <br> 'MinDifference '. | Inf |
| MinDifference | Smallest allowed parameter perturbation <br> when computing numerical derivatives. <br> Specified as a positive real value <br> <'MaxDifference '. | $0.01^{*}$ sqrt (eps ) |


| Field Name | Description | Default |
| :---: | :---: | :---: |
| DifferenceSch eme | Method for computing numerical derivatives with respect to the components of the parameters and/or the initial state(s) to form the Jacobian. Specified as one of the following: <br> - 'Auto' - Automatically chooses from the following methods. <br> - 'Central approximation' <br> - 'Forward approximation' <br> - 'Backward approximation' | 'Auto' |
| Type | Method used when computing derivatives (Jacobian) of the parameters or the initial states to be estimated. Specified as one of the following: <br> - 'Auto' - Automatically chooses from the following methods. <br> - 'Basic' - Individually computes all numerical derivatives required to form each column of the Jacobian. <br> - 'Refined ' - Simultaneously computes all numerical derivatives required to form each column of the Jacobian. | 'Auto ' |

To specify field values in GradientOptions, create a default nlgreyestOptions set and modify the fields using dot notation. Any fields that you do not modify retain their default values.

```
opt = nlgreyestOptions;
opt.GradientOptions.Type = 'Basic';
```

EstimateCovariance - Parameter covariance data generation setting
1 or true (default) | 0 or false
Controls whether parameter covariance data is generated, specified as true (1) or false (0).

## Display - Estimation progress display setting

'off' (default)|'on'
Estimation progress display setting, specified as the comma-separated pair consisting of 'Display' and one of the following:

- 'off' - No progress or results information is displayed.
- 'on ' - Information on model structure and estimation results are displayed in a progress-viewer window.


## Regularization - Options for regularized estimation of model parameters

structure
Options for regularized estimation of model parameters, specified as the comma-separated pair consisting of 'Regularization' and a structure with fields:

| Field <br> Name | Description | Default |
| :--- | :--- | :--- |
| Lambda | Bias versus variance trade-off constant, specified as a <br> nonnegative scalar. | 0 - Indicates no <br> regularization. |
| R | Weighting matrix, specified as a vector of nonnegative scalars <br> or a square positive semi-definite matrix. The length must be <br> equal to the number of free parameters in the model, np. Use <br> the nparams command to determine the number of model <br> parameters. | 1-Indicates a value of <br> eye (np). |
| Nominal | The nominal value towards which the free parameters are <br> pulled during estimation specified as one of the following: <br> - 'zero' - Pull parameters towards zero. <br> - 'model ' - Pull parameters towards pre-existing values in <br> the initial model. | 'zero' |

To specify field values in Regularization, create a default nlgreyestOptions set and modify the fields using dot notation. Any fields that you do not modify retain their default values.

```
opt = nlgreyestOptions;
opt.Regularization.Lambda = 1.2;
opt.Regularization.R = 0.5*eye(np);
```

Regularization is a technique for specifying model flexibility constraints, which reduce uncertainty in the estimated parameter values. For more information, see "Regularized Estimates of Model Parameters".

## SearchMethod - Numerical search method used for iterative parameter estimation <br> 'auto' (default)|'gn'|'gna'|'lm'|'grad'|'lsqnonlin'

Numerical search method used for iterative parameter estimation, specified as the comma-separated pair consisting of 'SearchMethod' and one of the following:

- 'auto ' - If Optimization Toolbox is available, ' lsqnonlin' is used. Otherwise, a combination of the line search algorithms, 'gn', 'lm', 'gna', and 'grad' methods is tried in sequence at each iteration. The first descent direction leading to a reduction in estimation cost is used.
- 'gn' - Subspace Gauss-Newton least squares search. Singular values of the Jacobian matrix less than GnPinvConstant*eps*max (size(J))*norm(J) are discarded when computing the search direction. $J$ is the Jacobian matrix. The Hessian matrix is approximated by $J^{T} J$. If there is no improvement in this direction, the function tries the gradient direction.
- 'gna' - Adaptive subspace Gauss-Newton search. Eigenvalues less than gamma*max(sv) of the Hessian are ignored, where $s v$ are the singular values of the Hessian. The Gauss-Newton direction is computed in the remaining subspace. gamma has the initial value InitialGnaTolerance (see Advanced in 'Search0ptions' for more information). This value is increased by the factor LMStep each time the search fails to find a lower value of the criterion in fewer than five bisections. This value is decreased by the factor $2 *$ LMStep each time a search is successful without any bisections.
- ' $1 m^{\prime}$ - Levenberg-Marquardt least squares search, where the next parameter value is - pinv ( H $+\mathrm{d} * \mathrm{I}) * \mathrm{grad}$ from the previous one. $H$ is the Hessian, $I$ is the identity matrix, and grad is the gradient. $d$ is a number that is increased until a lower value of the criterion is found.
- 'grad' - Steepest descent least squares search.
- 'lsqnonlin' - Trust-region-reflective algorithm of lsqnonlin. Requires Optimization Toolbox software.
- 'fmincon' - Constrained nonlinear solvers. You can use the sequential quadratic programming (SQP) and trust-region-reflective algorithms of the fmincon solver. If you have Optimization Toolbox software, you can also use the interior-point and active-set algorithms of the fmincon solver. Specify the algorithm in the Search0ptions. Algorithm option. The fmincon algorithms may result in improved estimation results in the following scenarios:
- Constrained minimization problems when there are bounds imposed on the model parameters.
- Model structures where the loss function is a nonlinear or non smooth function of the parameters.
- Multi-output model estimation. A determinant loss function is minimized by default for MIMO model estimation. fmincon algorithms are able to minimize such loss functions directly. The other available search methods such as ' 1 m ' and ' gn ' minimize the determinant loss function by alternately estimating the noise variance and reducing the loss value for a given noise variance value. Hence, the fmincon algorithms can offer better efficiency and accuracy for multi-output model estimations.


## SearchOptions - Option set for the search algorithm

search option set
Option set for the search algorithm, specified as the comma-separated pair consisting of
'Search0ptions ' and a search option set with fields that depend on the value of SearchMethod.

## SearchOptions Structure When SearchMethod Is Specified as 'lsqnonlin' or 'auto', When Optimization Toolbox Is Available

| Field <br> Name | Description | Default |
| :--- | :--- | :--- |
| Function <br> Toleranc <br> e | Termination tolerance on the loss function that the software minimizes to <br> determine the estimated parameter values, specified as a positive scalar. <br> The value of FunctionTolerance is the same as that of <br> opt. Search0ptions. Advanced. TolFun. | $1 \mathrm{e}-5$ |
| StepTole <br> rance | Termination tolerance on the estimated parameter values, specified as a <br> positive scalar. <br> The value of StepTolerance is the same as that of <br> opt. Search0ptions.Advanced. TolX. | $1 \mathrm{le}-6$ |
| MaxItera <br> tions | Maximum number of iterations during loss-function minimization, <br> specified as a positive integer. The iterations stop when MaxIterations <br> is reached or another stopping criterion is satisfied, such as <br> FunctionTolerance. <br> The value of MaxIterations is the same as that of <br> opt. Search0ptions. Advanced. MaxIter. | 20 |
| Advanced | Advanced search settings, specified as an option set for lsqnonlin. <br> For more information, see the Optimization Options table in "Optimization <br> Options" (Optimization Toolbox). | Use <br> optimset (' <br> lsqnonlin ' <br> ) to create a <br> default <br> option set. |

Search0ptions Structure When SearchMethod Is Specified as 'gn', 'gna', 'lm', 'grad', or 'auto ', When Optimization Toolbox Is Not Available

| Field <br> Name | Description | Default |
| :--- | :--- | :--- |
| Toleranc <br> e | Minimum percentage difference between the current value of the loss <br> function and its expected improvement after the next iteration, specified <br> as a positive scalar. When the percentage of expected improvement is less <br> than Tolerance, the iterations stop. The estimate of the expected loss- <br> function improvement at the next iteration is based on the Gauss-Newton <br> vector computed for the current parameter value. | $1 \mathrm{e}-5$ |
| MaxItera <br> tions | Maximum number of iterations during loss-function minimization, <br> specified as a positive integer. The iterations stop when MaxIterations <br> is reached or another stopping criterion is satisfied, such as Tolerance. | 20 |
| Setting MaxIterations = 0 returns the result of the start-up <br> procedure. | Use sys. Report. Termination. Iterations to get the actual number <br> of iterations during an estimation, where sys is an idtf model. |  |


| Field Name | Description |  | Default |
| :---: | :---: | :---: | :---: |
| Advanced | Advanced search settings, specified as a structure with the following fields: |  |  |
|  | Field Name | Description | Default |
|  | GnPinvCons tant | Jacobian matrix singular value threshold, specified as a positive scalar. Singular values of the Jacobian matrix that are smaller than GnPinvConstant*max(size(J)*norm(J)*eps) are discarded when computing the search direction. Applicable when SearchMethod is 'gn'. | 10000 |
|  | InitialGna Tolerance | Initial value of gamma, specified as a positive scalar. Applicable when SearchMethod is 'gna'. | 0.0001 |
|  | LMStartVal ue | Starting value of search-direction length $d$ in the Levenberg-Marquardt method, specified as a positive scalar. Applicable when SearchMethod is ' lm' | 0.001 |
|  | LMStep | Size of the Levenberg-Marquardt step, specified as a positive integer. The next value of the search-direction length $d$ in the Levenberg-Marquardt method is LMStep times the previous one. Applicable when SearchMethod is ' 1 m '. | 2 |
|  | MaxBisecti ons | Maximum number of bisections used for line search along the search direction, specified as a positive integer. | 25 |
|  | MaxFunctio nEvaluatio ns | Maximum number of calls to the model file, specified as a positive integer. Iterations stop if the number of calls to the model file exceeds this value. | Inf |
|  | MinParamet erChange | Smallest parameter update allowed per iteration, specified as a nonnegative scalar. | 0 |
|  | RelativeIm provement | Relative improvement threshold, specified as a nonnegative scalar. Iterations stop if the relative improvement of the criterion function is less than this value. | 0 |
|  | StepReduct ion | Step reduction factor, specified as a positive scalar that is greater than 1 . The suggested parameter update is reduced by the factor StepReduction after each try. This reduction continues until either MaxBisections tries are completed or a lower value of the criterion function is obtained. <br> StepReduction is not applicable for SearchMethod ' lm' (Levenberg-Marquardt method). | 2 |

SearchOptions Structure When SearchMethod is Specified as 'fmincon'

| Field Name | Description | Default |
| :---: | :---: | :---: |
| Algorithm | fmincon optimization algorithm, specified as one of the following: <br> - 'sqp' - Sequential quadratic programming algorithm. The algorithm satisfies bounds at all iterations, and it can recover from NaN or Inf results. It is not a large-scale algorithm. For more information, see "Large-Scale vs. MediumScale Algorithms" (Optimization Toolbox). <br> - 'trust-regionreflective' - Subspace trust-region method based on the interior-reflective Newton method. It is a largescale algorithm. <br> - 'interior-point' -Large-scale algorithm that requires Optimization Toolbox software. The algorithm satisfies bounds at all iterations, and it can recover from NaN or Inf results. <br> - 'active-set' - Requires Optimization Toolbox software. The algorithm can take large steps, which adds speed. It is not a large-scale algorithm. <br> For more information about the algorithms, see "Constrained Nonlinear Optimization Algorithms" (Optimization Toolbox) and "Choosing the Algorithm" (Optimization Toolbox). | 'sqp' |


| Field Name | Description | Default |
| :--- | :--- | :--- |
| FunctionTolerance | Termination tolerance on the <br> loss function that the software <br> minimizes to determine the <br> estimated parameter values, <br> specified as a positive scalar. | $1 \mathrm{e}-6$ |
| StepTolerance | Termination tolerance on the <br> estimated parameter values, <br> specified as a positive scalar. | $1 \mathrm{e}-6$ |
| MaxIterations | Maximum number of iterations <br> during loss function <br> minimization, specified as a <br> positive integer. The iterations <br> stop when MaxIterations is <br> reached or another stopping <br> criterion is satisfied, such as <br> FunctionTolerance. | 100 |

To specify field values in SearchOptions, create a default nlgreyestOptions set and modify the fields using dot notation. Any fields that you do not modify retain their default values.

```
opt = nlgreyestOptions('SearchMethod','gna');
opt.Search0ptions.MaxIterations = 50;
opt.SearchOptions.Advanced.RelImprovement = 0.5;
```


## OutputWeight - Weighting of prediction error in multi-output estimations

[] (default) | 'noise' | matrix
Weighting of prediction error in multi-output model estimations, specified as the comma-separated pair consisting of 'OutputWeight' and one of the following:

- [ ] - No weighting is used. Specifying as [] is the same as eye(Ny), where Ny is the number of outputs.
- 'noise' - Optimal weighting is automatically computed as the inverse of the estimated noise variance. This weighting minimizes $\operatorname{det}\left(E^{\prime} * E / N\right)$, where $E$ is the matrix of prediction errors and $N$ is the number of data samples. This option is not available when using 'lsqnonlin' as a 'SearchMethod'.
- A positive semidefinite matrix, W , of size equal to the number of outputs. This weighting minimizes trace ( $\left.E^{\prime} * E^{*} W / N\right)$, where $E$ is the matrix of prediction errors and $N$ is the number of data samples.


## Advanced - Additional advanced options

structure
Additional advanced options, specified as the comma-separated pair consisting of 'Advanced' and a structure with field:

$\left.$| Field Name | Description | Default |
| :--- | :--- | :--- |
| ErrorThreshold | Threshold for when to adjust the <br> weight of large errors from <br> quadratic to linear, specified as <br> a nonnegative scalar. Errors <br> larger than ErrorThreshold <br> times the estimated standard <br> deviation have a linear weight in | loass function. |
| the loss function. The standard |  |  |
| deviation is estimated robustly |  |  |
| as the median of the absolute |  |  |
| deviations from the median of |  |  |
| the prediction errors divided by |  |  |
| 0.7. If your estimation data |  |  |
| contains outliers, try setting |  |  |
| ErrorThreshold to 1.6. |  |  |$\quad \right\rvert\,$

To specify field values in Advanced, create a default nlgreyestOptions set and modify the fields using dot notation. Any fields that you do not modify retain their default values.

```
opt = nlgreyestOptions;
opt.Advanced.ErrorThreshold = 1.2;
```


## Output Arguments

opt - Option set for nlgreyest
nlgreyestOptions option set
Option set for $n l$ greyest, returned as an $n l$ greyestOptions option set.

## Compatibility Considerations

## Renaming of Estimation and Analysis Options

The names of some estimation and analysis options were changed in R2018a. Prior names still work. For details, see the R2018a release note "Renaming of Estimation and Analysis Options".

## See Also

nlgreyest

Introduced in R2015a

## nlhw

## Estimate Hammerstein-Wiener model

## Syntax

```
sys = nlhw(Data,Orders)
sys = nlhw(Data,Orders,InputNL,OutputNL)
sys = nlhw(Data,LinModel)
sys = nlhw(Data,LinModel,InputNL,OutputNL)
sys = nlhw(Data,sys0)
sys = nlhw(
```

$\qquad$

``` ,Options)
```


## Description

sys $=n l h w($ Data,Orders) creates and estimates a Hammerstein-Wiener model using the estimation data, model orders and delays, and default piecewise linear functions as input and output nonlinearity estimators.
sys = nlhw(Data,Orders,InputNL,OutputNL) specifies InputNL and OutputNL as the input and output nonlinearity estimators, respectively.
sys $=$ nlhw (Data, LinModel) uses a linear model to specify the model orders and delays, and default piecewise linear functions for the input and output nonlinearity estimators.
sys $=$ nlhw(Data,LinModel,InputNL,OutputNL) specifies InputNL and OutputNL as the input and output nonlinearity estimators, respectively.
sys $=n l h w($ Data,sys0) refines or estimates the parameters of a Hammerstein-Wiener model, sys0, using the estimation data.

Use this syntax to:

- Update the parameters of a previously estimated model to improve the fit to the estimation data. In this case, the estimation algorithm uses the parameters of sys0 as initial guesses.
- Estimate the parameters of a model previously created using the idnlhw constructor. Prior to estimation, you can configure the model properties using dot notation.
sys $=n l h w(\ldots$, Options ) specifies additional model estimation options. Use Options with any of the previous syntaxes.


## Examples

## Estimate a Hammerstein-Wiener Model

```
load iddata3
m1 = nlhw(z3,[4 2 1]);
```


## Estimate a Hammerstein Model with Saturation

Load data.
load twotankdata;
z = iddata(y,u,0.2,'Name','Two tank system');
z1 = z(1:1000);
Create a saturation object with lower limit of 0 and upper limit of 5 .
InputNL = saturation('LinearInterval',[0 5]);
Estimate model with no output nonlinearity.
m = nlhw(z1,[2 3 0],InputNL,[]);

## Estimate Hammerstein-Wiener Model with a Custom Network Nonlinearity

Generating a custom network nonlinearity requires the definition of a user-defined unit function.
Define the unit function and save it as gaussunit.m.

```
% Copyright 2015 The MathWorks, Inc.
function [f, g, a] = gaussunit(x)
f = exp(-x.*x);
if nargout>1
    g = -2*x.*f;
    a = 0.2;
end
```

Create a custom network nonlinearity using the gaussunit function.

```
H = @gaussunit;
```

CNet $=$ customnet (H);

Load the estimation data.
load twotankdata;
z = iddata(y,u,0.2,'Name','Two tank system');
z1 = z(1:1000);
Estimate a Hammerstein-Wiener model using the custom network.

```
m = nlhw(z1,[5 1 3],CNet,[]);
```


## Estimate Default Hammerstein-Wiener Model Using an Input-Output Polynomial Model of OE Structure

Estimate linear OE model.

```
load throttledata.mat
Tr = getTrend(ThrottleData);
Tr.OutputOffset = 15;
DetrendedData = detrend(ThrottleData, Tr);
opt = oeOptions('Focus','simulation');
LinearModel = oe(DetrendedData,[1 2 1],opt);
```

Estimate Hammerstein-Wiener model using OE model as its linear component and saturation as its output nonlinearity.

```
sys = nlhw(ThrottleData,LinearModel,[],'saturation');
```


## Estimate a Hammerstein-Wiener Model Using idnlhw to first Define the Model Properties

Load the estimation data.

```
load iddata1
```

Construct a Hammerstein-Wiener model using idnlhw to define the model properties B and F.

```
sys0 = idnlhw([2,2,0],[],'wavenet');
sys0.B{1} = [0.8,1];
sys0.F{1} = [1,-1.2,0.5];
```

Estimate the model.
sys = nlhw(z1,sys0);
Estimate a Hammerstein-Wiener model using $n l h w$ to define the model properties $B$ and $F$.

```
sys2 = nlhw(z1,[2,2,0],[],'wavenet','B',{[0.8,1]},'F',{[1,-1.2,0.5]});
```

Compare the two estimated models to see that they are equivalent.

```
compare(z1,sys,'g',sys2,'r--');
```

Simulated Response Comparison


## Refine a Hammerstein-Wiener Model Using Successive Calls of nlhw

Estimate a Hammerstein-Wiener Model.
load iddata3
sys = nlhw(z3,[4 2 1],'sigmoidnet','wavenet');
Refine the model, sys.
sys $=n l h w(z 3, s y s) ;$

## Estimate Hammerstein-Wiener Model Using an Estimation Option Set

Create estimation option set for $n l h w$ to view estimation progress and to set the maximum iteration steps to 50 .
opt = nlhwOptions;
opt.Display = 'on';
opt.SearchOptions.MaxIterations = 50;
Load data and estimate the model.

```
load iddata3
sys = nlhw(z3,[4 2 1],'sigmoidnet','deadzone',opt);
```


## Input Arguments

## Data - Time domain data

iddata object

Time-domain estimation data, specified as an iddata.

## Orders - Order and delays of the linear subsystem transfer function

[ $n b$ nf nk] vector of positive integers | [nb nf nk] vector of matrices
Order and delays of the linear subsystem transfer function, specified as a [ nb nf nk ] vector.
Dimensions of Orders:

- For a SISO transfer function, Orders is a vector of positive integers.
nb is the number of zeros plus $1, \mathrm{nf}$ is the number of poles, and nk is the input delay.
- For a MIMO transfer function with $\mathrm{n}_{\mathrm{u}}$ inputs and $\mathrm{n}_{\mathrm{y}}$ outputs, Orders is a vector of matrices.
$n b, n f$, and $n k$ are $n_{y}$-by- $n_{u}$ matrices whose $i$-jth entry specifies the orders and delay of the transfer function from the $j$ th input to the ith output.


## InputNL - Input static nonlinearity

```
'pwlinear' (default)|'sigmoidnet'| 'wavenet'|'saturation'|'deadzone'|'polyld' |
'unitgain' | nonlinearity estimator object | array of nonlinearity estimators
```

Input static nonlinearity estimator, specified as one of the following.
'pwlinear' or pwlinear object (default)
'sigmoidnet' or sigmoidnet object
'wavenet' or wavenet object
'saturation' or saturation object
'deadzone' or deadzone object
'poly1d' or poly1d object
'unitgain' or [] or unitgain object
customnet object

Piecewise linear function
Sigmoid network
Wavelet network
Saturation
Dead zone
One-dimensional polynomial
Unit gain
Custom network - Similar to sigmoidnet, but with a user-defined replacement for the sigmoid function.

Specifying a character vector, for example 'sigmoidnet ', creates a nonlinearity estimator object with default settings. Use object representation instead to configure the properties of a nonlinearity estimator.

InputNL = wavenet;
InputNL. NumberOfUnits = 10;

Alternatively, use the associated input nonlinearity estimator function with Name-Value pair arguments.

InputNL = wavenet('NumberOfUnits', 10);
For $n_{u}$ input channels, you can specify nonlinear estimators individually for each input channel by setting InputNL to an $n_{u}$-by-1 array of nonlinearity estimators.

InputNL $=$ [sigmoidnet('NumberofUnits',5); deadzone([-1,2])]
To specify the same nonlinearity for all inputs, specify a single input nonlinearity estimator.

## OutputNL - Output static nonlinearity

'pwlinear' (default)|'sigmoidnet'|'wavenet'|'saturation'|'deadzone'|'polyld'|
' unitgain ' | nonlinearity estimator object | array of nonlinearity estimators
Output static nonlinearity estimator, specified as one of the following:

| 'pwlinear' or pwlinear object | Piecewise linear function |
| :--- | :--- |
| (default) | Sigmoid network |
| 'sigmoidnet ' or sigmoidnet object | Wavelet network |
| 'wavenet' or wavenet object | Saturation |
| 'saturation' or saturation object | Dead zone |
| 'deadzone' or deadzone object | One-dimensional polynomial |
| 'polyld' or polyld object | Unit gain |
| 'unitgain' or [] or unitgain object | Custom network - Similar to <br> sigmoidnet, but with a user-defined <br> replacement for the sigmoid function. |

'pwlinear' or pwlinear object
'sigmoidnet' or sigmoidnet object
'wavenet ' or wavenet object
'saturation' or saturation object
'deadzone' or deadzone object
'polyld' or polyld object
'unitgain' or [] or unitgain object
customnet object

Piecewise linear function
Sigmoid network
Wavelet network
Saturation
Dead zone
One-dimensional polynomial
Unit gain
Custom network - Similar to replacement for the sigmoid function.

Specifying a character vector creates a nonlinearity estimator object with default settings. Use object representation instead to configure the properties of a nonlinearity estimator.

```
OutputNL = sigmoidnet;
OutputNL.NumberOfUnits = 10;
```

Alternatively, use the associated input nonlinearity estimator function with Name-Value pair arguments.

```
OutputNL = sigmoidnet('NumberOfUnits',10);
```

For $\mathrm{n}_{\mathrm{y}}$ output channels, you can specify nonlinear estimators individually for each output channel by setting OutputNL to an $\mathrm{n}_{\mathrm{y}}$-by-1 array of nonlinearity estimators. To specify the same nonlinearity for all outputs, specify a single output nonlinearity estimator.

## LinModel - Discrete time linear model

idpoly|idss with K = 0|idtf
Discrete-time linear model used to specify the linear subsystem, specified as one of the following:

- Input-output polynomial model of Output-Error (OE) structure (idpoly)
- State-space model with no disturbance component (idss with $K=0$ )
- Transfer function model (idtf)

Typically, you estimate the model using oe, n4sid, or tfest.

## sys0 - Hammerstein-Wiener model

idnlhw object
Hammerstein-Wiener model, specified as an idnlhw object. sys0 can be:

- A model previously created using idnlhw to specify model properties.
- A model previously estimated using nlhw, that you want to update using a new estimation data set.

You can also refine sys0 using the original estimation data set. If the previous estimation stopped when the numerical search was stuck at a local minima of the cost function, use init to first randomize the parameters of sys0. See sys0. Report. Termination for search stopping conditions. Using init does not guarantee a better solution on further refinement.

## Options - Estimation options

nlhwOptions option set
Estimation options for Hammerstein-Wiener model identification, specified as an nlhw0ptions option set.

## Output Arguments

## sys - Estimated Hammerstein-Wiener model

idnlhw object
Estimated Hammerstein-Wiener model, returned as an idnlhw object. The model is estimated using the specified model orders and delays, input and output nonlinearity estimators, and estimation options.

Information about the estimation results and options used is stored in the Report property of the model. Report has the following fields:

| Report <br> Field | Description |
| :--- | :--- |
| Status | Summary of the model status, which indicates whether the model was created by <br> construction or obtained by estimation. |
| Method | Estimation command used. |


| Report <br> Field | Description |  |
| :--- | :--- | :--- |
| Fit | Quantitative assessment of the estimation, returned as a structure. See "Loss Function <br> and Model Quality Metrics" for more information on these quality metrics. The <br> structure has the following fields: |  |
|  | Field | Description |
|  | FitPerce <br> nt | Normalized root mean squared error (NRMSE) measure of how well the <br> response of the model fits the estimation data, expressed as the <br> percentage fit = 100(1-NRMSE). |
|  | LossFcn | Value of the loss function when the estimation completes. |
|  | MSE <br> Mean squared error (MSE) measure of how well the response of the <br> model fits the estimation data. |  |
|  | FPE | Final prediction error for the model. |
|  | AIC | Raw Akaike Information Criteria (AIC) measure of model quality. |
|  | AICc | Small sample-size corrected AIC. |
| nAIC | Normalized AIC. |  |
|  | BIC | Bayesian Information Criteria (BIC). |
| Parameter <br> s | Estimated values of model parameters. <br> OptionsUs <br> edOption set used for estimation. If no custom options were configured, this is a set of <br> default options. See nlhw0ptions for more information. |  |
| RandState | State of the random number stream at the start of estimation. Empty, [ ], if <br> randomization was not used during estimation. For more information, see rng. |  |


| Report Field | Description |  |
| :---: | :---: | :---: |
| DataUsed | Attributes of the data used for estimation, returned as a structure with the following fields: |  |
|  | Field | Description |
|  | Name | Name of the data set. |
|  | Type | Data type. |
|  | Length | Number of data samples. |
|  | Ts | Sample time. |
|  | InterSam ple | Input intersample behavior, returned as one of the following values: <br> - 'zoh ' - Zero-order hold maintains a piecewise-constant input signal between samples. <br> - ' foh ' - First-order hold maintains a piecewise-linear input signal between samples. <br> - 'bl' - Band-limited behavior specifies that the continuous-time input signal has zero power above the Nyquist frequency. |
|  | InputOff set | Offset removed from time-domain input data during estimation. For nonlinear models, it is [ ]. |
|  | Output0f fset | Offset removed from time-domain output data during estimation. For nonlinear models, it is []. |
| Terminati on | Termination conditions for the iterative search used for prediction error minimization, returned as a structure with the following fields: |  |
|  | Field | Description |
|  | WhyStop | Reason for terminating the numerical search. |
|  | $\begin{aligned} & \text { Iteratio } \\ & \text { ns } \end{aligned}$ | Number of search iterations performed by the estimation algorithm. |
|  | First0rd erOptima lity | $\infty$-norm of the gradient search vector when the search algorithm terminates. |
|  | FcnCount | Number of times the objective function was called. |
|  | UpdateNo rm | Norm of the gradient search vector in the last iteration. Omitted when the search method is 'lsqnonlin' or 'fmincon'. |
|  | LastImpr ovement | Criterion improvement in the last iteration, expressed as a percentage. Omitted when the search method is 'lsqnonlin' or 'fmincon'. |
|  | $\begin{aligned} & \text { Algorith } \\ & \mathrm{m} \end{aligned}$ | Algorithm used by ' lsqnonlin' or 'fmincon' search method. Omitted when other search methods are used. |
|  | For estimation methods that do not require numerical search optimization, the Termination field is omitted. |  |

For more information, see "Estimation Report".

## Extended Capabilities

## Automatic Parallel Support

Accelerate code by automatically running computation in parallel using Parallel Computing Toolbox ${ }^{\mathrm{TM}}$.
Parallel computing support is available for estimation using the lsqnonlin search method (requires
Optimization Toolbox). To enable parallel computing, use nlhwOptions, set SearchMethod to
'lsqnonlin', and set Search0ptions.Advanced. UseParallel to true.
For example:
opt = nlhwOptions;
opt.SearchMethod = 'lsqnonlin';
opt.SearchOptions.Advanced.UseParallel = true;

## See Also

aic | customnet | deadzone | fpe | goodnessOfFit |idnlhw|idnlhw/findop|init|linapp| linearize|n4sid|nlhwOptions|oe|pem|polyld|pwlinear|saturation|sigmoidnet|
tfest|unitgain | wavenet

## Topics

"Estimate Multiple Hammerstein-Wiener Models"
"Estimate Hammerstein-Wiener Models Initialized Using Linear OE Models"
"Identifying Hammerstein-Wiener Models"
"Initialize Hammerstein-Wiener Estimation Using Linear Model"
"Loss Function and Model Quality Metrics"
"Regularized Estimates of Model Parameters"
"Estimation Report"

Introduced in R2007a

## nlhwOptions

Option set for nlhw

## Syntax

```
opt = nlhwOptions
opt = nlhwOptions(Name,Value)
```


## Description

opt $=n l h w 0 p t i o n s$ creates the default option set for $n l h w$. Use dot notation to customize the option set, if needed.
opt $=n l h w O p t i o n s($ Name, Value) creates an option set with options specified by one or more Name, Value pair arguments. The options that you do not specify retain their default value.

## Examples

## Estimate Hammerstein-Wiener Model Using an Estimation Option Set

Create estimation option set for $n$ lhw to view estimation progress and to set the maximum iteration steps to 50 .

```
opt = nlhwOptions;
opt.Display = 'on';
opt.SearchOptions.MaxIterations = 50;
```

Load data and estimate the model.

```
load iddata3
sys = nlhw(z3,[4 2 1],'sigmoidnet','deadzone',opt);
```


## Specify an Option Set for Hammerstein-Wiener Model Estimation

Create an options set for $n l h w$ where:

- Initial conditions are estimated from the estimation data.
- Subspace Gauss-Newton least squares method is used for estimation.

```
opt = nlhwOptions('InitialCondition','estimate','SearchMethod','gn');
```


## Input Arguments

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, ... , NameN, ValueN.
Example: nlhwOptions('InitialCondition','estimate')

## InitialCondition - Handling of initial conditions <br> 'zero' (default)|'estimate'

Handling of initial conditions during estimation using $n l h w$, specified as the comma-separated pair consisting of InitialCondition and one of the following:

- 'zero' - The initial conditions are set to zero.
- 'estimate' - The initial conditions are treated as independent estimation parameters.


## Display - Estimation progress display setting

'off' (default)|'on'
Estimation progress display setting, specified as the comma-separated pair consisting of 'Display' and one of the following:

- 'off ' - No progress or results information is displayed.
- 'on' - Information on model structure and estimation results are displayed in a progress-viewer window.


## OutputWeight - Weighting of prediction error in multi-output estimations

'noise' (default)| positive semidefinite matrix
Weighting of prediction error in multi-output model estimations, specified as the comma-separated pair consisting of 'OutputWeight' and one of the following:

- 'noise' - Optimal weighting is automatically computed as the inverse of the estimated noise variance. This weighting minimizes $\operatorname{det}\left(\mathrm{E}^{\prime} * \mathrm{E}\right)$, where E is the matrix of prediction errors. This option is not available when using 'lsqnonlin' as a 'SearchMethod'.
- A positive semidefinite matrix, W , of size equal to the number of outputs. This weighting minimizes $\operatorname{trace}\left(E^{\prime} * E^{*} W / N\right)$, where $E$ is the matrix of prediction errors and $N$ is the number of data samples.


## Regularization - Options for regularized estimation of model parameters structure

Options for regularized estimation of model parameters, specified as the comma-separated pair consisting of 'Regularization' and a structure with fields:

| Field <br> Name | Description | Default |
| :--- | :--- | :--- |
| Lambda | Bias versus variance trade-off constant, specified as a <br> nonnegative scalar. | 0 - Indicates no <br> regularization. |


| Field <br> Name | Description | Default |
| :--- | :--- | :--- |
| R | Weighting matrix, specified as a vector of nonnegative scalars <br> or a square positive semi-definite matrix. The length must be <br> equal to the number of free parameters in the model, np. Use <br> the nparams command to determine the number of model <br> parameters. | 1 - Indicates a value of <br> eye (np). |
| Nominal | The nominal value towards which the free parameters are <br> pulled during estimation, specified as one of the following: <br> - 'zero' - Pull parameters towards zero. <br> - 'model' - Pull parameters towards pre-existing values in <br> the initial model. Use this option only when you have a well- <br> initialized idnlhw model with finite parameter values. | 'zero ' |

To specify field values in Regularization, create a default nlhwOptions set and modify the fields using dot notation. Any fields that you do not modify retain their default values.

```
opt = nlhwOptions;
opt.Regularization.Lambda = 1.2;
opt.Regularization.R = 0.5*eye(np);
```

Regularization is a technique for specifying model flexibility constraints, which reduce uncertainty in the estimated parameter values. For more information, see "Regularized Estimates of Model Parameters".

## SearchMethod - Numerical search method used for iterative parameter estimation 'auto' (default)|'gn'|'gna'|'lm'|'grad'|'Isqnonlin'|'fmincon'

Numerical search method used for iterative parameter estimation, specified as the comma-separated pair consisting of 'SearchMethod' and one of the following:

- 'auto' - A combination of the line search algorithms, 'gn', 'lm', 'gna', and 'grad' methods is tried in sequence at each iteration. The first descent direction leading to a reduction in estimation cost is used.
- 'gn' - Subspace Gauss-Newton least squares search. Singular values of the Jacobian matrix less than GnPinvConstant*eps*max(size(J))*norm(J) are discarded when computing the search direction. $J$ is the Jacobian matrix. The Hessian matrix is approximated as $J^{T} J$. If there is no improvement in this direction, the function tries the gradient direction.
- 'gna' - Adaptive subspace Gauss-Newton search. Eigenvalues less than gamma*max(sv) of the Hessian are ignored, where $s v$ contains the singular values of the Hessian. The Gauss-Newton direction is computed in the remaining subspace. gamma has the initial value InitialGnaTolerance (see Advanced in 'Search0ptions' for more information). This value is increased by the factor LMStep each time the search fails to find a lower value of the criterion in fewer than five bisections. This value is decreased by the factor $2 *$ LMStep each time a search is successful without any bisections.
- ' lm' - Levenberg-Marquardt least squares search, where the next parameter value is - pinv (H $+\mathrm{d} * \mathrm{I}) * \mathrm{grad}$ from the previous one. $H$ is the Hessian, $I$ is the identity matrix, and grad is the gradient. $d$ is a number that is increased until a lower value of the criterion is found.
- 'grad' - Steepest descent least squares search.
- 'lsqnonlin' - Trust-region-reflective algorithm of lsqnonlin. Requires Optimization Toolbox software.
- 'fmincon ' - Constrained nonlinear solvers. You can use the sequential quadratic programming (SQP) and trust-region-reflective algorithms of the fmincon solver. If you have Optimization Toolbox software, you can also use the interior-point and active-set algorithms of the fmincon solver. Specify the algorithm in the SearchOptions.Algorithm option. The fmincon algorithms may result in improved estimation results in the following scenarios:
- Constrained minimization problems when there are bounds imposed on the model parameters.
- Model structures where the loss function is a nonlinear or non smooth function of the parameters.
- Multi-output model estimation. A determinant loss function is minimized by default for multioutput model estimation. fmincon algorithms are able to minimize such loss functions directly. The other search methods such as ' lm ' and ' gn ' minimize the determinant loss function by alternately estimating the noise variance and reducing the loss value for a given noise variance value. Hence, the fmincon algorithms can offer better efficiency and accuracy for multi-output model estimations.


## SearchOptions - Option set for the search algorithm

search option set
Option set for the search algorithm, specified as the comma-separated pair consisting of 'Search0ptions ' and a search option set with fields that depend on the value of SearchMethod.

## SearchOptions Structure When SearchMethod is Specified as 'gn', 'gna', 'lm', 'grad', or 'auto'

| Field <br> Name | Description | Default |
| :--- | :--- | :--- |
| Toleran <br> ce | Minimum percentage difference between the current value of the loss <br> function and its expected improvement after the next iteration, specified as <br> a positive scalar. When the percentage of expected improvement is less <br> than Tolerance, the iterations stop. The estimate of the expected loss- <br> function improvement at the next iteration is based on the Gauss-Newton <br> vector computed for the current parameter value. | $1 \mathrm{e}-5$ |
| MaxIter <br> ations | Maximum number of iterations during loss-function minimization, specified <br> as a positive integer. The iterations stop when MaxIterations is reached <br> or another stopping criterion is satisfied, such as Tolerance. | 20 |
| Setting MaxIterations = 0 returns the result of the start-up procedure. <br> Use sys. Report. Termination. Iterations to get the actual number <br> of iterations during an estimation, where sys is an idtf model. |  |  |


| Field Name | Description |  | Default |
| :---: | :---: | :---: | :---: |
| Advance <br> d | Advanced search settings, specified as a structure with the following fields: |  |  |
|  | Field Name | Description | Default |
|  | GnPinvCons tant | Jacobian matrix singular value threshold, specified as a positive scalar. Singular values of the Jacobian matrix that are smaller than GnPinvConstant*max(size(J)*norm(J)*eps) are discarded when computing the search direction. Applicable when SearchMethod is 'gn'. | 10000 |
|  | InitialGna Tolerance | Initial value of gamma, specified as a positive scalar. Applicable when SearchMethod is 'gna'. | 0.0001 |
|  | LMStartVal ue | Starting value of search-direction length $d$ in the Levenberg-Marquardt method, specified as a positive scalar. Applicable when SearchMethod is ' 1 m '. | 0.001 |
|  | LMStep | Size of the Levenberg-Marquardt step, specified as a positive integer. The next value of the search-direction length $d$ in the Levenberg-Marquardt method is LMStep times the previous one. Applicable when SearchMethod is ' lm '. | 2 |
|  | MaxBisecti ons | Maximum number of bisections used for line search along the search direction, specified as a positive integer. | 25 |
|  | MaxFunctio nEvaluatio ns | Maximum number of calls to the model file, specified as a positive integer. Iterations stop if the number of calls to the model file exceeds this value. | Inf |
|  | MinParamet erChange | Smallest parameter update allowed per iteration, specified as a nonnegative scalar. | 0 |
|  | RelativeIm provement | Relative improvement threshold, specified as a nonnegative scalar. Iterations stop if the relative improvement of the criterion function is less than this value. | 0 |
|  | StepReduct ion | Step reduction factor, specified as a positive scalar that is greater than 1 . The suggested parameter update is reduced by the factor StepReduction after each try. This reduction continues until MaxBisections tries are completed or a lower value of the criterion function is obtained. <br> StepReduction is not applicable for SearchMethod ' lm' (Levenberg-Marquardt method). | 2 |

SearchOptions Structure When SearchMethod is Specified as ' Isqnonlin'

| Field Name | Description | Default |
| :---: | :---: | :---: |
| Function Toleranc e | Termination tolerance on the loss function that the software minimizes to determine the estimated parameter values, specified as a positive scalar. <br> The value of FunctionTolerance is the same as that of opt.SearchOptions.Advanced.TolFun. | 1e-5 |
| StepTole rance | Termination tolerance on the estimated parameter values, specified as a positive scalar. <br> The value of StepTolerance is the same as that of opt. SearchOptions.Advanced.TolX. | 1e-6 |
| MaxItera tions | Maximum number of iterations during loss-function minimization, specified as a positive integer. The iterations stop when MaxIterations is reached or another stopping criterion is satisfied, such as FunctionTolerance. <br> The value of MaxIterations is the same as that of opt. SearchOptions.Advanced.MaxIter. | 20 |
| Advanced | Advanced search settings, specified as an option set for lsqnonlin. <br> For more information, see the Optimization Options table in "Optimization Options" (Optimization Toolbox). | Use optimset('lsqnonl in') to create a default option set. |

SearchOptions Structure When SearchMethod is Specified as 'fmincon'

| Field Name | Description | Default |
| :---: | :---: | :---: |
| Algorithm | fmincon optimization algorithm, specified as one of the following: <br> - 'sqp' - Sequential quadratic programming algorithm. The algorithm satisfies bounds at all iterations, and it can recover from NaN or Inf results. It is not a large-scale algorithm. For more information, see "Large-Scale vs. MediumScale Algorithms" (Optimization Toolbox). <br> - 'trust-regionreflective' - Subspace trust-region method based on the interior-reflective Newton method. It is a largescale algorithm. <br> - 'interior-point' -Large-scale algorithm that requires Optimization Toolbox software. The algorithm satisfies bounds at all iterations, and it can recover from NaN or Inf results. <br> - 'active-set' - Requires Optimization Toolbox software. The algorithm can take large steps, which adds speed. It is not a large-scale algorithm. <br> For more information about the algorithms, see "Constrained Nonlinear Optimization Algorithms" (Optimization Toolbox) and "Choosing the Algorithm" (Optimization Toolbox). | 'sqp' |


| Field Name | Description | Default |
| :--- | :--- | :--- |
| FunctionTolerance | Termination tolerance on the <br> loss function that the software <br> minimizes to determine the <br> estimated parameter values, <br> specified as a positive scalar. | $1 \mathrm{e}-6$ |
| StepTolerance | Termination tolerance on the <br> estimated parameter values, <br> specified as a positive scalar. | $1 \mathrm{e}-6$ |
| MaxIterations | Maximum number of iterations <br> during loss function <br> minimization, specified as a <br> positive integer. The iterations <br> stop when MaxIterations is <br> reached or another stopping <br> criterion is satisfied, such as <br> FunctionTolerance. | 100 |

To specify field values in SearchOptions, create a default nlhwOptions set and modify the fields using dot notation. Any fields that you do not modify retain their default values.

```
opt = nlhwOptions;
opt.SearchOptions.MaxIterations = 50;
opt.SearchOptions.Advanced.RelImprovement = 0.5;
```

Advanced - Additional advanced options
structure
Additional advanced options, specified as the comma-separated pair consisting of 'Advanced ' and a structure with fields:

| Field Name | Description | Default |
| :--- | :--- | :--- |
| ErrorThres <br> hold | Threshold for when to adjust the weight of large errors from <br> quadratic to linear, specified as a nonnegative scalar. Errors <br> larger than ErrorThreshold times the estimated standard <br> deviation have a linear weight in the loss function. The <br> standard deviation is estimated robustly as the median of the <br> absolute deviations from the median of the prediction errors, <br> divided by 0.7. If your estimation data contains outliers, try <br> setting ErrorThreshold to 1.6. | quadratic loss function. |
| MaxSize | Maximum number of elements in a segment when input- <br> output data is split into segments, specified as a positive <br> integer. | 250000 |

To specify field values in Advanced, create a default nlhwOptions set and modify the fields using dot notation. Any fields that you do not modify retain their default values.

```
opt = nlhwOptions;
opt.Advanced.ErrorThreshold = 1.2;
```


## Output Arguments

opt - Option set for nlhw
nlhwOptions option set
Option set for nlhw , returned as an nlhwOptions option set.

## Compatibility Considerations

## Renaming of Estimation and Analysis Options

The names of some estimation and analysis options were changed in R2018a. Prior names still work. For details, see the R2018a release note "Renaming of Estimation and Analysis Options".

## See Also

nlhw

Introduced in R2015a

## noise2meas

Noise component of model

## Syntax

noise_model = noise2meas(sys)
noise_model $=$ noise2meas(sys,noise)

## Description

noise_model = noise2meas(sys) returns the noise component, noise_model, of a linear identified model, sys. Use noise2meas to convert a time-series model (no inputs) to an input/output model. The converted model can be used for linear analysis, including viewing pole/zero maps, and plotting the step response.
noise_model = noise2meas(sys,noise) specifies the noise variance normalization method.

## Input Arguments

## sys

Identified linear model.
noise
Noise variance normalization method, specified as one of the following values:

- 'innovations ' - Noise sources are not normalized and remain as the innovations process.
- 'normalize' - Noise sources are normalized to be independent and of unit variance.

Default: 'innovations'

## Output Arguments

## noise_model

Noise component of sys.
sys represents the system

$$
y(t)=G u(t)+H e(t)
$$

$G$ is the transfer function between the measured input, $u(t)$, and the output, $y(t) . H$ is the noise model and describes the effect of the disturbance, $e(t)$, on the model's response.

An equivalent state-space representation of sys is

$$
\begin{aligned}
& \dot{x}(t)=A x(t)+B u(t)+K e(t) \\
& y(t)=C x(t)+D u(t)+e(t) \\
& e(t)=L v(t)
\end{aligned}
$$

$v(t)$ is white noise with independent channels and unit variances. The white-noise signal $e(t)$ represents the model's innovations and has variance $L L^{T}$. The noise-variance data is stored using the NoiseVariance property of sys.

- If noise is 'innovations', then noise2meas returns $H$ and noise_model represents the system

$$
y(t)=H e(t)
$$

An equivalent state-space representation of noise_model is

$$
\begin{aligned}
& \dot{x}(t)=A x(t)+K e(t) \\
& y(t)=C x(t)+e(t)
\end{aligned}
$$

noise2meas returns the noise channels of sys as the input channels of noise_model. The input channels are named using the format 'e@yk', where yk corresponds to the OutputName property of an output. The measured input channels of sys are discarded and the noise variance is set to zero.

- If noise is 'normalize', then noise2meas first normalizes

$$
e(t)=L v(t)
$$

noise_model represents the system

$$
y(t)=H L v(t)
$$

or, equivalently, in state-space representation

$$
\begin{aligned}
& \dot{x}(t)=A x(t)+K L v(t) \\
& y(t)=C x(t)+L v(t)
\end{aligned}
$$

The input channels are named using the format 'v@yk', where yk corresponds to the OutputName property of an output.

The model type of noise_model depends on the model type of sys.

- noise_model is an idtf model if sys is an idproc model.
- noise_model is an idss model if sys is an idgrey model.
- noise_model is the same type of model as sys for all other model types.

To obtain the model coefficients of noise_model in state-space form, use ssdata. Similarly, to obtain the model coefficients in transfer-function form, use tfdata.

## Examples

## Convert Noise Component of Linear Identified Model into Input/Output Model

Convert a time-series model to an input/output model that may be used by linear analysis tools.
Identify a time-series model.

```
load iddata9 z9
sys = ar(z9,4,'ls');
```

sys is an idpoly model with no inputs.
Convert sys to a measured model.
noise_model $=$ noise2meas(sys);
noise_model is an idpoly model with one input.
You can use noise_model for linear analysis functions such as step, iopzmap, etc.

## Normalizing Noise Variance

Convert an identified linear model to an input/output model, and normalize its noise variance.
Identify a linear model using data.

```
load twotankdata;
z = iddata(y,u,0.2);
sys = ssest(z,4);
```

sys is an idss model, with a noise variance of $6.6211 \mathrm{e}-06$. The value of $L$ is sqrt (sys. NoiseVariance), which is 0.0026 .

View the disturbance matrix.

```
sys.K
ans = 4×1
    0.2719
    1.6570
    0.6318
    0.2877
```

Obtain a model that absorbs the noise variance of sys.

```
noise_model_normalize = noise2meas(sys,'normalize');
noise_model_normalize is an idpoly model.
View the B matrix for noise_model_normalize
noise_model_normalize.B
ans = 4×1
    0.0007
    0.0043
    0.0016
    0.0007
```

As expected, noise_model_normalize.B is equal to L*sys.K.
Compare the bode response with a model that ignores the noise variance of sys.
noise_model_innovation = noise2meas(sys,'innovations');
bodemag(noise_model_normalize, noise_model_innovation);
legend('Normā̄ized noise variance','Ignorè noise variance');

## Bode Diagram



The difference between the bode magnitudes of the noise_model_innovation and noise_model_normalized is approximately 51 dB . As expected, the magnitude difference is approximately equal to $20 * \log 10(\mathrm{~L})$.

## See Also

idssdata|noisecnv|spectrum|tfdata|zpkdata

## Introduced in R2012a

## noisecnv

Transform identified linear model with noise channels to model with measured channels only

## Syntax

$\bmod 1=$ noisecnv(mod)
$\bmod 2=$ noisecnv(mod,'normalize')

## Description

mod1 $=$ noisecnv(mod) and mod2 = noisecnv(mod,'normalize') transform an identified linear model with noise channels to a model with measured channels only.
mod is any linear identified model, idproc, idtf, idgrey, idpoly, or idss.
The noise input channels in mod are converted as follows: Consider a model with both measured input channels $u$ ( $n u$ channels) and noise channels $e$ (ny channels) with covariance matrix $\Lambda$ :

$$
\begin{aligned}
& y=G u+H e \\
& \operatorname{cov}(e)=\Lambda=L L^{\prime}
\end{aligned}
$$

where $L$ is a lower triangular matrix. Note that mod. NoiseVariance $=\Lambda$. The model can also be described with unit variance, using a normalized noise source $v$ :

$$
\begin{aligned}
& y=G u+H L v \\
& \operatorname{cov}(v)=I
\end{aligned}
$$

- mod1 = noisecnv(mod) converts the model to a representation of the system [GH] with nu+ny inputs and ny outputs. All inputs are treated as measured, and mod1 does not have any noise model. The former noise input channels have names e@yname, where yname is the name of the corresponding output.
- mod2 = noisecnv(mod, 'norm') converts the model to a representation of the system [G HL] with nu $n$ ny inputs and ny outputs. All inputs are treated as measured, and mod 2 does not have any noise model. The former noise input channels have names v@yname, where yname is the name of the corresponding output. Note that the noise variance matrix factor $L$ typically is uncertain (has a nonzero covariance). This is taken into account in the uncertainty description of mod2.
- If mod is a time series, that is, $n u=0, \bmod 1$ is a model that describes the transfer function $H$ with measured input channels. Analogously, mod2 describes the transfer function HL.

Note the difference with subreferencing:

- $\bmod (:,[])$ gives a description of the noise model characteristics as a time-series model, that is, it describes $H$ and also the covariance of $e$. In contrast, noisecnv (m(:, [])) or noise2meas (m) describe just the transfer function $H$. To obtain a description of the normalized transfer function HL, use noisecnv(m(:, []),'normalize') or noise2meas('normalize').

Converting the noise channels to measured inputs is useful to study the properties of the individual transfer functions from noise to output. It is also useful for transforming identified linear models to representations that do not handle disturbance descriptions explicitly.

## Examples

Identify a model with a measured component ( $G$ ) and a non-trivial noise component ( $H$ ). Compare the amplitude of the measured component's frequency response to the noise component's spectrum amplitude. You must convert the noise component into a measured one by using noisecnv if you want to compare its behavior against a truly measured component.

```
load iddata2 z2
sys1 = armax(z2,[2 2 2 1]); % model with noise component
sys2 = tfest(z2,3); % model with a trivial noise component
sys1 = noisecnv(sys1);
sys2 = noisecnv(sys2);
bodemag(sys1,sys2)
```


## See Also

idssdata|noise2meas |tfdata|zpkdata

## Topics

"Treating Noise Channels as Measured Inputs"
Introduced before R2006a

## norm

Norm of linear model

## Syntax

```
n = norm(sys)
n = norm(sys,2)
n = norm(sys,Inf)
[n,fpeak] = norm(sys,Inf)
[n,fpeak] = norm(sys,Inf,tol)
```


## Description

$n=$ norm(sys) or $n=$ norm(sys,2) returns the root-mean-squares of the impulse response of the linear dynamic system model sys. This value is equivalent to the $H_{2}$ norm on page 1-939 of sys.
$\mathrm{n}=$ norm (sys, Inf) returns the $L_{\infty}$ norm (Control System Toolbox) of sys, which is the peak gain of the frequency response of sys across frequencies. For MIMO systems, this quantity is the peak gain over all frequencies and all input directions, which corresponds to the peak value of the largest singular value of sys. For stable systems, the $L_{\infty}$ norm is equivalent to the $H_{\infty}$ norm. For more information, see hinfnorm.
[n,fpeak] $=$ norm(sys,Inf) also returns the frequency fpeak at which the gain reaches its peak value.
[ $\mathrm{n}, \mathrm{fpeak}$ ] $=$ norm(sys,Inf, tol) sets the relative accuracy of the $L_{\infty}$ norm to tol.
This command requires a Control System Toolbox license.

## Examples

## Compute Norm of Discrete-Time Linear System

Compute the $\mathrm{H}_{2}$ and $L_{\infty}$ norms of the following discrete-time transfer function, with sample time 0.1 second.

$$
\operatorname{sys}(z)=\frac{z^{3}-2.841 z^{2}+2.875 z-1.004}{z^{3}-2.417 z^{2}+2.003 z-0.5488}
$$

Compute the $\mathrm{H}_{2}$ norm of the transfer function. The $\mathrm{H}_{2}$ norm is the root-mean-square of the impulse response of sys.

```
sys = tf([1 -2.841 2.875 -1.004],[1 -2.417 2.003 -0.5488],0.1);
n2 = norm(sys)
n2 = 1.2438
```

Compute the $L_{\infty}$ norm of the transfer function.

```
[ninf,fpeak] = norm(sys,Inf)
ninf = 2.5721
fpeak = 3.0178
```

Because sys is a stable system, ninf is the peak gain of the frequency response of sys, and fpeak is the frequency at which the peak gain occurs. Confirm these values using getPeakGain.

```
[gpeak,fpeak] = getPeakGain(sys)
gpeak = 2.5721
fpeak = 3.0178
```


## Input Arguments

## sys - Dynamic system

dynamic system model | model array
Input dynamic system, specified as any SISO or MIMO linear dynamic system model or model array. sys can be continuous-time or discrete-time.

## tol - Relative accuracy

0.01 (default) | positive real scalar

Relative accuracy of the $H_{\infty}$ norm, specified as a positive real scalar value.

## Output Arguments

n $-H_{2}$ or $L_{\infty}$ norm
scalar | array
$H_{2}$ norm or $L_{\infty}$ norm of sys, returned as a scalar or an array.

- If sys is a single model, then n is a scalar value.
- If sys is a model array, then $n$ is an array of the same size as sys, where $n(k)=$ norm(sys(:,: k)).


## fpeak - Frequency of peak gain

nonnegative real scalar | array of nonnegative real values
Frequency at which the gain achieves the peak value gpeak, returned as a nonnegative real scalar value or an array of nonnegative real values. The frequency is expressed in units of rad/TimeUnit, relative to the TimeUnit property of sys.

- If sys is a single model, then fpeak is a scalar.
- If sys is a model array, then fpeak is an array of the same size as sys, where fpeak( $k$ ) is the peak gain frequency of sys (: : : , k).


## More About

## H2 norm

The $H_{2}$ norm of a stable system $H$ is the root-mean-square of the impulse response of the system. The $H_{2}$ norm measures the steady-state covariance (or power) of the output response $y=H w$ to unit white noise inputs $w$ :

$$
\|H\|_{2}^{2}=\lim _{t \rightarrow \infty} E\left\{y(t)^{T} y(t)\right\}, \quad E\left(w(t) w(\tau)^{T}\right)=\delta(t-\tau) I
$$

The $H_{2}$ norm of a continuous-time system with transfer function $H(s)$ is given by:

$$
\|H\|_{2}=\sqrt{\frac{1}{2 \Pi} \int_{-\infty}^{\infty} \operatorname{Trace}\left[H(j \omega)^{H} H(j \omega)\right] d \omega}
$$

For a discrete-time system with transfer function $H(z)$, the $H_{2}$ norm is given by:

$$
\|H\|_{2}=\sqrt{\frac{1}{2 \Pi} \int_{-\Pi}^{\Pi} \operatorname{Trace}\left[H\left(e^{j \omega}\right)^{H} H\left(e^{j \omega}\right)\right] d \omega} .
$$

The $\mathrm{H}_{2}$ norm is infinite in the following cases:

- sys is unstable.
- sys is continuous and has a nonzero feedthrough (that is, nonzero gain at the frequency $\omega=\infty$ ).

Using norm(sys) produces the same result as sqrt(trace(covar(sys,1))).

## L-infinity norm

The $L_{\infty}$ norm of a SISO linear system is the peak gain of the frequency response. For a MIMO system, the $L_{\infty}$ norm is the peak gain across all input/output channels.

For a continuous-time system $H(s)$, this definition means:

$$
\begin{aligned}
& \|H(s)\|_{L_{\infty}}=\max _{\omega \in R}|H(j \omega)| \\
& \|H(s)\|_{L_{\infty}}=\max _{\omega \in R} \sigma_{\max }(H(j \omega))
\end{aligned}
$$

where $\sigma_{\max }(\cdot)$ denotes the largest singular value of a matrix.
For a discrete-time system $H(z)$, the definition means:

$$
\begin{aligned}
& \|H(z)\|_{L_{\infty}}=\max _{\theta \in[0,2 \pi]}\left|H\left(e^{j \theta}\right)\right| \\
& \|H(z)\|_{L_{\infty}}=\max _{\theta \in[0,2 \pi]} \sigma_{\max }\left(H\left(e^{j \theta}\right)\right)
\end{aligned}
$$

For stable systems, the $L_{\infty}$ norm is equivalent to the $H_{\infty}$ norm. For more information, see hinfnorm. For a system with unstable poles, the $H_{\infty}$ norm is infinite. For all systems, norm returns the $L_{\infty}$ norm, which is the peak gain without regard to system stability.

## Algorithms

After converting sys to a state space model, norm uses the same algorithm as covar for the $H_{2}$ norm. For the $L_{\infty}$ norm, norm uses the algorithm of [1]. norm computes the peak gain using the SLICOT library. For more information about the SLICOT library, see http://slicot.org.

## References

[1] Bruisma, N.A. and M. Steinbuch, "A Fast Algorithm to Compute the $\mathrm{H}_{\omega}$-Norm of a Transfer Function Matrix," System Control Letters, 14 (1990), pp. 287-293.

## See Also

freqresp|getPeakGain| hinfnorm| sigma

Introduced before R2006a

## nparams

Number of model parameters

## Syntax

np = nparams(sys)
$n p=n p a r a m s(s y s, ' f r e e ')$

## Description

$\mathrm{np}=$ nparams(sys) returns the number of parameters in the identified model sys.
$n p=$ nparams(sys,'free') returns the number free estimation parameters in the identified model sys.

Note Not all model coefficients are parameters, such as the leading entry of the denominator polynomials in idpoly and idtf models.

## Input Arguments

## sys

Identified linear model.

## Output Arguments

## np

Number of parameters of sys.
For the syntax $n p=n p a r a m s(s y s, ' f r e e '), n p$ is the number of free estimation parameters of sys.
idgrey models can contain non-scalar parameters. nparams accounts for each individual entry of the non-scalar parameters in the total parameter count.

## Examples

Obtain the number of parameters of a transfer function model.

```
sys = idtf(1,[1 2]);
np = nparams(sys);
```

Obtain the number of free estimation parameters of a transfer function model.

```
sys0 = idtf([1 0],[1 2 0]);
sys0.Structure.Denominator.Free(3) = false;
np = nparams(sys,'free');
```


## See Also

idfrd|idgrey|idpoly|idproc|idss|idtf|size
Introduced in R2012a

## nuderst

Set step size for numerical differentiation

## Syntax

nds = nuderst(pars)

## Description

Many estimation functions use numerical differentiation with respect to the model parameters to compute their values.

The step size used in these numerical derivatives is determined by the nuderst command. The output argument nds is a row vector whose kth entry gives the increment to be used when differentiating with respect to the kth element of the parameter vector pars.

The default version of nuderst uses a very simple method. The step size is the maximum of $10^{-4}$ times the absolute value of the current parameter and $10^{-7}$. You can adjust this to the actual value of the corresponding parameter by editing nuderst. Note that the nominal value, for example 0 , of a parameter might not reflect its normal size.

Introduced before R2006a

## nyquist

Nyquist plot of frequency response

## Syntax

```
nyquist(sys)
nyquist(sys,w)
nyquist(sys1,sys2,...,sysN)
nyquist(sys1,sys2,...,sysN,w)
nyquist(sys1,'PlotStyle1',...,sysN,'PlotStyleN')
[re,im,w] = nyquist(sys)
[re,im] = nyquist(sys,w)
[re,im,w,sdre,sdim] = nyquist(sys)
```


## Description

nyquist creates a Nyquist plot of the frequency response of a dynamic system model. When invoked without left-hand arguments, nyquist produces a Nyquist plot on the screen. Nyquist plots are used to analyze system properties including gain margin, phase margin, and stability.
nyquist (sys) creates a Nyquist plot of a dynamic system sys. This model can be continuous or discrete, and SISO or MIMO. In the MIMO case, nyquist produces an array of Nyquist plots, each plot showing the response of one particular I/O channel. The frequency points are chosen automatically based on the system poles and zeros.
nyquist (sys,w) explicitly specifies the frequency range or frequency points to be used for the plot. To focus on a particular frequency interval, set $w=\{w m i n, w m a x\}$. To use particular frequency points, set $w$ to the vector of desired frequencies. Use logspace to generate logarithmically spaced frequency vectors. Frequencies must be in rad/TimeUnit, where TimeUnit is the time units of the input dynamic system, specified in the TimeUnit property of sys.
nyquist(sys1,sys2,...,sysN) or nyquist(sys1,sys2, ...,sysN,w) superimposes the Nyquist plots of several LTI models on a single figure. All systems must have the same number of inputs and outputs, but may otherwise be a mix of continuous- and discrete-time systems. You can also specify a distinctive color, linestyle, and/or marker for each system plot with the syntax nyquist(sys1,'PlotStyle1',..., sysN, 'PlotStyleN').
[re,im,w] = nyquist(sys) and [re,im] = nyquist(sys,w) return the real and imaginary parts of the frequency response at the frequencies $w$ (in rad/TimeUnit). re and im are 3-D arrays (see "Arguments" below for details).
[re,im,w,sdre,sdim] = nyquist(sys) also returns the standard deviations of re and im for the identified system sys.

## Arguments

The output arguments re and im are 3-D arrays with dimensions
(number of outputs) $\times$ (number of inputs) $\times$ (length of w)

For SISO systems, the scalars $\mathrm{re}(1,1, k)$ and $\mathrm{im}(1,1, k)$ are the real and imaginary parts of the response at the frequency $\omega_{k}=\mathrm{w}(\mathrm{k})$.

$$
\begin{aligned}
& \operatorname{re}(1,1, k)=\operatorname{Re}\left(h\left(j \omega_{k}\right)\right) \\
& \operatorname{im}(1,1, k)=\operatorname{Im}\left(h\left(j w_{k}\right)\right)
\end{aligned}
$$

For MIMO systems with transfer function $H(s)$, re (: , : , k) and im (: , : , k) give the real and imaginary parts of $H\left(j \omega_{k}\right)$ (both arrays with as many rows as outputs and as many columns as inputs). Thus,

$$
\begin{aligned}
& \operatorname{re}(\mathrm{i}, \mathrm{j}, \mathrm{k})=\operatorname{Re}\left(h_{i j}\left(j \omega_{k}\right)\right) \\
& \operatorname{im}(\mathrm{i}, \mathrm{j}, \mathrm{k})=\operatorname{Im}\left(h_{i j}\left(j \omega_{k}\right)\right)
\end{aligned}
$$

where $h_{i j}$ is the transfer function from input $j$ to output $i$.

## Examples

## Nyquist Plot of Dynamic System

Create the following transfer function and plot its Nyquist response.
$H(s)=\frac{2 s^{2}+5 s+1}{s^{2}+2 s+3}$.
$H=t f\left(\left[\begin{array}{lll}2 & 5 & 1\end{array}\right],\left[\begin{array}{lll}1 & 2 & 3\end{array}\right]\right) ;$
nyquist(H)


The nyquist function can display a grid of $M$-circles, which are the contours of constant closed-loop magnitude. $M$-circles are defined as the locus of complex numbers where the following quantity is a constant value across frequency.
$T(j \omega)=\left|\frac{G(j \omega)}{1+G(j \omega)}\right|$.
Here, $\omega$ is the frequency in radians/TimeUnit, where TimeUnit is the system time units, and $G$ is the collection of complex numbers that satisfy the constant magnitude requirement.

To display the grid of $M$-circles, right-click in the plot and select Grid. Alternatively, use the grid command.
grid on


## Create Nyquist Plot of Identified Model With Response Uncertainty

Compute the standard deviations of the real and imaginary parts of the frequency response of an identified model. Use this data to create a $3 \sigma$ plot of the response uncertainty.

Load estimation data z2.
load iddata2 z2;
Identify a transfer function model using the data.
sys_p = tfest(z2,2);
Obtain the standard deviations for the real and imaginary parts of the frequency response for a set of 512 frequencies, $w$.
w = linspace(-10*pi,10*pi,512);
[re,im,wout,sdre,sdim] = nyquist(sys_p,w);
Here re and im are the real and imaginary parts of the frequency response, and sdre and sdim are their standard deviations, respectively. The frequencies in wout are the same as the frequencies you specified in $w$.

Create a Nyquist plot showing the response and its $3 \sigma$ uncertainty.

```
re = squeeze(re);
im = squeeze(im);
sdre = squeeze(sdre);
sdim = squeeze(sdim);
plot(re,im,'b',re+3*sdre,im+3*sdim,'k:',re-3*sdre,im-3*sdim,'k:')
xlabel('Real Axis');
ylabel('Imaginary Axis');
```



## Tips

- You can change the properties of your plot, for example the units. For information on the ways to change properties of your plots, see "Ways to Customize Plots" (Control System Toolbox). For the most flexibility in customizing plot properties, use the nyquistplot command instead of nyquist.
- There are two zoom options available from the right-click menu that apply specifically to Nyquist plots:
- Full View - Clips unbounded branches of the Nyquist plot, but still includes the critical point (-1, 0 ).
- Zoom on (-1,0) - Zooms around the critical point ( $-1,0$ ). (To access critical-point zoom programmatically, use nyquistplot instead.)
- To activate data markers that display the real and imaginary values at a given frequency, click anywhere on the curve. The following figure shows a nyquist plot with a data marker.



## Algorithms

See bode.

## See Also

Linear System Analyzer | bode | evalfr \| freqresp|nichols \| sigma
Introduced before R2006a

## nyquistoptions

List of Nyquist plot options

## Syntax

P = nyquistoptions
P = nyquistoptions('cstprefs')

## Description

$\mathrm{P}=$ nyquistoptions returns the default options for Nyquist plots. You can use these options to customize the Nyquist plot appearance using the command line.
$P=$ nyquistoptions('cstprefs') initializes the plot options with the options you selected in the Control System and System Identification Toolbox Preferences Editor. For more information about the editor, see "Toolbox Preferences Editor".

The following table summarizes the Nyquist plot options.

| Option | Description |
| :---: | :---: |
| Title, XLabel, YLabel | Label text and style |
| TickLabel | Tick label style |
| Grid | Show or hide the grid <br> Specified as one of the following values: 'off' \| 'on' <br> Default: 'off' |
| GridColor | Color of the grid lines <br> Specified as one of the following: Vector of RGB values in the range [0, 1] \| <br> character vector of color name \| ' none '. For example, for yellow color, specify as one of the following: $\left.\begin{array}{lll}1 & 1 & 0\end{array}\right]$, 'yellow', or ' $y$ '. <br> Default: [0.15, 0.15, 0.15] |
| XlimMode, YlimMode | Limit modes |
| Xlim, Ylim | Axes limits |
| IOGrouping | Grouping of input-output pairs <br> Specified as one of the following values: 'none'\|'inputs'|'outputs'|'all' <br> Default: ' none' |
| InputLabels, OutputLabels | Input and output label styles |
| InputVisible, OutputVisible | Visibility of input and output channels |


| Option | Description |
| :---: | :---: |
| FreqUnits | Frequency units, specified as one of the following values: <br> - 'Hz' <br> - 'rad/second' <br> - 'rpm' <br> - 'kHz' <br> - 'MHz' <br> - 'GHz' <br> - 'rad/nanosecond' <br> - 'rad/microsecond' <br> - 'rad/millisecond' <br> - 'rad/minute' <br> - 'rad/hour' <br> - 'rad/day' <br> - 'rad/week' <br> - 'rad/month' <br> - 'rad/year' <br> - 'cycles/nanosecond' <br> - 'cycles/microsecond' <br> - 'cycles/millisecond' <br> - 'cycles/hour' <br> - 'cycles/day' <br> - 'cycles/week' <br> - 'cycles/month' <br> - 'cycles/year' <br> Default: ' rad/s' <br> You can also specify ' auto' which uses frequency units rad/TimeUnit relative to system time units specified in the TimeUnit property. For multiple systems with different time units, the units of the first system are used. |
| MagUnits | Magnitude units <br> Specified as one of the following values: ' dB ' \| 'abs ' <br> Default: ' dB ' |
| PhaseUnits | Phase units <br> Specified as one of the following values: 'deg ' \| 'rad' <br> Default: 'deg' |
| ShowFullContour | Show response for negative frequencies Specified as one of the following values: 'on ' \| 'off' Default: ' on ' |


| Option | Description |
| :--- | :--- |
| ConfidenceRegionNumberS <br> D | Number of standard deviations to use to plotting the response confidence region <br> (identified models only). <br> Default: 1. |
| ConfidenceRegionDisplaySp <br> acing | The frequency spacing of confidence ellipses. For identified models only. <br> Default: 5, which means the confidence ellipses are shown at every 5th <br> frequency sample. |

## Examples

This example shows how to create a Nyquist plot displaying the full contour (the response for both positive and negative frequencies).

```
P = nyquistoptions;
P.ShowFullContour = 'on';
h = nyquistplot(tf(1,[1,.2,1]),P);
```


## See Also

getoptions | nyquist| nyquistplot | setoptions \| setoptions | showConfidence

Introduced in R2012a

## nyquistplot

Nyquist plot with additional plot customization options

## Syntax

```
h = nyquistplot(sys)
nyquistplot(sys,{wmin,wmax})
nyquistplot(sys,w)
nyquistplot(sys1,sys2,...,w)
nyquistplot(AX,...)
nyquistplot(..., plotoptions)
```


## Description

h = nyquistplot(sys) draws the Nyquist plot of the dynamic system model sys. It also returns the plot handle h. You can use this handle to customize the plot with the getoptions and setoptions commands. Type
help nyquistoptions
for a list of available plot options.
The frequency range and number of points are chosen automatically. See bode for details on the notion of frequency in discrete time.
nyquistplot(sys,\{wmin, wmax\}) draws the Nyquist plot for frequencies between wmin and wmax (in rad/TimeUnit, where TimeUnit is the time units of the input dynamic system, specified in the TimeUnit property of sys).
nyquistplot (sys,w) uses the user-supplied vector w of frequencies (in rad/TimeUnit, where TimeUnit is the time units of the input dynamic system, specified in the TimeUnit property of sys) at which the Nyquist response is to be evaluated. See logspace to generate logarithmically spaced frequency vectors.
nyquistplot (sys1, sys2,...,w) draws the Nyquist plots of multiple models sys1,sys2,... on a single plot. The frequency vector $w$ is optional. You can also specify a color, line style, and marker for each system, as in
nyquistplot(sys1,'r',sys2,'y--',sys3,'gx')
nyquistplot (AX,...) plots into the axes with handle AX.
nyquistplot(..., plotoptions) plots the Nyquist response with the options specified in plotoptions. Type
help nyquistoptions
for more details.

## Examples

## Customize Nyquist Plot Title

Create a Nyquist plot of a dynamic system model and store a handle to the plot.
sys $=t f(100,[1,2,1])$;
h = nyquistplot(sys);

Nyquist Diagram


Change the plot title to read "Nyquist Plot of sys." To do so, use getoptions to extract the existing plot options from the plot handle h .

```
opt = getoptions(h)
opt =
                        FreqUnits: 'rad/s'
                        MagUnits: 'dB'
            PhaseUnits: 'deg'
                ShowFullContour: 'on'
    ConfidenceRegionNumberSD: 1
ConfidenceRegionDisplaySpacing: 5
            IOGrouping: 'none'
            InputLabels: [1x1 struct]
            OutputLabels: [1x1 struct]
            InputVisible: {'on'}
            OutputVisible: {'on'}
```

```
            Title: [1x1 struct]
            XLabel: [1x1 struct]
            YLabel: [1x1 struct]
TickLabel: [1x1 struct]
            Grid: 'off'
GridColor: [0.1500 0.1500 0.1500]
            XLim: {[-20 100]}
            YLim: {[-80 80]}
XLimMode: {'auto'}
YLimMode: {'auto'}
```

The Title option is a structure with several fields.

```
opt.Title
ans = struct with fields:
            String: 'Nyquist Diagram'
        FontSize: 11
        FontWeight: 'bold'
        FontAngle: 'normal'
            Color: [0 0 0]
    Interpreter: 'tex'
```

Change the String field of the Title structure, and use setoptions to apply the change to the plot.

```
opt.Title.String = 'Nyquist Plot of sys';
```

setoptions(h,opt)


## Zoom on Critical Point

Plot the Nyquist frequency response of a dynamic system. Assign a variable name to the plot handle so that you can access it for further manipulation.
sys = tf(100,[1,2,1]);
h = nyquistplot(sys);


Zoom in on the critical point, ( $-1,0$ ). You can do so interactively by right-clicking on the plot and selecting Zoom on (-1,0). Alternatively, use the zoomcp command on the plot handle $h$.
zoomcp(h)


## Plot Identified Models with Confidence Regions at Selected Points

Compare the frequency responses of identified state-space models of order 2 and 6 along with their 1 -std confidence regions rendered at every 50th frequency sample.

```
load iddatal
sys1 = n4sid(z1,2); % discrete-time IDSS model of order 2
sys2 = n4sid(z1,6); % discrete-time IDSS model of order 6
```

Both models produce about 76\% fit to data. However, sys2 shows higher uncertainty in its frequency response, especially close to Nyquist frequency as shown by the plot. To see this, generate a Nyquist plot and show the confidence region at a subset of the points at which the Nyquist response is displayed.

```
w = linspace(10,10*pi,256);
h = nyquistplot(sys1,sys2,w);
setoptions(h,'ConfidenceRegionDisplaySpacing',50,...
    'ShowFullContour','off');
```

To turn on the confidence region display, right-click the plot and select Characteristics > Confidence Region.

## Nyquist Diagram



## Tips

- You can change the properties of your plot, for example the units. For information on the ways to change properties of your plots, see "Ways to Customize Plots" (Control System Toolbox).
- There are two zoom options available from the right-click menu that apply specifically to Nyquist plots:
- Full View - Clips unbounded branches of the Nyquist plot, but still includes the critical point $(-1,0)$.
- Zoom on (-1,0) - Zooms around the critical point ( $-1,0$ ). To access critical-point zoom programmatically, use the zoomcp command. See "Zoom on Critical Point" on page 1-956.
- To activate data markers that display the real and imaginary values at a given frequency, click anywhere on the curve. The following figure shows a nyquist plot with a data marker.



## See Also

getoptions|nyquist|setoptions|showConfidence

Introduced in R2012a

## 02

Estimate output-error polynomial model using time-domain or frequency-domain data

## Syntax

```
sys = oe(data,[nb nf nk])
sys = oe(data,[nb nf nk],Name,Value)
sys = oe(data,init_sys)
sys = oe(data,___,opt)
[sys,ic] = oe(
```

$\qquad$

``` )
```


## Description

Output-error (OE) models are a special configuration of polynomial models, having only two active polynomials- $B$ and $F$. OE models represent conventional transfer functions that relate measured inputs to outputs while also including white noise as an additive output disturbance. You can estimate OE models using time- and frequency-domain data. The tfest command offers the same functionality as oe. For tfest, you specify the model orders using number of poles and zeros rather than polynomial degrees. For continuous-time estimation, tfest provides faster and more accurate results, and is recommended.
sys = oe(data,[nb nf nk]) estimates an OE model sys, represented by

$$
y(t)=\frac{B(q)}{F(q)} u(t-n k)+e(t)
$$

$y(t)$ is the output, $u(t)$ is the input, and $e(t)$ is the error.
oe estimates sys using the measured input-output data data, which can be in the time or the frequency domain. The orders [ nb nf nk ] define the number of parameters in each component of the estimated polynomial.
sys = oe(data,[nb nf nk],Name,Value) specifies model structure attributes using additional options specified by one or more name-value pair arguments.
sys = oe(data,init_sys) uses the linear system init_sys to configure the initial parameterization of sys.
sys = oe(data,__, opt) estimates a polynomial model using the option set opt to specify estimation behavior. You can use this syntax with any of the previous input-argument combinations.
[sys,ic] = oe( ___) returns the estimated initial conditions as an initialCondition object. Use this syntax if you plan to simulate or predict the model response using the same estimation input data and then compare the response with the same estimation output data. Incorporating the initial conditions yields a better match during the first part of the simulation.

## Examples

## Estimate OE Polynomial Model

Estimate an OE polynomial from time-domain data using two methods to specify input delay.
Load the estimation data.
load iddatal z1
Set the orders of the $B$ and $F$ polynomials $n b$ and $n f$. Set the input delay $n k$ to one sample. Compute the model sys.

```
nb = 2;
nf = 2;
nk = 1;
sys = oe(z1,[nb nf nk]);
```

Compare the simulated model response with the measured output.
compare(z1,sys)


The plot shows that the fit percentage between the simulated model and the estimation data is greater than $70 \%$.

Instead of using nk, you can also use the name-value pair argument 'InputDelay ' to specify the one-sample delay.

```
nk = 0;
sys1 = oe(z1,[nb nf nk],'InputDelay',1);
```

figure
compare(z1,sys1)


The results are identical.
You can view more information about the estimation by exploring the idpoly property sys.Report.

```
sys.Report
ans =
        Status: 'Estimated using OE'
        Method: 'OE'
    InitialCondition: 'zero'
            Fit: [1x1 struct]
        Parameters: [1x1 struct]
        OptionsUsed: [1x1 idoptions.polyest]
            RandState: [1x1 struct]
            DataUsed: [1x1 struct]
            Termination: [1x1 struct]
```

For example, find out more information about the termination conditions.

```
sys.Report.Termination
ans = struct with fields:
    WhyStop: 'Near (local) minimum, (norm(g) < tol).'
        Iterations: 3
```

```
FirstOrderOptimality: 0.0708
            FcnCount: 7
    UpdateNorm: 1.4809e-05
LastImprovement: 5.1744e-06
```

The report includes information on the number of iterations and the reason the estimation stopped iterating.

## Estimate Continuous-Time OE Model Using Frequency Response

Load the estimation data.
load oe_datal data;
The idfrd object data contains the continuous-time frequency response for the following model:

$$
G(s)=\frac{s+3}{s^{3}+2 s^{2}+s+1}
$$

Estimate the model.
$\mathrm{nb}=2$;
$n f=3$;
sys = oe(data,[nb nf]);
Evaluate the goodness of fit.
compare(data,sys);


## Estimate OE Model Using Regularization

Estimate a high-order OE model from data collected by simulating a high-order system. Determine the regularization constants by trial and error and use the values for model estimation.

Load the data.
load regularizationExampleData.mat m0simdata
Estimate an unregularized OE model of order 30.
m1 = oe(m0simdata,[30 30 1]);
Obtain a regularized OE model by determining the Lambda value using trial and error.

```
opt = oeOptions;
```

opt. Regularization.Lambda = 1;
m2 = oe(m0simdata,[30 30 1],opt);

Compare the model outputs with the estimation data.

```
opt = compareOptions('InitialCondition','z');
compare(m0simdata,m1,m2,opt);
```

Simulated Response Comparison


The regularized model m 2 produces a better fit than the unregularized model m 1 .
Compare the variance in the model responses.

```
h = bodeplot(m1,m2);
opt = getoptions(h);
opt.PhaseMatching = 'on';
opt.ConfidenceRegionNumberSD = 3;
opt.PhaseMatching = 'on';
setoptions(h,opt);
showConfidence(h);
```



The regularized model m 2 has a reduced variance compared to the unregularized model m 1 .

## Estimate Continuous Model Using Band-Limited Discrete-Time Frequency-Domain Data

Load the estimation data data and sample time Ts.
load oe_data2.mat data Ts
An iddata object data contains the discrete-time frequency response for the following model:

$$
G(s)=\frac{1000}{s+500}
$$

View the estimation sample time Ts that you loaded.
Ts
Ts $=1.0000 \mathrm{e}-03$
This value matches the property data.Ts.
data.Ts
ans $=1.0000 \mathrm{e}-03$

You can estimate a continuous model from data by limiting the input and output frequency bands to the Nyquist frequency. To do so, specify the estimation prefilter option 'WeightingFilter' to define a passband from 0 to $0.5^{*} \mathrm{pi} / \mathrm{Ts} \mathrm{rad} / \mathrm{s}$. The software ignores any response values with frequencies outside of that passband.

```
opt = oeOptions('WeightingFilter',[0 0.5*pi/Ts]);
```

Set the Ts property to 0 to treat data as continuous-time data.
data.Ts = 0;
Estimate the continuous model.

```
nb = 1;
nf = 3;
sys = oe(data,[nb nf],opt);
```


## Obtain Initial Conditions

Load the data.

```
load iddatalic zli
```

Estimate an OE polynomial model sys and return the initial conditions in ic.

```
nb = 2;
nf = 2;
nk = 1;
[sys,ic] = oe(zli,[nb,nf,nk]);
ic
ic =
    initialCondition with properties:
        A: [2x2 double]
        X0: [2x1 double]
        C: [0.9428 0.4824]
        Ts: 0.1000
```

ic is an initialCondition object that encapsulates the free response of sys, in state-space form, to the initial state vector in X0. You can incorporate ic when you simulate sys with the z1i input signal and compare the response with the zli output signal.

## Input Arguments

## data - Estimation data

iddata object | frd object | idfrd object
Estimation data, specified as an iddata object, an frd object, or an idfrd object.
For time-domain estimation, data must be an iddata object containing the input and output signal values.

For frequency-domain estimation, data can be one of the following:

- Recorded frequency response data (frd or idfrd)
- iddata object with properties specified as follows:
- InputData - Fourier transform of the input signal
- OutputData - Fourier transform of the output signal
- Domain - 'Frequency'

Time-domain estimation data must be uniformly sampled. By default, the software sets the sample time of the model to the sample time of the estimation data.

For multiexperiment data, the sample times and intersample behavior of all the experiments must match.

You can compute discrete-time models from time-domain data or discrete-time frequency-domain data. Use tfest to compute continuous-time models.

## [nb nf nk] - OE model orders

integer row vector | row vector of integer matrices
OE model orders, specified as a 1-by-3 vector or a vector of integer matrices.
For a system represented by

$$
y(t)=\frac{B(q)}{F(q)} u(t-n k)+e(t)
$$

where $y(t)$ is the output, $u(t)$ is the input, and $e(t)$ is the error, the elements of [ nb nf nk ] are as follows:

- nb - Order of the $B(q)$ polynomial +1 , which is equivalent to the length of the $B(q)$ polynomial. nb is an $N_{y}$-by- $N_{u}$ matrix. $N_{y}$ is the number of outputs and $N_{u}$ is the number of inputs.
- nf - Order of the $F$ polynomial. nf is an $N_{y}$-by- $N_{u}$ matrix.
- nk - Input delay, expressed as the number of samples. nk is an $N_{y}$-by- $N_{u}$ matrix. The delay appears as leading zeros of the $B$ polynomial.

For estimation using continuous-time frequency-domain data, specify only [ $n b \mathrm{nf}$ ] and omit nk. For an example, see "Estimate Continuous-Time OE Model Using Frequency Response" on page 1-964.

## init_sys - Linear system

idpoly model | linear model | structure
Linear system that configures the initial parameterization of sys, specified as an idpoly model, another linear model, or a structure. You obtain init_sys either by performing an estimation using measured data or by direct construction.

If init_sys is an idpoly model of the OE structure, oe uses the parameter values of init_sys as the initial guess for estimating sys. The sample time of init_sys must match the sample time of the data.

Use the Structure property of init_sys to configure initial guesses and constraints for $B(q)$ and $F(q)$. For example:

- To specify an initial guess for the $F(q)$ term of init_sys, set init_sys.Structure.F.Value as the initial guess.
- To specify constraints for the $B(q)$ term of init_sys:
- Set init_sys.Structure.B.Minimum to the minimum $B(q)$ coefficient values.
- Set init_sys.Structure.B.Maximum to the maximum $B(q)$ coefficient values.
- Set init_sys.Structure.B.Free to indicate which $B(q)$ coefficients are free for estimation.

If init_sys is not a polynomial model of the OE structure, the software first converts init_sys to an OE structure model. oe uses the parameters of the resulting model as the initial guess for estimating sys.

If you do not specify opt and init_sys was obtained by estimation, then the software uses estimation options from init_sys.Report. OptionsUsed.

## opt - Estimation options

oe0ptions option set
Estimation options, specified as an oeOptions option set. Options specified by opt include:

- Estimation objective
- Handling of initial conditions
- Numerical search method and the associated options

For examples of specifying estimation options, see "Estimate Continuous Model Using Band-Limited Discrete-Time Frequency-Domain Data" on page 1-967.

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Example: 'InputDelay',1

## InputDelay - Input delays

0 (default) | positive integer vector | integer scalar
Input delays for each input channel, specified as the comma-separated pair consisting of 'InputDelay' and a numeric vector.

- For continuous-time models, specify 'InputDelay' in the time units stored in the TimeUnit property.
- For discrete-time models, specify 'InputDelay' in integer multiples of the sample time Ts. For example, setting 'InputDelay' to 3 specifies a delay of three sampling periods.

For a system with $N_{u}$ inputs, set InputDelay to an $N_{u}$-by- 1 vector. Each entry of this vector is a numerical value that represents the input delay for the corresponding input channel.

To apply the same delay to all channels, specify 'InputDelay' as a scalar.
For an example, see "Estimate OE Polynomial Model" on page 1-961.

## IODelay - Transport delays

0 (default) | scalar | numeric array
Transport delays for each input-output pair, specified as the comma-separated pair consisting of
'IODelay ' and a numeric array.

- For continuous-time models, specify 'IODelay ' in the time units stored in the TimeUnit property.
- For discrete-time models, specify 'IODelay ' in integer multiples of the sample time Ts. For example, setting 'IODelay' to 4 specifies a transport delay of four sampling periods.

For a system with $N_{u}$ inputs and $N_{y}$ outputs, set 'IODelay' to an $N_{y}$-by- $N_{u}$ matrix. Each entry is an integer value representing the transport delay for the corresponding input-output pair.

To apply the same delay to all channels, specify 'IODelay ' as a scalar.
You can specify 'IODelay' as an alternative to the nk value. Doing so simplifies the model structure by reducing the number of leading zeros in the $B$ polynomial. In particular, you can represent $\max (\mathrm{nk}-1,0)$ leading zeros as input-output delays using 'IODelay' instead.

## Output Arguments

## sys - OE polynomial model

idpoly object
OE polynomial model that fits the estimation data, returned as an idpoly model object. This model is created using the specified model orders, delays, and estimation options. The sample time of sys matches the sample time of the estimation data. Therefore, sys is always a discrete-time model when estimated from time-domain data. For continuous-time model identification using time-domain data, use tfest.

The Report property of the model stores information about the estimation results and options used. Report has the following fields.

| Report Field | Description |
| :---: | :---: |
| Status | Summary of the model status, which indicates whether the model was created by construction or obtained by estimation. |
| Method | Estimation command used. |
| InitialCo ndition | Handling of initial conditions during model estimation, returned as one of the following values: <br> - 'zero' - The initial conditions were set to zero. <br> - 'estimate' - The initial conditions were treated as independent estimation parameters. <br> - 'backcast' - The initial conditions were estimated using the best least squares fit. <br> This field is especially useful to view how the initial conditions were handled when the InitialCondition option in the estimation option set is 'auto'. |


| Report Field | Description |  |
| :---: | :---: | :---: |
| Fit | Quantitative assessment of the estimation, returned as a structure. See "Loss Function and Model Quality Metrics" for more information on these quality metrics. The structure has the following fields: |  |
|  | Field | Description |
|  | FitPerce nt | Normalized root mean squared error (NRMSE) measure of how well the response of the model fits the estimation data, expressed as the percentage fit $=100(1-$ NRMSE $)$. |
|  | LossFcn | Value of the loss function when the estimation completes. |
|  | MSE | Mean squared error (MSE) measure of how well the response of the model fits the estimation data. |
|  | FPE | Final prediction error for the model. |
|  | AIC | Raw Akaike Information Criteria (AIC) measure of model quality. |
|  | AICc | Small sample-size corrected AIC. |
|  | nAIC | Normalized AIC. |
|  | BIC | Bayesian Information Criteria (BIC). |
| Parameter <br> s | Estimated values of model parameters. |  |
| OptionsUs ed | Option set used for estimation. If no custom options were configured, this is a set of default options. See oeOptions for more information. |  |
| RandState | State of the random number stream at the start of estimation. Empty, [ ], if randomization was not used during estimation. For more information, see rng. |  |


| Report Field | Description |  |
| :---: | :---: | :---: |
| DataUsed | Attributes of the data used for estimation, returned as a structure with the following fields: |  |
|  | Field | Description |
|  | Name | Name of the data set. |
|  | Type | Data type. |
|  | Length | Number of data samples. |
|  | Ts | Sample time. |
|  | InterSam ple | Input intersample behavior, returned as one of the following values: <br> - 'zoh ' - Zero-order hold maintains a piecewise-constant input signal between samples. <br> - ' foh ' - First-order hold maintains a piecewise-linear input signal between samples. <br> - 'bl' - Band-limited behavior specifies that the continuous-time input signal has zero power above the Nyquist frequency. |
|  | Input0ff set | Offset removed from time-domain input data during estimation. For nonlinear models, it is []. |
|  | Output0f fset | Offset removed from time-domain output data during estimation. For nonlinear models, it is [ ]. |
| Terminati on | Termination conditions for the iterative search used for prediction error minimization, returned as a structure with the following fields: |  |
|  | Field | Description |
|  | WhyStop | Reason for terminating the numerical search. |
|  | Iteratio ns | Number of search iterations performed by the estimation algorithm. |
|  | First0rd erOptima lity | $\infty$-norm of the gradient search vector when the search algorithm terminates. |
|  | FcnCount | Number of times the objective function was called. |
|  | UpdateNo rm | Norm of the gradient search vector in the last iteration. Omitted when the search method is 'lsqnonlin' or 'fmincon'. |
|  | LastImpr ovement | Criterion improvement in the last iteration, expressed as a percentage. Omitted when the search method is 'lsqnonlin' or 'fmincon'. |
|  | $\begin{array}{\|l\|l\|} \hline \text { Algorith } \\ \hline \end{array}$ | Algorithm used by ' lsqnonlin' or 'fmincon' search method. Omitted when other search methods are used. |
|  | For estimation methods that do not require numerical search optimization, the Termination field is omitted. |  |

For more information on using Report, see "Estimation Report".

## ic - Initial conditions

initialCondition object| object array of initialCondition values
Estimated initial conditions, returned as an initialCondition object or an object array of initialCondition values.

- For a single-experiment data set, ic represents, in state-space form, the free response of the transfer function model ( $A$ and $C$ matrices) to the estimated initial states ( $x_{0}$ ).
- For a multiple-experiment data set with $N_{e}$ experiments, ic is an object array of length $N_{e}$ that contains one set of initialCondition values for each experiment.

If oe returns ic values of 0 and the you know that you have non-zero initial conditions, set the 'InitialCondition' option in oeOptions to 'estimate' and pass the updated option set to oe. For example:

```
opt = oeOptions('InitialCondition,'estimate')
[sys,ic] = oe(data,np,nz,opt)
```

The default 'auto' setting of 'InitialCondition' uses the 'zero' method when the initial conditions have a negligible effect on the overall estimation-error minimization process. Specifying 'estimate' ensures that the software estimates values for ic.

For more information, see initialCondition. For an example of using this argument, see "Obtain Initial Conditions" on page 1-968.

## More About

## Output-Error (OE) Model

The general output-error model structure is:

$$
y(t)=\frac{B(q)}{F(q)} u\left(t-n_{k}\right)+e(t)
$$

The orders of the output-error model are:

$$
\begin{aligned}
& n b: \quad B(q)=b_{1}+b_{2} q^{-1}+\ldots+b_{n b} q^{-n b+1} \\
& n f: \quad F(q)=1+f_{1} q^{-1}+\ldots+f_{n f} q^{-n f}
\end{aligned}
$$

## Continuous-Time Output-Error Model

If data is continuous-time frequency-domain data, oe estimates a continuous-time model with the following transfer function:

$$
G(s)=\frac{B(s)}{F(s)}=\frac{b_{n b} s^{(n b-1)}+b_{n b-1} s^{(n b-2)}+\ldots+b_{1}}{s^{n f}+f_{n f} s^{(n f-1)}+\ldots+f_{1}}
$$

The orders of the numerator and denominator are nb and nf , similar to the discrete-time case. However, the sample delay nk does not exist in the continuous case, and you should not specify nk when you command the estimation. Instead, express any system delay using the name-value pair argument ' IODelay ' along with the system delay in the time units that are stored in the property TimeUnit. For example, suppose that your continuous system has a delay of iod seconds. Use model = oe(data,[nb nf],'IODelay',iod).

## Extended Capabilities

## Automatic Parallel Support

Accelerate code by automatically running computation in parallel using Parallel Computing Toolbox ${ }^{\mathrm{TM}}$.
Parallel computing support is available for estimation using the lsqnonlin search method (requires Optimization Toolbox). To enable parallel computing, use oeOptions, set SearchMethod to ' lsqnonlin' , and set SearchOptions. Advanced. UseParallel to true.

For example:

```
opt = oeOptions;
opt.SearchMethod = 'lsqnonlin';
opt.SearchOptions.Advanced.UseParallel = true;
```


## See Also

armax|arx|bj|compare|iddata|idfrd|idpoly|iv4|n4sid|oeOptions|polyest|sim |tfest

## Topics

"What Are Polynomial Models?"
"Data Supported by Polynomial Models"
"Regularized Estimates of Model Parameters"
"Apply Initial Conditions when Simulating Identified Linear Models"
Introduced before R2006a

## oeOptions

Option set for oe

## Syntax

opt $=$ oe0ptions
opt $=$ oe0ptions(Name, Value)

## Description

opt = oeOptions creates the default options set for oe.
opt $=$ oeOptions (Name, Value) creates an option set with the options specified by one or more Name, Value pair arguments.

## Input Arguments

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, ... , NameN, ValueN.

## InitialCondition - Handling of initial conditions <br> 'auto' (default)|'zero'|'estimate'|'backcast'

Handling of initial conditions during estimation, specified as one of the following values:

- 'zero' - The initial conditions are set to zero.
- 'estimate' - The initial conditions are treated as independent estimation parameters.
- 'backcast' - The initial conditions are estimated using the best least squares fit.
- 'auto ' - The software chooses the method to handle initial conditions based on the estimation data.


## WeightingFilter - Weighting prefilter

[ ] (default) | vector | matrix | cell array | linear system
Weighting prefilter applied to the loss function to be minimized during estimation. To understand the effect of WeightingFilter on the loss function, see "Loss Function and Model Quality Metrics".

Specify WeightingFilter as one of the following values:

- [] - No weighting prefilter is used.
- Passbands - Specify a row vector or matrix containing frequency values that define desired passbands. You select a frequency band where the fit between estimated model and estimation data is optimized. For example, [ $w l, w h$ ] where $w l$ and wh represent lower and upper limits of a passband. For a matrix with several rows defining frequency passbands, $[w 1 l, w 1 h ; w 2 l, w 2 h ; w 3 l, w 3 h ; \ldots]$, the estimation algorithm uses the union of the frequency ranges to define the estimation passband.

Passbands are expressed in rad/TimeUnit for time-domain data and in FrequencyUnit for frequency-domain data, where TimeUnit and FrequencyUnit are the time and frequency units of the estimation data.

- SISO filter - Specify a single-input-single-output (SISO) linear filter in one of the following ways:
- A SISO LTI model
- $\{A, B, C, D\}$ format, which specifies the state-space matrices of a filter with the same sample time as estimation data.
- \{numerator, denominator\} format, which specifies the numerator and denominator of the filter as a transfer function with same sample time as estimation data.

This option calculates the weighting function as a product of the filter and the input spectrum to estimate the transfer function.

- Weighting vector - Applicable for frequency-domain data only. Specify a column vector of weights. This vector must have the same length as the frequency vector of the data set, Data. Frequency. Each input and output response in the data is multiplied by the corresponding weight at that frequency.


## EnforceStability - Control whether to enforce stability of model <br> false (default) | true

Control whether to enforce stability of estimated model, specified as the comma-separated pair consisting of 'EnforceStability' and either true or false.

Use this option when estimating models using frequency-domain data. Models estimated using timedomain data are always stable.
Data Types: logical

## EstimateCovariance - Control whether to generate parameter covariance data <br> true (default) | false

Controls whether parameter covariance data is generated, specified as true or false.
If EstimateCovariance is true, then use getcov to fetch the covariance matrix from the estimated model.

## Display - Specify whether to display the estimation progress

'off' (default) |'on'
Specify whether to display the estimation progress, specified as one of the following values:

- 'on' - Information on model structure and estimation results are displayed in a progress-viewer window.
- 'off' - No progress or results information is displayed.


## InputOffset - Removal of offset from time-domain input data during estimation

[ ] (default) | vector of positive integers | matrix
Removal of offset from time-domain input data during estimation, specified as the comma-separated pair consisting of 'InputOffset' and one of the following:

- A column vector of positive integers of length $N u$, where $N u$ is the number of inputs.
- [ ] - Indicates no offset.
- $N u$-by-Ne matrix - For multi-experiment data, specify Input0ffset as an $N u$-by- $N e$ matrix. $N u$ is the number of inputs, and $N e$ is the number of experiments.

Each entry specified by Input0ffset is subtracted from the corresponding input data.

## OutputOffset - Removal of offset from time-domain output data during estimation

## [ ] (default) | vector | matrix

Removal of offset from time-domain output data during estimation, specified as the comma-separated pair consisting of 'OutputOffset' and one of the following:

- A column vector of length $N y$, where $N y$ is the number of outputs.
- [ ] - Indicates no offset.
- $N y$-by-Ne matrix - For multi-experiment data, specify Output0ffset as a $N y$-by- $N e$ matrix. $N y$ is the number of outputs, and $N e$ is the number of experiments.

Each entry specified by Output0ffset is subtracted from the corresponding output data.

## Regularization - Options for regularized estimation of model parameters

structure
Options for regularized estimation of model parameters. For more information on regularization, see "Regularized Estimates of Model Parameters".

Regularization is a structure with the following fields:

- Lambda - Constant that determines the bias versus variance tradeoff.

Specify a positive scalar to add the regularization term to the estimation cost.
The default value of zero implies no regularization.
Default: 0

- R - Weighting matrix.

Specify a vector of nonnegative numbers or a square positive semi-definite matrix. The length must be equal to the number of free parameters of the model.

For black-box models, using the default value is recommended. For structured and grey-box models, you can also specify a vector of np positive numbers such that each entry denotes the confidence in the value of the associated parameter.

The default value of 1 implies a value of eye(npfree), where npfree is the number of free parameters.

Default: 1

- Nominal - The nominal value towards which the free parameters are pulled during estimation.

The default value of zero implies that the parameter values are pulled towards zero. If you are refining a model, you can set the value to 'model' to pull the parameters towards the parameter values of the initial model. The initial parameter values must be finite for this setting to work.

Default: 0

## SearchMethod - Numerical search method used for iterative parameter estimation <br> 'auto' (default)|'gn'|'gna'|'lm'|'grad'|'lsqnonlin' |'fmincon'

Numerical search method used for iterative parameter estimation, specified as the comma-separated pair consisting of 'SearchMethod' and one of the following:

- 'auto' - A combination of the line search algorithms, 'gn', 'lm', 'gna', and 'grad' methods is tried in sequence at each iteration. The first descent direction leading to a reduction in estimation cost is used.
- 'gn' - Subspace Gauss-Newton least squares search. Singular values of the Jacobian matrix less than GnPinvConstant*eps*max(size(J))*norm(J) are discarded when computing the search direction. $J$ is the Jacobian matrix. The Hessian matrix is approximated as $J^{T} J$. If there is no improvement in this direction, the function tries the gradient direction.
- 'gna' - Adaptive subspace Gauss-Newton search. Eigenvalues less than gamma*max(sv) of the Hessian are ignored, where $s v$ contains the singular values of the Hessian. The Gauss-Newton direction is computed in the remaining subspace. gamma has the initial value
InitialGnaTolerance (see Advanced in 'Search0ptions' for more information). This value is increased by the factor LMStep each time the search fails to find a lower value of the criterion in fewer than five bisections. This value is decreased by the factor $2 *$ LMStep each time a search is successful without any bisections.
- 'lm' - Levenberg-Marquardt least squares search, where the next parameter value is - pinv (H $\left.+d^{*} \mathrm{I}\right) *$ grad from the previous one. $H$ is the Hessian, $I$ is the identity matrix, and grad is the gradient. $d$ is a number that is increased until a lower value of the criterion is found.
- 'grad ' - Steepest descent least squares search.
- ' lsqnonlin' - Trust-region-reflective algorithm of lsqnonlin. Requires Optimization Toolbox software.
- 'fmincon' - Constrained nonlinear solvers. You can use the sequential quadratic programming (SQP) and trust-region-reflective algorithms of the fmincon solver. If you have Optimization Toolbox software, you can also use the interior-point and active-set algorithms of the fmincon solver. Specify the algorithm in the SearchOptions.Algorithm option. The fmincon algorithms may result in improved estimation results in the following scenarios:
- Constrained minimization problems when there are bounds imposed on the model parameters.
- Model structures where the loss function is a nonlinear or non smooth function of the parameters.
- Multi-output model estimation. A determinant loss function is minimized by default for multioutput model estimation. fmincon algorithms are able to minimize such loss functions directly. The other search methods such as ' lm ' and 'gn' minimize the determinant loss function by alternately estimating the noise variance and reducing the loss value for a given noise variance value. Hence, the fmincon algorithms can offer better efficiency and accuracy for multi-output model estimations.


## SearchOptions - Option set for the search algorithm

search option set
Option set for the search algorithm, specified as the comma-separated pair consisting of
'Search0ptions ' and a search option set with fields that depend on the value of SearchMethod.

| SearchOptions Structure When SearchMethod is Specified as 'gn ', 'gna', 'lm', 'grad', or |
| :--- |
| 'auto' |
| Field <br> Name Description Default <br> Toleran <br> ce Minimum percentage difference between the current value of the loss <br> function and its expected improvement after the next iteration, specified as <br> a positive scalar. When the percentage of expected improvement is less <br> than Tolerance, the iterations stop. The estimate of the expected loss- <br> function improvement at the next iteration is based on the Gauss-Newton <br> vector computed for the current parameter value. 0.01MaxIter <br> ationsMaximum number of iterations during loss-function minimization, specified <br> as a positive integer. The iterations stop when MaxIterations is reached <br> or another stopping criterion is satisfied, such as Tolerance. |
| Setting MaxIterations = 0 returns the result of the start-up procedure. <br> Use sys. Report. Termination. Iterations to get the actual number <br> of iterations during an estimation, where sys is an idtf model. |


| Field Name | Description |  | Default |
| :---: | :---: | :---: | :---: |
| Advance <br> d | Advanced search settings, specified as a structure with the following fields: |  |  |
|  | Field Name | Description | Default |
|  | GnPinvCons tant | Jacobian matrix singular value threshold, specified as a positive scalar. Singular values of the Jacobian matrix that are smaller than GnPinvConstant*max(size(J)*norm(J)*eps) are discarded when computing the search direction. Applicable when SearchMethod is 'gn'. | 10000 |
|  | InitialGna Tolerance | Initial value of gamma, specified as a positive scalar. Applicable when SearchMethod is 'gna'. | 0.0001 |
|  | LMStartVal ue | Starting value of search-direction length $d$ in the Levenberg-Marquardt method, specified as a positive scalar. Applicable when SearchMethod is ' 1 m '. | 0.001 |
|  | LMStep | Size of the Levenberg-Marquardt step, specified as a positive integer. The next value of the search-direction length $d$ in the Levenberg-Marquardt method is LMStep times the previous one. Applicable when SearchMethod is ' lm '. | 2 |
|  | MaxBisecti ons | Maximum number of bisections used for line search along the search direction, specified as a positive integer. | 25 |
|  | MaxFunctio nEvaluatio ns | Maximum number of calls to the model file, specified as a positive integer. Iterations stop if the number of calls to the model file exceeds this value. | Inf |
|  | MinParamet erChange | Smallest parameter update allowed per iteration, specified as a nonnegative scalar. | 0 |
|  | RelativeIm provement | Relative improvement threshold, specified as a nonnegative scalar. Iterations stop if the relative improvement of the criterion function is less than this value. | 0 |
|  | StepReduct ion | Step reduction factor, specified as a positive scalar that is greater than 1 . The suggested parameter update is reduced by the factor StepReduction after each try. This reduction continues until MaxBisections tries are completed or a lower value of the criterion function is obtained. <br> StepReduction is not applicable for SearchMethod ' lm' (Levenberg-Marquardt method). | 2 |

SearchOptions Structure When SearchMethod is Specified as ' Isqnonlin'

| Field Name | Description | Default |
| :---: | :---: | :---: |
| Function Toleranc e | Termination tolerance on the loss function that the software minimizes to determine the estimated parameter values, specified as a positive scalar. <br> The value of FunctionTolerance is the same as that of opt.SearchOptions.Advanced.TolFun. | 1e-5 |
| StepTole rance | Termination tolerance on the estimated parameter values, specified as a positive scalar. <br> The value of StepTolerance is the same as that of opt.SearchOptions.Advanced. TolX. | 1e-6 |
| MaxItera tions | Maximum number of iterations during loss-function minimization, specified as a positive integer. The iterations stop when MaxIterations is reached or another stopping criterion is satisfied, such as FunctionTolerance. <br> The value of MaxIterations is the same as that of opt.SearchOptions.Advanced.MaxIter. | 20 |
| Advanced | Advanced search settings, specified as an option set for lsqnonlin. <br> For more information, see the Optimization Options table in "Optimization Options" (Optimization Toolbox). | Use optimset('lsqnonl in') to create a default option set. |

SearchOptions Structure When SearchMethod is Specified as 'fmincon'

| Field Name | Description | Default |
| :---: | :---: | :---: |
| Algorithm | fmincon optimization algorithm, specified as one of the following: <br> 'sqp ' - Sequential quadratic programming algorithm. The algorithm satisfies bounds at all iterations, and it can recover from NaN or Inf results. It is not a large-scale algorithm. For more information, see "Large-Scale vs. MediumScale Algorithms" (Optimization Toolbox). <br> - 'trust-regionreflective' - Subspace trust-region method based on the interior-reflective Newton method. It is a largescale algorithm. <br> - 'interior-point' -Large-scale algorithm that requires Optimization Toolbox software. The algorithm satisfies bounds at all iterations, and it can recover from NaN or Inf results. <br> - 'active-set' - Requires Optimization Toolbox software. The algorithm can take large steps, which adds speed. It is not a large-scale algorithm. <br> For more information about the algorithms, see "Constrained Nonlinear Optimization Algorithms" (Optimization Toolbox) and "Choosing the Algorithm" (Optimization Toolbox). | 'sqp ' |


| Field Name | Description | Default |
| :--- | :--- | :--- |
| FunctionTolerance | Termination tolerance on the <br> loss function that the software <br> minimizes to determine the <br> estimated parameter values, <br> specified as a positive scalar. | $1 \mathrm{e}-6$ |
| StepTolerance | Termination tolerance on the <br> estimated parameter values, <br> specified as a positive scalar. | $1 \mathrm{e}-6$ |
| MaxIterations | Maximum number of iterations <br> during loss function <br> minimization, specified as a <br> positive integer. The iterations <br> stop when MaxIterations is <br> reached or another stopping <br> criterion is satisfied, such as <br> FunctionTolerance. | 100 |

## Advanced - Additional advanced options

structure
Additional advanced options, specified as a structure with the following fields:

- ErrorThreshold - Specifies when to adjust the weight of large errors from quadratic to linear.

Errors larger than ErrorThreshold times the estimated standard deviation have a linear weight in the loss function. The standard deviation is estimated robustly as the median of the absolute deviations from the median of the prediction errors, divided by 0.7. For more information on robust norm choices, see section 15.2 of [2].

ErrorThreshold $=0$ disables robustification and leads to a purely quadratic loss function. When estimating with frequency-domain data, the software sets ErrorThreshold to zero. For time-domain data that contains outliers, try setting ErrorThreshold to 1.6.

Default: 0

- MaxSize - Specifies the maximum number of elements in a segment when input-output data is split into segments.

MaxSize must be a positive integer.
Default: 250000

- StabilityThreshold - Specifies thresholds for stability tests.

StabilityThreshold is a structure with the following fields:

- s-Specifies the location of the right-most pole to test the stability of continuous-time models. A model is considered stable when its right-most pole is to the left of $s$.

Default: 0

- $z$ - Specifies the maximum distance of all poles from the origin to test stability of discretetime models. A model is considered stable if all poles are within the distance $z$ from the origin.

Default: $1+$ sqrt (eps)

- AutoInitThreshold - Specifies when to automatically estimate the initial condition.

The initial condition is estimated when

$$
\frac{\left\|y_{p, z}-y_{\text {meas }}\right\|}{\left\|y_{p, e}-y_{\text {meas }}\right\|}>\text { AutoInitThreshold }
$$

- $y_{\text {meas }}$ is the measured output.
- $y_{p, z}$ is the predicted output of a model estimated using zero initial conditions.
- $y_{p, e}$ is the predicted output of a model estimated using estimated initial conditions.

Applicable when InitialCondition is 'auto'.
Default: 1.05

## Output Arguments

opt - Options set for oe
oeOptions option set
Option set for oe, returned as an oeOptions option set.

## Examples

## Create Default Options Set for Output-Error Estimation

```
opt = oeOptions;
```


## Specify Options for Output-Error Estimation

Create an options set for oe using the 'backcast' algorithm to initialize the condition and set the Display to 'on'.

```
opt = oeOptions('InitialCondition','backcast','Display','on');
```

Alternatively, use dot notation to set the values of opt.

```
opt = oeOptions;
opt.InitialCondition = 'backcast';
opt.Display = 'on';
```


## Compatibility Considerations

## Renaming of Estimation and Analysis Options

The names of some estimation and analysis options were changed in R2018a. Prior names still work. For details, see the R2018a release note "Renaming of Estimation and Analysis Options".

## References

[1] Wills, Adrian, B. Ninness, and S. Gibson. "On Gradient-Based Search for Multivariable System Estimates". Proceedings of the 16th IFAC World Congress, Prague, Czech Republic, July 3-8, 2005. Oxford, UK: Elsevier Ltd., 2005.
[2] Ljung, L. System Identification: Theory for the User. Upper Saddle River, NJ: Prentice-Hall PTR, 1999.

## See Also <br> idfilt|oe

Topics
"Loss Function and Model Quality Metrics"

Introduced in R2012a

## idnlarx/operspec

Construct operating point specification object for idnlarx model

## Syntax

spec = operspec(nlsys)

## Description

spec $=$ operspec ( $n$ lsys) creates a default operating point specification object for the idnlarx model $n l$ sys. This object is used with findop and specifies constraints on the model input and output signal values. Modify the default specifications using dot notation.

## Input Arguments

## nlsys - Nonlinear ARX model

idnlarx object
Nonlinear ARX model, specified as an idnlarx object.

## Output Arguments

## spec - Operating point specification

operating point specification object
Operating point specification, used to determine an operating point of the idnlarx model using findop, returned as an object containing the following:

- Input - Structure with fields:

| Field | Description | Default <br> for <br> Each <br> Input |
| :--- | :--- | :--- |
| Value | Initial guesses or fixed levels for the values of the model inputs, specified as <br> a vector with length equal to the number of input signals. | 0 |
| Min | Minimum value constraints on the model inputs, specified as a vector with <br> length equal to the number of input signals. | -Inf |
| Max | Maximum value constraints on the model inputs, specified as a vector with <br> length equal to the number of input signals. | Inf |
| Known | Known value indicator, specified as a logical vector with length equal to the <br> number of input signals and with the following values: <br> - true - findop will set the corresponding input signal to Value. <br> - false - findop will estimate the corresponding input signal using <br> Value as an initial guess. | true |

- Output - Structure with fields:

| Field | Description | Default <br> for <br> Each <br> Output |
| :--- | :--- | :--- |
| Value | lnitial guesses for the values of the model outputs, specified as a vector <br> with length equal to the number of output signals. | 0 |
| Min | Minimum value constraints on the model outputs, specified as a vector with <br> length equal to the number of output signals. | -Inf |
| Max | Maximum value constraints on the model outputs, specified as a vector with <br> length equal to the number of output signals. | Inf |

## See Also

idnlarx/findop
Introduced in R2008a

## idnlhw/operspec

Construct operating point specification object for idnlhw model

## Syntax

spec = operspec(nlsys)

## Description

spec $=$ operspec (nlsys) creates a default operating point specification object for the idnlhw model $n l$ sys. This object is used with findop and specifies constraints on the model input and output signal values. Modify the default specifications using dot notation.

## Input Arguments

## nlsys - Nonlinear Hammerstein-Wiener model

idnlhw object
Nonlinear Hammerstein-Wiener model, specified as an idnlhw object.

## Output Arguments

## spec - Operating point specification

operating point specification object
Operating point specification, used to determine an operating point of the idnlhw model using findop, returned as an object containing the following:

- Input - Structure with fields:

| Field | Description | Default <br> for <br> Each <br> Input |
| :--- | :--- | :--- |
| Value | Initial guesses or fixed levels for the values of the model inputs, specified as <br> a vector with length equal to the number of input signals. | 0 |
| Min | Minimum value constraints on the model inputs, specified as a vector with <br> length equal to the number of input signals. | -Inf |
| Max | Maximum value constraints on the model inputs, specified as a vector with <br> length equal to the number of input signals. | Inf |
| Known | Known value indicator, specified as a logical vector with length equal to the <br> number of input signals and with the following values: <br> - true - findop will set the corresponding input signal to Value. <br> - false - findop will estimate the corresponding input signal using <br> Value as an initial guess. | true |

- Output - Structure with fields:

| Field | Description | Default <br> for <br> Each <br> Input |
| :--- | :--- | :--- |
| Value | Target values the model outputs, specified as a vector with length equal to <br> the number of output signals. | 0 |
| Min | Minimum value constraints on the model outputs, specified as a vector with <br> length equal to the number of output signals. | -Inf |
| Max | Maximum value constraints on the model outputs, specified as a vector with <br> length equal to the number of output signals. | Inf |
| Known | Known value indicator, specified as a logical vector with length equal to the <br> number of output signals and with the following values: <br> - true - findop will use Value as an estimation target for the <br> corresponding output. <br> false - findop will keep the corresponding output within the <br> constraints specified by Min and Max. | false |

## Note

1 If Input.Known is true for all model inputs, then the initial state values are determined using the input specifications only. In this case, findop ignores the specifications in the Output structure.
2 Otherwise, findop uses the output specifications to meet the objectives indicated by Output.Known.

## See Also

idnlhw/findop

## Introduced in R2008a

## order

Query model order

## Syntax

NS = order(sys)

## Description

NS = order(sys) returns the model order NS. The order of a dynamic system model is the number of poles (for proper transfer functions) or the number of states (for state-space models). For improper transfer functions, the order is defined as the minimum number of states needed to build an equivalent state-space model (ignoring pole/zero cancellations).
order(sys) is an overloaded method that accepts SS, TF, and ZPK models. For LTI arrays, NS is an array of the same size listing the orders of each model in sys.

## Caveat

order does not attempt to find minimal realizations of MIMO systems. For example, consider this 2-by-2 MIMO system:
s=tf('s');
$\mathrm{h}=\left[1,1 /(\mathrm{s} *(\mathrm{~s}+1)) ; 1 /(\mathrm{s}+2), 1 /\left(\mathrm{s}^{*}(\mathrm{~s}+1) *(\mathrm{~s}+2)\right)\right]$;
order (h)
ans $=$
6

Although h has a 3rd order realization, order returns 6. Use

```
order(ss(h,'min'))
```

to find the minimal realization order.

## See Also

balred | pole
Introduced in R2012a

## particleFilter

Particle filter object for online state estimation

## Description

A particle filter is a recursive, Bayesian state estimator that uses discrete particles to approximate the posterior distribution of an estimated state. It is useful for online state estimation when measurements and a system model, that relates model states to the measurements, are available. The particle filter algorithm computes the state estimates recursively and involves initialization, prediction, and correction steps.
particleFilter creates an object for online state estimation of a discrete-time nonlinear system using the discrete-time particle filter algorithm.

Consider a plant with states $x$, input $u$, output $m$, process noise $w$, and measurement $y$. Assume that you can represent the plant as a nonlinear system.


The algorithm computes the state estimates $\widehat{x}$ of the nonlinear system using the state transition and measurement likelihood functions you specify.

The software supports arbitrary nonlinear state transition and measurement models, with arbitrary process and measurement noise distributions.

To perform online state estimation, create the nonlinear state transition function and measurement likelihood function. Then construct the particleFilter object using these nonlinear functions. After you create the object:

1 Initialize the particles using the initialize command.
2 Predict state estimates at the next step using the predict command.
3 Correct the state estimates using the correct command.
The prediction step uses the latest state to predict the next state based on the state transition model you provide. The correction step uses the current sensor measurement to correct the state estimate. The algorithm optionally redistributes, or resamples, the particles in the state space to match the posterior distribution of the estimated state. Each particle represents a discrete state hypothesis of these state variables. The set of all particles is used to help determine the state estimate.

## Creation

## Syntax

```
pf = particleFilter(StateTransitionFcn,MeasurementLikelihoodFcn)
```


## Object Description

pf = particleFilter(StateTransitionFcn, MeasurementLikelihoodFcn) creates a particle filter object for online state estimation of a discrete-time nonlinear system. StateTransitionFcn is a function that calculates the particles (state hypotheses) at the next time step, given the state vector at a time step. MeasurementLikelihoodFcn is a function that calculates the likelihood of each particle based on sensor measurements.

After creating the object, use the initialize command to initialize the particles with a known mean and covariance or uniformly distributed particles within defined bounds. Then, use the correct and predict commands to update particles (and hence the state estimate) using sensor measurements.

## Input Arguments

## StateTransitionFcn - State transition function

function handle
State transition function, specified as a function handle, determines the transition of particles (state hypotheses) between time steps. Also a property of the particleFilter object. For more information, see "Properties" on page 1-993.

## MeasurementLikelihoodFcn - Measurement likelihood function

function handle
Measurement likelihood function, specified as a function handle, is used to calculate the likelihood of particles (state hypotheses) from sensor measurements. Also a property of the particleFilter object. For more information, see "Properties" on page 1-993.

## Properties

## NumStateVariables - Number of state variables

[ ] (default) | scalar
Number of state variables, specified as a scalar. This property is read-only and is set using initialize. The number of states is implicit based on the specified matrices for the initial mean of particles, or the state bounds.

## NumParticles - Number of particles used in the filter

[ ] (default) | scalar
Number of particles used in the filter, specified as a scalar. Each particle represents a state hypothesis. You specify this property only by using initialize.

## StateTransitionFcn - State transition function

function handle

State transition function, specified as a function handle, determines the transition of particles (state hypotheses) between time steps. This function calculates the particles at the next time step, including the process noise, given particles at a time step.

In contrast, the state transition function for the extendedKalmanFilter and unscentedKalmanFilter generates a single state estimate at a given time step.

You write and save the state transition function for your nonlinear system, and specify it as a function handle when constructing the particleFilter object. For example, if
vdpParticleFilterStateFcn.m is the state transition function, specify StateTransitionFcn as @vdpParticleFilterStateFcn. You can also specify StateTransitionFcn as a function handle to an anonymous function.

The function signature is as follows:
function predictedParticles = myStateTransitionFcn(previousParticles,varargin)
The StateTransitionFcn function accepts at least one input argument. The first argument is the set of particles previousParticles that represents the state hypotheses at the previous time step. The optional use of varargin in the function enables you to input any extra parameters that are relevant for predicting the next state, using predict, as follows:
predict(pf,arg1,arg2)
If StateOrientation is 'column', then previousParticles is a NumStateVariables-byNumParticles array. If StateOrientation is 'row', then previousParticles is a NumParticles-by-NumStateVariables array.

StateTransitionFcn must return exactly one output, predictedParticles, which is the set of predicted particle locations for the current time step (array with same dimensions as previousParticles).

StateTransitionFcn must include the random process noise (from any distribution suitable for your application) in the predictedParticles.

To see an example of a state transition function with the StateOrientation property set to 'column', type edit vdpParticleFilterStateFcn at the command line.

## MeasurementLikelihoodFcn - Measurement likelihood function

function handle
Measurement likelihood function, specified as a function handle, is used to calculate the likelihood of particles (state hypotheses) using the sensor measurements. For each state hypothesis (particle), the function first calculates an N -element measurement hypothesis vector. Then the likelihood of each measurement hypothesis is calculated based on the sensor measurement and the measurement noise probability distribution.

In contrast, the measurement function for extendedKalmanFilter and unscentedKalmanFilter takes a single state hypothesis and returns a single measurement estimate.

You write and save the measurement likelihood function based on your measurement model, and use it to construct the object. For example, if vdpMeasurementLikelihoodFcn.m is the measurement likelihood function, specify MeasurementLikelihoodFcn as @vdpMeasurementLikelihoodFcn. You can also specify MeasurementLikelihoodFcn as a function handle to an anonymous function.

The function signature is as follows:
function likelihood = myMeasurementLikelihoodFcn(predictedParticles,measurement,varargin)
The MeasurementLikelihoodFcn function accepts at least two input arguments. The first argument is the set of particles predictedParticles that represents the predicted state hypothesis. If StateOrientation is 'column', then predictedParticles is a NumStateVariables-by-NumParticles array. If StateOrientation is 'row', then predictedParticles is a NumParticles-by-NumStateVariables array. The second argument, measurement, is the N -element sensor measurement at the current time step. You can provide additional input arguments using varargin.

The MeasurementLikelihoodFcn must return exactly one output, likelihood, a vector with NumParticles length, which is the likelihood of the given measurement for each particle (state hypothesis).

To see an example of a measurement likelihood function, type edit vdpMeasurementLikelihoodFcn at the command line.

## IsStateVariableCircular - Whether the state variables have a circular distribution [ ] (default) | logical array

Whether the state variables have a circular distribution, specified as a logical array.
This is a read-only property and is set using initialize.
Circular (or angular) distributions use a probability density function with a range of [-pi, pi]. IsStateVariableCircular is a row-vector with NumStateVariables elements. Each vector element indicates whether the associated state variable is circular.

## ResamplingPolicy - Policy settings that determine when to trigger resampling particleResamplingPolicy object

Policy settings that determine when to trigger resampling, specified as a particleResamplingPolicy object.

The resampling of particles is a vital step in estimating states using a particle filter. It enables you to select particles based on the current state, instead of using the particle distribution given at initialization. By continuously resampling the particles around the current estimate, you can get more accurate tracking and improve long-term performance.

You can trigger resampling either at fixed intervals or dynamically, based on the number of effective particles. The minimum effective particle ratio is a measure of how well the current set of particles approximates the posterior distribution. The number of effective particles is calculated by:

$$
N_{\text {eff }}=\frac{1}{\sum_{i=1}^{N}\left(w^{i}\right)^{2}}
$$

In this equation, N is the number of particles, and w is the normalized weight of each particle. The effective particle ratio is then $\mathrm{N}_{\text {eff }}$ / NumParticles. Therefore, the effective particle ratio is a function of the weights of all the particles. After the weights of the particles reach a low enough value, they are not contributing to the state estimation. This low value triggers resampling, so the particles are closer to the current state estimation and have higher weights.

The following properties of the particleResamplingPolicy object can be modified to control when resampling is triggered:

Description
P
b
p
e
r
t
y
$\mathbb{T}^{2}$ is a method to determine when resampling occurs, based on the value chosen. The 'interval ' talue triggers resampling at regular time steps of the particle filter operation. The 'ratio' value triggers resampling based on the ratio of effective total particles.



## Vescription

Wixed interval between resampling, specified as a scalar. This interval determines during which Gorrection steps the resampling is executed. For example, a value of 2 means the resampling is dxecuted every second correction step. A value of inf means that resampling is never executed. b his property only applies with the TriggerMethod is set to 'interval'.
i
n
g
t
n
t
r
a
1
d is the minimum desired ratio of the effective number of particles to the total number of particles NumParticles. The effective number of particles is a measure of how well the current set of $\overline{\bar{q}}$ articles approximates the posterior distribution. A lower effective particle ratio implies that a lower Fumber of particles are contributing to the estimation and resampling is required.
fit
If the ratio of the effective number of particles to the total number of particles NumParticles falls below the MinEffectiveParticleRatio, a resampling step is triggered.

## ResamplingMethod - Method used for particle resampling

'multinomial' (default)|'residual'|'stratified'|'systematic'

Method used for particle resampling, specified as one of the following:

- 'multinomial'
- 'residual'
- 'stratified'
- 'systematic'


## StateEstimationMethod - Method used for extracting a state estimate from particles

'mean' (default)|'maxweight'
Method used for extracting a state estimate from particles, specified as one of the following:

- 'mean' - The object outputs the weighted mean of the particles, depending on the properties Weights and Particles, as the state estimate.
- 'maxweight ' - The object outputs the particle with the highest weight as the state estimate.


## Particles - Array of particle values

[ ] (default) | array
Array of particle values, specified as an array based on the StateOrientation property:

- If StateOrientation is 'row' then Particles is an NumParticles-by-NumStateVariables array.
- If StateOrientation is 'column' then Particles is an NumStateVariables-byNumParticles array.

Each row or column corresponds to a state hypothesis (a single particle).

## Weights - Particle weights

[ ] (default) | vector
Particle weights, defined as a vector based on the value of the StateOrientation property:

- If StateOrientation is 'row' then Weights is a NumParticles-by-1 vector, where each weight is associated with the particle in the same row in the Particles property.
- If StateOrientation is 'column' then Weights is a 1-by-NumParticles vector, where each weight is associated with the particle in the same column in the Particles property.


## State - Current state estimate

[ ] (default) | vector
Current state estimate, defined as a vector based on the value of the StateOrientation property:

- If StateOrientation is 'row' then State is a 1-by-NumStateVariables vector
- If StateOrientation is 'column' then State is a NumStateVariables-by-1 vector

State is a read-only property, and is derived from Particles based on the
StateEstimationMethod property. Refer to "StateEstimationMethod" on page 1-0 for details on how the value of State is determined.

State along with StateCovariance can also be determined using getStateEstimate.

## StateCovariance - Current estimate of state estimation error covariance

NumStateVariables-by-NumStateVariables array (default)|[ ]| array

Current estimate of state estimation error covariance, defined as an NumStateVariables-byNumStateVariables array. StateCovariance is a read-only property and is calculated based on the StateEstimationMethod. If you specify a state estimation method that does not support covariance, then the function returns StateCovariance as [ ].

StateCovariance and State can be determined together using getStateEstimate.

## Object Functions

| initialize | Initialize the state of the particle filter |
| :--- | :--- |
| predict | Predict state and state estimation error covariance at next time step using <br> extended or unscented Kalman filter, or particle filter |
| correct | Correct state and state estimation error covariance using extended or unscented <br> Kalman filter, or particle filter and measurements |
| getStateEstimate | Extract best state estimate and covariance from particles <br> clone |
| Copy online state estimation object |  |

## Examples

## Create Particle Filter Object for Online State Estimation

To create a particle filter object for estimating the states of your system, create appropriate state transition function and measurement likelihood function for the system.

In this example, the function vdpParticleFilterStateFcn describes a discrete-time approximation to van der Pol oscillator with nonlinearity parameter, mu, equal to 1. In addition, it models Gaussian process noise. vdpMeasurementLikelihood function calculates the likelihood of particles from the noisy measurements of the first state, assuming a Gaussian measurement noise distribution.

Create the particle filter object. Use function handles to provide the state transition and measurement likelihood functions to the object.

```
myPF = particleFilter(@vdpParticleFilterStateFcn,@vdpMeasurementLikelihoodFcn);
```

To initialize and estimate the states and state estimation error covariance from the constructed object, use the initialize, predict, and correct commands.

## Estimate States Online using Particle Filter

Load the van der Pol ODE data, and specify the sample time.
vdpODEdata.mat contains a simulation of the van der Pol ODE with nonlinearity parameter mu=1, using ode45, with initial conditions [2;0]. The true state was extracted with sample time $\mathrm{dt}=$ 0.05 .
load ('vdpODEdata.mat','xTrue','dt')
tSpan = 0:dt:5;
Get the measurements. For this example, a sensor measures the first state with a Gaussian noise with standard deviation 0.04.

```
sqrtR = 0.04;
yMeas = xTrue(:,1) + sqrtR*randn(numel(tSpan),1);
```

Create a particle filter, and set the state transition and measurement likelihood functions.
myPF = particleFilter(@vdpParticleFilterStateFcn,@vdpMeasurementLikelihoodFcn);
Initialize the particle filter at state [2; 0] with unit covariance, and use 1000 particles.

```
initialize(myPF,1000,[2;0],eye(2));
```

Pick the mean state estimation and systematic resampling methods.

```
myPF.StateEstimationMethod = 'mean';
```

myPF.ResamplingMethod = 'systematic';

Estimate the states using the correct and predict commands, and store the estimated states.

```
xEst = zeros(size(xTrue));
for k=1:size(xTrue,1)
    xEst(k,:) = correct(myPF,yMeas(k));
    predict(myPF);
end
```

Plot the results, and compare the estimated and true states.

```
figure(1)
plot(xTrue(:,1),xTrue(:,2),'x',xEst(:,1),xEst(:,2),'ro')
legend('True','Estimated')
```



## Extended Capabilities

## C/C++ Code Generation

Generate C and $\mathrm{C}++$ code using MATLAB® Coder $^{\mathrm{TM}}$.
Usage notes and limitations:
For more information, see "Generate Code for Online State Estimation in MATLAB".
Supports MATLAB Function block: No

## See Also

## Functions

clone|correct|extendedKalmanFilter|initialize|predict|unscentedKalmanFilter

## Topics

"Nonlinear State Estimation Using Unscented Kalman Filter and Particle Filter"
"Generate Code for Online State Estimation in MATLAB"
"What Is Online Estimation?"
"Validate Online State Estimation at the Command Line"
"Troubleshoot Online State Estimation"
External Websites
Understanding Kalman Filters: Nonlinear State Estimators - MATLAB Video Series
Introduced in R2017b

## pe

Prediction error for identified model

## Syntax

```
err = pe(sys,data,K)
err = pe(sys,data,K,opt)
[err,ice,sys_pred] = pe( ___ )
pe(sys,data,K,___)
pe(sys,Linespec,data,K,___)
pe(sys1,...,sysN,data,K,
```

$\qquad$

```
pe(sys1,Linespec1,...,sysN,LinespecN,data,K,
```

$\qquad$

``` )
```


## Description

err = pe(sys,data,K) returns the K-step prediction error for the output of the identified model sys. The prediction error is determined by subtracting the K-step ahead predicted response from the measured output. The prediction error is calculated for the time span covered by data. For more information on the computation of predicted response, see predict.
err = pe(sys,data, K, opt) returns the prediction error using the option set, opt, to specify prediction error calculation behavior.
[err,ice,sys_pred] = pe( ___ ) also returns the estimated initial conditions, ice, and a predictor system, sys_pred.
pe(sys,data,K, $\qquad$ ) plots the prediction error. Use with any of the previous input argument combinations. To change display options in the plot, right-click the plot to access the context menu. For more details about the menu, see "Tips" on page 1-1005.
pe(sys, Linespec , data, K, ___) uses Linespec to specify the line type, marker symbol, and color.
pe(sys1, ...,sysN, data, K,__) plots the prediction errors for multiple identified models. pe automatically chooses colors and line styles.
pe(sys1,Linespec1, ...,sysN, LinespecN,data, K, $\qquad$ ) uses the line type, marker symbol, and color specified for each model.

## Input Arguments

## sys

Identified model.

## data

Measured input-output history.

If sys is a time-series model, which has no input signals, then specify data as an iddata object with no inputs. In this case, you can also specify data as a matrix of the past time-series values.

## K

Prediction horizon.
Specify K as a positive integer that is a multiple of the data sample time. Use $K=$ Inf to compute the pure simulation error.

## Default: 1

opt
Prediction options.
opt is an option set, created using pe0ptions, that configures the computation of the predicted response. Options that you can specify include:

- Handling of initial conditions
- Data offsets


## Linespec

Line style, marker, and color
Line style, marker, and color, specified as a character vector. For example, 'b ' or 'b+: '.
For more information about configuring Linespec, see plot.

## Output Arguments

err
Prediction error.
err is returned as an iddata object or matrix, depending on how you specify data. For example, if data is an iddata object, then so is err.

Outputs up to the time $\mathrm{t}-\mathrm{K}$ and inputs up to the time instant t are used to calculate the prediction error at the time instant t .

When $K=$ Inf, the predicted output is a pure simulation of the system.
For multi-experiment data, err contains the prediction error data for each experiment. The time span of the prediction error matches that of the observed data.

## ice

Estimated initial conditions.
ice is returned as a column vector of initial states for state-space systems and as an initialCondition object for transfer function and polynomial systems.

## sys_pred

Predictor system.
sys_pred is a dynamic system. When you simulate sys_pred, using [data.OutputData data. InputData] as the input, the output, $y p$, is such that err.OutputData = data. OutputData - yp. For state-space models, the software uses $x 0 \mathrm{e}$ as the initial condition when simulating sys_pred.

For discrete-time data, sys_pred is always a discrete-time model.
For multi-experiment data, sys_pred is an array of models, with one entry for each experiment.

## Examples

## Compute Prediction Error for an ARIX Model

Compute the prediction error for an ARIX model.
Use the error data to compute the variance of the noise source $e(t)$.
Obtain noisy data.

```
noise = [(1:150)';(151:-1:2)'];
load iddatal z1;
z1.y = z1.y+noise;
```

noise is a triangular wave that is added to the output signal of $z 1$, an iddata object.
Estimate an ARIX model for the noisy data.

```
sys = arx(z1,[2 2 1],'IntegrateNoise',true);
```

Compute the prediction error of the estimated model.

```
K = 1;
err = pe(z1,sys,K);
```

pe computes the one-step prediction error for the output of the identified model, sys.
Compute the variance of the noise source, $e(t)$.
noise_var = err.y'*err.y/(299-nparams(sys)-order(sys));
Compare the computed value with model's noise variance.
sys.NoiseVariance
The output of sys.NoiseVariance matches the computed variance.

## Plot Prediction Error for Multiple Models

Load the estimation data.

```
load iddatal;
data = zl;
```

Estimate an ARX model of order [2 2 1].
sys1 = arx(data,[2 2 1]);
Estimate a transfer function with 2 poles.

```
sys2 = tfest(data,2);
```

Plot the prediction error for the estimated models. Specify prediction horizon as 10, and specify the line styles for plotting the prediction error of each system.

```
pe(sys1,'r--',sys2,'b',data,10);
```



To change the display options, right-click the plot to access the context menu. For example, to view the estimation data, select Show Validation Data from the context menu. To view the predicted outputs, select Predicted Response Plot.

## Tips

- Right-clicking the plot of the prediction error opens the context menu, where you can access the following options:
- Systems - Select systems to view prediction error. By default, the prediction error of all systems is plotted.
- Data Experiment - For multi-experiment data only. Toggle between data from different experiments.
- Characteristics - View the following data characteristics:
- Peak Value - View the absolute peak value of the data. Applicable for time-domain data only.
- Peak Response - View peak response of the data. Applicable for frequency-response data only.
- Mean Value - View mean value of the data. Applicable for time-domain data only.
- Show - For frequency-domain and frequency-response data only.
- Magnitude - View magnitude of frequency response of the system.
- Phase - View phase of frequency response of the system.
- Show Validation Data - Plot data used to compute the prediction error.
- I/O Grouping - For datasets containing more than one input or output channel. Select grouping of input and output channels on the plot.
- None - Plot input-output channels in their own separate axes.
- All - Group all input channels together and all output channels together.
- I/O Selector - For datasets containing more than one input or output channel. Select a subset of the input and output channels to plot. By default, all output channels are plotted.
- Grid - Add grids to the plot.
- Normalize - Normalize the y-scale of all data in the plot.
- Full View - Return to full view. By default, the plot is scaled to full view.
- Prediction Horizon - Set the prediction horizon, or choose simulation.
- Initial Condition - Specify handling of initial conditions. Not applicable for frequencyresponse data.

Specify as one of the following:

- Estimate - Treat the initial conditions as estimation parameters.
- Zero - Set all initial conditions to zero.
- Absorb delays and estimate - Absorb nonzero delays into the model coefficients and treat the initial conditions as estimation parameters. Use this option for discrete-time models only.
- Predicted Response Plot - Plot the predicted model response.
- Prediction Error Plot - Plot the error between the model response and prediction data. By default, the error plot is shown.
- Properties - Open the Property Editor dialog box to customize plot attributes.


## See Also

ar|arx|compare|iddata|n4sid|peOptions|predict|resid|sim

Introduced before R2006a

## peOptions

Option set for pe

## Syntax

```
opt = peOptions
opt = peOptions(Name,Value)
```


## Description

opt $=$ peOptions creates the default options set for pe.
opt $=$ peOptions (Name, Value) creates an option set with the options specified by one or more Name, Value pair arguments.

## Input Arguments

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, ... , NameN, ValueN.

## InitialCondition

Handling of initial conditions.
Specify InitialCondition as one of the following:

- 'z' - Zero initial conditions.
- 'e' - Estimate initial conditions such that the prediction error for observed output is minimized.

For nonlinear grey-box models, only those initial states ithat are designated as free in the model (sys.InitialStates(i).Fixed $=$ false) are estimated. To estimate all the states of the model, first specify all the Nx states of the idnlgrey model sys as free.

```
for i = 1:Nx
sys.InitialStates(i).Fixed = false;
end
```

Similarly, to fix all the initial states to values specified in sys.InitialStates, first specify all the states as fixed in the sys. InitialStates property of the nonlinear grey-box model.

- 'd' - Similar to 'e', but absorbs nonzero delays into the model coefficients. The delays are first converted to explicit model states, and the initial values of those states are also estimated and returned.

Use this option for linear models only.

- Vector or Matrix - Initial guess for state values, specified as a numerical column vector of length equal to the number of states. For multi-experiment data, specify a matrix with Ne columns, where
$N e$ is the number of experiments. Otherwise, use a column vector to specify the same initial conditions for all experiments. Use this option for state-space (idss and idgrey) and nonlinear models (idnlarx, idnlhw, and idnlgrey) only.
- initialCondition object - initialCondition object that represents a model of the free response of the system to initial conditions. For multiexperiment data, specify a 1-by- $N_{e}$ array of objects, where $N_{e}$ is the number of experiments.

Use this option for linear models only.

- Structure with the following fields, which contain the historical input and output values for a time interval immediately before the start time of the data used by pe:

| Field | Description |
| :--- | :--- |
| Input | Input history, specified as a matrix with $N u$ columns, where $N u$ is the number of <br> input channels. For time series models, use [ ]. The number of rows must be <br> greater than or equal to the model order. |
| Output | Output history, specified as a matrix with $N y$ columns, where $N y$ is the number of <br> output channels. The number of rows must be greater than or equal to the model <br> order. |

For multi-experiment data, configure the initial conditions separately for each experiment by specifying InitialCondition as a structure array with $N e$ elements. To specify the same initial conditions for all experiments, use a single structure.

The software uses data2state to map the historical data to states. If your model is not idss, idgrey, idnlgrey, or idnlarx, the software first converts the model to its state-space representation and then maps the data to states. If conversion of your model to idss is not possible, the estimated states are returned empty.

- x0obj - Specification object created using idpar. Use this object for discrete-time state-space (idss and idgrey) and nonlinear grey-box (idnlgrey) models only. Use x0obj to impose constraints on the initial states by fixing their value or specifying minimum or maximum bounds.

Default: 'e'

## InputOffset

Removes offset from time domain input data during prediction-error calculation.
Specify as a column vector of length $N u$, where $N u$ is the number of inputs.
For multi-experiment data, specify Input0ffset as an $N u$-by- $N e$ matrix. $N u$ is the number of inputs, and $N e$ is the number of experiments.

Each entry specified by Input0ffset is subtracted from the corresponding input data.
Specify input offset for only time domain data.
Default: []
OutputOffset
Removes offset from time domain output data during prediction-error calculation.
Specify as a column vector of length $N y$, where $N y$ is the number of outputs.

In case of multi-experiment data, specify OutputOffset as a $N y$-by-Ne matrix. $N y$ is the number of outputs, and $N e$ is the number of experiments.

Each entry specified by OutputOffset is subtracted from the corresponding output data.
Specify output offset for only time domain data.

## Default: []

## OutputWeight

Weight of output for initial condition estimation.
OutputWeight takes one of the following:

- [] - No weighting is used. This value is the same as using eye(Ny) for the output weight, where $N y$ is the number of outputs.
- ' noise' - Inverse of the noise variance stored with the model.
- matrix - A positive, semidefinite matrix of dimension $N y$-by- $N y$, where $N y$ is the number of outputs.

Default: []

## Output Arguments

opt
Option set containing the specified options for pe.

## Examples

## Create Default Options Set for Prediction-Error Calculation

opt = peOptions;

## Specify Options for Prediction-Error Calculation

Create an options set for pe using zero initial conditions, and set the input offset to 5 .

```
opt = peOptions('InitialCondition','z','InputOffset',5);
```

Alternatively, use dot notation to set the values of opt.

```
opt = peOptions;
opt.InitialCondition = 'z';
opt.InputOffset = 5;
```


## See Also

idpar|pe

Introduced in R2012a

## pem

Prediction error minimization for refining linear and nonlinear models

## Syntax

sys = pem(data,init_sys)
sys = pem(data,init_sys,opt)

## Description

sys = pem(data,init_sys) updates the parameters of an initial model to fit the estimation data. The function uses prediction-error minimization algorithm to update the parameters of the initial model. Use this command to refine the parameters of a previously estimated model.
sys $=$ pem(data,init_sys,opt) specifies estimation options using an option set.

## Examples

## Refine Estimated State-Space Model

Estimate a discrete-time state-space model using the subspace method. Then, refine it by minimizing the prediction error.

Estimate a discrete-time state-space model using n4sid, which applies the subspace method.

```
load iddata7 z7;
z7a = z7(1:300);
opt = n4sidOptions('Focus','simulation');
init_sys = n4sid(z7a,4,opt);
init_sys provides a 73.85% fit to the estimation data.
init_sys.Report.Fit.FitPercent
ans = 73.8490
```

Use pem to improve the closeness of the fit.
sys = pem(z7a,init_sys);
Analyze the results.

```
compare(z7a,sys,init_sys);
```

Simulated Response Comparison

sys provides a $74.54 \%$ fit to the estimation data.

## Estimate Nonlinear Grey-Box Model

Estimate the parameters of a nonlinear grey-box model to fit DC motor data.
Load the experimental data, and specify the signal attributes such as start time and units.

```
load(fullfile(matlabroot,'toolbox','ident','iddemos','data','dcmotordata'));
data = iddata(y, u, 0.1);
data.Tstart = 0;
data.TimeUnit = 's';
```

Configure the nonlinear grey-box model (idnlgrey) model.
For this example, use dcmotor_m.m file. To view this file, type edit dcmotor_m.m at the MATLAB® command prompt.

```
file_name = 'dcmotor m';
order = [2 1 2];
parameters = [1;0.28];
initial_states = [0;0];
Ts = 0;
init_sys = idnlgrey(file_name,order,parameters,initial_states,Ts);
init_sys.TimeUnit = 's';
```

```
setinit(init_sys,'Fixed',{false false});
```

init_sys is a nonlinear grey-box model with its structure described by dcmotor_m.m. The model has one input, two outputs and two states, as specified by order.
setinit(init_sys,'Fixed', \{false false\}) specifies that the initial states of init_sys are free estimation parameters.

Estimate the model parameters and initial states.
sys = pem(data,init_sys);
sys is an idnlgrey model, which encapsulates the estimated parameters and their covariance.
Analyze the estimation result.

```
compare(data,sys,init_sys);
```


sys provides a $98.34 \%$ fit to the estimation data.

## Configure Estimation Using Process Model

Create a process model structure and update its parameter values to minimize prediction error.

Initialize the coefficients of a process model.

```
init sys = idproc('P2UDZ');
init_sys.Kp = 10;
init sys.Tw = 0.4;
init_sys.Zeta = 0.5;
init_sys.Td = 0.1;
init_sys.Tz = 0.01;
```

The Kp, Tw, Zeta, Td, and Tz coefficients of init_sys are configured with their initial guesses.
Use init_sys to configure the estimation of a prediction error minimizing model using measured data. Because init_sys is an idproc model, use procest0ptions to create the option set.

```
load iddata1 z1;
opt = procestOptions('Display','on','SearchMethod','lm');
sys = pem(z1,init_sys,opt);
Process Model Identification
Estimation data: Time domain data z1
Data has 1 outputs, 1 inputs and 300 samples.
Model Type:
    {'P2DUZ'}
```

Algorithm: Levenberg-Marquardt search
<br>

<br>

|  | Norm of | First-order |
| :--- | :---: | :---: |
| 21.2201 | - | 414 |
| 19.4048 | 1.15 | 323 |
| 14.8743 | 2.48 | 814 |
| 6.84305 | 0.873 | 451 |
| 5.20355 | 0.977 | $1.49 \mathrm{e}+03$ |
| 1.83911 | 0.973 | 473 |
| 1.67582 | 0.225 | 20.3 |
| 1.67335 | 0.062 | 6.57 |
| 1.67334 | 0.00494 | 0.0555 |


| Improvement | $(\%)<b r>$ | Iteration |
| :---: | :---: | :---: |
| 3.8 | - | - |
| 3.8 | 8.55 | 7 |
| 4.41 | 23.3 | 0 |
| 4.43 | 54 | 11 |
| 8.75 | 24 | 7 |
| 13 | 64.7 | 0 |
| 4.98 | 8.88 | 0 |
| 0.0829 | 0.147 | 0 |
| 0.000374 | 0.000648 | 0 |

Termination condition: Near (local) minimum, (norm(g) < tol)..
Number of iterations: 8, Number of function evaluations: 42
Status: Estimated using PEM
Fit to estimation data: 70.63\%, FPE: 1.73006
Examine the model fit.
sys.Report.Fit.FitPercent
ans $=70.6330$
sys provides a $70.63 \%$ fit to the measured data.

## Input Arguments

## data - Estimation data

iddata|idfrd
Estimation data that contains measured input-output data, specified as an iddata or idfrd object. You can use frequency-domain data only when init_sys is a linear model.

The input-output dimensions of data and init_sys must match.

## init_sys - Identified model that configures the initial parameterization of sys

linear model | nonlinear model
Identified model that configures the initial parameterization of sys, specified as a linear, or nonlinear model. You can obtain init_sys by performing an estimation using measured data or by direct construction.
init_sys must have finite parameter values. You can configure initial guesses, specify minimum/ maximum bounds, and fix or free for estimating any parameter of init_sys:

- For linear models, use the Structure property. For more information, see "Imposing Constraints on Model Parameter Values".
- For nonlinear grey-box models, use the InitialStates and Parameters properties. Parameter constraints cannot be specified for nonlinear ARX and Hammerstein-Wiener models.


## opt - Estimation options

option set
Estimation options that configure the algorithm settings, handling of estimation focus, initial conditions, and data offsets, specified as an option set. The command used to create the option set depends on the initial model type:

| Model Type | Use |
| :--- | :--- |
| idss | ssest0ptions |
| idtf | tfest0ptions |
| idproc | procest0ptions |
| idpoly | polyest0ptions |
| idgrey | greyest0ptions |
| idnlarx | nlarx0ptions |
| idnlhw | nlhwOptions |
| idnlgrey | nlgreyest0ptions |

## Output Arguments

## sys - Identified model

linear model | nonlinear model
Identified model, returned as the same model type as init_sys. The model is obtained by estimating the free parameters of init_sys using the prediction error minimization algorithm.

## Algorithms

PEM uses numerical optimization to minimize the cost function, a weighted norm of the prediction error, defined as follows for scalar outputs:

$$
V_{N}(G, H)=\sum_{t=1}^{N} e^{2}(t)
$$

where $e(t)$ is the difference between the measured output and the predicted output of the model. For a linear model, the error is defined as:

$$
e(t)=H^{-1}(q)[y(t)-G(q) u(t)]
$$

where $e(t)$ is a vector and the cost function $V_{N}(G, H)$ is a scalar value. The subscript $N$ indicates that the cost function is a function of the number of data samples and becomes more accurate for larger values of $N$. For multiple-output models, the previous equation is more complex. For more information, see chapter 7 in System Identification: Theory for the User, Second Edition, by Lennart Ljung, Prentice Hall PTR, 1999.

## Alternative Functionality

You can achieve the same results as pem by using dedicated estimation commands for the various model structures. For example, use ssest (data, init_sys) for estimating state-space models.

```
See Also
armax|bj|greyest|n4sid|nlarx|nlgreyest|nlhw|oe|polyest|procest| ssest|
tfest
```


## Topics

"Refine Linear Parametric Models"
Introduced before R2006a

## pexcit

Level of excitation of input signals

## Syntax

Ped = pexcit(Data)
[Ped.Maxnr] = pexcit(Data,Maxnr,Threshold)

## Description

Ped = pexcit(Data) tests the degree of persistence of excitation for the input. Data is an iddata object with time- or frequency-domain signals. Ped is the degree or order of excitation of the inputs in Data and is a row vector of integers with as many components as there are inputs in Data. The intuitive interpretation of the degree of excitation in an input is the order of a model that the input is capable of estimating in an unambiguous way.
[Ped.Maxnr] = pexcit(Data,Maxnr,Threshold) specifies the maximum order tested and threshold level used to measure which singular values are significant. Default value of Maxnr is $\mathrm{min}(\mathrm{N} / 3,50)$, where N is the number of input data. Default value of Threshold is 1e-9.

## References

Section 13.2 in Ljung (1999).

## See Also

advice|feedback|iddata|idnlarx
Introduced before R2006a

## plot

Plot input and output channels of iddata object

## Syntax

```
plot(data)
plot(data,LineSpec)
plot(datal,...,dataN)
plot(data1,LineSpec1...,dataN,LineSpecN)
plot(axes_handle,
```

$\qquad$

``` )
plot(___,plotoptions)
h = plot(
```

$\qquad$

``` )
```


## Description

plot (data) plots the input and output channels of an iddata object. The function plots the outputs on the top axes and the inputs on the bottom axes.

- For time-domain data, the input and output signals are plotted as a function of time. Depending on the InterSample property of the iddata object, the input signals are plotted as linearly interpolated curves or as staircase plots. For example, if data. InterSample = 'zoh', the input is piecewise constant between sampling points, and is plotted accordingly.
- For frequency-domain data, the magnitude and phase of each input and output signal are plotted over the available frequency span.

To plot a subset of the data, use subreferencing:

- plot(data(201:300)) plots the samples 201 to 300 in the data set data.
- plot(data(201:300,'Altitude', \{'Angle_of_attack','Speed'\})) plots the specified samples of the output named Altitude and the inputs named Angle_of_attack and Speed.
- plot(data(:, [3 4],[3:7])) plots all samples of output channel numbers 3 and 4 and input numbers 3 through 7.
plot (data, LineSpec) specifies the color, line style, and marker symbol for the dataset.
plot (data1, ..., dataN) plots multiple datasets. The number of plot axes is determined by the number of unique input and output names among all the datasets.
plot (data1,LineSpec1..., dataN, LineSpecN) specifies the line style, marker type, and color for each dataset. You can specify options for only some data sets. For example, plot (data1, data2, 'k', data3) specifies black as the plot color for data2.
plot (axes_handle, ___ ) plots into the axes with the handle axes_handle instead of into the current axes (gca). Use this syntax with any of the input argument combinations in the previous syntaxes.
plot( $\qquad$ , plotoptions) specifies the plot options.
h = plot( $\qquad$ ) returns the handle to the plot. You can use this handle to customize the plot with getoptions and setoptions.


## Examples

## Plot Time-Domain Input-Output Data

Load the data.
load iddatal $\mathrm{z1}$;
Plot the data.
plot(z1)


The function plots the output on the top axes and the input on the bottom axes.
Plot the first 100 samples.
plot(z1(1:100))

## Input-Output Data



Only the first 100 samples appear in the plot.
You can right-click the plot to explore characteristics such as peak and mean values.

## Plot Frequency-Domain Input-Output Data

Load the data.
load iddata1 z1
Convert the data to the frequency domain.
$z f=f f t(z 1) ;$
Plot the data.
plot(zf);

## Input-Output Data



Plot Input Data, Output Data, and Input-Output Data
Generate input data.
u = idinput([100 1 20],'sine',[],[],[5 10 1]);
u = iddata([],u,1,'per',100);
Generate output data.
sys = idtf(1,[12 1]);
y = sim(sys,u);
Plot only the input.
plot(u)


Plot only the output.
plot(y)

Input-Output Data


Plot the input and output together.
plot(y, u)

## Input-Output Data



Alternatively, you can use plot(iddata(y,u)).

## Plot Multiple Data Sets

Load two data sets.
load iddata1 z1
load iddata2
z2
Plot both datasets.
plot(z1,z2)

## Input-Output Data



Because the data sets use the same input and output names, the function plots both data sets together.

Specify unique input and output names.
z1.InputName = "z1 input";
z2.InputName = "z2_input";
z1.OutputName = "z1_output";
z2.OutputName = "z2 output";
Plot both datasets.
plot(z1,z2)


The function plots the data sets separately.

## Plot Multiexperiment Data

Create a multiexperiment data set.
load iddata1 z1
load iddata2 z2
zm = merge(z1,z2);
Plot the data.
plot(zm)
legend('show')

## Input-Output Data



For multiexperiment data, each experiment is treated as a separate data set. You can right-click the plots to view their characteristics.

## Specify Line Style, Marker Symbol, and Color

Load two data sets.

```
load iddatal z1;
load iddata2 z2;
```

Specify the line style for both data sets.

```
plot(z1,'y:*',z2,'b')
```


## Input-Output Data



## Specify Axes Handle

Create a figure with two subplots and return the handles for each subplot axes in s .
figure \% new figure
s(1) $=$ subplot(1,2,1); \% left subplot
s(2) = subplot(1,2,2); \% right subplot


Load the data sets.
load iddata1;
load iddata2;
Create a data plot in each axes using the handles.
plot(s(1), z1)



## Get and Use Axes Handle

Get the handle to your current plot and modify an axis property.
Load and plot the data.
load iddatal z1
plot(z1)


Get the axes handle for the plot.

```
ah = gca
ah =
    Axes (u1) with properties:
                        XLim: [0.1000 30]
                YLim: [-1 1]
            XScale: 'linear'
            YScale: 'linear'
        GridLineStyle: '-'
            Position: [0.1300 0.1100 0.7750 0.3480]
            Units: 'normalized'
```

Show all properties

The display shows the properties of the axes handle.
The scale of the x -axis xScale is 'linear'. Change $x$ Scale to ' $\log$ '.
ah. XScale = 'log';

## Input-Output Data



The x-axis now displays a log scale.

## Specify Plot Options

Configure a time plot.
opt = iddataPlotOptions('time');
Specify minutes as the time unit of the plot.
opt.TimeUnits = 'minutes';
Turn the grid on.
opt.Grid = 'on';
Create the plot with the options specified by opt.
load iddatal z1
plot(z1, opt);


## Change Plot Properties Using Handle

Create a data plot and return the handle.
load iddatal;
h = plot(z1);

## Input-Output Data



Set the time unit of the plot.

```
setoptions(h,'TimeUnits','minutes');
```



## Change Orientation of Input-Output Data Axes

Generate data with two inputs and one output.
$z=i d d a t a(r a n d n(100,1), r a n d(100,2)) ;$
Configure a time plot.
opt = iddataPlotOptions('time');
Plot the data.
h = plot(z,opt);


Change the orientation of the plots such that all inputs are plotted in one column, and all outputs are in a second column.

```
opt.Orientation = 'two-column';
```

h = plot(z,opt);


Alternatively, use setoptions.
setoptions(h,'Orientation','two-column')
You can also change the orientation by right-clicking the plot and choosing Orientation in the context menu.

## Input Arguments

## data - Input-output data

iddata object
Input-output data, specified as an iddata object. The data can be in the time domain or the frequency domain. It can be a single-channel or multichannel data, and single-experiment or multiexperiment data.

## LineSpec - Line style, marker symbol, and color

character vector
Line style, marker symbol, and color, specified as a character vector. LineSpec takes values such as ' b ' and ' $b+$ : '. For more information, see the plot reference page in the MATLAB documentation. For an example of using LineSpec, see "Specify Line Style, Marker Symbol, and Color" on page 11028.

## axes_handle - Axes handle <br> handle

Axes handle, specified as a handle, and which is the reference to an axes object. Use the gca function to get the handle to the current axes, for example, axes handle= gca. For an example of using axes_handle to apply a specific set of axes to the current plot, see "Specify Axes Handle" on page 1-1029. For an example of using gca to get your plot axes and then modifying the axes properties, see "Get and Use Axes Handle" on page 1-1032.

## plotoptions - Plot options

structure
Plot options, specified as an option set created using iddataPlotOptions. For an example of using plotoptions, see "Specify Plot Options" on page 1-1034.

## Output Arguments

## h - Plot handle

scalar | vector
Plot handle, returned as a scalar or vector. Handles are unique identifiers that you can use to query and modify properties of a specific plot. For an example, see "Change Plot Properties Using Handle" on page 1-1035.

## Tips

Right-clicking the plot opens the context menu, where you can access the following options and plot controls.

| Option | Description and Suboptions |
| :--- | :--- |
| Datasets | View the datasets used in the plot. |
| Characteristics | Peak Value - View the peak value of the data. <br> This value is useful for transient data. <br> Mean Value - View the mean value of the data. <br> This value is useful for steady-state data. |


| Option | Description and Suboptions |
| :--- | :--- |
| Orientation | For data with one input and one output channel: <br> Single row - Plot all inputs and outputs in <br> one row. <br> Single column- Plot all inputs and outputs <br> in one column. <br> For data with more than one input or output <br> channel: <br> Output row and input row - Plot all <br> outputs in one row and all inputs in a second <br> row. <br> Output column and input column - Plot <br> all outputs in one column and all inputs in a <br> second column. |
| I/O Grouping | Group input and output channels on the plot. <br> Use this option with datasets with more than one <br> input or output channel. |
| I/O Selector | Select a subset of the input and output channels <br> to plot. By default, all input and output channels <br> are plotted. |
| Grid | Use this option with data sets with more than one <br> input or output channel. |
| Normalize | Add grids to your plot. |
| Properties | Normalize the y-scale of all data in the plot. <br> can customize plot attributes. |

## See Also <br> iddata|iddataPlotOptions|identpref

Introduced in R2014a

## idnlarx/plot

Plot nonlinearity of nonlinear ARX model

## Syntax

```
plot(model)
plot(model,color)
plot(model1,...,modelN)
plot(model1,color1...,modelN,colorN)
plot(
```

$\qquad$

``` ,'NumberofSamples',N)
```


## Description

plot (model) plots the nonlinearity of a nonlinear ARX model on a nonlinear ARX plot on page 11046. The plot shows the nonlinearity for all outputs of the model as a function of its input regressors.
plot (model, color) specifies the color to use.
plot (model1, ..., modelN) generates the plot for multiple models.
plot (model1, color1. . . modelN, colorN) specifies the color for each model. You do not need to specify the color for all models.
plot( $\qquad$ , 'NumberofSamples',$N$ ) specifies the number of samples to use to grid the regressor space on each axis. This syntax can include any of the input argument combinations in the previous syntaxes.

## Examples

Plot Nonlinearity of a Nonlinear ARX Model
Estimate a nonlinear ARX model and plot its nonlinearity.

```
load iddata1
model1 = nlarx(z1,[4 2 1],'wave','nlr',[1:3]);
plot(model1)
```



In the plot window, you can choose:

- The regressors to use on the plot axes, and specify the center points for the other regressors in the configuration panel. For multi-output models, each output is plotted separately.
- The output to view from the drop-down list located at the top of the plot.


## Specify Line Style for Multiple Models

```
load iddata1
model1 = nlarx(z1,[4 2 1],'wave','nlr',[1:3]);
model2 = nlarx(z1,[4 2 1],'sigmoidnet','nlr',[1:3]);
plot(model1,'b', model2, 'g')
```



## Specify Number of Samples

load iddatal
model1 = nlarx(z1,[4 2 1],'wave','nlr',[1:3]);
model2 = nlarx(z1,[4 2 1],'sigmoidnet','nlr',[1:3]);
plot(model1,'b', model2, 'g','NumberofSamples',50)


## Input Arguments

model - Estimated nonlinear ARX model
idnlarx model
Estimated nonlinear ARX model, specified as an idnlarx model object. Use nlarx to estimate the model.

## color - Color to use

character vector of color name | vector of doubles
Color to use to plot the regressors, specified as one of the following:

- Character vector of color name, specified as one of the following:
- 'b'
- 'y'
- 'm'
- 'c'
- 'r'
- 'g'
- 'w'
- 3-element double vector of RGB values

By default, the colors are automatically chosen.
Data Types: double | char

## N - Number of points

20 (default) | positive integer
Number of points used on the regressor axis to display the regressor samples, specified as a positive integer.

```
Data Types: double
```


## More About

## What is a Nonlinear ARX Plot?

A nonlinear ARX plot displays the evaluated model nonlinearity for a chosen model output as a function of one or two model regressors. For a model M, the model nonlinearity (M. Nonlinearity) is a nonlinearity estimator function, such as wavenet, sigmoidnet, or treepartition, that uses model regressors as its inputs.


To understand what is plotted, suppose that $\{r 1, r 2, \ldots, r N\}$ are the $N$ regressors used by a nonlinear ARX model $M$ with nonlinearity $n l$ corresponding to a model output. You can use getreg ( $M$ ) to view these regressors. The expression Nonlin $=$ evaluate ( $n \mathrm{l},[\mathrm{v} 1, \mathrm{v} 2, \ldots, \mathrm{vN}]$ ) returns the model
output for given values of these regressors, that is, $r 1=v 1, r 2=v 2, \ldots, r N=v N$. For plotting the nonlinearities, you select one or two of the $N$ regressors, for example, rsub $=\{r 1, r 4\}$. The software varies the values of these regressors in a specified range, while fixing the value of the remaining regressors, and generates the plot of Nonlin vs. rsub. By default, the software sets the values of the remaining fixed regressors to their estimated means, but you can change these values. The regressor means are stored in the Nonlinearity. Parameters. RegressorMean property of the model.

Examining a nonlinear ARX plot can help you gain insight into which regressors have the strongest effect on the model output. Understanding the relative importance of the regressors on the output can help you decide which regressors to include in the nonlinear function for that output. If the shape of the plot looks like a plane for all the chosen regressor values, then the model is probably linear in those regressors. In this case, you can remove the corresponding regressors from nonlinear block, and repeat the estimation.

Furthermore, you can create several nonlinear models for the same data using different nonlinearity estimators, such a wavenet network and treepartition, and then compare the nonlinear surfaces of these models. Agreement between plots for various models increases the confidence that these nonlinear models capture the true dynamics of the system.

To learn more about configuring the plot, see "Configuring a Nonlinear ARX Plot".

## See Also

evaluate | getreg|idnlarx|nlarx

## Topics

"Structure of Nonlinear ARX Models"
"Validate Nonlinear ARX Models"

## Introduced in R2014a

## idnlhw/plot

Plot input and output nonlinearity, and linear responses of Hammerstein-Wiener model

## Syntax

```
plot(model)
plot(model,LineSpec)
plot(model1,...,modelN)
plot(model1,LineSpec1...,modelN,LineSpecN)
plot(
``` \(\qquad\)
``` ,Name, Value)
```


## Description

plot (model) plots the input and output nonlinearity, and linear responses of a Hammerstein-Wiener model on a Hammerstein-Wiener plot on page 1-1054. The plot shows the responses of the input and output nonlinearity, and linear blocks that represent the model.
plot (model, LineSpec) specifies the line style.
plot (model1, . . . , modelN) generates the plot for multiple models.
plot(model1,LineSpec1..., modelN,LineSpecN) specifies the line style for each model. You do not need to specify the line style for all models.
plot( $\qquad$ ,Name, Value) specifies plot properties using additional options specified by one or more Name, Value pair arguments. This syntax can include any of the input argument combinations in the previous syntaxes.

## Examples

## Plot Input and Output Nonlinearity and Linear Response of a Hammerstein-Wiener Model

Estimate a Hammerstein-Wiener Model and plot responses of its input and output nonlinearity and linear blocks.

```
load iddata3
model1 = nlhw(z3,[4 2 1],'sigmoidnet','deadzone');
plot(model1)
```



Explore the various plots in the plot window by clicking one of the three blocks that represent the model:

- uNL - Input nonlinearity, representing the static nonlinearity at the input ( model.InputNonlinearity) to the LinearBlock.
- Linear Block - Step, impulse,Bode and pole-zero plots of the embedded linear model ( model.LinearModel ). By default, a step plot is displayed.
- yNL - Output nonlinearity, representing the static nonlinearity at the output ( model.OutputNonlinearity) of the Linear Block.


## Specify Line Style for Multiple Hammerstein-Weiner Models

load iddata3
model1 = nlhw(z3,[4 2 1],'sigmoidnet','deadzone');
model2 = nlhw(z3, [4 2 1],[],'sigmoidnet');
plot(model1,'b-',model2,'g')
File Edit View Insert Tools Desktop Window Help

## Specify Number of Samples, Time Samples, and Range of Input Nonlinearity

load iddata3
modell = nlhw(z3,[4 2 1],'sigmoidnet','deadzone');
model2 = nlhw(z3, [4 2 1],[],'sigmoidnet');
plot(model1,'b-',model2,'g','NumberOfSamples',50,'time',10,'InputRange',[-2 2]);


## Specify Time Samples, Frequency, and Range of Output Nonlinearity

```
load iddata3
model1 = nlhw(z3,[4 2 1],'sigmoidnet','deadzone');
model2 = nlhw(z3, [4 2 1],[],'sigmoidnet');
plot(model1,model2,'time',1:500,'freq',{0.01,100},'OutputRange',[0 1000]);
```

Sele Edit View Insert Tools Desktop Window Help

## Input Arguments

## model - Estimated Hammerstein-Wiener model

idnlhw model
Estimated Hammerstein-Wiener model, specified as an idnlhw model object. Use nlhw to estimate the model.

## LineSpec - Line style, marker symbol, and color

character vector

Line style, marker symbol, and color, specified as a character vector. LineSpec takes values such as 'b', 'b+:'. For more information, see the plot reference page in the MATLAB documentation.

## Data Types: char

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.
Example: plot (model, 'NumberofSamples',10) specifies to use 10 data points for the input regressors.

## NumberOfSamples - Number of data points to use for input regressors

100 (default) | positive integer
Number of data points to use for the input regressors when evaluating the nonlinearities at individual input or output channels, specified as a positive integer. This property does not affect the plots of the linear block.

Data Types: double
InputRange - Minimum and maximum regressor values for evaluating input nonlinearities range of regressor values used during each model's estimation. (default) | positive integer | vector

Minimum and maximum regressor values to use when evaluating the nonlinearities at each input channel, specified as positive integers or [min max] vector, where minimum value is less than the maximum value.

You can use 'uRange' as a shortcut name for this property.
Data Types: double
OutputRange - Minimum and maximum regressor values for evaluating output nonlinearities
range of regressor values used during each model's estimation (default) | positive integer | vector
Minimum and maximum regressor values to use when evaluating the nonlinearities at each output channel, specified as positive integers or [min max] vector, where minimum value is less than the maximum value.

You can use 'yRange' as a shortcut name for this property.
Data Types: double

## Time - Time samples to compute transient responses of the linear block

each model's dynamics determine the time samples used (default) | positive scalar | vector
The time samples at which the transient responses (step and impulse) of the linear block of the idnlhw model must be computed, specified as one of the following values:

- Positive scalar - Denotes end time for transient responses of all models. For example, 10.
- Vector of time instants - A double vector of equi-sampled values denotes the time samples at which the transient response must be computed. For example, [0:0.1:10].

This property takes the same values as the step command on the model.

## Frequency - Frequencies at which to compute the Bode response

automatically chosen inside the Nyquist frequency range (default) | [min max] range of positive scalars | vector of positive integers

Frequencies at which to compute the Bode response, specified as one of the following values:

- [Wmin Wmax] range - Frequency interval between Wmin and Wmax (in units rad/ (model.TimeUnit)) covered using logarithmically placed points.
- Vector of non-negative frequency values - Allows computation of bode response at those frequencies.

By default, the response is computed at some automatically chosen frequencies inside the Nyquist frequency range. Frequencies above Nyquist frequency (pi/model.Ts) are ignored.

This property takes the same values as the bode command on the model.

## More About

## What is a Hammerstein-Wiener Plot?

A Hammerstein-Wiener plot displays the static input and output nonlinearities and linear responses of a Hammerstein-Wiener model.

Examining a Hammerstein-Wiener plot can help you determine whether you have selected a complicated nonlinearity for modeling your system. For example, suppose you use a piecewise-linear input nonlinearity to estimate your model, but the plot indicates saturation behavior. You can estimate a new model using the simpler saturation nonlinearity instead. For multivariable systems, you can use the Hammerstein-Wiener plot to determine whether to exclude nonlinearities for specific channels. If the nonlinearity for a specific input or output channel does not exhibit strong nonlinear behavior, you can estimate a new model after setting the nonlinearity at that channel to unit gain.

You can generate these plots in the System Identification app and at the command line. In the plot window, you can view the nonlinearities and linear responses by clicking one of the three blocks that represent the model:

- $u_{\mathrm{NL}}$ (input nonlinearity)— Click this block to view the static nonlinearity at the input to the Linear Block. The plot displays evaluate(M. InputNonlinearity, u) where M is the HammersteinWiener model, and $u$ is the input to the input nonlinearity block. For information about the blocks, see "Structure of Hammerstein-Wiener Models".
- Linear Block - Click this block to view the Step, impulse, Bode, and pole-zero response plots of the embedded linear model (M.LinearModel). By default, a step plot of the linear model is displayed.
- $y_{\mathrm{NL}}$ (output nonlinearity) - Click this block to view the static nonlinearity at the output of the Linear Block. The plot displays evaluate(M.OutputNonlinearity, x ), where x is the output of the linear block.

To learn more about how to configure the linear and nonlinear blocks plots, see "Configuring a Hammerstein-Wiener Plot".

## See Also

idnlhw|nlhw

## Topics

"Structure of Hammerstein-Wiener Models"
"Validating Hammerstein-Wiener Models"
Introduced in R2014a

## pole

Poles of dynamic system

## Syntax

P = pole(sys)
P = pole(sys,J1,..., JN)

## Description

$P=$ pole(sys) returns the poles of the SISO or MIMO dynamic system model sys. The output is expressed as the reciprocal of the time units specified in sys.TimeUnit. The poles of a dynamic system determine the stability and response of the system.

An open-loop linear time-invariant system is stable if:

- In continuous-time, all the poles of the transfer function have negative real parts. When the poles are visualized on the complex s-plane, then they must all lie in the left-half plane (LHP) to ensure stability.
- In discrete-time, all the poles must have a magnitude strictly smaller than one, that is they must all lie inside the unit circle.
$P=$ pole(sys, $J 1, \ldots, J N$ ) returns the poles $P$ of the entries in model array sys with subscripts (J1,..., JN).


## Examples

## Poles of Discrete-Time Transfer Function

Compute the poles of the following discrete-time transfer function:
$\operatorname{sys}(z)=\frac{0.0478 z-0.0464}{z^{2}-1.81 z+0.9048}$
sys $=t f([0.047980 .0464],[1-1.810 .9048], 0.1) ;$
P = pole(sys)
P = 2×1 complex
$0.9050+0.2929 i$
0.9050-0.2929i

For stable discrete systems, all their poles must have a magnitude strictly smaller than one, that is they must all lie inside the unit circle. The poles in this example are a pair of complex conjugates, and lie inside the unit circle. Hence, the system sys is stable.

## Poles of Transfer Function

Calculate the poles of following transfer function:
$\operatorname{sys}(s)=\frac{4.2 s^{2}+0.25 s-0.004}{s^{2}+9.6 s+17}$

```
sys = tf([4.2,0.25,-0.004],[1,9.6,17]);
P = pole(sys)
P = 2\times1
```

    -7.2576
    -2.3424
    For stable continuous systems, all their poles must have negative real parts. sys is stable since the poles are negative, that is, they lie in the left half of the complex plane.

## Poles of Models in an Array

For this example, load invertedPendulumArray.mat, which contains a 3-by-3 array of inverted pendulum models. The mass of the pendulum varies as you move from model to model along a single column of sys, and the length of the pendulum varies as you move along a single row. The mass values used are $100 \mathrm{~g}, 200 \mathrm{~g}$ and 300 g , and the pendulum lengths used are $3 \mathrm{~m}, 2 \mathrm{~m}$ and 1 m respectively.

Column 1 Column 2 Column 3
Row $1 \quad 100 \mathrm{~g}, 3 \mathrm{~m} \quad 100 \mathrm{~g}, 2 \mathrm{~m} \quad 100 \mathrm{~g}, 1 \mathrm{~m}$
Row $2200 \mathrm{~g}, 3 \mathrm{~m} 200 \mathrm{~g}, 2 \mathrm{~m} 200 \mathrm{~g}, 1 \mathrm{~m}$
Row 3 300g, $3 \mathrm{~m} 300 \mathrm{~g}, 2 \mathrm{~m} 300 \mathrm{~g}, 1 \mathrm{~m}$
load('invertedPendulumArray.mat','sys'); size(sys)
$3 \times 3$ array of transfer functions.
Each model has 1 outputs and 1 inputs.
Find poles of the model array.

```
P = pole(sys);
P(:,:,2,1)
ans = 3\times1
    2.1071
    -2.1642
    -0.1426
```

$P(:,:, 2,1)$ corresponds to the poles of the model with 200 g pendulum weight and 3 m length.

## Input Arguments

## sys - Dynamic system

dynamic system model | model array
Dynamic system, specified as a SISO or MIMO dynamic system model, or an array of SISO or MIMO dynamic system models. Dynamic systems that you can use include continuous-time or discrete-time numeric LTI models such as tf , zpk , or ss models.

If sys is a generalized state-space model genss or an uncertain state-space model uss, pole returns the poles of the current or nominal value of sys. If sys is an array of models, pole returns the poles of the model corresponding to its subscript J1, . . , JN in sys. For more information on model arrays, see "Model Arrays" (Control System Toolbox).

## J1, ..., JN - Indices of models in array whose poles you want to extract positive integer

Indices of models in array whose poles you want to extract, specified as a positive integer. You can provide as many indices as there are array dimensions in sys. For example, if sys is a 4 -by- 5 array of dynamic system models, the following command extracts the poles for entry $(2,3)$ in the array.

```
P = pole(sys,2,3);
```


## Output Arguments

## P - Poles of the dynamic system

## column vector | array

Poles of the dynamic system, returned as a scalar or an array. If sys is:

- A single model, then $P$ is a column vector of poles of the dynamic system model sys
- A model array, then P is an array of poles of each model in sys
$P$ is expressed as the reciprocal of the time units specified in sys.TimeUnit. For example, pole is expressed in 1 /minute if sys.TimeUnit = 'minutes'.

Depending on the type of system model, poles are computed in the following way:

- For state-space models, the poles are the eigenvalues of the $A$ matrix, or the generalized eigenvalues of $A-\lambda E$ in the descriptor case.
- For SISO transfer functions or zero-pole-gain models, the poles are the denominator roots. For more information, see roots.
- For MIMO transfer functions (or zero-pole-gain models), the poles are returned as the union of the poles for each SISO entry. If some I/O pairs have a common denominator, the roots of such I/O pair denominator are counted only once.


## Limitations

- Multiple poles are numerically sensitive and cannot be computed with high accuracy. A pole $\lambda$ with multiplicity $m$ typically results in a cluster of computed poles distributed on a circle with center $\lambda$ and radius of order

$$
\rho \approx \varepsilon^{1 / m}
$$

where $\varepsilon$ is the relative machine precision (eps).
For more information on multiple poles, see "Sensitivity of Multiple Roots" (Control System Toolbox).

- If sys has internal delays, poles are obtained by first setting all internal delays to zero so that the system has a finite number of poles, thereby creating a zero-order Padé approximation. For some systems, setting delays to zero creates singular algebraic loops, which result in either improper or ill-defined, zero-delay approximations. For these systems, pole returns an error.

To assess the stability of models with internal delays, use step or impulse.

## See Also

damp |impulse | pzmap | pzplot | step | zero

## Topics

"Pole and Zero Locations" (Control System Toolbox)
"Sensitivity of Multiple Roots" (Control System Toolbox)
Introduced in R2012a

## polydata

Access polynomial coefficients and uncertainties of identified model

## Syntax

[A,B,C,D,F] = polydata(sys)
[A, B, C, D, F, dA, dB, dC, dD, dF] = polydata(sys)
[___] = polydata(sys,J1,...,JN)
[___] = polydata(___,'cell')

## Description

$[A, B, C, D, F]=$ polydata(sys) returns the coefficients of the polynomials $A, B, C, D$, and $F$ that describe the identified model sys. The polynomials describe the idpoly representation of sys as follows.

- For discrete-time sys:

$$
A\left(q^{-1}\right) y(t)=\frac{B\left(q^{-1}\right)}{F\left(q^{-1}\right)} u(t-n k)+\frac{C\left(q^{-1}\right)}{D\left(q^{-1}\right)} e(t) .
$$

$u(t)$ are the inputs to sys. $y(t)$ are the outputs. $e(t)$ is a white noise disturbance.

- For continuous-time sys:

$$
A(s) Y(s)=\frac{B(s)}{F(s)} U(s) e^{-\tau s}+\frac{C(s)}{D(s)} E(s)
$$

$U(s)$ are the Laplace transformed inputs to sys. $Y(s)$ are the Laplace transformed outputs. $E(s)$ is the Laplace transform of a white noise disturbance.

If sys is an identified model that is not an idpoly model, polydata converts sys to idpoly form to extract the polynomial coefficients.
$[A, B, C, D, F, d A, d B, d C, d D, d F]=p o l y d a t a(s y s)$ also returns the uncertainties $d A, d B, d C, d D$, and dF of each of the corresponding polynomial coefficients of sys.
[___] = polydata(sys, J1, ..., JN ) returns the polynomial coefficients for the J1, ..., JN entry in the array sys of identified models.
[___] = polydata(__ , 'cell') returns all polynomials as cell arrays of double vectors, regardless of the input and output dimensions of sys.

## Input Arguments

## sys

Identified model or array of identified models. sys can be continuous-time or discrete-time. sys can be SISO or MIMO.

## J1,...., JN

Indices selecting a particular model from an N -dimensional array sys of identified models.

## Output Arguments

## A,B,C,D,F

Polynomial coefficients of the idpoly representation of sys.

- If sys is a SISO model, each of A, B, C, D, and F is a row vector. The length of each row vector is the order of the corresponding polynomial.
- For discrete-time sys, the coefficients are ordered in ascending powers of $q^{-1}$. For example, B $=\left[\begin{array}{lll}1 & -4 & 9\end{array}\right]$ means that $B\left(q^{-1}\right)=1-4 q^{-1}+9 q^{-2}$.
- For continuous-time sys, the coefficients are ordered in descending powers of $s$. For example, $B=\left[\begin{array}{lll}1 & -4 & 9\end{array}\right]$ means that $B(s)=s^{2}-4 s+9$.
- If sys is a MIMO model, each of $A, B, C, D$, and $F$ is a cell array. The dimensions of the cell arrays are determined by the input and output dimensions of sys as follows:
- A $-N_{y}$-by- $N_{y}$ cell array
- B, F $-N_{y}$-by- $N_{u}$ cell array
- C, D $-N_{y}$-by- 1 cell array
$N_{y}$ is the number of outputs of sys, and $N_{u}$ is the number of inputs.
Each entry in a cell array is a row vector that contains the coefficients of the corresponding polynomial. The polynomial coefficients are ordered the same way as the SISO case.


## $d A, d B, d C, d D, d F$

Uncertainties in the estimated polynomial coefficients of sys.
$d A, d B, d C, d D$, and $d F$ are row vectors or cell arrays whose dimensions exactly match the corresponding $\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}$, and F outputs.

Each entry in $\mathrm{dA}, \mathrm{dB}, \mathrm{dC}, \mathrm{dD}$, and dF gives the standard deviation of the corresponding estimated coefficient. For example, $\mathrm{dA}\{1,1\}(2)$ gives the standard deviation of the estimated coefficient returned at $A\{1,1\}(2)$.

## Examples

## Extract Polynomial Coefficients and Uncertainties from Identified Model

Load system data and estimate a 2 -input, 2-output model.

```
load iddata1 z1
load iddata2 z2
data = [z1 z2(1:300)];
nk = [1 1; 1 0];
na = [2 2; 1 3];
```

```
nb = [2 3; 1 4];
nc = [2;3];
nd = [1;2];
nf = [2 2;2 1];
sys = polyest(data,[na nb nc nd nf nk]);
```

The data loaded into $z 1$ and $z 2$ is discrete-time iddata with a sample time of 0.1 s . Therefore, sys is a two-input, two-output discrete-time idpoly model of the form:

$$
A\left(q^{-1}\right) y(t)=\frac{B\left(q^{-1}\right)}{F\left(q^{-1}\right)} u(t-n k)+\frac{C\left(q^{-1}\right)}{D\left(q^{-1}\right)} e(t)
$$

The inputs to polyest set the order of each polynomial in sys.
Access the estimated polynomial coefficients of sys and the uncertainties in those coefficients.
[A, B , C , D, F, dA , dB, dC, dD, dF] = polydata(sys);
The outputs $A, B, C, D$, and $F$ are cell arrays of coefficient vectors. The dimensions of the cell arrays are determined by the input and output dimensions of sys. For example, $A$ is a 2 -by- 2 cell array because sys has two inputs and two outputs. Each entry in A is a row vector containing identified polynomial coefficients. For example, examine the second diagonal entry in A.

```
A{2,2}
ans = 1\times4
    1.0000 -0.8825 -0.2030 0.4364
```

For discrete-time sys, the coefficients are arranged in order of increasing powers of $q^{-1}$. Therefore, A $\{2,2\}$ corresponds to the polynomial $1-0.8682 q^{-1}-0.2244 q^{-2}+0.4467 q^{-3}$.

The dimensions of dA match those of A. Each entry in dA gives the standard deviation of the corresponding estimated polynomial coefficient in A. For example, examine the uncertainties of the second diagonal entry in A .
dA\{2,2\}
ans $=1 \times 4$

$$
\begin{array}{llll}
0 & 0.2849 & 0.4269 & 0.2056
\end{array}
$$

The lead coefficient of $A\{2,2\}$ is fixed at 1 , and therefore has no uncertainty. The remaining entries in $\mathrm{dA}\{2,2\}$ are the uncertainties in the $q^{-1}, q^{-2}$, and $q^{-3}$ coefficients, respectively.

## See Also

iddata|idpoly|idssdata|polyest|tfdata|zpkdata
Introduced before R2006a

## polyest

Estimate polynomial model using time- or frequency-domain data

## Syntax

```
sys = polyest(data,[na nb nc nd nf nk])
sys = polyest(data,[na nb nc nd nf nk],Name,Value)
sys = polyest(data,init_sys)
sys = polyest(__, , opt)
[sys,ic] = polyest(
```

$\qquad$

## Description

sys $=$ polyest(data, [na nb nc nd nf nk]) estimates a polynomial model, sys, using the time- or frequency-domain data, data.
sys is of the form

$$
A(q) y(t)=\frac{B(q)}{F(q)} u(t-n k)+\frac{C(q)}{D(q)} e(t)
$$

$A(q), B(q), F(q), C(q)$ and $D(q)$ are polynomial matrices. $u(t)$ is the input, and nk is the input delay. $y(t)$ is the output and $e(t)$ is the disturbance signal. na $n \mathrm{nb}, \mathrm{nc}$, nd and nf are the orders of the $A(q), B(q)$, $C(q), D(q)$ and $F(q)$ polynomials, respectively.
sys = polyest(data, [na nb nc nd nf nk], Name, Value) estimates a polynomial model with additional attributes of the estimated model structure specified by one or more Name, Value pair arguments.
sys = polyest(data,init_sys) estimates a polynomial model using the linear system init_sys to configure the initial parameterization.
sys = polyest( $\qquad$ , opt) estimates a polynomial model using the option set, opt, to specify estimation behavior.
[sys,ic] = polyest( $\qquad$ ) returns the estimated initial conditions as an initialCondition object. Use this syntax if you plan to simulate or predict the model response using the same estimation input data and then compare the response with the same estimation output data. Incorporating the initial conditions yields a better match during the first part of the simulation.

## Input Arguments

data
Estimation data.
For time-domain estimation, data is an iddata object containing the input and output signal values.
You can estimate only discrete-time models using time-domain data. For estimating continuous-time models using time-domain data, see tfest.

For frequency-domain estimation, data can be one of the following:

- Recorded frequency response data (frd or idfrd)
- iddata object with its properties specified as follows:
- InputData - Fourier transform of the input signal
- OutputData - Fourier transform of the output signal
- Domain - 'Frequency'

It may be more convenient to use oe or tfest to estimate a model for frequency-domain data.
na
Order of the polynomial $A(q)$.
na is an $N y$-by- $N y$ matrix of nonnegative integers. $N y$ is the number of outputs, and $N u$ is the number of inputs.
na must be zero if you are estimating a model using frequency-domain data.

## nb

Order of the polynomial $B(q)+1$.
nb is an $N y$-by- $N u$ matrix of nonnegative integers. $N y$ is the number of outputs, and $N u$ is the number of inputs.
nc
Order of the polynomial $C(q)$.
nc is a column vector of nonnegative integers of length $N y . N y$ is the number of outputs.
nc must be zero if you are estimating a model using frequency-domain data.
nd
Order of the polynomial $D(q)$.
nd is a column vector of nonnegative integers of length $N y . N y$ is the number of outputs.
nd must be zero if you are estimating a model using frequency-domain data.
nf
Order of the polynomial $F(q)$.
nf is an $N y$-by- $N u$ matrix of nonnegative integers. $N y$ is the number of outputs, and $N u$ is the number of inputs.
nk
Input delay in number of samples, expressed as fixed leading zeros of the $B$ polynomial.
$n k$ is an $N y$-by-Nu matrix of nonnegative integers.
nk must be zero when estimating a continuous-time model.
opt
Estimation options.
opt is an options set, created using polyest0ptions, that specifies estimation options including:

- Estimation objective
- Handling of initial conditions
- Numerical search method to be used in estimation


## init_sys

Linear system that configures the initial parameterization of sys.
You obtain init_sys by either performing an estimation using measured data or by direct construction.

If init_sys is an idpoly model, polyest uses the parameters and constraints defined in init_sys as the initial guess for estimating sys.

Use the Structure property of init_sys to configure initial guesses and constraints for $A(q), B(q)$, $F(q), C(q)$, and $D(q)$. For example:

- To specify an initial guess for the $A(q)$ term of init_sys, set init_sys.Structure.A.Value as the initial guess.
- To specify constraints for the $B(q)$ term of init_sys:
- Set init_sys.Structure.B.Minimum to the minimum $B(q)$ coefficient values.
- Set init_sys.Structure.B.Maximum to the maximum $B(q)$ coefficient values.
- Set init_sys.Structure.B.Free to indicate which $B(q)$ coefficients are free for estimation.

If init_sys is not an idpoly model, the software first converts init_sys to a polynomial model. polyest uses the parameters of the resulting model as the initial guess for estimation.

If opt is not specified, and init_sys is created by estimation, then the estimation options from init_sys.Report.OptionsUsed are used.

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, ... , NameN, ValueN.

## IODelay

Transport delays. IODelay is a numeric array specifying a separate transport delay for each input/ output pair.

For continuous-time systems, specify transport delays in the time unit stored in the TimeUnit property. For discrete-time systems, specify transport delays in integer multiples of the sample time, Ts.

For a MIMO system with Ny outputs and Nu inputs, set IODelay to a Ny-by-Nu array. Each entry of this array is a numerical value that represents the transport delay for the corresponding input/output pair. You can also set IODelay to a scalar value to apply the same delay to all input/output pairs.

Default: 0 for all input/output pairs

## InputDelay

Input delay for each input channel, specified as a scalar value or numeric vector. For continuous-time systems, specify input delays in the time unit stored in the TimeUnit property. For discrete-time systems, specify input delays in integer multiples of the sample time Ts. For example, InputDelay = 3 means a delay of three sample times.

For a system with Nu inputs, set InputDelay to an Nu-by-1 vector. Each entry of this vector is a numerical value that represents the input delay for the corresponding input channel.

You can also set InputDelay to a scalar value to apply the same delay to all channels.

## Default: 0

## IntegrateNoise

Logical vector specifying integrators in the noise channel.
IntegrateNoise is a logical vector of length $N y$, where $N y$ is the number of outputs.
Setting IntegrateNoise to true for a particular output results in the model:

$$
A(q) y(t)=\frac{B(q)}{F(q)} u(t-n k)+\frac{C(q)}{D(q)} \frac{e(t)}{1-q^{-1}}
$$

Where, $\frac{1}{1-q^{-1}}$ is the integrator in the noise channel, $e(t)$.
Use IntegrateNoise to create an ARIMAX model.
For example,
load iddatal z1;
z1 = iddata(cumsum(z1.y), cumsum(z1.u),z1.Ts,'InterSample','foh');
sys = polyest(z1, [2 22001$], ' I n t e g r a t e N o i s e ', t r u e) ;$

## Output Arguments

## sys

Polynomial model, returned as an idpoly model. This model is created using the specified model orders, delays, and estimation options.

If data.Ts is zero, sys is a continuous-time model representing:

$$
Y(s)=\frac{B(s)}{F(s)} U(s)+E(s)
$$

$Y(s), U(s)$ and $E(s)$ are the Laplace transforms of the time-domain signals $y(t), u(t)$ and $e(t)$, respectively.

Information about the estimation results and options used is stored in the Report property of the model. Report have the following fields:

| Report Field | Description |  |
| :---: | :---: | :---: |
| Status | Summary of the model status, which indicates whether the model was created by construction or obtained by estimation. |  |
| Method | Estimation command used. |  |
| InitialCo ndition | Handling of initial conditions during model estimation, returned as one of the following values: <br> - 'zero' - The initial conditions were set to zero. <br> - 'estimate' - The initial conditions were treated as independent estimation parameters. <br> - 'backcast' - The initial conditions were estimated using the best least squares fit. <br> This field is especially useful to view how the initial conditions were handled when the InitialCondition option in the estimation option set is 'auto'. |  |
| Fit | Quantitative assessment of the estimation, returned as a structure. See "Loss Function and Model Quality Metrics" for more information on these quality metrics. The structure has the following fields: |  |
|  | Field | Description |
|  | FitPerce <br> nt | Normalized root mean squared error (NRMSE) measure of how well the response of the model fits the estimation data, expressed as the percentage fit = 100(1-NRMSE). |
|  | LossFcn | Value of the loss function when the estimation completes. |
|  | MSE | Mean squared error (MSE) measure of how well the response of the model fits the estimation data. |
|  | FPE | Final prediction error for the model. |
|  | AIC | Raw Akaike Information Criteria (AIC) measure of model quality. |
|  | AICc | Small sample-size corrected AIC. |
|  | nAIC | Normalized AIC. |
|  | BIC | Bayesian Information Criteria (BIC). |
| Parameter <br> s | Estimated values of model parameters. |  |
| OptionsUs ed | Option set used for estimation. If no custom options were configured, this is a set of default options. See polyestOptions for more information. |  |
| RandState | State of the random number stream at the start of estimation. Empty, [ ], if randomization was not used during estimation. For more information, see rng. |  |


| Report Field | Description |  |
| :---: | :---: | :---: |
| DataUsed | Attributes of the data used for estimation, returned as a structure with the following fields: |  |
|  | Field | Description |
|  | Name | Name of the data set. |
|  | Type | Data type. |
|  | Length | Number of data samples. |
|  | Ts | Sample time. |
|  | InterSam ple | Input intersample behavior, returned as one of the following values: <br> - 'zoh ' - Zero-order hold maintains a piecewise-constant input signal between samples. <br> - 'foh ' - First-order hold maintains a piecewise-linear input signal between samples. <br> - 'bl' - Band-limited behavior specifies that the continuous-time input signal has zero power above the Nyquist frequency. |
|  | Input0ff set | Offset removed from time-domain input data during estimation. For nonlinear models, it is [ ]. |
|  | Output0f fset | Offset removed from time-domain output data during estimation. For nonlinear models, it is []. |
| Terminati on | Termination conditions for the iterative search used for prediction error minimization, returned as a structure with the following fields: |  |
|  | Field | Description |
|  | WhyStop | Reason for terminating the numerical search. |
|  | Iteratio ns | Number of search iterations performed by the estimation algorithm. |
|  | First0rd erOptima lity | $\infty$-norm of the gradient search vector when the search algorithm terminates. |
|  | FcnCount | Number of times the objective function was called. |
|  | UpdateNo rm | Norm of the gradient search vector in the last iteration. Omitted when the search method is 'lsqnonlin' or 'fmincon'. |
|  | LastImpr ovement | Criterion improvement in the last iteration, expressed as a percentage. Omitted when the search method is 'lsqnonlin' or 'fmincon'. |
|  | $\begin{aligned} & \text { Algorith } \\ & \mathrm{m} \end{aligned}$ | Algorithm used by 'lsqnonlin' or 'fmincon' search method. Omitted when other search methods are used. |
|  | For estimation methods that do not require numerical search optimization, the Termination field is omitted. |  |

For more information on using Report, see "Estimation Report".

## ic

Estimated initial conditions, returned as an initialCondition object or an object array of initialCondition values.

- For a single-experiment data set, ic represents, in state-space form, the free response of the transfer function model ( $A$ and $C$ matrices) to the estimated initial states ( $x_{0}$ ).
- For a multiple-experiment data set with $N_{e}$ experiments, ic is an object array of length $N_{e}$ that contains one set of initialCondition values for each experiment.

If polyest returns ic values of 0 and the you know that you have non-zero initial conditions, set the 'InitialCondition' option in polyestOptions to 'estimate' and pass the updated option set to polyest. For example:

```
opt = polyestOptions('InitialCondition,'estimate')
[sys,ic] = polyest(data,[nb nc nd nf nk],opt)
```

The default 'auto' setting of 'InitialCondition' uses the 'zero' method when the initial conditions have a negligible effect on the overall estimation-error minimization process. Specifying 'estimate' ensures that the software estimates values for ic.

For more information, see initialCondition. For an example of using this argument, see "Obtain Initial Conditions" on page 1-1072.

## Examples

## Estimate Polynomial Model with Redundant Parameterization

Estimate a model with redundant parameterization. That is, a model with all polynomials ( $A, B, C, D$, and $F$ ) active.

Load estimation data.

```
load iddata2 z2;
```

Specify the model orders and delays.

```
na = 2;
nb = 2;
nc = 3;
nd = 3;
nf = 2;
nk = 1;
```

Estimate the model.
sys = polyest(z2,[na nb nc nd nf nk]);

## Estimate Polynomial Model Using Regularization

Estimate a regularized polynomial model by converting a regularized ARX model.
Load estimation data.
load regularizationExampleData.mat m0simdata;
Estimate an unregularized polynomial model of order 20.
m1 = polyest(m0simdata(1:150),[0 20202020 1]);
Estimate a regularized polynomial model of the same order. Determine the Lambda value by trial and error.

```
opt = polyestOptions;
opt.Regularization.Lambda = 1;
m2 = polyest(m0simdata(1:150),[0 20 20 20 20 1],opt);
```

Obtain a lower-order polynomial model by converting a regularized ARX model and reducing its order. Use arxregul to determine the regularization parameters.

```
[L,R] = arxRegul(m0simdata(1:150),[30 30 1]);
opt1 = arxOptions;
opt1.Regularization.Lambda = L;
opt1.Regularization.R = R;
m0 = arx(m0simdata(1:150),[30 30 1],opt1);
mr = idpoly(balred(idss(m0),7));
```

Compare the model outputs against the data.

```
opt2 = compareOptions('InitialCondition','z');
```

compare(m0simdata(150:end),m1,m2,mr,opt2);


## Estimate ARIMAX model

Load input/output data and create cumulative sum input and output signals for estimation.

```
load iddatal z1
data = iddata(cumsum(z1.y),cumsum(z1.u),z1.Ts,'InterSample','foh');
```

Specify the model polynomial orders. Set the orders of the inactive polynomials, $D$ and $F$, to 0 .

```
na = 2;
nb = 2;
nc = 2;
nd = 0;
nf = 0;
nk = 1;
```

Identify an ARIMAX model by setting the 'IntegrateNoise' option to true.
sys = polyest(data,[na nb nc nd nf nk],'IntegrateNoise',true);

## Estimate Multi-Output ARMAX Model

Estimate a multi-output ARMAX model for a multi-input, multi-output data set.
Load estimation data.

```
load iddata1 z1
load iddata2 z2
data = [z1 z2(1:300)];
```

data is a data set with 2 inputs and 2 outputs. The first input affects only the first output. Similarly, the second input affects only the second output.

Specify the model orders and delays. The F and D polynomials are inactive.

```
na = [2 2; 2 2];
nb = [2 2; 3 4];
nk = [1 1; 0 0];
nc = [2;2];
nd = [0;0];
nf = [0 0; 0 0];
```

Estimate the model.
sys = polyest(data,[na nb nc nd nf nk]);
In the estimated ARMAX model, the cross terms, which model the effect of the first input on the second output and vice versa, are negligible. If you assigned higher orders to those dynamics, their estimation would show a high level of uncertainty.

Analyze the results.

```
h = bodeplot(sys);
showConfidence(h,3)
```


## Bode Diagram



The responses from the cross terms show larger uncertainty.

## Obtain Initial Conditions

Load the data.
load iddatalic z1i
Estimate a polynomial model sys and return the initial conditions in ic.

```
na = 2;
nb = 2;
nc = 3;
nd = 3;
nf = 2;
nk = 1;
[sys,ic] = polyest(zli,[na nb nc nd nf nk]);
ic
ic =
    initialCondition with properties:
            A: [7\times7 double]
            X0: [7x1 double]
            C: [0 0 0 0 0 0 1]
```

Ts: 0.1000
ic is an initialCondition object that encapsulates the free response of sys, in state-space form, to the initial state vector in X0. You can incorporate ic when you simulate sys with the z1i input signal and compare the response with the zli output signal.

## Tips

- In most situations, all the polynomials of an identified polynomial model are not simultaneously active. Set one or more of the orders na, nc, nd and nf to zero to simplify the model structure.

For example, you can estimate an Output-Error (OE) model by specifying na, nc and nd as zero.
Alternatively, you can use a dedicated estimating function for the simplified model structure. Linear polynomial estimation functions include oe, bj, arx and armax.

## Alternatives

- To estimate a polynomial model using time-series data, use ar.
- Use polyest to estimate a polynomial of arbitrary structure. If the structure of the estimated polynomial model is known, that is, you know which polynomials will be active, then use the appropriate dedicated estimating function. For examples, for an ARX model, use arx. Other polynomial model estimating functions include, oe, armax, and bj .
- To estimate a continuous-time transfer function, use tfest. You can also use oe, but only with continuous-time frequency-domain data.


## Extended Capabilities

## Automatic Parallel Support

Accelerate code by automatically running computation in parallel using Parallel Computing Toolbox ${ }^{\mathrm{TM}}$.
Parallel computing support is available for estimation using the lsqnonlin search method (requires Optimization Toolbox). To enable parallel computing, use polyestOptions, set SearchMethod to 'lsqnonlin', and set SearchOptions.Advanced.UseParallel to true.

For example:

```
opt = polyestOptions;
opt.SearchMethod = 'lsqnonlin';
opt.SearchOptions.Advanced.UseParallel = true;
```


## See Also

ar|armax|arx|bj|forecast|iddata|idpoly|oe|pem|polyest0ptions|procest|
ssest|tfest

## Topics

"Regularized Estimates of Model Parameters"
"Apply Initial Conditions when Simulating Identified Linear Models"

Introduced in R2012a

## polyestOptions

Option set for polyest

## Syntax

```
opt = polyestOptions
opt = polyestOptions(Name,Value)
```


## Description

opt $=$ polyestOptions creates the default option set for polyest.
opt = polyestOptions(Name,Value) creates an option set with the options specified by one or more Name, Value pair arguments.

## Input Arguments

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

## InitialCondition - Handling of initial conditions <br> 'auto' (default)|'zero'|'estimate'|'backcast'

Handling of initial conditions during estimation, specified as one of the following values:

- 'zero' - The initial condition is set to zero.
- 'estimate' - The initial state is treated as an independent estimation parameter.
- 'backcast' - The initial state is estimated using the best least squares fit.
- 'auto' - The software chooses the method to handle initial states based on the estimation data.


## Focus - Error to be minimized

```
'prediction' (default)|'simulation'
```

Error to be minimized in the loss function during estimation, specified as the comma-separated pair consisting of 'Focus' and one of the following values:

- 'prediction ' - The one-step ahead prediction error between measured and predicted outputs is minimized during estimation. As a result, the estimation focuses on producing a good predictor model.
- 'simulation ' - The simulation error between measured and simulated outputs is minimized during estimation. As a result, the estimation focuses on making a good fit for simulation of model response with the current inputs.

The Focus option can be interpreted as a weighting filter in the loss function. For more information, see "Loss Function and Model Quality Metrics".

## WeightingFilter - Weighting prefilter

[ ] (default) | vector | matrix | cell array | linear system
Weighting prefilter applied to the loss function to be minimized during estimation. To understand the effect of WeightingFilter on the loss function, see "Loss Function and Model Quality Metrics".

Specify WeightingFilter as one of the following values:

- [] - No weighting prefilter is used.
- Passbands - Specify a row vector or matrix containing frequency values that define desired passbands. You select a frequency band where the fit between estimated model and estimation data is optimized. For example, [ $w l, w h$ ] where $w l$ and wh represent lower and upper limits of a passband. For a matrix with several rows defining frequency passbands, [w1l,w1h;w2l,w2h;w3l,w3h; ...], the estimation algorithm uses the union of the frequency ranges to define the estimation passband.

Passbands are expressed in rad/TimeUnit for time-domain data and in FrequencyUnit for frequency-domain data, where TimeUnit and FrequencyUnit are the time and frequency units of the estimation data.

- SISO filter - Specify a single-input-single-output (SISO) linear filter in one of the following ways:
- A SISO LTI model
- $\{A, B, C, D\}$ format, which specifies the state-space matrices of a filter with the same sample time as estimation data.
- \{numerator, denominator\} format, which specifies the numerator and denominator of the filter as a transfer function with same sample time as estimation data.

This option calculates the weighting function as a product of the filter and the input spectrum to estimate the transfer function.

- Weighting vector - Applicable for frequency-domain data only. Specify a column vector of weights. This vector must have the same length as the frequency vector of the data set, Data. Frequency. Each input and output response in the data is multiplied by the corresponding weight at that frequency.


## EnforceStability - Control whether to enforce stability of model <br> false (default) | true

Control whether to enforce stability of estimated model, specified as the comma-separated pair consisting of 'EnforceStability' and either true or false.

This option is not available for multi-output models with a non-diagonal $A$ polynomial array.

## Data Types: logical

## EstimateCovariance - Control whether to generate parameter covariance data true (default) | false

Controls whether parameter covariance data is generated, specified as true or false.
If EstimateCovariance is true, then use getcov to fetch the covariance matrix from the estimated model.

## Display - Specify whether to display the estimation progress <br> 'off' (default)|'on'

Specify whether to display the estimation progress, specified as one of the following values:

- 'on' - Information on model structure and estimation results are displayed in a progress-viewer window.
- 'off' - No progress or results information is displayed.


## InputOffset - Removal of offset from time-domain input data during estimation

[ ] (default) | vector of positive integers | matrix
Removal of offset from time-domain input data during estimation, specified as the comma-separated pair consisting of 'InputOffset' and one of the following:

- A column vector of positive integers of length $N u$, where $N u$ is the number of inputs.
- [ ] - Indicates no offset.
- $N u$-by-Ne matrix - For multi-experiment data, specify Input0ffset as an $N u$-by-Ne matrix. $N u$ is the number of inputs, and $N e$ is the number of experiments.

Each entry specified by InputOffset is subtracted from the corresponding input data.

## OutputOffset - Removal of offset from time-domain output data during estimation

[ ] (default) | vector | matrix
Removal of offset from time-domain output data during estimation, specified as the comma-separated pair consisting of 'OutputOffset' and one of the following:

- A column vector of length $N y$, where $N y$ is the number of outputs.
- [ ] - Indicates no offset.
- $N y$-by-Ne matrix - For multi-experiment data, specify OutputOffset as a $N y$-by-Ne matrix. $N y$ is the number of outputs, and $N e$ is the number of experiments.

Each entry specified by OutputOffset is subtracted from the corresponding output data.

## Regularization - Options for regularized estimation of model parameters

structure
Options for regularized estimation of model parameters. For more information on regularization, see "Regularized Estimates of Model Parameters".

Regularization is a structure with the following fields:

- Lambda - Constant that determines the bias versus variance tradeoff.

Specify a positive scalar to add the regularization term to the estimation cost.
The default value of zero implies no regularization.
Default: 0

- R - Weighting matrix.

Specify a vector of nonnegative numbers or a square positive semi-definite matrix. The length must be equal to the number of free parameters of the model.

For black-box models, using the default value is recommended. For structured and grey-box models, you can also specify a vector of np positive numbers such that each entry denotes the confidence in the value of the associated parameter.

The default value of 1 implies a value of eye (npfree), where npfree is the number of free parameters.

Default: 1

- Nominal - The nominal value towards which the free parameters are pulled during estimation.

The default value of zero implies that the parameter values are pulled towards zero. If you are refining a model, you can set the value to 'model' to pull the parameters towards the parameter values of the initial model. The initial parameter values must be finite for this setting to work.

Default: 0

## SearchMethod - Numerical search method used for iterative parameter estimation <br> 'auto' (default) | 'gn'| 'gna'|'lm'|'grad'|'lsqnonlin'|'fmincon'

Numerical search method used for iterative parameter estimation, specified as the comma-separated pair consisting of 'SearchMethod' and one of the following:

- 'auto' - A combination of the line search algorithms, 'gn', 'lm', 'gna', and 'grad' methods is tried in sequence at each iteration. The first descent direction leading to a reduction in estimation cost is used.
- 'gn' - Subspace Gauss-Newton least squares search. Singular values of the Jacobian matrix less than GnPinvConstant*eps*max(size(J))*norm(J) are discarded when computing the search direction. $J$ is the Jacobian matrix. The Hessian matrix is approximated as $J^{T} J$. If there is no improvement in this direction, the function tries the gradient direction.
- 'gna' - Adaptive subspace Gauss-Newton search. Eigenvalues less than gamma*max(sv) of the Hessian are ignored, where $s v$ contains the singular values of the Hessian. The Gauss-Newton direction is computed in the remaining subspace. gamma has the initial value InitialGnaTolerance (see Advanced in 'Search0ptions' for more information). This value is increased by the factor LMStep each time the search fails to find a lower value of the criterion in fewer than five bisections. This value is decreased by the factor $2 *$ LMStep each time a search is successful without any bisections.
- 'lm' - Levenberg-Marquardt least squares search, where the next parameter value is - pinv (H $+\mathrm{d} * \mathrm{I}) * \mathrm{grad}$ from the previous one. $H$ is the Hessian, $I$ is the identity matrix, and grad is the gradient. $d$ is a number that is increased until a lower value of the criterion is found.
- 'grad ' - Steepest descent least squares search.
- 'lsqnonlin' - Trust-region-reflective algorithm of lsqnonlin. Requires Optimization Toolbox software.
- 'fmincon' - Constrained nonlinear solvers. You can use the sequential quadratic programming (SQP) and trust-region-reflective algorithms of the fmincon solver. If you have Optimization Toolbox software, you can also use the interior-point and active-set algorithms of the fmincon solver. Specify the algorithm in the SearchOptions. Algorithm option. The fmincon algorithms may result in improved estimation results in the following scenarios:
- Constrained minimization problems when there are bounds imposed on the model parameters.
- Model structures where the loss function is a nonlinear or non smooth function of the parameters.
- Multi-output model estimation. A determinant loss function is minimized by default for multioutput model estimation. fmincon algorithms are able to minimize such loss functions directly. The other search methods such as ' lm ' and ' gn ' minimize the determinant loss function by alternately estimating the noise variance and reducing the loss value for a given noise variance value. Hence, the fmincon algorithms can offer better efficiency and accuracy for multi-output model estimations.


## SearchOptions - Option set for the search algorithm

search option set
Option set for the search algorithm, specified as the comma-separated pair consisting of
'SearchOptions ' and a search option set with fields that depend on the value of SearchMethod.

| SearchOptions Structure When SearchMethod is Specified as 'gn ', 'gna', 'lm', 'grad', or |
| :--- |
| 'auto' |
| Field <br> Name Description Default <br> Toleran <br> ce Minimum percentage difference between the current value of the loss <br> function and its expected improvement after the next iteration, specified as <br> a positive scalar. When the percentage of expected improvement is less <br> than Tolerance, the iterations stop. The estimate of the expected loss- <br> function improvement at the next iteration is based on the Gauss-Newton <br> vector computed for the current parameter value. 0.01MaxIter <br> ationsMaximum number of iterations during loss-function minimization, specified <br> as a positive integer. The iterations stop when MaxIterations is reached <br> or another stopping criterion is satisfied, such as Tolerance. |
| Setting MaxIterations = 0 returns the result of the start-up procedure. <br> Use sys. Report. Termination. Iterations to get the actual number <br> of iterations during an estimation, where sys is an idtf model. |


| Field Name | Description |  | Default |
| :---: | :---: | :---: | :---: |
|  | Advanced search settings, specified as a structure with the following fields: |  |  |
|  | Field Name | Description | Default |
|  | GnPinvCons tant | Jacobian matrix singular value threshold, specified as a positive scalar. Singular values of the Jacobian matrix that are smaller than <br> GnPinvConstant*max(size(J)*norm(J)*eps) are discarded when computing the search direction. Applicable when SearchMethod is 'gn'. | 10000 |
|  | InitialGna Tolerance | Initial value of gamma, specified as a positive scalar. Applicable when SearchMethod is 'gna'. | 0.0001 |
|  | LMStartVal ue | Starting value of search-direction length $d$ in the Levenberg-Marquardt method, specified as a positive scalar. Applicable when SearchMethod is ' lm '. | 0.001 |
|  | LMStep | Size of the Levenberg-Marquardt step, specified as a positive integer. The next value of the search-direction length $d$ in the Levenberg-Marquardt method is LMStep times the previous one. Applicable when SearchMethod is ' 1 m '. | 2 |
|  | MaxBisecti ons | Maximum number of bisections used for line search along the search direction, specified as a positive integer. | 25 |
|  | MaxFunctio nEvaluatio ns | Maximum number of calls to the model file, specified as a positive integer. Iterations stop if the number of calls to the model file exceeds this value. | Inf |
|  | MinParamet erChange | Smallest parameter update allowed per iteration, specified as a nonnegative scalar. | 0 |
|  | RelativeIm provement | Relative improvement threshold, specified as a nonnegative scalar. Iterations stop if the relative improvement of the criterion function is less than this value. | 0 |
|  | StepReduct ion | Step reduction factor, specified as a positive scalar that is greater than 1. The suggested parameter update is reduced by the factor StepReduction after each try. This reduction continues until MaxBisections tries are completed or a lower value of the criterion function is obtained. <br> StepReduction is not applicable for SearchMethod ' lm' (Levenberg-Marquardt method). | 2 |

SearchOptions Structure When SearchMethod is Specified as ' Isqnonlin'

| Field Name | Description | Default |
| :---: | :---: | :---: |
| Function Toleranc e | Termination tolerance on the loss function that the software minimizes to determine the estimated parameter values, specified as a positive scalar. <br> The value of FunctionTolerance is the same as that of opt.SearchOptions.Advanced.TolFun. | 1e-5 |
| StepTole rance | Termination tolerance on the estimated parameter values, specified as a positive scalar. <br> The value of StepTolerance is the same as that of opt.SearchOptions.Advanced.TolX. | 1e-6 |
| MaxItera tions | Maximum number of iterations during loss-function minimization, specified as a positive integer. The iterations stop when MaxIterations is reached or another stopping criterion is satisfied, such as FunctionTolerance. <br> The value of MaxIterations is the same as that of opt.SearchOptions.Advanced.MaxIter. | 20 |
| Advanced | Advanced search settings, specified as an option set for lsqnonlin. <br> For more information, see the Optimization Options table in "Optimization Options" (Optimization Toolbox). | Use optimset('lsqnonl in') to create a default option set. |

SearchOptions Structure When SearchMethod is Specified as 'fmincon'

| Field Name | Description | Default |
| :---: | :---: | :---: |
| Algorithm | fmincon optimization algorithm, specified as one of the following: <br> 'sqp ' - Sequential quadratic programming algorithm. The algorithm satisfies bounds at all iterations, and it can recover from NaN or Inf results. It is not a large-scale algorithm. For more information, see "Large-Scale vs. MediumScale Algorithms" (Optimization Toolbox). <br> - 'trust-regionreflective' - Subspace trust-region method based on the interior-reflective Newton method. It is a largescale algorithm. <br> - 'interior-point' -Large-scale algorithm that requires Optimization Toolbox software. The algorithm satisfies bounds at all iterations, and it can recover from NaN or Inf results. <br> - 'active-set' - Requires Optimization Toolbox software. The algorithm can take large steps, which adds speed. It is not a large-scale algorithm. <br> For more information about the algorithms, see "Constrained Nonlinear Optimization Algorithms" (Optimization Toolbox) and "Choosing the Algorithm" (Optimization Toolbox). | 'sqp ' |


| Field Name | Description | Default |
| :--- | :--- | :--- |
| FunctionTolerance | Termination tolerance on the <br> loss function that the software <br> minimizes to determine the <br> estimated parameter values, <br> specified as a positive scalar. | $1 \mathrm{e}-6$ |
| StepTolerance | Termination tolerance on the <br> estimated parameter values, <br> specified as a positive scalar. | $1 \mathrm{e}-6$ |
| MaxIterations | Maximum number of iterations <br> during loss function <br> minimization, specified as a <br> positive integer. The iterations <br> stop when MaxIterations is <br> reached or another stopping <br> criterion is satisfied, such as <br> FunctionTolerance. | 100 |

## Advanced - Additional advanced options

structure
Additional advanced options, specified as a structure with the following fields:

- ErrorThreshold - Specifies when to adjust the weight of large errors from quadratic to linear.

Errors larger than ErrorThreshold times the estimated standard deviation have a linear weight in the loss function. The standard deviation is estimated robustly as the median of the absolute deviations from the median of the prediction errors, divided by 0.7. For more information on robust norm choices, see section 15.2 of [2].

ErrorThreshold $=0$ disables robustification and leads to a purely quadratic loss function. When estimating with frequency-domain data, the software sets ErrorThreshold to zero. For time-domain data that contains outliers, try setting ErrorThreshold to 1.6.

Default: 0

- MaxSize - Specifies the maximum number of elements in a segment when input-output data is split into segments.

MaxSize must be a positive integer.
Default: 250000

- StabilityThreshold - Specifies thresholds for stability tests.

StabilityThreshold is a structure with the following fields:

- s-Specifies the location of the right-most pole to test the stability of continuous-time models. A model is considered stable when its right-most pole is to the left of $s$.

Default: 0

- $z$ - Specifies the maximum distance of all poles from the origin to test stability of discretetime models. A model is considered stable if all poles are within the distance $z$ from the origin.

Default: 1+sqrt(eps)

- AutoInitThreshold - Specifies when to automatically estimate the initial condition.

The initial condition is estimated when

$$
\frac{\left\|y_{p, z}-y_{\text {meas }}\right\|}{\left\|y_{p, e}-y_{\text {meas }}\right\|}>\text { AutoInitThreshold }
$$

- $y_{\text {meas }}$ is the measured output.
- $y_{p, z}$ is the predicted output of a model estimated using zero initial states.
- $y_{p, e}$ is the predicted output of a model estimated using estimated initial states.

Applicable when InitialCondition is 'auto'.
Default: 1.05

## Output Arguments

## opt - Options set for polyest

polyestOptions option set
Option set for polyest, returned as an polyestOptions option set.

## Examples

## Create Default Option Set for Polynomial Estimation

```
opt = polyestOptions;
```


## Specify Options for Polynomial Estimation

Create an option set for polyest where you enforce model stability and set the Display to 'on '.
opt = polyestOptions('EnforceStability',true,'Display','on');
Alternatively, use dot notation to set the values of opt.

```
opt = polyestOptions;
opt.EnforceStability = true;
opt.Display = 'on';
```


## Compatibility Considerations

## Renaming of Estimation and Analysis Options

The names of some estimation and analysis options were changed in R2018a. Prior names still work. For details, see the R2018a release note "Renaming of Estimation and Analysis Options".

## References

[1] Wills, Adrian, B. Ninness, and S. Gibson. "On Gradient-Based Search for Multivariable System Estimates". Proceedings of the 16th IFAC World Congress, Prague, Czech Republic, July 3-8, 2005. Oxford, UK: Elsevier Ltd., 2005.
[2] Ljung, L. System Identification: Theory for the User. Upper Saddle River, NJ: Prentice-Hall PTR, 1999.

See Also<br>polyest<br>Topics<br>"Loss Function and Model Quality Metrics"<br>Introduced in R2012a

## polynomialRegressor

Specify polynomial regressor for nonlinear ARX model

## Description

Polynomial regressors are polynomials that are composed of delayed input and output variables. For example, $y(t-1)^{2}$ and $y(t-1) u(t-1)$ are both polynomial regressors with orders of 2 and variable delays of one sample. A polynomialRegressor object encapsulates a set of polynomial regressors. Use polynomialRegressor objects when you create nonlinear ARX models using idnlarx or nlarx. You can specify polynomialRegressor objects along with linearRegressor and customRegressor objects and combine them into a single combined regressor set.

## Creation

## Syntax

```
pReg = polynomialRegressor(Variables,Lags)
pReg = polynomialRegressor(Variables,Lags,Order)
pReg = polynomialRegressor(Variables,Lags,Order,UseAbsolute)
pReg = polynomialRegressor(Variables,Lags,Order,UseAbsolute,AllowVariableMix)
pReg =
polynomialRegressor(Variables,Lags,Order,UseAbsolute,AllowVariableMix,AllowLa
gMix)
```


## Description

pReg = polynomialRegressor(Variables,Lags) creates a polynomialRegressor object of order 2 that contains output and input names in Variables and the corresponding lags in Lags. For example, if Variables contains ' $y$ ' and lags contains the corresponding lag vector [2 4], then the regressors that use ' $y$ ' are $y(t-2)^{2}$ and $y(t-4)^{2}$.
pReg = polynomialRegressor(Variables,Lags,Order) creates a polynomialRegressor object of order Order .
pReg = polynomialRegressor(Variables,Lags,Order,UseAbsolute) specifies in UseAbsolute whether to use the absolute values of the variables to create the regressors.
pReg = polynomialRegressor(Variables,Lags,Order,UseAbsolute,AllowVariableMix) specifies in AllowVariableMix whether to allow multiple variables in the regressor formulas. For example, if Variables is equal to \{'y', 'u'\}, Lags is equal to $\{1,1\}$, and $\operatorname{Order}$ is equal to 2 , then a value of true for AllowVariableMix results in the inclusion of the mixed-variable regressor $y(t-1) u(t-1)$, along with the single-variable regressors $y(t-1)^{2}$ and $u(t-1)^{2}$.
pReg =
polynomialRegressor(Variables,Lags,Order, UseAbsolute,AllowVariableMix, AllowLa gMix) specifies in AllowLagMix whether to allow different lags in the regressor formulas. For example, if Variables is equal to \{'y', 'u'\}, Lags is equal to $\{2,[03]\}$, Order is equal to 2 ,
and AllowVariableMix is equal to false, then a value of true for AllowLagMix results in the inclusion of the mixed-lag regressor $u(t) u(t-3)$, along with the unique-lag regressors $y(t-2)^{2}, u(t)^{2}$, and $u(t-3)^{2}$. Note that if you set AllowVariableMix to true, then the regressor set will also include $y(t-$ $2) u(t)$ and $y(t-2) u(t-3)$.

## Properties

## Variables - Output and input variable names

cell array of strings | iddata object properties
Output and input variable names, specified as a cell array of strings or a cell array that references the OutputName and InputName properties of an iddata object. Each entry must be a string with no special characters other than white space. For an example of using this property, see "Estimate Nonlinear ARX Model with Polynomial Regressors" on page 1-1089.
Example: \{'y1', 'u1'\}
Example: [z.OutputName; z.InputName]'

## Lags - Lags in each variable

cell array of non-negative integers
Lags in each variable, specified as a 1-by- $n_{\mathrm{v}}$ cell array of non-negative integer row vectors, where $n_{\mathrm{v}}$ is the total number of regressor variables. Each row vector contains $n_{r}$ integers that specify the $n_{r}$ regressor lags for the corresponding variable. For instance, suppose that you want the following regressors:

- Output variable $y_{1}: y_{1}(t-1)^{2}$ and $y_{1}(t-2)^{2}$
- Input variable $u_{1}: u_{1}(t-3)^{2}$

To obtain these lags, set Lags to $\left\{\left[\begin{array}{ll}12] & 3\end{array}\right]\right.$.
If a lag corresponds to an output variable of an idnlarx model, the minimum lag must be greater than or equal to 1 .

For an example of using this property, see "Estimate Nonlinear ARX Model with Polynomial Regressors" on page 1-1089.
Example: \{1 1\}
Example: \{[1 2],[1,3,4]\}

## UseAbsolute - Absolute value indicator

false (default) | logical vector
Absolute value indicator that determines whether to use the absolute value of a regressor variable instead of the signed value, specified as a logical vector with a length equal to the number of variables.

For an example of setting this property, see "Use Absolute Value in Polynomial Regressor Set" on page 1-1091.

Example: [true,false]

## AllowVariableMix - Mixed variables indicator

false (default) | logical vector

Mixed variables indicator that determines whether to use multiple variables in regressor formulas such as $y(t-1) u(t-1)$, specified as a logical vector with a length equal to the number of variables.

For an example of setting this property, see "Use Multiple Variables in Polynomial Regressor Term" on page 1-1091.
Example: [true,false]

## AllowLagMix - Mixed lag indicator

false (default) | logical vector
Mixed lag indicator that determines whether to use different lags in regressor formulas such as $u(t) u(t-3)$, specified as a logical vector with a length equal to the number of variables.

To set this property for an existing nonlinear ARX model sys, use dot notation, as shown in the following command.

For an example of setting this property, see "Use Mixed Lags in Polynomial Regressor Term" on page 1-1092.

Example: [true,false]

## TimeVariable - Name of time variable

't ' (default) | character array | string
Name of the time variable, specified as a valid MATLAB variable name that is distinct from values in Variables.

```
Example: 'ClockTime'
```


## Examples

## Estimate Nonlinear ARX Model with Polynomial Regressors

Load the data and create an iddata object $z$ with a sample time of 0.1 seconds.
load twotankdata y u
z = iddata(y,u,'Ts',0.1);
Specify polynomial regressors that have the forms $u(t-2)^{2}, u(t-4)^{2}$, and $y(t-1)^{2}$.
Use the properties of $z$ to specify the variable names.

```
Variables = [z.OutputName;z.InputName];
```

Specify the lags.
Lags $=\{1,[24]\} ;$
Create the regressor. The default order is 2.

```
pReg = polynomialRegressor(Variables,Lags)
pReg =
Order 2 regressors in variables y1, ul
    Order: 2
```

```
            Variables: {'y1' 'ul'}
                        Lags: {[1] [2 4]}
            UseAbsolute: [0 0]
        AllowVariableMix: 0
            AllowLagMix: 0
    TimeVariable: 't'
Regressors described by this set
```

Use pReg to estimate the nonlinear ARX model.

```
sys = nlarx(z,pReg)
sys =
Nonlinear ARX model with 1 output and 1 input
    Inputs: u1
    Outputs: y1
Regressors:
    Order 2 regressors in variables y1, u1
    List of all regressors
Output function: Wavelet Network with 66 units
Sample time: 0.1 seconds
Status:
Estimated using NLARX on time domain data "z".
Fit to estimation data: 95.97% (prediction focus)
FPE: 5.843e-05, MSE: 5.569e-05
```

View the regressors.

```
getreg(sys)
```

```
ans = 3\times1 cell
    {'y1(t-1)^2'}
    {'u1(t-2)^2'}
    {'u1(t-4)^2'}
```


## Specify Order for Polynomial Regressor

Specify the third-order polynomial regressor $u_{1}(t-2)^{3}$.

```
Variables = 'u1';
Lags = 2;
Order = 3;
pReg = polynomialRegressor(Variables,Lags,Order)
pReg =
Order 3 regressors in variables u1
            Order: 3
        Variables: {'u1'}
            Lags: {[2]}
    UseAbsolute: 0
```

AllowVariableMix: 0
AllowLagMix: 0
TimeVariable: 't'
Regressors described by this set

## Use Absolute Value in Polynomial Regressor Set

Create a second-order polynomial regressor set that uses lags of 3,10 , and 100 in variable y1 and lags of 0 and 4 in variable u1.

```
vars = {'y1','u1'};
lags = {[3 10 100],[0,4]};
```

Specify that the y1 regressor use the absolute value of y 1 .

```
UseAbs = [true,false];
```

Create the polynomial regressor.

```
reg = polynomialRegressor(vars,lags,2,UseAbs)
reg =
Order 2 regressors in variables y1, u1
                    Order: 2
        Variables: {'y1' 'ul'}
                            Lags: {[3 10 100] [0 4]}
            UseAbsolute: [1 0]
        AllowVariableMix: 0
            AllowLagMix: 0
    TimeVariable: 't'
    Regressors described by this set
```


## Use Multiple Variables in Polynomial Regressor Term

Create a polynomial regressor set that includes the terms $y_{1}(t-1)^{2}, u_{1}(t-1)^{2}$, and $y_{1}(t-1) u_{1}(t-1)$.
Specify the variables and lags.

```
vars = {'y1','u1'};
lags = {1, 1};
```

Specify that mixed-variable regressors be created.

```
mixvar = true;
```

Create a second-order polynomial regressor using mixvar. Set the fourth position, which represents the UseAbsolute property, to false.

```
reg = polynomialRegressor(vars,lags,2,false,mixvar)
reg =
Order 2 regressors in variables y1, u1
```

Order: 2
Variables: \{'yl' 'ul'\}
Lags: \{[1] [1]\}
UseAbsolute: [0 0]
AllowVariableMix: 1
AllowLagMix: 0
TimeVariable: 't'

```
Regressors described by this set
```

As an alternative, you can create the regressor specification first using the variables and lags and set the AllowVariableMix property afterward using dot notation.

```
reg1 = polynomialRegressor(vars,lags);
reg1.AllowVariablemix = true
reg1 =
Order 2 regressors in variables yl, u1
            Order: 2
            Variables: {'y1' 'ul'}
                Lags: {[1] [1]}
            UseAbsolute: [0 0]
    AllowVariableMix: 1
        AllowLagMix: 0
        TimeVariable: 't'
    Regressors described by this set
```

Use reg1 in a nonlinear ARX model.

```
load twotankdata y u;
z = iddata(y,u,'Ts',0.1);
sys = nlarx(z,regl);
```

View the regressors.

```
getreg(sys)
ans = 3\times1 cell
    {'y1(t-1)^2' }
    {'ul(t-1)^2' }
    {'y1(t-1)*u1(t-1)'}
```

The regressors include mixed-variable terms.

## Use Mixed Lags in Polynomial Regressor Term

Specify a polynomial regressor set that includes a term of the form $u(t) u(t-3)$.
Specify the variable names and the lags.

```
vars = {'y1','u1'};
lags = {2,[0 3]};
```

Initialize a second-order polynomial regressor.

```
reg = polynomialRegressor(vars,lags);
```

Specify that the regressor use mixed lags.

```
reg.AllowLagMix = true;
```

Use the regressor set in a nonlinear ARX model.
load twotankdata y u;
z = iddata(y,u,'Ts',0.1);
sys = nlarx(z,reg);
View the regressors.

```
getreg(sys)
ans = 4\times1 cell
    {'y1(t-2)^2' }
    {'u1(t)^2' }
    {'u1(t-3)^2' }
    {'u1(t)*u1(t-3)'}
```

The regressors include the mixed-lag term.

## Estimate Nonlinear ARX Model with Polynomial and Linear Regressors

Load the data and create an iddata object $z$.

```
load twotankdata y u
z = iddata(y,u,'Ts',0.1);
```

Specify polynomial regressors that have the forms $u(t-2)^{2}$ and $u(t-4)^{2}$. Also specify a linear regressor of the form $y(t-1)$.

Specify the input lag.
uLags $=\left\{\left[\begin{array}{ll}2 & 4\end{array}\right]\right\} ;$
Specify the polynomial regressors. The default regressor order is 2.

```
pReg = polynomialRegressor(z.InputName,uLags);
```

Specify the output lag and specify the linear regressor.

```
lLags = 1;
lReg = linearRegressor(z.OutputName,lLags);
Estimate a nonlinear ARX model.
reg = [pReg;lReg]
reg=2\times1 object
[2 1] array of polynomialRegressor, linearRegressor objects.
1. Order 2 regressors in variables ul
```

```
                Order: 2
                Variables: {'u1'}
                            Lags: {[2 4]}
        UseAbsolute: 0
        AllowVariableMix: 0
        AllowLagMix: 0
        TimeVariable: 't'
2. Linear regressors in variables y1
            Variables: {'y1'}
                Lags: {[1]}
        UseAbsolute: 0
        TimeVariable: 't'
    Regressors described by this set
sys = nlarx(z,reg)
sys =
Nonlinear ARX model with 1 output and 1 input
    Inputs: u1
    Outputs: y1
Regressors:
    1. Linear regressors in variables y1
    2. Order 2 regressors in variables u1
    List of all regressors
Output function: Wavelet Network with }21\mathrm{ units
Sample time: 0.1 seconds
Status:
Estimated using NLARX on time domain data "z".
Fit to estimation data: 96.56% (prediction focus)
FPE: 4.133e-05, MSE: 4.059e-05
```

View the regressors.

```
getreg(sys)
```

```
ans = 3\times1 cell
    {'u1(t-2)^2'}
    {'u1(t-4)^2'}
    {'y1(t-1)' }
```


## Specify Linear, Polynomial, and Custom Regressors

Load the estimation data $z 1$, which has one input and one output, and obtain the output and input names.
load iddatal z1;
names $=$ [z1.OutputName z1.InputName]

```
names = 1\times2 cell
    {'y1'} {'ul'}
```

Specify $L$ as the set of linear regressors that represents $y_{1}(t-1), u_{1}(t-2)$, and $u_{1}(t-5)$.
$\mathrm{L}=$ linearRegressor(names,\{1,[2 5]\});
Specify $P$ as the polynomial regressor $y_{1}(t-1)^{2}$.
$P=$ polynomialRegressor(names(1),1,2);
Specify C as the custom regressor $y_{1}(t-2) u_{1}(t-3)$. Use an anonymous function handle to define this function.

```
C = customRegressor(names,{2 3},@(x,y)x.*y)
C =
Custom regressor: y1(t-2).*ul(t-3)
    VariablesToRegressorFcn: @(x,y)x.*y
            Variables: {'y1' 'ul'}
                Lags: {[2] [3]}
            Vectorized: 1
            TimeVariable: 't'
    Regressors described by this set
```

Combine the regressors in the column vector $R$.
$R=[L ; P ; C]$
$\mathrm{R}=3 \times 1$ object
[3 1] array of linearRegressor, polynomialRegressor, customRegressor objects.

1. Linear regressors in variables y1, u1

Variables: \{'y1' 'ul'\}
Lags: \{[1] [2 5]\}
UseAbsolute: [0 0]
TimeVariable: 't'
2. Order 2 regressors in variables y1 Order: 2
Variables: \{'yl'\}
Lags: \{[1]\}
UseAbsolute: 0
AllowVariableMix: 0
AllowLagMix: 0
TimeVariable: 't'
3. Custom regressor: y1(t-2).*ul(t-3)

VariablesToRegressorFcn: @(x,y)x.*y
Variables: \{'yl' 'ul'\}
Lags: \{[2] [3]\}
Vectorized: 1
TimeVariable: 't'

Regressors described by this set

Estimate a nonlinear ARX model with R.

```
sys = nlarx(z1,R)
sys =
Nonlinear ARX model with 1 output and 1 input
    Inputs: ul
    Outputs: y1
Regressors:
    1. Linear regressors in variables y1, ul
    2. Order 2 regressors in variables y1
    3. Custom regressor: yl(t-2).*ul(t-3)
    List of all regressors
Output function: Wavelet Network with 1 unit
Sample time: 0.1 seconds
Status:
Estimated using NLARX on time domain data "z1"
Fit to estimation data: 59.73% (prediction focus)
FPE: 3.356, MSE: 3.147
```

View the full regressor set.
getreg(sys)
ans $=5 \times 1$ cell
\{'y1(t-1)' \}
\{'u1(t-2)' \}
\{'ul(t-5)' \}
\{'y1(t-1)^2' \}
\{'y1(t-2).*u1(t-3)'\}

See Also<br>customRegressor|getreg|idnlarx|linearRegressor|nlarx<br>Introduced in R2021a

## polyreg

(Not recommended) Powers and products of standard regressors

Note polyreg is not recommended. Use polynomialRegressor instead to create polynomial regressor objects, and them add them direclty to the regressor idnlarx Regressors property. For more information, see "Compatibility Considerations".

## Syntax

$R=$ polyreg(model)
$R=$ polyreg(model,'MaxPower', $n$ )
$R=$ polyreg(model,'MaxPower', $n, '$ CrossTerm',CrossTermVal)

## Description

$R=$ polyreg(model) creates an array $R$ of polynomial regressors up to the power 2 . If a model order has input $u$ and output $y, n a=n b=2$, and delay $n k=1$, polynomial regressors are $y(t-1)^{2}, u(t$ $-1)^{2}, y(t-2)^{2}, u(t-2)^{2}$. model is an idnlarx object. You must add these regressors to the model by assigning the CustomRegressors model property or by using addreg.
$R=$ polyreg(model,'MaxPower',$n$ ) creates an array $R$ of polynomial regressors up to the power $n$. Excludes terms of power 1 and cross terms, such as $y(t-1) * u(t-1)$.
$R=$ polyreg(model,'MaxPower', $n$, 'CrossTerm',CrossTermVal) creates an array $R$ of polynomial regressors up to the power $n$ and includes cross terms (products of standards regressors) when CrossTermVal is 'on'. By default, CrossTermVal is 'off'.

## Examples

## Create Polynomial Regressors Up To Power 2

Estimate a nonlinear ARX model with $n a=2, n b=2$, and $n k=1$, and nonlinearity estimator wavenet.
load iddatal
m = nlarx(z1,[2 2 1]);
Create polynomial regressors.
$R=$ polyreg(m);
Estimate the model.
m = nlarx(z1,[2 2 1],'wavenet','CustomReg',R);
View all model regressors (standard and custom).
getreg(m)

```
ans = 8x1 cell
    {'y1(t-1)'
    {'y1(t-2)'
    {'u1(t-1)' }
    {'ul(t-2)' }
    {'y1(t-1).^2'}
    {'y1(t-2).^2'}
    {'u1(t-1).^2'}
    {'u1(t-2).^2'}
```


## Create Polynomial Regressors Up To Power 3

Estimate a nonlinear ARX model with $n a=2, n b=1$, and $n k=1$, and nonlinearity estimator wavenet.
load iddatal
m = nlarx(z1,[2 1 1]);
Create polynomial regressors.
$R=$ polyreg(m,'MaxPower', 3, 'CrossTerm','on')
16x1 array of Custom Regressors with fields: Function, Arguments, Delays, Vectorized
If the model m has three standard regressors $\mathrm{a}, \mathrm{b}$ and c , then R includes the terms $a^{2}, b^{2}, c^{2}, a b, a c$, $b c, a^{2} b, a^{2} c, a b^{2}, a b c, a c^{2}, b^{2} c, b c^{2}, a^{3}, b^{3}$, and $c^{3}$.

Estimate the model.

```
m = nlarx(z1,[2 1 1],'wavenet','CustomReg',R);
```


## Compatibility Considerations

## polyreg is not recommended

Not recommended starting in R2021a
Starting in R2021a, the polyreg command is not recommended. Use the polynomialRegressor command instead to construct polynomial regressors. Doing so improves the computation speed and the accuracy of results, reduces the memory footprint of the idnlarx object, and improves code generation in Simulink.

After creating a polynomial regressor, add it directly to the idnlarx model Regressor property by using the syntax model.Regressors(end+1) = new_polymomial_regressor_object.

There are no plans to remove polyreg at this time.

```
See Also
getreg|idnlarx|nlarx|polynomialRegressor
```


## Topics

```
"Identifying Nonlinear ARX Models"
```

Introduced in R2007a

1-1099

## poly1d

Class representing single-variable polynomial nonlinear estimator for Hammerstein-Wiener models

## Syntax

t=poly1d('Degree', n)
t=polyld('Coefficients', C)
$\mathrm{t}=\mathrm{polyld}(\mathrm{n})$

## Description

poly1d is an object that stores the single-variable polynomial nonlinear estimator for HammersteinWiener models.

You can use the constructor to create the nonlinearity object, as follows:
t=poly1d('Degree',n) creates a polynomial nonlinearity estimator object of nth degree.
t=polyld('Coefficients', C) creates a polynomial nonlinearity estimator object with coefficients C.
$\mathrm{t}=\mathrm{polyld}(\mathrm{n})$ a polynomial nonlinearity estimator object of $n$th degree.
Use evaluate $(p, x)$ to compute the value of the function defined by the poly1d object $p$ at $x$.

## poly1d Properties

After creating the object, you can use get or dot notation to access the object property values. For example:
\% List all property values
get (p)
\% Get value of Coefficients property
p.Coefficients

| Property Name | Description |
| :--- | :--- |
| Degree | Positive integer specifies the degree of the polynomial <br> Default=1. <br> For example: <br> poly1d( ' Degree ' ' 3) |
| Coefficients | $1-$ by-( $n+1$ ) matrix containing the polynomial coefficients. |

## Examples

Use poly1s to specify the single-variable polynomial nonlinearity estimator in Hammerstein-Wiener models. For example:

```
m=nlhw(Data,Orders,poly1d('deg',3),[]);
```

where 'deg' is an abbreviation for the property 'Degree'.

## Tips

Use polyld to define a nonlinear function $y=F(x)$, where $F$ is a single-variable polynomial function of $x$ :

$$
F(x)=c(1) x^{n}+c(2) x^{(n-1)}+\ldots+c(n) x+c(n+1)
$$

## See Also

nlhw
Introduced in R2007b

## predict

Predict K-step-ahead model output

## Syntax

```
yp = predict(sys,data,K)
yp = predict(sys,data,K,opt)
[yp,ic,sys_pred] = predict(
```

$\qquad$

```
predict(sys,data,K,
``` \(\qquad\)
``` )
predict(sys,Linespec,data,k
predict(sys1,...,sysN,data,K,_
predict(sys1,Linespec1,...,sysN,LinespecN,data,K,
```

$\qquad$

``` )
```


## Description

This predict command computes the K-step-ahead output of an identified model using measured input-output data. To identify the model, you first collect all the input-output data and then estimate the model parameters offline. To perform online state estimation of a nonlinear system using realtime data, use the predict command for extended and unscented Kalman filters instead.
$y p=p r e d i c t(s y s, d a t a, K)$ predicts the output of an identified model sys, $K$ steps ahead using the measured input-output data.
predict command predicts the output response over the time span of measured data. In contrast, forecast performs prediction into the future in a time range beyond the last instant of measured data. Use predict to validate sys over the time span of measured data.
yp = predict(sys,data, K,opt) uses the option set opt to specify additional prediction options such as handling of initial conditions and data offsets.
[yp,ic,sys_pred] = predict( __ ) also returns the estimated values for initial conditions ic and a predictor model sys_pred. Use this syntax with any of the previous input argument combinations.
predict(sys,data, K, __ ) plots the predicted output. Use with any of the previous input argument combinations. To change display options in the plot, right-click the plot to access the context menu. For more details about the menu, see "Tips" on page 1-1113.

You can also plot the predicted model response using the compare command. The compare command compares the prediction results with observed data and displays a quantitative goodness of fit.
predict(sys, Linespec, data, K, ___ ) uses Linespec to specify the line type, marker symbol, and color.
predict (sys1, ...,sysN, data, K, __ ) plots the predicted outputs for multiple identified models. predict automatically chooses colors and line styles.
predict(sys1,Linespec1,...,sysN,LinespecN,data,K, $\qquad$ ) uses the line type, marker symbol, and color specified for each model.

## Examples

## Predict Time Series Model Response

Simulate time-series data.

```
init_sys = idpoly([1 -0.99],[],[1 -1 0.2]);
opt = simOptions('AddNoise',true);
u = iddata([],zeros(400,0),1);
data = sim(init_sys,u,opt);
```

data is an iddata object containing the simulated response data of a time series model.
Estimate an ARMAX model by using data as estimation data.

```
na = 1;
nb = 2;
sys = armax(data(1:200),[na nb]);
```

Predict the output of the model using a prediction horizon of 4.
$\mathrm{K}=4$;
yp = predict(sys,data,K);
yp is an iddata object. The predicted output is returned in the OutputData property of the object.
Compare the predicted and estimated data outputs.

```
plot(data(201:400),yp(201:400));
legend('Estimation data','Predicted data');
```


## Input-Output Data



Alternatively, to plot the predicted response and estimation data, use compare(sys,data,K).

## Plot Predicted Output for Multiple Models

Load the estimation data.
load iddatal;
data = z1;
Estimate an ARX model of order [2 21 1].
sys1 = arx(data,[2 2 1]);
Estimate a transfer function with 2 poles.

```
sys2 = tfest(data,2);
```

Create a predict option set to specify zero initial conditions for prediction.
opt = predictOptions('InitialCondition','z');
Plot the predicted outputs for the estimated models. Use the specified prediction option set, opt, and specify prediction horizon as 10 . Specify line styles for plotting the predicted output of each system.
predict(sys1,'r--',sys2,'b',data,10,opt);


To change the display options, right-click the plot to access the context menu. For example, to view the estimation data, select Show Validation Data from the context menu. To view the prediction error, select Prediction Error Plot.

You can also plot the predicted response using the compare command. To do so, first create an option set for compare to specify the use of zero initial conditions.

```
opt = compareOptions('InitialCondition','z');
compare(data,sys1,'r--',sys2,'b',10,opt);
```



## Reproduce Prediction Results by Simulation

Use estimation data to estimate a model, and then compute the predicted model output and predictor model using the predict command. Simulate the predictor model to reproduce the predicted output.

Load estimation data.

```
load iddata3 z3
data = z3;
```

Estimate a polynomial model from the data.
sys = polyest(z3,[2 22001$])$;
Predict the system response using prediction horizon 4.
K = 4;
[yp,ic,sysp] = predict(sys,data,K);
$y p$ is the predicted model response, ic contains the estimated initial conditions, and sysp is the predictor model.

Simulate the predictor model with inputs [data.OutputData, data.InputData] and initial conditions ic.

```
opt = simOptions;
opt.InitialCondition = ic;
ys = sim(sysp,[data.OutputData,data.InputData],opt);
```

Plot the predicted and simulated outputs.
$\mathrm{t}=\mathrm{yp}$.SamplingInstants;
plot(t,yp.OutputData,'b',t,ys,'.r');
legend('Predicted Output','Simulated Output')


## Predict Model Using Initial Conditions Obtained During Estimation

Incorporate initial conditions that you obtained previously into your model prediction.
Load the data.
load iddatalic z1i
Specify the ARMAX estimation option to estimate the initial state.
estimOpt = armax0ptions('InitialCondition','estimate');
Estimate an ARMAX model and return an initialCondition object ic that encapsulates the initial conditions in state-space form.

```
na = 2;
nb = 2;
nc = 2;
nk = 1;
[sys,ic] = armax(z1i,[na nb nc nk],estimOpt);
```

Specify the initial conditions for prediction.

```
predictOpt = predictOptions('InitialCondition',ic);
```

Predict the model and obtain the model response. Plot the response $y$ with the measured data.

```
y = predict(sys,z1i,predictOpt);
plot(zli,y)
legend('Measured Data','Predicted Response')
```

Input-Output Data


The measured and predicted responses show good agreement at the start of the prediction.

## Understand Use of Historical Data for Model Prediction

Perform model prediction using historical data to specify initial conditions. You first predict using the predict command and specify the historical data using the predictOptions option set. You then reproduce the predicted response by manually mapping the historical data to initial states.

Load a two-input, one-output dataset.

```
load iddata7 z7
```

Identify a fifth-order state-space model using the data.

```
sys = n4sid(z7,5);
```

Split the dataset into two parts.

```
zA = z7(1:15);
zB = z7(16:end);
```

Suppose that you want to compute the 10 -step-ahead prediction of the response of the identified system for data $z B$. For initial conditions, use the signal values in $z A$ as the historical record. That is, the input and output values for the time immediately preceding data in zB .

```
IO = struct('Input',zA.InputData,'Output',zA.OutputData);
opt = predictOptions('InitialCondition',IO);
```

Generate the 10 -step-ahead prediction for data $z B$ using the specified initial conditions and predict.

```
[yp,x0,Predictor] = predict(sys,zB,10,opt);
```

$y p$ is the predicted model response, $x 0$ are the initial states corresponding to the predictor model Predictor. You can simulate Predictor using $x 0$ as initial conditions to reproduce yp.OutputData.

Now reproduce the output by manually mapping the historical data to initial states. To do so, minimize 1-step prediction errors over the time span of $z A$.
x0est = data2state(sys,zA);
x0est contains the values of the five states of sys at the time instant immediately after the most recent data sample in zA .

The Predictor has more states than the original system due to the 10 -step prediction horizon. Specify the additional states induced by the horizon to zero initial values, and then append x 0 e es.

```
x0Predictor = zeros(order(Predictor),1);
x0Predictor(end-4:end) = x0est;
```

Simulate the predictor using [zB.OutputData,zB.InputData] as the input signal and x0Predictor as initial conditions.

```
uData = [zB.OutputData,zB.InputData]; % signals required for prediction
[ysim,t,xsim] = lsim(Predictor,uData,[],x0Predictor);
```

Plot the predicted output of the predict command yp.OutputData and the manually computed results ysim.

```
plot(t,yp.OutputData,t,ysim, '.')
```


ysim is the same as yp.OutputData.

## Input Arguments

## sys - Identified model

linear model | nonlinear model
Identified model whose output is to be predicted, specified as one of the following:

- Linear model - idpoly, idproc,idss,idtf, or idgrey
- Nonlinear model - idnlgrey, idnlhw, or idnlarx

When sys is an idnlhw or idnlgrey model, the predicted output yp is the same as the simulated response computed using data. InputData as input.

If a model is unavailable, estimate sys from data using commands such as ar, armax, tfest, nlarx, and ssest.

## data - Measured input-output data

iddata object | matrix of doubles
Measured input-output data, specified as one of the following:

- iddata object - Use observed input and output signals to create an iddata object. For timeseries data (no inputs), specify as an iddata object with no inputs iddata (output, []).
- Matrix of doubles - For models with $N u$ inputs and $N y$ outputs, specify data as an $N$-by- $(N y+N u)$ matrix. Where, $N$ is the number of observations.

For time series data, specify as an $N$-by-Ny matrix.

## K - Prediction horizon

1 (default) | positive integer | Inf
Prediction horizon, specified as one of the following:

- Positive integer - Output yp is calculated K steps into the future, where K represents a multiple of data sample time.

The output at time instant $t$ is calculated using previously measured outputs up to time $t-K$ and inputs up to the time instant t .

- Inf - No previous outputs are used in the computation, and predict returns the same result as simulation using the sim command.

For Output-Error models, there is no difference between the K step-ahead predictions and the simulated output. This is because Output-Error models only use past inputs to predict future outputs.

Note For careful model validation, a one-step-ahead prediction ( $\mathrm{K}=1$ ) is usually not a good test for validating the model sys over the time span of measured data. Even the trivial one step-ahead predictor, $\widehat{y}(t)=y(t-1)$, can give good predictions. So a poor model may look fine for one-stepahead prediction of data that has a small sample time. Prediction with K = Inf, which is the same as performing simulation with sim command, can lead to diverging outputs because low-frequency disturbances in the data are emphasized, especially for models with integration. Use a K value between 1 and Inf to capture the mid-frequency behavior of the measured data.

## opt - Prediction options

predict0ptions option set
Prediction options, specified as a predictOptions option set. Use the option set to specify prediction options such as handling of initial conditions and data offsets.

## Linespec - Line style, marker, and color

## character vector

Line style, marker, and color, specified as a character vector. For example, 'b' or 'b+: '.
For more information about configuring Linespec, see the Linespec argument of plot.

## Output Arguments

## yp - Predicted output response

iddata object | matrix of doubles
Predicted output response, returned as one of the following:

- iddata object - When data is an iddata object. The OutputData property of yp stores the values of the predicted output. The time variable takes values in the range represented by data. SamplingInstants.
- Matrix of doubles - When data is a matrix of doubles.

The output at time instant t is calculated using previously measured outputs up to time $\mathrm{t}-\mathrm{K}$ and inputs up to the time instant $t$. In other words, the predicted response at time point $r$ of measured data is stored in the $r+K-1$ sample of $y p$. Note that at time $r$, the future inputs $u(r+1), u(r+2), \ldots$, $u(r+K)$ required for prediction are assumed to be known. For multi-experiment data, yp contains a predicted data set for each experiment. The time span of the predicted outputs matches that of the observed data.

When sys is specified using an idnlhw or idnlgrey model, yp is the same as the simulated response computed using data. InputData as input.

## ic - Estimated initial conditions

column vector | initialCondition object | cell array
Estimated initial conditions corresponding to the predictor model sys_pred, returned as a column vector, an initialCondition object, or a cell array.

- If sys is a linear transfer function or polynomial model, then ic is an initialCondition object. The initialCondition object encapsulates the free response of sys, in state-space form, with the corresponding initial state vector.
- If sys is any other type of linear or nonlinear dynamic model, then ic is an initial state vector, returned as a column vector of size equal to the number of states.
- If data contains multiexperiment data, then ic is a cell array of size $N e$, where $N e$ is the number of experiments.

To reproduce prediction results, you can simulate sys_pred using ic as the initial conditions. For an example, see "Reproduce Prediction Results by Simulation" on page 1-1106.

If sys is an idnlarx model, ic is returned empty.

## sys_pred - Predictor model

dynamic system model | array of models
Predictor model, returned as a dynamic system model. For multi-experiment data, sys_pred is an array of models, with one entry for each experiment. You can use the predictor model sys_pred and estimated initial conditions ic to reproduce the results of prediction:

- If sys is a linear model, the predictor model is returned as either a model of the same type as sys or as a state-space version of the model (idss). To reproduce the results of prediction, simulate sys_pred using [data.OutputData data.InputData] as the input and ic as the initial conditions. The simulation output is the same as the predicted output yp.OutputData. For an example, see "Reproduce Prediction Results by Simulation" on page 1-1106.
- When sys is a nonlinear grey-box model (idnlgrey) or Hammerstein-Wiener model (idnlhw), the noise-component of the model is trivial, and so the predictor model is the same as the model. sys_pred is returned empty. To reproduce the results of prediction, simulate sys using initial conditions ic. For a definition of the states of idnlhw models, see "Definition of idnlhw States" on page 1-601.
- If sys is a nonlinear ARX model (idnlarx), sys_pred and ic are returned empty. You cannot reproduce the prediction results by simulation.

For discrete-time data that is time-domain or frequency-domain data with sample time Ts greater than zero, sys_pred is a discrete-time model, even if sys is a continuous-time model.

## Tips

- Right-clicking the plot of the predicted output opens the context menu, where you can access the following options:
- Systems - Select systems to view predicted response. By default, the response of all systems is plotted.
- Data Experiment - For multi-experiment data only. Toggle between data from different experiments.
- Characteristics - View the following data characteristics:
- Peak Value - View the absolute peak value of the data. Applicable for time-domain data only.
- Peak Response - View peak response of the data. Applicable for frequency-response data only.
- Mean Value - View mean value of the data. Applicable for time-domain data only.
- Show - For frequency-domain and frequency-response data only.
- Magnitude - View magnitude of frequency response of the system.
- Phase - View phase of frequency response of the system.
- Show Validation Data - Plot data used to predict the model response.
- I/O Grouping - For datasets containing more than one input or output channel. Select grouping of input and output channels on the plot.
- None - Plot input-output channels in their own separate axes.
- All - Group all input channels together and all output channels together.
- I/O Selector - For datasets containing more than one input or output channel. Select a subset of the input and output channels to plot. By default, all output channels are plotted.
- Grid - Add grids to the plot.
- Normalize - Normalize the $y$-scale of all data in the plot.
- Full View - Return to full view. By default, the plot is scaled to full view.
- Prediction Horizon - Set the prediction horizon, or choose simulation.
- Initial Condition - Specify handling of initial conditions. Not applicable for frequencyresponse data.

Specify as one of the following:

- Estimate - Treat the initial conditions as estimation parameters.
- Zero - Set all initial conditions to zero.
- Absorb delays and estimate - Absorb nonzero delays into the model coefficients and treat the initial conditions as estimation parameters. Use this option for discrete-time models only.
- Predicted Response Plot - Plot the predicted model response. By default, the response plot is shown.
- Prediction Error Plot - Plot the error between the model response and prediction data.
- Properties - Open the Property Editor dialog box to customize plot attributes.

See Also<br>compare|forecast|iddata|pe|predict0ptions|sim|simsd<br>\section*{Topics}<br>"Simulate and Predict Identified Model Output"<br>"Simulation and Prediction at the Command Line"<br>Introduced before R2006a

## predict

Predict state and state estimation error covariance at next time step using extended or unscented Kalman filter, or particle filter

## Syntax

[PredictedState,PredictedStateCovariance] = predict(obj)
[PredictedState,PredictedStateCovariance] = predict(obj,Us1,...Usn)

## Description

The predict command predicts the state and state estimation error covariance of an extendedKalmanFilter, unscentedKalmanFilter or particleFilter object at the next time step. To implement the extended or unscented Kalman filter algorithms, use the predict and correct commands together. If the current output measurement exists, you can use predict and correct. If the measurement is missing, you can only use predict. For information about the order in which to use the commands, see "Using predict and correct Commands" on page 1-1121.

Use this predict command for online state estimation using real-time data. When data is not available in real time, to compute the K-step ahead output of an identified model, use predict for offline estimation.
[PredictedState,PredictedStateCovariance] = predict(obj) predicts state estimate and state estimation error covariance of an extended or unscented Kalman filter, or particle filter object obj at the next time step.

You create obj using the extendedKalmanFilter, unscentedKalmanFilter or particleFilter commands. You specify the state transition function and measurement function of your nonlinear system in obj. You also specify whether the process and measurement noise terms are additive or nonadditive in these functions. The State property of the object stores the latest estimated state value. Assume that at time step $k$, obj. State is $\widehat{x}[k \mid k]$. This value is the state estimate for time $k$, estimated using measured outputs until time $k$. When you use the predict command, the software returns $\widehat{x}[k+1 \mid k]$ in the PredictedState output. Where $\widehat{x}[k+1 \mid k]$ is the state estimate for time $k+1$, estimated using measured output until time $k$. The command returns the state estimation error covariance of $\widehat{x}[k+1 \mid k]$ in the PredictedStateCovariance output. The software also updates the State and StateCovariance properties of obj with these corrected values.

Use this syntax if the state transition function $f$ that you specified in obj. StateTransitionFcn has one of the following forms:

- $x(k)=f(x(k-1))-$ for additive process noise.
- $x(k)=f(x(k-1), w(k-1))-$ for nonadditive process noise.

Where x and w are the state and process noise of the system. The only inputs to $f$ are the states and process noise.
[PredictedState,PredictedStateCovariance] = predict(obj,Us1,...Usn) specifies additional input arguments, if the state transition function of the system requires these inputs. You can specify multiple arguments.

Use this syntax if your state transition function $f$ has one of the following forms:

- $x(k)=f(x(k-1), U s 1, \ldots$ Usn $)-$ for additive process noise.
- $x(k)=f(x(k-1), w(k-1), U s 1, \ldots$ Usn $)-$ for nonadditive process noise.


## Examples

## Estimate States Online Using Unscented Kalman Filter

Estimate the states of a van der Pol oscillator using an unscented Kalman filter algorithm and measured output data. The oscillator has two states and one output.

Create an unscented Kalman filter object for the oscillator. Use previously written and saved state transition and measurement functions, vdpStateFcn.m and vdpMeasurementFcn.m. These functions describe a discrete-approximation to a van der Pol oscillator with nonlinearity parameter, mu , equal to 1 . The functions assume additive process and measurement noise in the system. Specify the initial state values for the two states as $[1 ; 0]$. This is the guess for the state value at initial time k , using knowledge of system outputs until time $\mathrm{k}-1, \hat{\chi}[k \mid k-1]$.
obj $=$ unscentedKalmanFilter(@vdpStateFcn,@vdpMeasurementFcn,[1;0]);
Load the measured output data, y , from the oscillator. In this example, use simulated static data for illustration. The data is stored in the vdp_data.mat file.
load vdp_data.mat y
Specify the process noise and measurement noise covariances of the oscillator.

```
obj.ProcessNoise = 0.01;
obj.MeasurementNoise = 0.16;
```

Initialize arrays to capture results of the estimation.

```
residBuf = [];
xcorBuf = [];
xpredBuf = [];
```

Implement the unscented Kalman filter algorithm to estimate the states of the oscillator by using the correct and predict commands. You first correct $\hat{\chi}[k \mid k-1]$ using measurements at time $k$ to get $\hat{x}[k \mid k]$. Then, you predict the state value at next time step, $\hat{x}[k+1 \mid k]$, using $\hat{x}[k \mid k]$, the state estimate at time step k that is estimated using measurements until time k .

To simulate real-time data measurements, use the measured data one time step at a time. Compute the residual between the predicted and actual measurement to assess how well the filter is performing and converging. Computing the residual is an optional step. When you use residual, place the command immediately before the correct command. If the prediction matches the measurement, the residual is zero.

After you perform the real-time commands for the time step, buffer the results so that you can plot them after the run is complete.

```
for k = 1:size(y)
    [Residual,ResidualCovariance] = residual(obj,y(k));
    [CorrectedState,CorrectedStateCovariance] = correct(obj,y(k));
```

```
[PredictedState,PredictedStateCovariance] = predict(obj);
residBuf(k,:) = Residual;
xcorBuf(k,:) = CorrectedState';
xpredBuf(k,:) = PredictedState';
```

end
When you use the correct command, obj. State and obj. StateCovariance are updated with the corrected state and state estimation error covariance values for time step k, CorrectedState and CorrectedStateCovariance. When you use the predict command, obj. State and obj. StateCovariance are updated with the predicted values for time step $\mathrm{k}+1$, PredictedState and PredictedStateCovariance.

In this example, you used correct before predict because the initial state value was $\hat{x}[k \mid k-1]$, a guess for the state value at initial time k using system outputs until time k -1. If your initial state value is $\hat{x}[k-1 \mid k-1]$, the value at previous time $k-1$ using measurement until $k-1$, then use the predict command first. For more information about the order of using predict and correct, see "Using predict and correct Commands" on page 1-1121.

Plot the estimated states, using postcorrection values.
plot(xcorBuf(:,1), xcorBuf(:,2))
title('Estimated States')

Estimated States


Plot the actual measurement, the corrected estimated measurement, and the residual. For the measurement function in vdpMeasurementFcn, the measurement is the first state.

```
M = [y,xcorBuf(:,1),residBuf];
plot(M)
grid on
title('Actual and Estimated Measurements, Residual')
legend('Measured','Estimated','Residual')
```



The estimate tracks the measurement closely. After the initial transient, the residual remains relatively small throughout the run.

## Estimate States Online using Particle Filter

Load the van der Pol ODE data, and specify the sample time.
vdpODEdata.mat contains a simulation of the van der Pol ODE with nonlinearity parameter mu=1, using ode45, with initial conditions [2;0]. The true state was extracted with sample time $\mathrm{dt}=$ 0.05 .

```
load ('vdpODEdata.mat','xTrue','dt')
```

tSpan $=0: d t: 5$;

Get the measurements. For this example, a sensor measures the first state with a Gaussian noise with standard deviation 0.04.
sqrtR = 0.04;
yMeas $=x \operatorname{True}(:, 1)+$ sqrtR* randn(numel(tSpan),1);

Create a particle filter, and set the state transition and measurement likelihood functions.

```
myPF = particleFilter(@vdpParticleFilterStateFcn,@vdpMeasurementLikelihoodFcn);
```

Initialize the particle filter at state [2; 0] with unit covariance, and use 1000 particles.

```
initialize(myPF,1000,[2;0],eye(2));
```

Pick the mean state estimation and systematic resampling methods.

```
myPF.StateEstimationMethod = 'mean';
```

myPF.ResamplingMethod = 'systematic';

Estimate the states using the correct and predict commands, and store the estimated states.

```
xEst = zeros(size(xTrue));
for k=1:size(xTrue,1)
    xEst(k,:) = correct(myPF,yMeas(k));
    predict(myPF);
end
```

Plot the results, and compare the estimated and true states.
figure(1)
plot(xTrue(:, 1), xTrue(:,2), 'x',xEst(:,1),xEst(:, 2), 'ro')
legend('True','Estimated')


## Specify State Transition and Measurement Functions with Additional Inputs

Consider a nonlinear system with input $u$ whose state $x$ and measurement $y$ evolve according to the following state transition and measurement equations:

$$
\begin{aligned}
& x[k]=\sqrt{x[k-1]+u[k-1]}+w[k-1] \\
& y[k]=x[k]+2 * u[k]+v[k]^{2}
\end{aligned}
$$

The process noise w of the system is additive while the measurement noise v is nonadditive.
Create the state transition function and measurement function for the system. Specify the functions with an additional input $u$.

```
f = @(x,u)(sqrt(x+u));
h = @(x,v,u)(x+2*u+v^2);
```

f and h are function handles to the anonymous functions that store the state transition and measurement functions, respectively. In the measurement function, because the measurement noise is nonadditive, v is also specified as an input. Note that v is specified as an input before the additional input u.

Create an extended Kalman filter object for estimating the state of the nonlinear system using the specified functions. Specify the initial value of the state as 1 and the measurement noise as nonadditive.
obj = extendedKalmanFilter(f,h,1,'HasAdditiveMeasurementNoise',false);
Specify the measurement noise covariance.
obj. MeasurementNoise $=0.01$;
You can now estimate the state of the system using the predict and correct commands. You pass the values of $u$ to predict and correct, which in turn pass them to the state transition and measurement functions, respectively.

Correct the state estimate with measurement $\mathrm{y}[\mathrm{k}]=0.8$ and input $\mathrm{u}[\mathrm{k}]=0.2$ at time step k .
correct(obj,0.8,0.2)
Predict the state at the next time step, given $u[k]=0.2$.

```
predict(obj,0.2)
```

Retrieve the error, or residual, between the prediction and the measurement.
[Residual, ResidualCovariance] = residual(obj, 0.8,0.2);

## Input Arguments

obj - Extended or unscented Kalman filter, or particle filter object extendedKalmanFilter object | unscentedKalmanFilter object | particleFilter object

Extended or unscented Kalman filter, or particle filter object for online state estimation, created using one of the following commands:

- extendedKalmanFilter - Uses the extended Kalman filter algorithm.
- unscentedKalmanFilter - Uses the unscented Kalman filter algorithm.
- particleFilter - Uses the particle filter algorithm.


## Us1, ... Usn - Additional input arguments to state transition function input arguments of any type

Additional input arguments to state transition function, specified as input arguments of any type. The state transition function, $f$, is specified in the StateTransitionFcn property of the object. If the function requires input arguments in addition to the state and process noise values, you specify these inputs in the predict command syntax.

For example, suppose that your state transition function calculates the predicted state $x$ at time step k using system inputs $u(k-1)$ and time $k-1$, in addition to the state $\times(k-1)$ :
$x(k)=f(x(k-1), u(k-1), k-1)$
Then when you perform online state estimation at time step $k$, specify these additional inputs in the predict command syntax:
[PredictedState, PredictedStateCovariance] = predict(obj,u(k-1),k-1);

## Output Arguments

## PredictedState - Predicted state estimate

vector
Predicted state estimate, returned as a vector of size $M$, where $M$ is the number of states of the system. If you specify the initial states of obj as a column vector then $M$ is returned as a column vector, otherwise $M$ is returned as a row vector.

For information about how to specify the initial states of the object, see the extendedKalmanFilter, unscentedKalmanFilter and particleFilter reference pages.

## PredictedStateCovariance - Predicted state estimation error covariance

 matrixPredicted state estimation error covariance, returned as an $M$-by- $M$ matrix, where $M$ is the number of states of the system.

## More About

## Using predict and correct Commands

After you have created an extended or unscented Kalman filter, or particle filter object, obj, to implement the estimation algorithms, use the correct and predict commands together.

At time step $k$, correct command returns the corrected value of states and state estimation error covariance using measured system outputs $y[k]$ at the same time step. If your measurement function has additional input arguments $\mathrm{U}_{\mathrm{m}}$, you specify these as inputs to the correct command. The command passes these values to the measurement function.
[CorrectedState,CorrectedCovariance] $=\operatorname{correct(obj,y,Um)~}$

The correct command updates the State and StateCovariance properties of the object with the estimated values, CorrectedState and CorrectedCovariance.

The predict command returns the prediction of state and state estimation error covariance at the next time step. If your state transition function has additional input arguments $U_{s}$, you specify these as inputs to the predict command. The command passes these values to the state transition function.
[PredictedState,PredictedCovariance] = predict(obj,Us)
The predict command updates the State and StateCovariance properties of the object with the predicted values, PredictedState and PredictedCovariance.

If the current output measurement exists at a given time step, you can use correct and predict. If the measurement is missing, you can only use predict. For details about how these commands implement the algorithms, see "Extended and Unscented Kalman Filter Algorithms for Online State Estimation".

The order in which you implement the commands depends on the availability of measured data $\mathrm{y}, \mathrm{U}_{\mathrm{s}}$, and $\mathrm{U}_{\mathrm{m}}$ for your system:

- correct then predict - Assume that at time step k, the value of obj. State is $\widehat{x}[k \mid k-1]$. This value is the state of the system at time $k$, estimated using measured outputs until time $k-1$. You also have the measured output $y[k]$ and inputs $U_{s}[k]$ and $U_{m}[k]$ at the same time step.

Then you first execute the correct command with measured system data y [k] and additional inputs $U_{m}[k]$. The command updates the value of obj. State to be $\widehat{x}[k \mid k]$, the state estimate for time $k$, estimated using measured outputs up to time $k$. When you then execute the predict command with input $U_{s}[k]$, obj. State now stores $\widehat{x}[k+1 \mid k]$. The algorithm uses this state value as an input to the correct command in the next time step.

- predict then correct - Assume that at time step k, the value of obj. State is $\widehat{x}[k-1 \mid k-1]$. You also have the measured output $y[k]$ and input $U_{m}[k]$ at the same time step but you have $\mathrm{U}_{\mathrm{s}}[\mathrm{k}-1]$ from the previous time step.

Then you first execute the predict command with input $U_{s}[k-1]$. The command updates the value of obj. State to $\widehat{x}[k \mid k-1]$. When you then execute the correct command with input arguments $y[k]$ and $U_{m}[k]$, obj. State is updated with $\widehat{x}[k \mid k]$. The algorithm uses this state value as an input to the predict command in the next time step.

Thus, while in both cases the state estimate for time $k, \widehat{x}[k \mid k]$ is the same, if at time $k$ you do not have access to the current state transition function inputs $U_{s}[k]$, and instead have $U_{s}[k-1]$, then use predict first and then correct.

For an example of estimating states using the predict and correct commands, see "Estimate States Online Using Unscented Kalman Filter" on page 1-1116 or "Estimate States Online using Particle Filter" on page 1-1118.

## See Also

clone|correct|extendedKalmanFilter|initialize|particleFilter|residual| unscentedKalmanFilter

## Topics

"Nonlinear State Estimation Using Unscented Kalman Filter and Particle Filter"
"Generate Code for Online State Estimation in MATLAB"
"What Is Online Estimation?"
"Extended and Unscented Kalman Filter Algorithms for Online State Estimation"

Introduced in R2016b

## predictOptions

Option set for predict

## Syntax

```
opt = predictOptions
opt = predictOptions(Name,Value)
```


## Description

opt $=$ predictOptions creates the default option set for predict. Use dot notation to modify this option set. Any options that you do not modify retain their default values.
opt = predictOptions(Name, Value) creates an option set with options specified by one or more Name, Value pair arguments.

## Examples

## Specify Output Offset for Predicting Model Response

Create a default option set for model prediction.
opt = predictOptions;
Specify the output offsets for a two-output model as 2 and 5, respectively.
opt.OutputOffset $=$ [2;5];
The software subtracts the offset value OutputOffset (i) from the $i$ th output signal before using the output to predict the model response. The software then adds back these offsets to the predicted response to give the final response.

## Specify Zero Initial Conditions for Model Prediction

Create an option set for predict using zero initial conditions.
opt = predictOptions('InitialCondition','z');

## Use Historical Data to Specify Initial Conditions for Model Prediction

Load a two-input, one-output dataset.
load iddata7 z7
Identify a fifth-order state-space model using the data.
sys $=n 4 s i d(z 7,5)$;
Split the dataset into two parts.

```
zA = z7(1:15);
zB = z7(16:end);
```

Suppose that you want to compute the 10-step-ahead prediction of the response of the identified system for data $z B$. For initial conditions, use the signal values in $z A$ as the historical record. That is, the input and output values for the time immediately preceding data in $z B$.

```
IO = struct('Input',zA.InputData,'Output',zA.OutputData);
opt = predictOptions('InitialCondition',IO);
```

Generate the 10-step-ahead prediction for data zB using the specified initial conditions.

```
[yp,x0,Predictor] = predict(sys,zB,10,opt);
```

yp is the predicted model response, $x 0$ are the initial states corresponding to the predictor model Predictor. You can simulate Predictor using x0 as initial conditions to reproduce yp.OutputData.

To understand how the past data is mapped to the initial states of the model, see "Understand Use of Historical Data for Model Prediction" on page 1-1108.

## Input Arguments

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Example: predictOptions('InitialCondition', 'z') specifies zero initial conditions for the measured input-output data.

InitialCondition - Handling of initial conditions
'e' (default)|'z'|'d'|column vector | matrix | initialCondition object | object array |
structure | idpar object x00bj
Handling of initial conditions, specified as the comma-separated pair consisting of 'InitialCondition' and one of the following values:

- 'z' - Zero initial conditions.
- 'e' - Estimate initial conditions such that the prediction error for observed output is minimized.

For nonlinear grey-box models, only those initial states $i$ that are designated as free in the model (sys.InitialStates(i).Fixed $=$ false) are estimated. To estimate all the states of the model, first specify all the Nx states of the idnlgrey model sys as free.

```
for i = 1:Nx
sys.InitialStates(i).Fixed = false;
end
```

Similarly, to fix all the initial states to values specified in sys. InitialStates, first specify all the states as fixed in the sys. InitialStates property of the nonlinear grey-box model.

- 'd' - Similar to 'e', but absorbs nonzero delays into the model coefficients. The delays are first converted to explicit model states, and the initial values of those states are also estimated and returned.

Use this option for linear models only.

- Vector or Matrix - Initial guess for state values, specified as a numerical column vector of length equal to the number of states. For multi-experiment data, specify a matrix with Ne columns, where $N e$ is the number of experiments. Otherwise, use a column vector to specify the same initial conditions for all experiments. Use this option for state-space (idss and idgrey) and nonlinear models (idnlarx, idnlhw, and idnlgrey) only.
- initialCondition object - initialCondition object that represents a model of the free response of the system to initial conditions. For multiexperiment data, specify a 1-by- $N_{e}$ array of objects, where $N_{e}$ is the number of experiments.

Use this option for linear models only.

- Structure with the following fields, which contain the historical input and output values for a time interval immediately before the start time of the data used in the prediction:

| Field | Description |
| :--- | :--- |
| Input | Input history, specified as a matrix with $N u$ columns, where $N u$ is the number of <br> input channels. For time series models, use []. The number of rows must be <br> greater than or equal to the model order. |
| Output | Output history, specified as a matrix with $N y$ columns, where $N y$ is the number of <br> output channels. The number of rows must be greater than or equal to the model <br> order. |

For an example, see "Use Historical Data to Specify Initial Conditions for Model Prediction" on page 1-1124.

For multi-experiment data, configure the initial conditions separately for each experiment by specifying InitialCondition as a structure array with $N e$ elements. To specify the same initial conditions for all experiments, use a single structure.

The software uses data2state to map the historical data to states. If your model is not idss, idgrey, idnlgrey, or idnlarx, the software first converts the model to its state-space representation and then maps the data to states. If conversion of your model to idss is not possible, the estimated states are returned empty.

- x0obj - Specification object created using idpar. Use this object for discrete-time state-space (idss and idgrey) and nonlinear grey-box (idnlgrey) models only. Use x0obj to impose constraints on the initial states by fixing their value or specifying minimum or maximum bounds.


## InputOffset - Input signal offset

[ ] (default) | column vector | matrix
Input signal offset for time-domain data, specified as the comma-separated pair consisting of 'Input0ffset' and one of the following values:

- [] - No input offsets.
- A column vector of length $N u$, where $N u$ is the number of inputs. The software subtracts the offset value InputOffset(i) from the ith input signal before using the input to predict the model response.
- $N u$-by-Ne matrix - For multi-experiment data, specify InputOffset as an $N u$-by-Ne matrix, where $N e$ is the number of experiments. The software subtracts the offset value InputOffset ( $\mathrm{i}, \mathrm{j}$ ) from the $i$ th input signal of the $j$ th experiment before prediction.

If you specify a column vector of length $N u$, then the offset value InputOffset (i) is subtracted from the $i$ th input signal of all the experiments.

## OutputOffset - Output signal offset

[] (default) | column vector | matrix
Output signal offset for time-domain data, specified as the comma-separated pair consisting of
'Output0ffset' and one of the following values:

- [] - No output offsets.
- A column vector of length $N y$, where $N y$ is the number of outputs. The software subtracts the offset value OutputOffset(i) from the ith output signal before using the output to predict the model response. After prediction, the software adds the offsets to the predicted response to give the final predicted response.
- Ny-by-Ne matrix - For multi-experiment data, specify OutputOffset as an $N y$-by-Ne matrix, where $N e$ is the number of experiments. The software subtracts the offset value OutputOffset $(i, j)$ from the $i$ th output signal of the $j$ th experiment before prediction.

If you specify a column vector of length $N y$, then the offset value OutputOffset (i) is subtracted from the ith output signal of all the experiments.

After prediction, the software adds the removed offsets to the predicted response to give the final predicted response.

## OutputWeight - Weight of output for initial condition estimation

[] (default) | 'noise' | matrix
Weight of output for initial condition estimation, specified as the comma-separated pair consisting of 'OutputWeight ' and one of the following values:

- [] - No weighting is used by the software for initial condition estimation. This option is the same as using eye ( Ny ) for the output weight, where $N y$ is the number of outputs.
- 'noise' - The software uses the inverse of the NoiseVariance property of the model as the weight.
- A positive, semidefinite matrix of dimension $N y$-by- $N y$, where $N y$ is the number of outputs.

OutputWeight is relevant only for multi-output models.

## Output Arguments

opt - Option set for predict
predict0ptions option set
Option set for predict, retuned as a predict0ptions option set.

## See Also

absorbDelay|idpar|predict

Introduced in R2012a

## present

Display model information, including estimated uncertainty

## Syntax

present(m)

## Description

present ( $m$ ) displays the linear or nonlinear identified model $m$ and the following information:

- Estimated one standard deviation of the parameters, which gives $68.27 \%$ confidence region
- Termination conditions for iterative estimation algorithms
- Status of the model - whether the model was constructed or estimated
- Fit to estimation data
- Akaike's Final Prediction Error (FPE) criterion
- Mean-square error (MSE)


## Examples

## Display Information About Identified Model

Estimate a transfer function model.

```
load iddatal z1;
np = 2;
sys = tfest(z1,np);
Display model information.
```

```
present(sys)
```

present(sys)
sys =
From input "u1" to output "y1":
2.455 (+/- 1.101) s + 177 (+/- 10.73)
s^2 + 3.163 (+/- 0.2522) s + 23.16 (+/- 1.115)
Continuous-time identified transfer function.
Parameterization:
Number of poles: 2 Number of zeros: 1
Number of free coefficients: 4
Use "tfdata", "getpvec", "getcov" for parameters and their uncertainties.
Status:
Termination condition: Near (local) minimum, (norm(g) < tol)..

```
```

Number of iterations: 1, Number of function evaluations: 3
Estimated using TFEST on time domain data "z1".
Fit to estimation data: 70.77%
FPE: 1.725, MSE: 1.658
More information in model's "Report" property.

```

\section*{See Also}
frdata|getcov|getpvec|idssdata|polydata|ssdata|tfdata|zpkdata
Topics
"Estimation Report"
"Loss Function and Model Quality Metrics"
Introduced before R2006a

\section*{procest}

Estimate process model using time or frequency data

\section*{Syntax}
sys = procest(data,type)
sys \(=\) procest(data,type,'InputDelay',InputDelay)
sys = procest(data,init_sys)
sys = procest( \(\qquad\) , opt)
[sys,offset] = procest( \(\qquad\) )
[sys,offset,ic] = procest( \(\qquad\)

\section*{Description}
sys \(=\) procest (data, type) estimates a process model, sys, using time or frequency-domain data, data. type defines the structure of sys.
sys \(=\) procest(data, type, 'InputDelay',InputDelay) specifies the input delay InputDelay.
sys = procest(data,init_sys) uses the process model init_sys to configure the initial parameterization.
sys = procest( \(\qquad\) , opt) specifies additional model estimation options. Use opt with any of the previous syntaxes.
[sys, offset] \(=\) procest (___ ) returns the estimated value of the offset in input signal. Input offset is automatically estimated when the model contains an integrator, or when you set the InputOffset estimation option to 'estimate' using procest0ptions. Use offset with any of the previous syntaxes.
[sys,offset,ic] = procest (__ ) returns the estimated initial conditions as an initialCondition object. Use this syntax if you plan to simulate or predict the model response using the same estimation input data and then compare the response with the same estimation output data. Incorporating the initial conditions yields a better match during the first part of the simulation.

\section*{Input Arguments}

\section*{data - Estimation data}
iddata|idfrd|frd
Estimation data, specified as an iddata object containing the input and output signal values, for time-domain estimation. For frequency-domain estimation, data can be one of the following:
- Recorded frequency response data (frd or idfrd)
- iddata object with its properties specified as follows:
- InputData - Fourier transform of the input signal
- OutputData - Fourier transform of the output signal
- Domain - 'Frequency'
data must have at least one input and one output.
Time-series models, which are models that contain no measured inputs, cannot be estimated using procest. Use ar, arx, or armax for time-series models instead.

\section*{type - Process model structure}
character vector \(\mid\) string \(\mid\) cell array of character vectors \(\mid\) string array
Process model structure, specified for SISO models as a character vector or string representing an acronym for the model structure, such as 'P1D' or 'P2DZ'. The acronym is made up of:
- P - All 'Type' acronyms start with this letter.
- \(0,1,2\), or \(3-\) Number of time constants (poles) to be modeled. Possible integrations (poles in the origin) are not included in this number.
- I - Integration is enforced (self-regulating process).
- D - Time delay (dead time).
- Z - Extra numerator term, a zero.
- \(\quad U\) - Underdamped modes (complex-valued poles) permitted. If \(U\) is not included in type, all poles must be real. The number of poles must be 2 or 3 .

For MIMO models, use an Ny-by-Nu cell array of character vectors or string array, with one entry for each input-output pair. Here Ny is the number of inputs and Nu is the number of outputs.

For information regarding how type affects the structure of a process model, see idproc.

\section*{InputDelay - Input delays}

0 for all input channels (default) | numeric vector
Input delays, specified as a numeric vector specifying a time delay for each input channel. Specify input delays in the time unit stored in the TimeUnit property.

For a system with Nu inputs, set InputDelay to an Nu-by-1 vector. Each entry of this vector is a numerical value that represents the input delay for the corresponding input channel. You can also set InputDelay to a scalar value to apply the same delay to all channels.

\section*{init_sys - System for configuring initial parametrization \\ idproc object}

System for configuring initial parametrization of sys, specified as an idproc object. You obtain init_sys by either performing an estimation using measured data or by direct construction using idproc. The software uses the parameters and constraints defined in init_sys as the initial guess for estimating sys.

Use the Structure property of init_sys to configure initial guesses and constraints for \(K p, T_{p 1}\), \(T_{p 2}, T_{p 3}, T_{w}\) Zeta, \(T_{d}\), and \(T_{z}\). For example:
- To specify an initial guess for the \(T_{p 1}\) parameter of init_sys, set
init_sys. Structure. Tp1. Value as the initial guess.
- To specify constraints for the \(T_{p 2}\) parameter of init_sys:
- Set init_sys.Structure.Tp2. Minimum to the minimum \(T_{p 2}\) value.
- Set init_sys.Structure.Tp2. Maximum to the maximum \(T_{p 2}\) value.
- Set init_sys.Structure.Tp2. Free to indicate if \(T_{p 2}\) is a free parameter for estimation.

If opt is not specified, and init_sys was obtained by estimation, then the estimation options from init_sys.Report.OptionsUsed are used.

\section*{opt - Estimation options}
procest0ptions option set
Estimation options, specified as an procestOptions option set. The estimation options include:
- Estimation objective
- Handling on initial conditions and disturbance component
- Numerical search method to be used in estimation

\section*{Output Arguments}

\section*{sys - Identified process model}
idproc model
Identified process model, returned as an idproc model of a structure defined by type.
Information about the estimation results and options used is stored in the model's Report property. Report has the following fields:
\begin{tabular}{|l|l|}
\hline Report Field & Description \\
\hline Status & \begin{tabular}{l} 
Summary of the model status, which indicates whether the model was \\
created by construction or obtained by estimation.
\end{tabular} \\
\hline Method & Estimation command used. \\
\hline InitialCondition & \begin{tabular}{l} 
Handling of initial conditions during model estimation, returned as one \\
of the following values: \\
- 'zero' - The initial conditions were set to zero. \\
- 'estimate ' - The initial conditions were treated as independent \\
estimation parameters. \\
' backcast ' - The initial conditions were estimated using the best \\
least squares fit. \\
This field is especially useful to view how the initial conditions were \\
handled when the InitialCondition option in the estimation option \\
set is 'auto'.
\end{tabular} \\
\hline
\end{tabular}
\begin{tabular}{|l|l|l|}
\hline Report Field & \multicolumn{2}{|l|}{ Description } \\
\hline \multicolumn{1}{|l|}{ Fit } & \begin{tabular}{l} 
Quantitative assessment of the estimation, returned as a structure. See \\
"Loss Function and Model Quality Metrics" for more information on \\
these quality metrics. The structure has the following fields:
\end{tabular} \\
& \begin{tabular}{ll} 
Field & Description \\
& \begin{tabular}{l} 
FitPerc \\
ent
\end{tabular} \\
& \begin{tabular}{l} 
Normalized root mean squared error (NRMSE) measure of \\
how well the response of the model fits the estimation data, \\
expressed as the percentage fit = 100(1-NRMSE).
\end{tabular} \\
\hline & LossFcn
\end{tabular} \begin{tabular}{l} 
Value of the loss function when the estimation completes.
\end{tabular} \\
\hline & \begin{tabular}{l} 
MSE \\
of the model fits the estimation data.
\end{tabular} \\
\hline & FPE & Final prediction error for the model. \\
\hline & AIC & \begin{tabular}{l} 
Raw Akaike Information Criteria (AIC) measure of model \\
quality.
\end{tabular} \\
\hline & AICc & Small sample-size corrected AIC. \\
\hline & nAIC & Normalized AIC. \\
\hline & BIC & Bayesian Information Criteria (BIC). \\
\hline Parameters & Estimated values of model parameters. \\
\hline OptionsUsed & \begin{tabular}{l} 
Option set used for estimation. If no custom options were configured, \\
this is a set of default options. See procest0ptions for more \\
information.
\end{tabular} \\
\hline RandState & \begin{tabular}{l} 
State of the random number stream at the start of estimation. Empty, [ ], \\
if fandomization was not used during estimation. For more information, \\
see rng.
\end{tabular} \\
\hline
\end{tabular}
\begin{tabular}{|c|c|c|}
\hline Report Field & \multicolumn{2}{|l|}{Description} \\
\hline DataUsed & \multicolumn{2}{|l|}{Attributes of the data used for estimation. Structure with the following fields:} \\
\hline & Field & Description \\
\hline & Name & Name of the data set. \\
\hline & Type & Data type. For idnlarx models, this is set to 'Time domain data'. \\
\hline & Length & Number of data samples. \\
\hline & Ts & Sample time. This is equivalent to Data.Ts. \\
\hline & InterSa mple & \begin{tabular}{l}
Input intersample behavior. One of the following values: \\
- 'zoh' - Zero-order hold maintains a piecewise-constant input signal between samples. \\
- ' foh' - First-order hold maintains a piecewise-linear input signal between samples. \\
- 'bl' - Band-limited behavior specifies that the continuous-time input signal has zero power above the Nyquist frequency. \\
The value of Intersample has no effect on estimation results for discrete-time models.
\end{tabular} \\
\hline & Input0f fset & Empty, [ ], for nonlinear estimation methods. \\
\hline & Output0 ffset & Empty, [ ], for nonlinear estimation methods. \\
\hline
\end{tabular}
\begin{tabular}{|l|}
\hline Report Field \\
\hline Termination \\
\\
\\
\hline
\end{tabular}
\begin{tabular}{|c|c|}
\hline \multicolumn{2}{|l|}{Description} \\
\hline \multicolumn{2}{|l|}{Termination conditions for the iterative search used for prediction error minimization, returned as a structure with the following fields:} \\
\hline Field & Description \\
\hline WhyStop & Reason for terminating the numerical search. \\
\hline Iterati ons & Number of search iterations performed by the estimation algorithm. \\
\hline First0r derOpti mality & \(\infty\)-norm of the gradient search vector when the search algorithm terminates. \\
\hline \[
\begin{array}{|l|}
\hline \begin{array}{l}
\text { FcnCoun } \\
\mathrm{t}
\end{array} \\
\hline
\end{array}
\] & Number of times the objective function was called. \\
\hline UpdateN orm & Norm of the gradient search vector in the last iteration. Omitted when the search method is ' lsqnonlin' or 'fmincon'. \\
\hline LastImp rovemen t & Criterion improvement in the last iteration, expressed as a percentage. Omitted when the search method is 'lsqnonlin' or 'fmincon'. \\
\hline Algorit hm & Algorithm used by 'lsqnonlin' or 'fmincon' search method. Omitted when other search methods are used. \\
\hline
\end{tabular}

For estimation methods that do not require numerical search optimization, the Termination field is omitted.

For more information on using Report, see "Estimation Report".

\section*{offset - Estimated value of input offset}
vector
Estimated value of input offset, returned as a vector. When data has multiple experiments, offset is a matrix where each column corresponds to an experiment.

\section*{ic - Initial conditions}
initialCondition object | object array of initialCondition values
Estimated initial conditions, returned as an initialCondition object or an object array of initialCondition values.
- For a single-experiment data set, ic represents, in state-space form, the free response of the transfer function model ( \(A\) and \(C\) matrices) to the estimated initial states ( \(x_{0}\) ).
- For a multiple-experiment data set with \(N_{e}\) experiments, ic is an object array of length \(N_{e}\) that contains one set of initialCondition values for each experiment.

If procest returns ic values of 0 and the you know that you have non-zero initial conditions, set the
'InitialCondition' option in procestOptions to 'estimate' and pass the updated option set to procest. For example:
```

opt = procestOptions('InitialCondition,'estimate')
[sys,offset,ic] = procest(data,np,nz,opt)

```

The default 'auto' setting of 'InitialCondition' uses the 'zero' method when the initial conditions have a negligible effect on the overall estimation-error minimization process. Specifying 'estimate' ensures that the software estimates values for ic.

For more information, see initialCondition. For an example of using this argument, see "Obtain Initial Conditions" on page 1-1145.

\section*{Examples}

\section*{Estimate a First Order Plus Dead Time Model}

Obtain the measured input-output data.
```

load iddemo_heatexchanger_data;
data = iddata(pt,ct,Ts);
data.InputName = '\Delta CTemp';
data.InputUnit = 'C';
data.OutputName = '\Delta PTemp';
data.OutputUnit = 'C';
data.TimeUnit = 'minutes';

```

Estimate a first-order plus dead time process model.
```

type = 'P1D';

```
sysP1D = procest(data,type);

Compare the model with the data.
```

compare(data,sysP1D)

```

\section*{Simulated Response Comparison}


Plot the model residuals.
```

figure
resid(data,sysP1D);

```


The figure shows that the residuals are correlated. To account for that, add a first order ARMA disturbance component to the process model.
opt = procestOptions('DisturbanceModel','ARMA1');
sysP1D_noise = procest(data,'pld',opt);
Compare the models.
compare(data, sysP1D, sysP1D_noise)

\section*{Simulated Response Comparison}


Plot the model residuals.
```

figure
resid(data,sysP1D_noise);

```

\section*{Residue Correlation}


The residues of sysP1D_noise are uncorrelated.

\section*{Estimate Over-parameterized Process Model Using Regularization}

Use regularization to estimate parameters of an over-parameterized process model.
Assume that gain is known with a higher degree of confidence than other model parameters.
Load data.
load iddatal z1;
Estimate an unregularized process model.
```

m = idproc('P3UZ','K',7.5,'Tw',0.25,'Zeta',0.3,'Tp3',20,'Tz',0.02);
m1 = procest(z1,m);

```

Estimate a regularized process model.
```

opt = procestOptions;
opt.Regularization.Nominal = 'model';
opt.Regularization.R = [100;1;1;1;1];
opt.Regularization.Lambda = 0.1;
m2 = procest(z1,m,opt);

```

Compare the model outputs with data.
compare(z1,m1,m2);

Simulated Response Comparison


Regularization helps steer the estimation process towards the correct parameter values.

\section*{Specify Parameter Initial Values for Estimated Process Model}

Estimate a process model after specifying initial guesses for parameter values and bounding them.
Obtain input/output data.
```

data = idfrd(idtf([10 2],[1 1.3 1.2],'iod',0.45),logspace(-2,2,256));

```

Specify the parameters of the estimation initialization model.
```

type = 'P2UZD';
init_sys = idproc(type);
init_sys.Structure.Kp.Value = 1;
init_sys.Structure.Tw.Value = 2;
init_sys.Structure.Zeta.Value = 0.1;
init_sys.Structure.Td.Value = 0;
init_sys.Structure.Tz.Value = 1;
init_sys.Structure.Kp.Minimum = 0.1;

```
```

init_sys.Structure.Kp.Maximum = 10;
init_sys.Structure.Td.Maximum = 1;
init_sys.Structure.Tz.Maximum = 10;

```

Specify the estimation options.
```

opt = procestOptions('Display','full','InitialCondition','Zero');
opt.SearchMethod = 'lm';
opt.SearchOptions.MaxIterations = 100;
Estimate the process model.
sys = procest(data,init_sys,opt);

```

Since the 'Display ' option is specified as 'full', the estimation progress is displayed in a separate Plant Identification Progress window.

Compare the data to the estimated model.
```

compare(data,sys,init_sys);

```

Frequency Response Comparison


\section*{Detect Overparameterization of Estimated Model}

Obtain input/output data.
```

load iddatal z1
load iddata2 z2
data = [z1 z2(1:300)];

```
data is a data set with 2 inputs and 2 outputs. The first input affects only the first output. Similarly, the second input affects only the second output.

In the estimated process model, the cross terms, modeling the effect of the first input on the second output and vice versa, should be negligible. If higher orders are assigned to those dynamics, their estimations show a high level of uncertainty.

Estimate the process model.
```

type = 'P2UZ';
sys = procest(data,type);

```

The type variable denotes a model with complex-conjugate pair of poles, a zero, and a delay.
To evaluate the uncertainties, plot the frequency response.
```

w = linspace(0,20*pi,100);
h = bodeplot(sys,w);
showConfidence(h);

```

\section*{Bode Diagram}


\section*{Return Input Offsets Estimated During Process Model Estimation}
```

load iddata1
[sys,offset] = procest(z1,'P1DI');
offset
offset = 0.0412

```

\section*{Obtain Initial Conditions}

Load the data.
load iddatalic zli
Estimate a first-order plus dead time process model sys and return the initial conditions in ic. First specify 'estimate' for 'InitialCondition' to force the software to estimate ic. The default 'auto' setting uses the 'estimate' method only when the influence of the initial conditions on the overall model error exceed a threshold. When the initial conditions have a negligible effect on the overall estimation-error minimization process, the 'auto' setting uses 'zero'.
```

opt = procestOptions('InitialCondition','estimate');
[sys,offset,ic] = procest(zli,'P1D',opt);
ic
iC =
initialCondition with properties:
A: -3.8997
X0: - 1.0871
C: 4.5652
Ts: 0

```
ic is an initialCondition object that encapsulates the free response of sys, in state-space form, to the initial state vector in X0. You can incorporate ic when you simulate sys with the zli input signal and compare the response with the zli output signal.

\section*{Extended Capabilities}

\section*{Automatic Parallel Support}

Accelerate code by automatically running computation in parallel using Parallel Computing Toolbox \({ }^{\mathrm{TM}}\).
Parallel computing support is available for estimation using the lsqnonlin search method (requires Optimization Toolbox). To enable parallel computing, use procestOptions, set SearchMethod to 'lsqnonlin', and set SearchOptions.Advanced.UseParallel to true.

For example:
```

opt = procestOptions;
opt.SearchMethod = 'lsqnonlin';
opt.Search0ptions.Advanced.UseParallel = true;

```

\section*{See Also}
ar|arx|bj|idproc|oe|polyest|procestOptions|ssest|tfest

\section*{Topics}
"What Is a Process Model?"
"Regularized Estimates of Model Parameters"
"Apply Initial Conditions when Simulating Identified Linear Models"
Introduced in R2012a

\section*{procestOptions}

Options set for procest

\section*{Syntax}
opt \(=\) procestOptions
opt \(=\) procestOptions(Name,Value)

\section*{Description}
opt \(=\) procestOptions creates the default options set for procest.
opt = procestOptions(Name,Value) creates an option set with the options specified by one or more Name, Value pair arguments.

\section*{Input Arguments}

\section*{Name-Value Pair Arguments}

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

\section*{InitialCondition - Handling of initial conditions}
'auto' (default)|'zero'|'estimate'|'backcast'
Handling of initial conditions during estimation, specified as one of the following values:
- 'zero' - The initial condition is set to zero.
- 'estimate' - The initial condition is treated as an independent estimation parameter.
- 'backcast ' - The initial condition is estimated using the best least squares fit.
- 'auto ' - The software chooses the method to handle initial condition based on the estimation data.

\section*{DisturbanceModel - Handling of additive noise}
```

'estimate' (default)|'none' | 'ARMA1' | 'ARMA2' | 'fixed'

```

Handling of additive noise \((H)\) during estimation for the model
\[
y=G(s) u+H(s) e
\]
\(e\) is white noise, \(u\) is the input and \(y\) is the output.
\(H(s)\) is stored in the NoiseTF property of the numerator and denominator of idproc models.
DisturbanceModel is specified as one of the following values:
- 'none' \(-H\) is fixed to one.
- 'estimate' \(-H\) is treated as an estimation parameter. The software uses the value of the NoiseTF property as the initial guess.
- 'ARMA1 ' - The software estimates \(H\) as a first-order ARMA model
\(\frac{1+c s}{1+d s}\)
- 'ARMA2 ' - The software estimates \(H\) as a second-order ARMA model
\(\frac{1+c_{1} s+c_{2} s^{2}}{1+d_{1} s+d_{2} s^{2}}\)
- ' fixed ' - The software fixes the value of the NoiseTF property of the idproc model as the value of \(H\).

Note A noise model cannot be estimated using frequency domain data.

\section*{Focus - Error to be minimized}
'prediction' (default) | 'simulation'
Error to be minimized in the loss function during estimation, specified as the comma-separated pair consisting of 'Focus' and one of the following values:
- 'prediction' - The one-step ahead prediction error between measured and predicted outputs is minimized during estimation. As a result, the estimation focuses on producing a good predictor model.
- 'simulation' - The simulation error between measured and simulated outputs is minimized during estimation. As a result, the estimation focuses on making a good fit for simulation of model response with the current inputs.

The Focus option can be interpreted as a weighting filter in the loss function. For more information, see "Loss Function and Model Quality Metrics".

\section*{WeightingFilter - Weighting prefilter}
[ ] (default) | vector | matrix | cell array | linear system
Weighting prefilter applied to the loss function to be minimized during estimation. To understand the effect of WeightingFilter on the loss function, see "Loss Function and Model Quality Metrics".

Specify WeightingFilter as one of the following values:
- [ ] - No weighting prefilter is used.
- Passbands - Specify a row vector or matrix containing frequency values that define desired passbands. You select a frequency band where the fit between estimated model and estimation data is optimized. For example, [ \(\mathrm{wl} \mathrm{l}, \mathrm{wh}\) ] where wl and wh represent lower and upper limits of a passband. For a matrix with several rows defining frequency passbands,
[w1l,w1h;w2l,w2h;w3l,w3h;..], the estimation algorithm uses the union of the frequency ranges to define the estimation passband.

Passbands are expressed in rad/TimeUnit for time-domain data and in FrequencyUnit for frequency-domain data, where TimeUnit and FrequencyUnit are the time and frequency units of the estimation data.
- SISO filter - Specify a single-input-single-output (SISO) linear filter in one of the following ways:
- A SISO LTI model
- \(\{A, B, C, D\}\) format, which specifies the state-space matrices of a filter with the same sample time as estimation data.
- \{numerator, denominator\} format, which specifies the numerator and denominator of the filter as a transfer function with same sample time as estimation data.

This option calculates the weighting function as a product of the filter and the input spectrum to estimate the transfer function.
- Weighting vector - Applicable for frequency-domain data only. Specify a column vector of weights. This vector must have the same length as the frequency vector of the data set, Data. Frequency. Each input and output response in the data is multiplied by the corresponding weight at that frequency.

\section*{EstimateCovariance - Control whether to generate parameter covariance data true (default) | false}

Controls whether parameter covariance data is generated, specified as true or false.
If EstimateCovariance is true, then use getcov to fetch the covariance matrix from the estimated model.

\section*{Display - Specify whether to display the estimation progress}
'off' (default)|'on'
Specify whether to display the estimation progress, specified as one of the following values:
- 'on ' - Information on model structure and estimation results are displayed in a progress-viewer window.
- 'off' - No progress or results information is displayed.

\section*{InputOffset - Removal of offset from time-domain input data \\ 'auto' (default)|'estimate' | vector | matrix | object | []}

Removal of offset from time-domain input data during estimation, specified as one of the following values:
- 'estimate' - The software treats the input offsets as an estimation parameter.
- 'auto' - The software chooses the method to handle input offsets based on the estimation data and the model structure. The estimation either assumes zero input offset or estimates the input offset.

For example, the software estimates the input offset for a model that contains an integrator.
- A column vector of length \(N u\), where \(N u\) is the number of inputs.

Use [] to specify no offsets.
In case of multi-experiment data, specify InputOffset as a \(N u\)-by- \(N e\) matrix. \(N u\) is the number of inputs, and \(N e\) is the number of experiments.

Each entry specified by Input0ffset is subtracted from the corresponding input data.
- A parameter object, constructed using param. Continuous, that imposes constraints on how the software estimates the input offset.

For example, create a parameter object for a 2 -input model estimation. Specify the first input offset as fixed to zero and the second input offset as an estimation parameter.
```

opt = procestOptions;
u0 = param.Continuous('u0',[0;NaN]);
u0.Free(1) = false;
opt.Inputoffset = u0;

```

OutputOffset - Removal of offset from time-domain output data during estimation
[] (default) | vector | matrix
Removal of offset from time-domain output data during estimation, specified as the comma-separated pair consisting of 'OutputOffset' and one of the following:
- A column vector of length \(N y\), where \(N y\) is the number of outputs.
- [ ] - Indicates no offset.
- \(N y\)-by-Ne matrix - For multi-experiment data, specify OutputOffset as a \(N y\)-by-Ne matrix. \(N y\) is the number of outputs, and \(N e\) is the number of experiments.

Each entry specified by OutputOffset is subtracted from the corresponding output data.

\section*{OutputWeight - Weighting of prediction errors in multi-output estimations}
[] (default) | 'noise' | positive semidefinite symmetric matrix
Weighting of prediction errors in multi-output estimations, specified as one of the following values:
- 'noise ' - Minimize \(\operatorname{det}\left(E^{\prime *} E / N\right)\), where \(E\) represents the prediction error and \(N\) is the number of data samples. This choice is optimal in a statistical sense and leads to maximum likelihood estimates if nothing is known about the variance of the noise. It uses the inverse of the estimated noise variance as the weighting function.

Note OutputWeight must not be 'noise' if SearchMethod is 'lsqnonlin'.
- Positive semidefinite symmetric matrix ( W ) - Minimize the trace of the weighted prediction error matrix trace (E'*E*W/N) where:
- \(E\) is the matrix of prediction errors, with one column for each output, and \(W\) is the positive semidefinite symmetric matrix of size equal to the number of outputs. Use \(W\) to specify the relative importance of outputs in multiple-output models, or the reliability of corresponding data.
- \(N\) is the number of data samples.
- [ ] - The software chooses between the 'noise' or using the identity matrix for W.

This option is relevant for only multi-output models.

\section*{Regularization - Options for regularized estimation of model parameters \\ structure}

Options for regularized estimation of model parameters. For more information on regularization, see "Regularized Estimates of Model Parameters".

Regularization is a structure with the following fields:
- Lambda - Constant that determines the bias versus variance tradeoff.

Specify a positive scalar to add the regularization term to the estimation cost.
The default value of zero implies no regularization.
Default: 0
- R - Weighting matrix.

Specify a vector of nonnegative numbers or a square positive semi-definite matrix. The length must be equal to the number of free parameters of the model.

For black-box models, using the default value is recommended. For structured and grey-box models, you can also specify a vector of \(n p\) positive numbers such that each entry denotes the confidence in the value of the associated parameter.

The default value of 1 implies a value of eye(npfree), where npfree is the number of free parameters.

\section*{Default: 1}
- Nominal - The nominal value towards which the free parameters are pulled during estimation.

The default value of zero implies that the parameter values are pulled towards zero. If you are refining a model, you can set the value to 'model' to pull the parameters towards the parameter values of the initial model. The initial parameter values must be finite for this setting to work.

Default: 0

\section*{SearchMethod - Numerical search method used for iterative parameter estimation}
'auto' (default)|'gn'|'gna'|'lm'|'grad'|'lsqnonlin' |'fmincon'
Numerical search method used for iterative parameter estimation, specified as the comma-separated pair consisting of 'SearchMethod' and one of the following:
- 'auto' - A combination of the line search algorithms, 'gn', 'lm', 'gna', and 'grad' methods is tried in sequence at each iteration. The first descent direction leading to a reduction in estimation cost is used.
- 'gn' - Subspace Gauss-Newton least squares search. Singular values of the Jacobian matrix less than GnPinvConstant*eps*max(size(J))*norm(J) are discarded when computing the search direction. \(J\) is the Jacobian matrix. The Hessian matrix is approximated as \(J^{T} J\). If there is no improvement in this direction, the function tries the gradient direction.
- 'gna' - Adaptive subspace Gauss-Newton search. Eigenvalues less than gamma*max (sv) of the Hessian are ignored, where \(s v\) contains the singular values of the Hessian. The Gauss-Newton direction is computed in the remaining subspace. gamma has the initial value InitialGnaTolerance (see Advanced in 'SearchOptions' for more information). This value is increased by the factor LMStep each time the search fails to find a lower value of the criterion in fewer than five bisections. This value is decreased by the factor \(2 *\) LMStep each time a search is successful without any bisections.
- ' lm' - Levenberg-Marquardt least squares search, where the next parameter value is - pinv (H \(\left.+\mathrm{d}^{*} \mathrm{I}\right) * \mathrm{grad}\) from the previous one. \(H\) is the Hessian, \(I\) is the identity matrix, and grad is the gradient. \(d\) is a number that is increased until a lower value of the criterion is found.
- 'grad' - Steepest descent least squares search.
- 'lsqnonlin' - Trust-region-reflective algorithm of lsqnonlin. Requires Optimization Toolbox software.
- 'fmincon ' - Constrained nonlinear solvers. You can use the sequential quadratic programming (SQP) and trust-region-reflective algorithms of the fmincon solver. If you have Optimization Toolbox software, you can also use the interior-point and active-set algorithms of the fmincon solver. Specify the algorithm in the SearchOptions.Algorithm option. The fmincon algorithms may result in improved estimation results in the following scenarios:
- Constrained minimization problems when there are bounds imposed on the model parameters.
- Model structures where the loss function is a nonlinear or non smooth function of the parameters.
- Multi-output model estimation. A determinant loss function is minimized by default for multioutput model estimation. fmincon algorithms are able to minimize such loss functions directly. The other search methods such as ' lm ' and ' gn ' minimize the determinant loss function by alternately estimating the noise variance and reducing the loss value for a given noise variance value. Hence, the fmincon algorithms can offer better efficiency and accuracy for multi-output model estimations.

\section*{SearchOptions - Option set for the search algorithm}
search option set
Option set for the search algorithm, specified as the comma-separated pair consisting of 'Search0ptions ' and a search option set with fields that depend on the value of SearchMethod.

\section*{SearchOptions Structure When SearchMethod is Specified as 'gn', 'gna', 'lm', 'grad', or 'auto'}
\begin{tabular}{|l|l|l|}
\hline \begin{tabular}{l} 
Field \\
Name
\end{tabular} & Description & Default \\
\hline \begin{tabular}{l} 
Toleran \\
ce
\end{tabular} & \begin{tabular}{l} 
Minimum percentage difference between the current value of the loss \\
function and its expected improvement after the next iteration, specified as \\
a positive scalar. When the percentage of expected improvement is less \\
than Tolerance, the iterations stop. The estimate of the expected loss- \\
function improvement at the next iteration is based on the Gauss-Newton \\
vector computed for the current parameter value.
\end{tabular} & 0.01 \\
\hline \begin{tabular}{l} 
MaxIter \\
ations
\end{tabular} & \begin{tabular}{l} 
Maximum number of iterations during loss-function minimization, specified \\
as a positive integer. The iterations stop when MaxIterations is reached \\
or another stopping criterion is satisfied, such as Tolerance.
\end{tabular} & 20 \\
\begin{tabular}{l} 
Setting MaxIterations = 0 returns the result of the start-up procedure. \\
Use sys. Report. Termination. Iterations to get the actual number \\
of iterations during an estimation, where sys is an idtf model.
\end{tabular} & \\
\hline
\end{tabular}
\begin{tabular}{|c|c|c|c|}
\hline Field Name & \multicolumn{2}{|l|}{Description} & Default \\
\hline \multirow[t]{11}{*}{\begin{tabular}{l}
Advance \\
d
\end{tabular}} & \multicolumn{3}{|l|}{Advanced search settings, specified as a structure with the following fields:} \\
\hline & Field Name & Description & Default \\
\hline & GnPinvCons tant & Jacobian matrix singular value threshold, specified as a positive scalar. Singular values of the Jacobian matrix that are smaller than GnPinvConstant*max(size(J)*norm(J)*eps) are discarded when computing the search direction. Applicable when SearchMethod is 'gn'. & 10000 \\
\hline & InitialGna Tolerance & Initial value of gamma, specified as a positive scalar. Applicable when SearchMethod is 'gna'. & 0.0001 \\
\hline & LMStartVal ue & Starting value of search-direction length \(d\) in the Levenberg-Marquardt method, specified as a positive scalar. Applicable when SearchMethod is ' 1 m '. & 0.001 \\
\hline & LMStep & Size of the Levenberg-Marquardt step, specified as a positive integer. The next value of the search-direction length \(d\) in the Levenberg-Marquardt method is LMStep times the previous one. Applicable when SearchMethod is ' lm '. & 2 \\
\hline & MaxBisecti ons & Maximum number of bisections used for line search along the search direction, specified as a positive integer. & 25 \\
\hline & MaxFunctio nEvaluatio ns & Maximum number of calls to the model file, specified as a positive integer. Iterations stop if the number of calls to the model file exceeds this value. & Inf \\
\hline & MinParamet erChange & Smallest parameter update allowed per iteration, specified as a nonnegative scalar. & 0 \\
\hline & RelativeIm provement & Relative improvement threshold, specified as a nonnegative scalar. Iterations stop if the relative improvement of the criterion function is less than this value. & 0 \\
\hline & StepReduct ion & \begin{tabular}{l}
Step reduction factor, specified as a positive scalar that is greater than 1 . The suggested parameter update is reduced by the factor StepReduction after each try. This reduction continues until MaxBisections tries are completed or a lower value of the criterion function is obtained. \\
StepReduction is not applicable for SearchMethod ' lm' (Levenberg-Marquardt method).
\end{tabular} & 2 \\
\hline
\end{tabular}

\section*{SearchOptions Structure When SearchMethod is Specified as ' Isqnonlin'}
\begin{tabular}{|l|l|l|}
\hline \begin{tabular}{l} 
Field \\
Name
\end{tabular} & Description & Default \\
\hline \begin{tabular}{l} 
Function \\
Toleranc \\
e
\end{tabular} & \begin{tabular}{l} 
Termination tolerance on the loss function that the software \\
minimizes to determine the estimated parameter values, \\
specified as a positive scalar.
\end{tabular} & \(1 \mathrm{e}-5\) \\
\hline \begin{tabular}{l} 
The value of FunctionTolerance is the same as that of \\
opt.Search0ptions. Advanced. TolFun.
\end{tabular} & (e-6 \\
\hline \begin{tabular}{l} 
StepTole \\
rance
\end{tabular} & \begin{tabular}{l} 
Termination tolerance on the estimated parameter values, \\
specified as a positive scalar.
\end{tabular} & \begin{tabular}{l} 
The value of StepTolerance is the same as that of \\
opt. SearchOptions . Advanced. TolX.
\end{tabular}
\end{tabular}

SearchOptions Structure When SearchMethod is Specified as 'fmincon'
\begin{tabular}{|c|c|c|}
\hline Field Name & Description & Default \\
\hline Algorithm & \begin{tabular}{l}
fmincon optimization algorithm, specified as one of the following: \\
- 'sqp' - Sequential quadratic programming algorithm. The algorithm satisfies bounds at all iterations, and it can recover from NaN or Inf results. It is not a large-scale algorithm. For more information, see "Large-Scale vs. MediumScale Algorithms" (Optimization Toolbox). \\
- 'trust-regionreflective' - Subspace trust-region method based on the interior-reflective Newton method. It is a largescale algorithm. \\
- 'interior-point' -Large-scale algorithm that requires Optimization Toolbox software. The algorithm satisfies bounds at all iterations, and it can recover from NaN or Inf results. \\
- 'active-set' - Requires Optimization Toolbox software. The algorithm can take large steps, which adds speed. It is not a large-scale algorithm. \\
For more information about the algorithms, see "Constrained Nonlinear Optimization Algorithms" (Optimization Toolbox) and "Choosing the Algorithm" (Optimization Toolbox).
\end{tabular} & 'sqp' \\
\hline
\end{tabular}
\begin{tabular}{|l|l|l|}
\hline Field Name & Description & Default \\
\hline FunctionTolerance & \begin{tabular}{l} 
Termination tolerance on the \\
loss function that the software \\
minimizes to determine the \\
estimated parameter values, \\
specified as a positive scalar.
\end{tabular} & \(1 \mathrm{e}-6\) \\
\hline StepTolerance & \begin{tabular}{l} 
Termination tolerance on the \\
estimated parameter values, \\
specified as a positive scalar.
\end{tabular} & \(1 \mathrm{e}-6\) \\
\hline MaxIterations & \begin{tabular}{l} 
Maximum number of iterations \\
during loss function \\
minimization, specified as a \\
positive integer. The iterations \\
stop when MaxIterations is \\
reached or another stopping \\
criterion is satisfied, such as \\
FunctionTolerance.
\end{tabular} & 100 \\
\hline
\end{tabular}

\section*{Advanced - Additional advanced options}
structure
Advanced is a structure with the following fields:
- ErrorThreshold - Specifies when to adjust the weight of large errors from quadratic to linear.

Errors larger than ErrorThreshold times the estimated standard deviation have a linear weight in the loss function. The standard deviation is estimated robustly as the median of the absolute deviations from the median of the prediction errors, divided by 0.7 . For more information on robust norm choices, see section 15.2 of [1].

ErrorThreshold \(=0\) disables robustification and leads to a purely quadratic loss function. When estimating with frequency-domain data, the software sets ErrorThreshold to zero. For time-domain data that contains outliers, try setting ErrorThreshold to 1.6.

Default: 0
- MaxSize - Specifies the maximum number of elements in a segment when input-output data is split into segments.

MaxSize must be a positive integer.
Default: 250000
- StabilityThreshold - Specifies thresholds for stability tests.

StabilityThreshold is a structure with the following fields:
- \(s\) - Specifies the location of the right-most pole to test the stability of continuous-time models. A model is considered stable when its right-most pole is to the left of \(s\).

\section*{Default: 0}
- \(z\) - Specifies the maximum distance of all poles from the origin to test stability of discretetime models. A model is considered stable if all poles are within the distance \(z\) from the origin.

Default: \(1+\) sqrt(eps)
- AutoInitThreshold - Specifies when to automatically estimate the initial condition.

The initial condition is estimated when
\[
\frac{\left\|y_{p, z}-y_{\text {meas }}\right\|}{\left\|y_{p, e}-y_{\text {meas }}\right\|}>\text { AutoInitThreshold }
\]
- \(y_{\text {meas }}\) is the measured output.
- \(y_{p, z}\) is the predicted output of a model estimated using zero initial states.
- \(y_{p, e}\) is the predicted output of a model estimated using estimated initial states.

Applicable when InitialCondition is 'auto'.
Default: 1.05

\section*{Output Arguments}

\section*{opt - Option set for procest}
procestOptions option set
Option set for procest, returned as a procestOptions option set.

\section*{Examples}

\section*{Create Default Option Set for Process Model Estimation}
```

opt = procestOptions;

```

\section*{Specify Options for Process Model Estimation}

Create an option set for procest setting Focus to 'simulation' and turning on the Display.
opt = procestOptions('Focus','simulation','Display','on');
Alternatively, use dot notation to set the values of opt.
```

opt = procestOptions;
opt.Focus = 'simulation';
opt.Display = 'on';

```

\section*{Compatibility Considerations}

\section*{Renaming of Estimation and Analysis Options}

The names of some estimation and analysis options were changed in R2018a. Prior names still work. For details, see the R2018a release note "Renaming of Estimation and Analysis Options".

\section*{References}
[1] Ljung, L. System Identification: Theory for the User. Upper Saddle River, NJ: Prentice-Hall PTR, 1999.
[2] Wills, Adrian, B. Ninness, and S. Gibson. "On Gradient-Based Search for Multivariable System Estimates". Proceedings of the 16th IFAC World Congress, Prague, Czech Republic, July 3-8, 2005. Oxford, UK: Elsevier Ltd., 2005.

\section*{See Also}
idfilt|idproc|procest
Topics
"Loss Function and Model Quality Metrics"
Introduced in R2012a

\section*{pwlinear}

Create a piecewise-linear nonlinearity estimator object

\section*{Syntax}
```

NL = pwlinear
NL = pwlinear(Name,Value)

```

\section*{Description}

NL = pwlinear creates a default piecewise-linear nonlinearity estimator object with 10 break points for estimating Hammerstein-Wiener models. The value of the nonlinearity at the break points are set to []. The initial value of the nonlinearity is determined from the estimation data range during estimation using \(n l h w\). Use dot notation to customize the object properties, if needed.

NL = pwlinear(Name, Value) creates a piecewise-linear nonlinearity estimator object with properties specified by one or more Name, Value pair arguments. The properties that you do not specify retain their default value.

\section*{Object Description}
pwlinear is an object that stores the piecewise-linear nonlinearity estimator for estimating Hammerstein-Wiener models.

Use pwlinear to define a nonlinear function \(y=F(x, \theta)\), where \(y\) and \(x\) are scalars, and \(\theta\) represents the parameters specifying the number of break points and the value of nonlinearity at the break points.

The nonlinearity function, \(F\), is a piecewise-linear (affine) function of \(x\). There are n breakpoints \(\left(x_{k}, y_{k}\right), k=1, \ldots, n\), such that \(y_{k}=F\left(x_{k}\right) . F\) is linearly interpolated between the breakpoints.
\(F\) is also linear to the left and right of the extreme breakpoints. The slope of these extensions is a function of \(x_{i}\) and \(y_{i}\) breakpoints. The breakpoints are ordered by ascending \(x\)-values, which is important when you set a specific breakpoint to a different value.

There are minor difference between the breakpoint values you set and the values stored in the object because the toolbox has a different internal representation of breakpoints.

For example, in the following plot, the breakpoints are \(x_{k}=[-2,1,4]\) and the corresponding nonlinearity values are \(y_{k}=[4,3,5]\).


The value \(F(x)\) is computed by evaluate \((N L, x)\), where NL is the pwlinear object. When using evaluate, the break points have to be initialized manually.

For pwlinear object properties, see "Properties" on page 1-1162.

\section*{Examples}

\section*{Create a Default Piecewise-Linear Nonlinearity Estimator}

NL = pwlinear;
Specify the number of break points.
NL. NumberOfUnits = 5;

\section*{Estimate a Hammerstein Model with Piecewise-Linear Nonlinearity}

Load estimation data.
```

load twotankdata;
z = iddata(y,u,0.2,'Name','Two tank system');
z1 = z(1:1000);

```

Create a pwlinear object, and specify the breakpoints.
InputNL = pwlinear('BreakPoints', [-2,1,4]);
Since BreakPoints is specified as a vector, the specified vector is interpreted as the \(x\)-values of the break points. The \(y\)-values of the break points are set to 0 , and are determined during model estimation.

Estimate model with no output nonlinearity.
sys = nlhw(z1,[2 3 0],InputNL,[]);

\section*{Input Arguments}

\section*{Name-Value Pair Arguments}

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Use Name, Value arguments to specify additional properties on page 1-594 of pwlinear nonlinearity. For example, NL= pwlinear('NumberofUnits',5) creates a piecewise-linear nonlinearity estimator object with 5 breakpoints.

\section*{Properties}
pwlinear object properties include:

\section*{NumberofUnits}

Number of breakpoints, specified as an integer.
Default: 10

\section*{BreakPoints}

Break points, \(x_{k}\), and the corresponding nonlinearity values at the breakpoints, \(y_{k}\), specified as one of the following:
- 2-by- \(n\) matrix - The \(x\) and \(y\) values for each of the \(n\) break points are specified as \(\left[x_{1}, x_{2}, \ldots ., x_{n} ; y_{1}\right.\), \(\left.y_{2}, \ldots, y_{n}\right]\).
- 1-by- \(n\) vector - The specified vector is interpreted as the \(x\) values of the break points: \(x_{1}, x_{2}, \ldots ., x_{n}\). All the \(y\) values of the break points are set to 0 .

When the nonlinearity object is created, the breakpoints are ordered by ascending \(x\)-values. This is important to consider if you set a specific breakpoint to a different value after creating the object.

\section*{Default: []}

\section*{Output Arguments}

\section*{NL - Piecewise-linear nonlinearity estimator object}
pwlinear object

Piecewise-linear nonlinearity estimator object, returned as a pwlinear object.

\section*{See Also}
nlhw
Introduced in R2007a

\section*{pzmap}

Pole-zero plot of dynamic system

\section*{Syntax}
```

pzmap(sys)
pzmap(sys1,sys2,...,sysN)
[p,z] = pzmap(sys)

```

\section*{Description}
pzmap (sys) creates a pole-zero plot of the continuous or discrete-time dynamic system model sys. x and o indicates the poles and zeros respectively, as shown in the following figure.


From the figure above, an open-loop linear time-invariant system is stable if:
- In continuous-time, all the poles on the complex s-plane must be in the left-half plane (blue region) to ensure stability. The system is marginally stable if distinct poles lie on the imaginary axis, that is, the real parts of the poles are zero.
- In discrete-time, all the poles in the complex z-plane must lie inside the unit circle (blue region). The system is marginally stable if it has one or more poles lying on the unit circle.
pzmap (sys1, sys2, ... sysN) creates the pole-zero plot of multiple models on a single figure. The models can have different numbers of inputs and outputs and can be a mix of continuous and discrete systems. For SISO systems, pzmap plots the system poles and zeros. For MIMO systems, pzmap plots the system poles and transmission zeros.
\([p, z]=p z m a p(s y s)\) returns the system poles and transmission zeros as column vectors \(p\) and \(z\). The pole-zero plot is not displayed on the screen.

\section*{Examples}

\section*{Pole-Zero Plot of Dynamic System}

Plot the poles and zeros of the continuous-time system represented by the following transfer function:
\[
H(s)=\frac{2 s^{2}+5 s+1}{s^{2}+3 s+5} .
\]
\(H=t f\left(\left[\begin{array}{lll}2 & 5 & 1\end{array}\right],\left[\begin{array}{lll}1 & 3 & 5\end{array}\right]\right)\);
pzmap(H)
grid on


Turning on the grid displays lines of constant damping ratio (zeta) and lines of constant natural frequency (wn). This system has two real zeros, marked by o on the plot. The system also has a pair of complex poles, marked by x.

\section*{Pole-Zero Plot of Identified System}

Plot the pole-zero map of a discrete time identified state-space (idss) model. In practice you can obtain an idss model by estimation based on input-output measurements of a system. For this example, create one from state-space data.
\(A=[0.10 ; 0.2-0.9] ;\)
\(B=[.1\); 0.1];
C = [10 5];
D = [0];
sys \(=\) idss(A,B,C,D,'Ts',0.1);
Examine the pole-zero map.
pzmap(sys)


System poles are marked by \(x\), and zeros are marked by 0 .

\section*{Pole-Zero Map of Multiple Models}

For this example, load a 3-by-1 array of transfer function models.
load('tfArray.mat','sys');
size(sys)
\(3 \times 1\) array of transfer functions.
Each model has 1 outputs and 1 inputs.
Plot the poles and zeros of each model in the array with distinct colors. For this example, use red for the first model, green for the second and blue for the third model in the array.
```

pzmap(sys(:,:,1),'r',sys(:,:,2),'g',sys(:,:,3),'b')
sgrid

```

sgrid plots lines of constant damping ratio and natural frequency in the s-plane of the pole-zero plot.

\section*{Poles and Zeros of Transfer Function}

Use pzmap to calculate the poles and zeros of the following transfer function:
\(\operatorname{sys}(s)=\frac{4.2 s^{2}+0.25 s-0.004}{s^{2}+9.6 s+17}\)
sys \(=\mathrm{tf}([4.2,0.25,-0.004],[1,9.6,17])\);
[p,z] = pzmap(sys)
\(p=2 \times 1\)
- 7.2576
-2.3424
\(z=2 \times 1\)
-0.0726
0.0131

\section*{Identify Near-Cancelling Pole-Zero Pairs}

This example uses a model of a building with eight floors, each with three degrees of freedom: two displacements and one rotation. The I/O relationship for any one of these displacements is represented as a 48 -state model, where each state represents a displacement or its rate of change (velocity).

Load the building model.
load('building.mat');
size(G)
State-space model with 1 outputs, 1 inputs, and 48 states.
Plot the poles and zeros of the system.
pzmap(G)


From the plot, observe that there are numerous near-canceling pole-zero pairs that could be potentially eliminated to simplify the model, with no effect on the overall model response. pzmap is useful to visually identify such near-canceling pole-zero pairs to perform pole-zero simplification.

\section*{Input Arguments}

\section*{sys - Dynamic system}
dynamic system model | model array
Dynamic system, specified as a dynamic system model or model array. Dynamic systems that you can use include continuous-time or discrete-time numeric LTI models such as tf, zpk, or ss models.

If sys is an array of models, pzmap plots all the poles and zeros of every model in the array on the same plot.

\section*{Output Arguments}

\section*{p-Poles of the system}
column vector
Poles of the system, returned as a column vector, in order of its increasing natural frequency. p is the same as the output of pole(sys), except for the order.

\section*{z - Transmission zeros of the system}
column vector
Transmission zeros of the system, returned as a column vector. \(z\) is the same as the output of tzero(sys).

\section*{Tips}
- Use the functions sgrid or zgrid to plot lines of constant damping ratio and natural frequency in the \(s\) - or \(z\)-plane on the pole-zero plot.
- For MIMO models, pzmap displays all system poles and transmission zeros on a single plot. To map poles and zeros for individual I/O pairs, use iopzmap.
- For additional options to customize the appearance of the pole-zero plot, use pzplot.

\section*{See Also}
damp|dsort|esort|iopzmap|pole|pzplot|rlocus|sgrid|zero|zgrid

\section*{Introduced before R2006a}

\section*{pzoptions}

Create list of pole/zero plot options

\section*{Syntax}
\(\mathrm{P}=\) pzoptions
P = pzoption('cstprefs')

\section*{Description}
\(P=\) pzoptions returns a list of available options for pole/zero plots (pole/zero, input-output pole/ zero and root locus) with default values set.. You can use these options to customize the pole/zero plot appearance from the command line.

P = pzoption('cstprefs') initializes the plot options with the options you selected in the Control System and System Identification Toolbox Preferences Editor. For more information about the editor, see "Toolbox Preferences Editor".

This table summarizes the available pole/zero plot options.
\begin{tabular}{|c|c|}
\hline Option & Description \\
\hline Title, XLabel, YLabel & Label text and style \\
\hline TickLabel & Tick label style \\
\hline Grid & Show or hide the grid, specified as one of the following values: 'off'|'on' Default: ' off' \\
\hline GridColor & Color of the grid lines, specified as one of the following: Vector of RGB values in the range \([0,1] \mid\) character vector of color name |' none For example, for yellow color, specify as one of the following: \(\left.\begin{array}{lll}1 & 1 & 0\end{array}\right]\), 'yellow', or ' \(y\) '. Default: [0.15, 0. 15, 0.15] \\
\hline XlimMode, YlimMode & Limit modes \\
\hline Xlim, Ylim & Axes limits \\
\hline IOGrouping & Grouping of input-output pairs, specified as one of the following values: ' none' |'inputs'|'outputs'|'all' Default: 'none' \\
\hline InputLabels, OutputLabels & Input and output label styles \\
\hline InputVisible, OutputVisible & Visibility of input and output channels \\
\hline
\end{tabular}
\begin{tabular}{|c|c|}
\hline Option & Description \\
\hline FreqUnits & \begin{tabular}{l}
Frequency units, specified as one of the following values: \\
- 'Hz' \\
- 'rad/second' \\
- 'rpm' \\
- 'kHz' \\
- 'MHz' \\
- 'GHz' \\
- 'rad/nanosecond' \\
- 'rad/microsecond' \\
- 'rad/millisecond' \\
- 'rad/minute' \\
- 'rad/hour' \\
- 'rad/day' \\
- 'rad/week' \\
- 'rad/month' \\
- 'rad/year' \\
- 'cycles/nanosecond' \\
- 'cycles/microsecond' \\
- 'cycles/millisecond' \\
- 'cycles/hour' \\
- 'cycles/day' \\
- 'cycles/week' \\
- 'cycles/month' \\
- 'cycles/year' \\
Default: ' rad/s ' \\
You can also specify 'auto ' which uses frequency units rad/TimeUnit relative to system time units specified in the TimeUnit property. For multiple systems with different time units, the units of the first system are used.
\end{tabular} \\
\hline
\end{tabular}
\begin{tabular}{|c|c|}
\hline Option & Description \\
\hline TimeUnits & \begin{tabular}{l}
Time units, specified as one of the following values: \\
- 'nanoseconds' \\
- 'microseconds' \\
- 'milliseconds' \\
- 'seconds' \\
- 'minutes' \\
- 'hours' \\
- 'days' \\
- 'weeks' \\
- 'months' \\
- 'years' \\
Default: ' seconds ' \\
You can also specify 'auto' which uses time units specified in the TimeUnit property of the input system. For multiple systems with different time units, the units of the first system is used.
\end{tabular} \\
\hline ConfidenceRegionNumberSD & Number of standard deviations to use when displaying the confidence region characteristic for identified models (valid only iopzplot). \\
\hline
\end{tabular}

\section*{Examples}

\section*{Display Grid in Root Locus Plot}

Create a default option set for root locus plots.
opt = pzoptions;
In the plot options, specify the grid as visible.
opt.Grid = 'on';
Use the specified options to plot the root locus of a transfer function.
```

h = rlocusplot(tf(1,[1,.2,1,0]),opt);

```


The root locus plot is generated with the grid enabled.

\section*{See Also \\ getoptions|iopzplot|pzplot|setoptions \\ Introduced in R2012a}

\section*{pzplot}

Pole-zero plot of dynamic system model with plot customization options

\section*{Syntax}
```

h = pzplot(sys)
pzplot(sys)
pzplot(sys1,sys2,...,sysN)
pzplot(ax,...)
pzplot(...,plotoptions)

```

\section*{Description}
\(\mathrm{h}=\mathrm{pzplot}(\) sys) plots the poles and transmission zeros of the dynamic system model sys and returns the plot handle \(h\) to the plot. Use this function to generate a pole-zero map with customizable plot options such as FreqUnits, TimeUnits, and IOGrouping. For more information about using and interpreting pole-zero maps, see pzmap.
pzplot(sys) plots the poles and transmission zeros of the dynamic system model sys. \(x\) and o indicates poles and zeros respectively.
pzplot(sys1,sys2,...sysN) displays the poles and transmission zeros of multiple models on a single plot. You can specify distinct colors for each model individually.
pzplot (ax, ...) plots into the axes specified by ax instead of the current axis gca.
pzplot(...,plotoptions) plots the poles and transmission zeros with the options specified in plotoptions. For more information on the ways to change properties of your plots, see "Ways to Customize Plots" (Control System Toolbox).

\section*{Examples}

\section*{Pole-Zero Plot with Custom Plot Title}

Plot the poles and zeros of the continuous-time system represented by the following transfer function:
\(\operatorname{sys}(s)=\frac{2 s^{2}+5 s+1}{s^{2}+3 s+5}\).
sys = tf([2 5 1],[1 3 5]);
h = pzplot(sys);
grid on


Turning on the grid displays lines of constant damping ratio (zeta) and lines of constant natural frequency (wn). This system has two real zeros, marked by o on the plot. The system also has a pair of complex poles, marked by x.

Change the color of the plot title. To do so, use the plot handle, h.
```

p = getoptions(h);
p.Title.Color = [1,0,0];
setoptions(h,p);

```


\section*{Pole-Zero Plot of Multiple Models}

For this example, load a 3-by-1 array of transfer function models.
load('tfArrayMargin.mat','sys');
size(sys)
\(3 \times 1\) array of transfer functions.
Each model has 1 outputs and 1 inputs.
Plot the poles and zeros of the model array. Define the colors for each model. For this example, use red for the first model, green for the second and blue for the third model in the array.
pzplot(sys(:,:,1),'r',sys(:,:,2),'g',sys(:,:,3),'b');


\section*{Pole-Zero Plot with Custom Options}

Plot the poles and zeros of the continuous-time system represented by the following transfer function with a custom option set:
\[
\operatorname{sys}(s)=\frac{2 s^{2}+5 s+1}{s^{2}+3 s+5} .
\]

Create the custom option set using pzoptions.
plotoptions = pzoptions;
For this example, specify the grid to be visible.
plotoptions.Grid = 'on';
Use the specified options to create a pole-zero map of the transfer function.



Turning on the grid displays lines of constant damping ratio (zeta) and lines of constant natural frequency (wn). This system has two real zeros, marked by o on the plot. The system also has a pair of complex poles, marked by \(x\).

\section*{Input Arguments}

\section*{sys - Dynamic system}
dynamic system model \| model array
Dynamic system, specified as a SISO or MIMO dynamic system model, or an array of SISO or MIMO dynamic system models. Dynamic systems that you can use include continuous-time or discrete-time numeric LTI models such as tf, zpk, or ss models.

If sys is a generalized state-space model genss or an uncertain state-space model uss, pzplot returns the poles and transmission of the current or nominal value of sys. If sys is an array of models, pzplot plots the poles and zeros of each model in the array on the same diagram.

\section*{ax - Axes handle}
axes object
Axes handle, specified as an axes object. If you do not specify the axes object, then pzplot uses the current axes gca to plot the poles and zeros of the system.
plotoptions - Pole-zero plot options
options object

Pole-zero plot options, specified as an options object. See pzoptions for a list of available plot options.

\section*{Output Arguments}

\section*{h - Pole-zero plot options handle}
scalar
Pole-zero plot options handle, returned as a scalar. Use \(h\) to query and modify properties of your polezero plot. You can use this handle to customize the plot with the getoptions and setoptions commands.

\section*{Tips}
- Use sgrid or zgrid to plot lines of constant damping ratio and natural frequency in the \(s\) - or \(z\) plane.

\section*{See Also}
getoptions |iopzplot | pzmap|pzoptions| setoptions

\section*{Topics}
"Ways to Customize Plots" (Control System Toolbox)
Introduced before R2006a

\section*{rarmax}
(To be removed) Estimate recursively parameters of ARMAX or ARMA models

Note rarmax will be removed in a future release. Use recursiveARMA or recursiveARMAX instead.

\section*{Syntax}
thm \(=\operatorname{rarmax}(z, n n, a d m, a d g)\)
[thm,yhat, P,phi,psi] = rarmax(z,nn,adm,adg,th0,P0,phi0,psi0)

\section*{Description}

The parameters of the ARMAX model structure
\[
A(q) y(t)=B(q) u(t-n k)+C(q) e(t)
\]
are estimated using a recursive prediction error method.
The input-output data is contained in \(z\), which is either an iddata object or a matrix \(z=[y \quad u]\) where \(y\) and \(u\) are column vectors. \(n n\) is given as
nn = [na nb nc nk]
where na, \(n b\), and \(n c\) are the orders of the ARMAX model, and \(n k\) is the delay. Specifically,
\[
n a: \quad A(q)=1+a_{1} q^{-1}+\ldots+a_{n a} q^{-n a}
\]
\(n b: \quad B(q)=b_{1}+b_{2} q^{-1}+\ldots+b_{n b} q^{-n b+1}\)
\[
n c: \quad \mathrm{C}(q)=1+c_{1} q^{-1}+\ldots+c_{n c} q^{-n c}
\]

See "What Are Polynomial Models?" for more information.
If \(z\) represents a time series \(y\) and \(n n=\) [na \(n c\) ], rarmax estimates the parameters of an ARMA model for y .
\[
A(q) y(t)=C(q) e(t)
\]

Only single-input, single-output models are handled by rarmax. Use rpem for the multiple-input case.
The estimated parameters are returned in the matrix thm. The kth row of thm contains the parameters associated with time \(k\); that is, they are based on the data in the rows up to and including row k in z . Each row of thm contains the estimated parameters in the following order:
```

thm(k,:) = [a1,a2,...,ana,b1,...,bnb,c1,...,cnc]

```
yhat is the predicted value of the output, according to the current model; that is, row \(k\) of yhat contains the predicted value of \(y(k)\) based on all past data.

The actual algorithm is selected with the two arguments adm and adg. These are described under rarx.

The input argument th0 contains the initial value of the parameters, a row vector consistent with the rows of thm. The default value of th0 is all zeros.

The arguments P 0 and P are the initial and final values, respectively, of the scaled covariance matrix of the parameters. See rarx. The default value of P0 is \(10^{4}\) times the unit matrix. The arguments phi0, psi0, phi, and psi contain initial and final values of the data vector and the gradient vector, respectively. The sizes of these depend on the chosen model orders. The normal choice of phi0 and psi0 is to use the outputs from a previous call to rarmax with the same model orders. (This call could be a dummy call with default input arguments.) The default values of phi0 and psi0 are all zeros.

Note that the function requires that the delay \(n k\) be larger than 0 . If you want \(n k=0\), shift the input sequence appropriately and use \(\mathrm{nk}=1\).

\section*{Examples}

Compute and plot, as functions of time, the four parameters in a second-order ARMA model of a time series given in the vector \(y\). The forgetting factor algorithm with a forgetting factor of 0.98 is applied.
```

thm = rarmax(y,[2 2],'ff',0.98);
plot(thm)

```

\section*{Algorithms}

The general recursive prediction error algorithm (11.44), (11.47) through (11.49) of Ljung (1999) is implemented. See "Recursive Algorithms for Online Parameter Estimation" for more information.

\section*{See Also}
nkshift | recursiveARMA|recursiveARMAX | rpem | rplr

\section*{Topics}
"Recursive Algorithms for Online Parameter Estimation"

\section*{Introduced before R2006a}

\section*{rarx}
(To be removed) Estimate parameters of ARX or AR models recursively

Note rarx will be removed in a future release. Use recursiveAR or recursiveARX instead.

\section*{Syntax}
thm \(=\) rarx \((z, n n, a d m, a d g)\)
[thm, yhat, P, phi] = rarx(z,nn,adm,adg,th0,P0,phi0)

\section*{Description}
thm \(=\operatorname{rarx}(\mathrm{z}, \mathrm{nn}, \operatorname{adm}, \mathrm{adg})\) estimates the parameters thm of single-output ARX model from inputoutput data \(z\) and model orders \(n n\) using the algorithm specified by adm and adg. If \(z\) is a time series y and \(\mathrm{nn}=\mathrm{na}\), rarx estimates the parameters of a single-output AR model.
[thm,yhat, P,phi] = rarx(z,nn,adm,adg,th0, P0, phi0) estimates the parameters thm, the predicted output yhat, final values of the scaled covariance matrix of the parameters \(P\), and final values of the data vector phi of single-output ARX model from input-output data \(z\) and model orders \(n n\) using the algorithm specified by adm and adg. If \(z\) is a time series \(y\) and \(n n=n a, ~ r a r x\) estimates the parameters of a single-output AR model.

\section*{Input Arguments}
z
Name of the matrix iddata object that represents the input-output data or a matrix \(z=[y u]\), where \(y\) and \(u\) are column vectors.

For multiple-input models, the u matrix contains each input as a column vector:
\(u=[u 1\)... unu]
nn
For input-output models, specifies the structure of the ARX model as:
nn = [na nb nk]
where na and nb are the orders of the ARX model, and nk is the delay.
For multiple-input models, nb and nk are row vectors that define orders and delays for each input.

For time-series models, \(n n=\) na, where na is the order of the AR model.

Note The delay nk must be larger than 0. If you want nk \(=0\), shift the input sequence appropriately and use \(n k=1\) (see nkshift).
adm and adg
\(\operatorname{adm}=\) 'ff' and adg = lam specify the forgetting factor algorithm with the forgetting factor \(\lambda=l a m\). This algorithm is also known as recursive least squares (RLS). In this case, the matrix \(P\) has the following interpretation: \(R_{2} / 2 * \mathrm{P}\) is approximately equal to the covariance matrix of the estimated parameters. \(R_{2}\) is the variance of the innovations (the true prediction errors \(e(t)\) ).
adm \(=\) 'ug' and adg = gam specify the unnormalized gradient algorithm with gain gamma= gam. This algorithm is also known as the normalized least mean squares (LMS).
adm \(=\) ' \(n g\) ' and adg = gam specify the normalized gradient or normalized least mean squares (NLMS) algorithm. In these cases, P is not applicable.
adm \(=\) ' kf ' and adg \(=\mathrm{R} 1\) specify the Kalman filter based algorithm with \(R_{2}=1\) and \(R_{1}=\mathrm{R} 1\). If the variance of the innovations \(e(t)\) is not unity but \(R_{2}\); then \(R_{2} * P\) is the covariance matrix of the parameter estimates, while \(R_{1}=\mathrm{R} 1 / R_{2}\) is the covariance matrix of the parameter changes. th0

Initial value of the parameters in a row vector, consistent with the rows of thm.
Default: All zeros.
P0
Initial values of the scaled covariance matrix of the parameters.
Default: \(10^{4}\) times the identity matrix.
phi0
The argument phi0 contains the initial values of the data vector:
\(\varphi(t)=[y(t-1), \ldots, y(t-n a), u(t-1), \ldots, u(t-n b-n k+1)]\)
If \(z=[y(1), u(1) ; \ldots ; y(N), u(N)], p h i 0=\varphi(1)\) and phi \(=\varphi(N)\). For online use of rarx, use phi0, th0, and P0 as the previous outputs phi, thm (last row), and P.

Default: All zeros.

\section*{Output Arguments}
thm
Estimated parameters of the model. The kth row of thm contains the parameters associated with time k; that is, the estimate parameters are based on the data in rows up to and including row \(k\) in \(z\). Each row of thm contains the estimated parameters in the following order:
thm(k,:) \(=[a 1, a 2, \ldots, a n a, b 1, \ldots, b n b]\)
For a multiple-input model, the \(b\) are grouped by input. For example, the \(b\) parameters associated with the first input are listed first, and the \(b\) parameters associated with the second input are listed next.
yhat
Predicted value of the output, according to the current model; that is, row \(k\) of yhat contains the predicted value of \(y(k)\) based on all past data.

P
Final values of the scaled covariance matrix of the parameters.
phi
phi contains the final values of the data vector:
\[
\varphi(t)=[y(t-1), \ldots, y(t-n a), u(t-1), \ldots, u(t-n b-n k+1)]
\]

\section*{Examples}

Adaptive noise canceling: The signal \(y\) contains a component that originates from a known signal \(r\). Remove this component by recursively estimating the system that relates \(r\) to \(y\) using a sixth-order FIR model and the NLMS algorithm.
```

z = [y r];
[thm,noise] = rarx(z,[0 6 1],'ng',0.1);
% noise is the adaptive estimate of the noise
% component of y
plot(y-noise)

```

If this is an online application, you can plot the best estimate of the signal y - noise at the same time as the data \(y\) and \(u\) become available, use the following code:
```

phi = zeros(6,1);
P=1000*eye(6);
th = zeros(1,6);
axis([0 100 -2 2]);
plot(0,0,'*'), hold on
% Use a while loop
while ~abort
[y,r,abort] = readAD(time);
[th,ns,P,phi] = rarx([y r],'ff',0.98,th,P,phi);
plot(time,y-ns,'*')
time = time + Dt
end

```

This example uses a forgetting factor algorithm with a forgetting factor of 0.98 . readAD is a function that reads the value of an A/D converter at the indicated time instant.

\section*{More About}

\section*{ARX Model Structure}

The general ARX model structure is:
\[
A(q) y(t)=B(q) u(t-n k)+e(t)
\]

The orders of the ARX model are:
\[
\begin{gathered}
n a: \quad A(q)=1+a_{1} q^{-1}+\ldots+a_{n a} q^{-n a} \\
n b: \quad B(q)=b_{1}+b_{2} q^{-1}+\ldots+b_{n b} q^{-n b+1}
\end{gathered}
\]

Models with several inputs are defined, as follows:
\(A(q) y(t)=B_{1}(q) u_{1}\left(t-n k_{1}\right)+\ldots+B_{\mathrm{nu}} u_{\mathrm{nu}}\left(t-n k_{\mathrm{nu}}\right)+e(t)\)

\section*{See Also}
nkshift | recursiveAR \| recursiveARX | rpem | rplr
Topics
"Recursive Algorithms for Online Parameter Estimation"
Introduced before R2006a

\section*{rbj}
(To be removed) Estimate recursively parameters of Box-Jenkins models

Note rbj will be removed in a future release. Use recursiveBJ instead.

\section*{Syntax}
thm \(=\) rbj \((z, n n, a d m, a d g)\)
[thm,yhat,P,phi,psi] = rbj(z,nn,adm,adg,th0,P0,phi0,psi0)

\section*{Description}

The parameters of the Box-Jenkins model structure
\[
y(t)=\frac{B(q)}{F(q)} u(t-n k)+\frac{C(q)}{D(q)} e(t)
\]
are estimated using a recursive prediction error method.
The input-output data is contained in \(z\), which is either an iddata object or a matrix \(z=\left[\begin{array}{ll}y & u\end{array}\right]\) where \(y\) and \(u\) are column vectors. \(n n\) is given as
nn = [nb nc nd nf nk]
where \(n b, n c, n d\), and \(n f\) are the orders of the Box-Jenkins model, and \(n k\) is the delay. Specifically,
\[
\begin{aligned}
n b: & B(q)=b_{1}+b_{2} q^{-1}+\ldots+b_{n b} q^{-n b+1} \\
n c: & C(q)=1+c_{1} q^{-1}+\ldots+c_{n c} q^{-n c} \\
n d: \quad & D(q)=1+d_{1} q^{-1}+\ldots+d_{n d} q^{-n d} \\
n f: & F(q)=1+f_{1} q^{-1}+\ldots+f_{n f} q^{-n f}
\end{aligned}
\]

See "What Are Polynomial Models?" for more information.
Only single-input, single-output models are handled by rbj. Use rpem for the multiple-input case.
The estimated parameters are returned in the matrix thm. The kth row of thm contains the parameters associated with time k; that is, they are based on the data in the rows up to and including row k in z . Each row of thm contains the estimated parameters in the following order.
```

thm(k,:) = [b1,...,bnb,c1,...,cnc,d1,...,dnd,f1,...,fnf]

```
yhat is the predicted value of the output, according to the current model; that is, row \(k\) of yhat contains the predicted value of \(y(k)\) based on all past data.

The actual algorithm is selected with the two arguments adm and adg. These are described under rarx.

The input argument th0 contains the initial value of the parameters, a row vector consistent with the rows of thm. The default value of th0 is all zeros.

The arguments P0 and \(P\) are the initial and final values, respectively, of the scaled covariance matrix of the parameters. See rarx. The default value of P0 is \(10^{4}\) times the unit matrix. The arguments phi0, psi0, phi, and psi contain initial and final values of the data vector and the gradient vector, respectively. The sizes of these depend on the chosen model orders. The normal choice of phi0 and psi0 is to use the outputs from a previous call to rbj with the same model orders. (This call could be a dummy call with default input arguments.) The default values of phi0 and psi0 are all zeros.

Note that the function requires that the delay \(n k\) be larger than 0 . If you want \(n k=0\), shift the input sequence appropriately and use \(\mathrm{nk}=1\).

\section*{Algorithms}

The general recursive prediction error algorithm (11.44) of Ljung (1999) is implemented. See also "Recursive Algorithms for Online Parameter Estimation".

\section*{See Also}
nkshift | recursiveBJ | rpem | rplr

\section*{Topics}
"Recursive Algorithms for Online Parameter Estimation"

\section*{Introduced before R2006a}

\section*{realdata}

Determine whether iddata is based on real-valued signals

\section*{Syntax}
realdata(data)

\section*{Description}
realdata returns 1 if
- data contains only real-valued signals.
- data contains frequency-domain signals, obtained by Fourier transformation of real-valued signals.

Otherwise realdata returns 0 .

\section*{Examples}

\section*{Determine if Data is Based on Real-Valued Signals}

Load data.
load iddatal
Transform the data to frequency domain.
zf = fft(z1);
Determine if the time-domain data values are real.
isreal(z1)
ans \(=1\)
Determine if the transformed data values are real.
isreal(zf)
ans \(=0\)
Determine if the data is based on real-valued signals.
```

realdata(zf)
ans = logical
1

```

Add negative frequencies to zf and rerun the command.
```

zf = complex(zf);
realdata(zf)
ans = logical
1

```

The command still returns 1 .

\section*{Introduced before R2006a}

\section*{recursiveAR}

Create System object for online parameter estimation of AR model

\section*{Syntax}
```

obj = recursiveAR
obj = recursiveAR(na)
obj = recursiveAR(na,A0)
obj = recursiveAR(

```
\(\qquad\)
``` ,Name, Value)
```


## Description

Use the recursiveAR command for parameter estimation with real-time data. If all data necessary for estimation is available at once, and you are estimating a time-invariant model, use the offline estimation command, ar.
obj = recursiveAR creates a System object for online parameter estimation of a default single output AR model structure on page 1-1199. The default model structure has a polynomial of order 1 and initial polynomial coefficient value eps.

After creating the object, use the step command to update model parameter estimates using recursive estimation algorithms and real-time data.
obj = recursiveAR(na) specifies the polynomial order of the AR model to be estimated.
obj = recursiveAR(na,A0) specifies the polynomial order and initial values of the polynomial coefficients.
obj = recursiveAR( __ , Name, Value) specifies additional attributes of the AR model structure and recursive estimation algorithm using one or more Name, Value pair arguments.

## Object Description

recursiveAR creates a System object for online parameter estimation of single output AR models using a recursive estimation algorithm.

A System object is a specialized MATLAB object designed specifically for implementing and simulating dynamic systems with inputs that change over time. System objects use internal states to store past behavior, which is used in the next computational step.

After you create a System object, you use commands to process data or obtain information from or about the object. System objects use a minimum of two commands to process data - a constructor to create the object and the step command to update object parameters using real-time data. This separation of declaration from execution lets you create multiple, persistent, reusable objects, each with different settings.

You can use the following commands with the online estimation System objects in System Identification Toolbox:
\(\left.$$
\begin{array}{|l|l|}\hline \text { Command } & \text { Description } \\
\hline \text { step } & \begin{array}{l}\text { Update model parameter estimates using } \\
\text { recursive estimation algorithms and real-time } \\
\text { data. } \\
\text { step puts the object into a locked state. In a } \\
\text { locked state, you cannot change any nontunable } \\
\text { properties or input specifications, such as model } \\
\text { order, data type, or estimation algorithm. During } \\
\text { execution, you can only change tunable } \\
\text { properties. }\end{array} \\
\hline \text { release } & \begin{array}{l}\text { Unlock the System object. Use this command to } \\
\text { enable setting of nontunable parameters. }\end{array} \\
\hline \text { reset } & \begin{array}{l}\text { Reset the internal states of a locked System } \\
\text { object to the initial values, and leave the object } \\
\text { locked. }\end{array} \\
\hline \text { clone } & \begin{array}{l}\text { Create another System object with the same } \\
\text { object property values. }\end{array}
$$ <br>
Do not create additional objects using syntax <br>

obj2 = obj. Any changes made to the\end{array}\right\}\)| properties of the new object created this way |
| :--- |
| (obj2) also change the properties of the original |
| object (obj). |

Use the recursiveAR command to create an online estimation System object. Then estimate the AR model parameter, $A$, and output using the step command with output data $y$.

```
[A,EstimatedOutput] = step(obj,y)
```

For recursiveAR object properties, see "Properties" on page 1-1193.

## Examples

## Estimate AR Model Online

Create a System object ${ }^{T M}$ for online parameter estimation of an AR model using recursive estimation algorithms.

```
obj = recursiveAR;
```

The AR model has a default structure with polynomial of order 1 and initial polynomial coefficient values, eps.

Load the time-series estimation data. In this example, use a static data set for illustration.

```
load iddata9 z9;
output = z9.y;
```

Estimate AR model parameters online using step.
for $\mathrm{i}=1$ : numel(output)
[A,EstimatedOutput] = step(obj,output(i));
end
View the current estimated values of polynomial A coefficients.
obj.A
ans $=1 \times 2$
$1.0000-0.9592$

View the current covariance estimate of the parameters.
obj. ParameterCovariance
ans $=1.6204 \mathrm{e}-04$
View the current estimated output.
EstimatedOutput
EstimatedOutput $=0.7830$

## Create Online Estimation System Object for AR Model With Known Polynomial Order

Specify AR model polynomial order.
na $=2$;
Create a System object ${ }^{\mathrm{TM}}$ for online estimation of an AR model with the specified polynomial order.
obj = recursiveAR(na);

## Create Online Estimation System Object for AR Model With Known Initial Parameters

Specify AR model order.
na $=2$;
Create a System object for online estimation of AR model with known initial polynomial coefficients.
A0 = [1 0.5 0.3];
obj $=$ recursiveAR(na,A0);
Specify the initial parameter covariance.
obj.InitialParameterCovariance = 0.1;
InitialParameterCovariance represents the uncertainty in your guess for the initial parameters. Typically, the default InitialParameterCovariance (10000) is too large relative to the parameter
values. This results in initial guesses being given less importance during estimation. If you have confidence in the initial parameter guesses, specify a smaller initial parameter covariance.

## Specify Estimation Method for Online Estimation of AR Model

Create a System object that uses the normalized gradient algorithm for online parameter estimation of an AR model.

```
obj = recursiveAR(2,'EstimationMethod','NormalizedGradient');
```


## Input Arguments

na - Model order
positive integer
Model order of the polynomial $A(q)$ of an AR model on page 1-1199, specified as a positive integer.

## A0 - Initial value of polynomial coefficients

row vector of real values | [ ]
Initial value of coefficients of the polynomial $A(q)$, specified as a 1 -by-(na+1) row vector of real values with 1 as the first element. Specify the coefficients in order of ascending powers of $q^{-1}$.

Specifying as [], uses the default value of eps for the polynomial coefficients.

Note If the initial parameter values are much smaller than InitialParameterCovariance, these initial values are given less importance during estimation. Specify a smaller initial parameter covariance if you have high confidence in the initial parameter values. This statement applies only for infinite-history estimation. Finite-history estimation does not use InitialParameterCovariance.

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name,Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Use Name, Value arguments to specify writable properties on page 1-1193 of recursiveAR System object during object creation. For example, obj = recursiveAR(2,'EstimationMethod','Gradient') creates a System object to estimate an AR model using the 'Gradient ' recursive estimation algorithm.

## Properties

recursiveAR System object properties consist of read-only and writable properties. The writable properties are tunable and nontunable properties. The nontunable properties cannot be changed when the object is locked, that is, after you use the step command.

Use Name, Value arguments to specify writable properties of recursiveAR objects during object creation. After object creation, use dot notation to modify the tunable properties.

```
obj = recursiveAR;
```

obj.ForgettingFactor $=0.99$;

## A

Estimated coefficients of polynomial $A(q)$, returned as a row vector of real values specified in order of ascending powers of $q^{-1}$.

A is a read-only property and is initially empty after you create the object. It is populated after you use the step command for online parameter estimation.

## InitialA

Initial values for the coefficients of polynomial $A(q)$ of order na, specified as a row vector of length na +1 , with 1 as the first element. Specify the coefficients in order of ascending powers of $q^{-1}$.

If the initial guesses are much smaller than the default InitialParameterCovariance, 10000, the initial guesses are given less importance during estimation. In that case, specify a smaller initial parameter covariance.

InitialA is a tunable property. You can change it when the object is in a locked state.
Default: [1 eps]

## InitialOutputs

Initial values of the outputs buffer in finite-history estimation, specified as 0 or as a ( $W+n a$ )-by- 1 vector, where $W$ is the window length and $n a$ is the polynomial order you specify during object construction.

The InitialOutputs property provides a means of controlling the initial behavior of the algorithm.
When InitialOutputs is set to 0, the object populates the buffer with zeros.
If the initial buffer is set to 0 or does not contain enough information, you see a warning message during the initial phase of your estimation. The warning should clear after a few cycles. The number of cycles it takes for sufficient information to be buffered depends upon the order of your polynomials and your input delays. If the warning persists, you should evaluate the content of your signals.

Specify InitialOutputs only when History is Finite.
InitialOutputs is a tunable property. You can change InitialOutputs when the object is in a locked state.

## Default: 0

## ParameterCovariance

Estimated covariance P of the parameters, returned as an N -by- N symmetric positive-definite matrix. $N$ is the number of parameters to be estimated. The software computes P assuming that the residuals (difference between estimated and measured outputs) are white noise, and the variance of these residuals is 1 .

ParameterCovariance is applicable only when EstimationMethod is 'ForgettingFactor' or 'KalmanFilter' or when History is Finite.

The interpretation of P depends on your settings for the History and EstimationMethod properties.

- If History is Infinite, then your EstimationMethod selection results in one of the following:
- 'ForgettingFactor' $-\left(R_{2} / 2\right) \mathrm{P}$ is approximately equal to the covariance matrix of the estimated parameters, where $R_{2}$ is the true variance of the residuals.
- 'KalmanFilter' $-R_{2} \mathrm{P}$ is the covariance matrix of the estimated parameters, and $R_{1} / R_{2}$ is the covariance matrix of the parameter changes. Here, $R_{1}$ is the covariance matrix that you specify in ProcessNoiseCovariance.
- If History is Finite (sliding-window estimation) $-R_{2} \mathrm{P}$ is the covariance of the estimated parameters. The sliding-window algorithm does not use this covariance in the parameterestimation process. However, the algorithm does compute the covariance for output so that you can use it for statistical evaluation.

ParameterCovariance is a read-only property and is initially empty after you create the object. It is populated after you use the step command for online parameter estimation.

## InitialParameterCovariance

Covariance of the initial parameter estimates, specified as one of the following:

- Real positive scalar, $\alpha$ - Covariance matrix is an $N$-by- $N$ diagonal matrix, with $\alpha$ as the diagonal elements. $N$ is the number of parameters to be estimated.
- Vector of real positive scalars, $\left[\alpha_{1}, \ldots, \alpha_{N}\right]$ - Covariance matrix is an $N$-by- $N$ diagonal matrix, with [ $\alpha_{1}, \ldots, \alpha_{N}$ ] as the diagonal elements.
- $N$-by- $N$ symmetric positive-definite matrix.

InitialParameterCovariance represents the uncertainty in the initial parameter estimates. For large values of InitialParameterCovariance, less importance is placed on the initial parameter values and more on the measured data during beginning of estimation using step.

Use only when EstimationMethod is 'ForgettingFactor' or 'KalmanFilter'.
InitialParameterCovariance is a tunable property. You can change it when the object is in a locked state.

## Default: 10000

## EstimationMethod

Recursive estimation algorithm used for online estimation of model parameters, specified as one of the following values:

- 'ForgettingFactor' - Algorithm used for parameter estimation
- 'KalmanFilter' - Algorithm used for parameter estimation
- 'NormalizedGradient' - Algorithm used for parameter estimation
- 'Gradient ' - Unnormalized gradient algorithm used for parameter estimation

Forgetting factor and Kalman filter algorithms are more computationally intensive than gradient and unnormalized gradient methods. However, they have better convergence properties. For information about these algorithms, see "Recursive Algorithms for Online Parameter Estimation".

These methods all use an infinite data history, and are available only when History is 'Infinite'.
EstimationMethod is a nontunable property. You cannot change it during execution, that is, after the object is locked using the step command.

Default: Forgetting Factor
ForgettingFactor
Forgetting factor, $\lambda$, relevant for parameter estimation, specified as a scalar in the range $(0,1]$.
Suppose that the system remains approximately constant over $T_{0}$ samples. You can choose $\lambda$ such that:

$$
T_{0}=\frac{1}{1-\lambda}
$$

- Setting $\lambda=1$ corresponds to "no forgetting" and estimating constant coefficients.
- Setting $\lambda<1$ implies that past measurements are less significant for parameter estimation and can be "forgotten". Set $\lambda<1$ to estimate time-varying coefficients.

Typical choices of $\lambda$ are in the range [0.98 0.995].
Use only when EstimationMethod is 'ForgettingFactor'.
ForgettingFactor is a tunable property. You can change it when the object is in a locked state.

## Default: 1

## EnableAdapation

Enable or disable parameter estimation, specified as one of the following:

- true or 1- The step command estimates the parameter values for that time step and updates the parameter values.
- false or 0 - The step command does not update the parameters for that time step and instead outputs the last estimated value. You can use this option when your system enters a mode where the parameter values do not vary with time.

Note If you set EnableAdapation to false, you must still execute the step command. Do not skip step to keep parameter values constant, because parameter estimation depends on current and past I/O measurements. step ensures past I/O data is stored, even when it does not update the parameters.

EnableAdapation is a tunable property. You can change it when the object is in a locked state.
Default: true

## DataType

Floating point precision of parameters, specified as one of the following values:

- 'double' - Double-precision floating point
- 'single' - Single-precision floating point

Setting DataType to 'single' saves memory, but leads to loss of precision. Specify DataType based on the precision required by the target processor where you will deploy generated code.

DataType is a nontunable property. It can only be set during object construction using Name, Value arguments and cannot be changed afterward.

Default: 'double'

## ProcessNoiseCovariance

Covariance matrix of parameter variations, specified as one of the following:

- Real nonnegative scalar, $\alpha$ - Covariance matrix is an $N$-by- $N$ diagonal matrix, with $\alpha$ as the diagonal elements.
- Vector of real nonnegative scalars, $\left[\alpha_{1}, \ldots, \alpha_{N}\right]$ - Covariance matrix is an $N$-by- $N$ diagonal matrix, with $\left[\alpha_{1}, \ldots, \alpha_{N}\right]$ as the diagonal elements.
- $N$-by- $N$ symmetric positive semidefinite matrix.
$N$ is the number of parameters to be estimated.
ProcessNoiseCovariance is applicable when EstimationMethod is 'KalmanFilter'.
Kalman filter algorithm treats the parameters as states of a dynamic system and estimates these parameters using a Kalman filter. ProcessNoiseCovariance is the covariance of the process noise acting on these parameters. Zero values in the noise covariance matrix correspond to estimating constant coefficients. Values larger than 0 correspond to time-varying parameters. Use large values for rapidly changing parameters. However, the larger values result in noisier parameter estimates.

ProcessNoiseCovariance is a tunable property. You can change it when the object is in a locked state.

Default: 0.1

## AdaptationGain

Adaptation gain, $\gamma$, used in gradient recursive estimation algorithms, specified as a positive scalar.
AdaptationGain is applicable when EstimationMethod is 'Gradient' or 'NormalizedGradient'.

Specify a large value for AdaptationGain when your measurements have a high signal-to-noise ratio.

AdaptationGain is a tunable property. You can change it when the object is in a locked state.
Default: 1

## NormalizationBias

Bias in adaptation gain scaling used in the 'NormalizedGradient ' method, specified as a nonnegative scalar.

NormalizationBias is applicable when EstimationMethod is 'NormalizedGradient'.

The normalized gradient algorithm divides the adaptation gain at each step by the square of the twonorm of the gradient vector. If the gradient is close to zero, this can cause jumps in the estimated parameters. NormalizationBias is the term introduced in the denominator to prevent these jumps. Increase NormalizationBias if you observe jumps in estimated parameters.

NormalizationBias is a tunable property. You can change it when the object is in a locked state.

## Default: eps

## History

Data history type defining which type of recursive algorithm you use, specified as:

- 'Infinite' - Use an algorithm that aims to minimize the error between the observed and predicted outputs for all time steps from the beginning of the simulation.
- 'Finite' - Use an algorithm that aims to minimize the error between the observed and predicted outputs for a finite number of past time steps.

Algorithms with infinite history aim to produce parameter estimates that explain all data since the start of the simulation. These algorithms still use a fixed amount of memory that does not grow over time. The object provides multiple algorithms of the 'Infinite' History type. Specifying this option activates the EstimationMethod property with which you specify an algorithm.

Algorithms with finite history aim to produce parameter estimates that explain only a finite number of past data samples. This method is also called sliding-window estimation. The object provides one algorithm of the 'Finite' type. Specifying this option activates the WindowLength property that sizes the window.

For more information on recursive estimation methods, see "Recursive Algorithms for Online Parameter Estimation".

History is a nontunable property. It can be set only during object construction using Name, Value arguments and cannot be changed afterward.

## Default: 'Infinite'

## WindowLength

Window size determining the number of time samples to use for the sliding-window estimation method, specified as a positive integer. Specify WindowLength only when History is Finite.

Choose a window size that balances estimation performance with computational and memory burden. Sizing factors include the number and time variance of the parameters in your model. Always specify Window Length in samples, even if you are using frame-based input processing.

WindowLength must be greater than or equal to the number of estimated parameters.
Suitable window length is independent of whether you are using sample-based or frame-based input processing (see InputProcessing). However, when using frame-based processing, your window length must be greater than or equal to the number of samples (time steps) contained in the frame.

WindowLength is a nontunable property. It can be set only during object construction using Name, Value arguments and cannot be changed afterward.

Default: 200

## InputProcessing

Option for sample-based or frame-based input processing, specified as a character vector or string.

- Sample-based processing operates on signals streamed one sample at a time.
- Frame-based processing operates on signals containing samples from multiple time steps. Many machine sensor interfaces package multiple samples and transmit these samples together in frames. Frame-based processing allows you to input this data directly without having to first unpack it.

Your InputProcessing specification impacts the dimensions for the input and output signals when using the step command:
[theta,EstimatedOutput] = step(obj,y)

- Sample-based
- y and EstimatedOutput are scalars.
- Frame-based with $M$ samples per frame
- y and EstimatedOutput are $M$-by- 1 vectors.

InputProcessing is a nontunable property. It can be set only during object construction using Name, Value arguments and cannot be changed afterward.

Default: 'Sample-based'

## Output Arguments

## obj - System object for online parameter estimation of AR model

recursiveAR System object
System object for online parameter estimation of AR model, returned as a recursiveAR System object. This object is created using the specified model orders and properties. Use step command to estimate the coefficients of the AR model polynomials. You can then access the estimated coefficients and parameter covariance using dot notation. For example, type obj. A to view the estimated $A$ polynomial coefficients.

## More About

## AR Model Structure

The AR model structure is:

$$
A(q) y(t)=e(t)
$$

where,

$$
A(q)=1+a_{1} q^{-1}+\ldots+a_{n_{a}} q^{-n_{a}}
$$

Here,

- $y(t)$ - Output at time $t$. Data is a time series that has no input channels and one output channel.
- na - Number of $A$ polynomial coefficients.
- $e(t)$ - White-noise disturbance value at time $t$.
- $q^{-1}$ - Time-shift operator.


## Tips

- Starting in R2016b, instead of using the step command to update model parameter estimates, you can call the System object with input arguments, as if it were a function. For example, [A,EstimatedOutput] = step(obj,y) and [A,EstimatedOutput] = obj(y) perform equivalent operations.


## Extended Capabilities

C/C++ Code Generation
Generate C and C++ code using MATLAB® Coder ${ }^{\mathrm{TM}}$.
Usage notes and limitations:

- For Simulink based workflows, use Recursive Polynomial Model Estimator.
- For limitations, see "Generate Code for Online Parameter Estimation in MATLAB".

Supports MATLAB Function block: No

## See Also

Recursive Polynomial Model Estimator | ar |clone |isLocked|recursiveARMA | recursiveARMAX | recursiveARX|recursiveBJ|recursiveLS|recursive0E|release| reset|step

## Topics

"Perform Online Parameter Estimation at the Command Line"
"Validate Online Parameter Estimation at the Command Line"
"What Is Online Estimation?"
"Recursive Algorithms for Online Parameter Estimation"
Introduced in R2015b

## recursiveARMA

Create System object for online parameter estimation of ARMA model

## Syntax

```
obj = recursiveARMA
obj = recursiveARMA(Orders)
obj = recursiveARMA(Orders,A0,C0)
obj = recursiveARMA( __ ,Name,Value)
```


## Description

Use recursiveARMA command for parameter estimation with real-time data. If all data necessary for estimation is available at once, and you are estimating a time-invariant model, use the offline estimation command, armax.
obj = recursiveARMA creates a System object for online parameter estimation of a default single output ARMA model structure on page 1-1209. The default model structure has polynomials of order 1 and initial polynomial coefficient values eps.

After creating the object, use the step command to update model parameter estimates using recursive estimation algorithms and real-time data.
obj $=$ recursiveARMA(Orders) specifies the polynomial orders of the ARMA model to be estimated.
obj $=$ recursiveARMA(Orders,A0,C0) specifies the polynomial orders and initial values of the polynomial coefficients. Specify initial values to potentially avoid local minima during estimation. If the initial values are small compared to the default InitialParameterCovariance property value, and you have confidence in your initial values, also specify a smaller
InitialParameterCovariance.
obj = recursiveARMA(__ ,Name,Value) specifies additional attributes of the ARMA model structure and recursive estimation algorithm using one or more Name, Value pair arguments.

## Object Description

recursiveARMA creates a System object for online parameter estimation of single output ARMA models using a recursive estimation algorithm.

A System object is a specialized MATLAB object designed specifically for implementing and simulating dynamic systems with inputs that change over time. System objects use internal states to store past behavior, which is used in the next computational step.

After you create a System object, you use commands to process data or obtain information from or about the object. System objects use a minimum of two commands to process data - a constructor to create the object and the step command to update object parameters using real-time data. This separation of declaration from execution lets you create multiple, persistent, reusable objects, each with different settings.

You can use the following commands with the online estimation System objects in System Identification Toolbox:

| Command | Description |
| :--- | :--- |
| step | Update model parameter estimates using <br> recursive estimation algorithms and real-time <br> data. <br> step puts the object into a locked state. In a <br> locked state, you cannot change any nontunable <br> properties or input specifications, such as model <br> order, data type, or estimation algorithm. During <br> execution, you can only change tunable <br> properties. |
| release | Unlock the System object. Use this command to <br> enable setting of nontunable parameters. |
| reset | Reset the internal states of a locked System <br> object to the initial values, and leave the object <br> locked. |
| clone | Create another System object with the same <br> object property values. |
|  | Do not create additional objects using syntax <br> obj2 = obj. Any changes made to the <br> properties of the new object created this way <br> (obj2) also change the properties of the original <br> object (obj). |
| isLocked | Query locked status for input attributes and <br> nontunable properties of the System object. |

Use the recursiveARMA command to create an online estimation System object. Then estimate the ARMA model parameters (A and C) and output using the step command with output data y.

```
[A,C,EstimatedOutput] = step(obj,y)
```

For recursiveARMA object properties, see "Properties" on page 1-1205.

## Examples

## Estimate ARMA Model Online

Create a System object for online parameter estimation of an ARMA model.
obj = recursiveARMA;
The ARMA model has a default structure with polynomials of order 1 and initial polynomial coefficient values, eps.

Load the time-series estimation data. In this example, use a static data set for illustration.

```
load iddata9 z9;
output = z9.y;
Estimate ARMA model parameters online using step.
```

```
for i = 1:numel(output)
```

for i = 1:numel(output)
[A,C,EstimatedOutput] = step(obj,output(i));
[A,C,EstimatedOutput] = step(obj,output(i));
end

```
end
```

View the current estimated values of polynomial C coefficients.

```
obj.C
```

ans $=1 \times 2$
$1.0000 \quad 0.2315$

View the current covariance estimate of the parameters.

```
obj.ParameterCovariance
```

ans $=2 \times 2$
$10^{-3} \times$

$$
0.6372-0.0257
$$

$$
-0.0257 \quad 0.0017
$$

View the current estimated output.
Estimated0utput
EstimatedOutput = 11.8121

## Create Online Estimation System Object for ARMA Model With Known Orders

Specify ARMA model orders.
na $=2$;
nc = 1;
Create a System object for online estimation of an ARMA model with the specified orders.
obj $=$ recursiveARMA([na nc]);

## Create Online Estimation System Object for ARMA Model With Known Initial Parameters

Specify ARMA model orders.
na $=2$;
nc = 1;

Create a System object for online estimation of ARMA model with known initial polynomial coefficients.
$\mathrm{A} 0=\left[\begin{array}{lll}1 & 0.5 & 0.3\end{array}\right] ;$
C0 = [1 0.7];
obj = recursiveARMA([na nc],A0,C0);
Specify the initial parameter covariance.
obj.InitialParameterCovariance = 0.1;
InitialParameterCovariance represents the uncertainty in your guess for the initial parameters. Typically, the default InitialParameterCovariance (10000) is too large relative to the parameter values. This results in initial guesses being given less importance during estimation. If you have confidence in the initial parameter guesses, specify a smaller initial parameter covariance.

## Specify Estimation Method for Online Estimation of ARMA Model

Create a System object that uses the unnormalized gradient algorithm for online parameter estimation of an ARMA model.

```
obj = recursiveARMA([2 1],'EstimationMethod','Gradient');
```


## Input Arguments

## Orders - Model orders

1-by-2 vector of integers
Model orders of an ARMA model on page 1-1209, specified as a 1-by-2 vector of integers, [na nc].

- na - Order of the polynomial $A(q)$, specified as a nonnegative integer.
- nc - Order of the polynomial $C(q)$, specified as a nonnegative integer.


## A0, C0 - Initial value of polynomial coefficients <br> row vectors of real values | [ ]

Initial value of polynomial coefficients, specified as row vectors of real values with elements in order of ascending powers of $q^{-1}$.

- A0 - Initial guess for the coefficients of the polynomial $A(q)$, specified as a 1-by-(na+1) vector with 1 as the first element.
- C0 - Initial guess for the coefficients of the polynomial $C(q)$, specified as a 1-by-(nc+1) vector with 1 as the first element.

The coefficients in C0 must define a stable discrete-time polynomial with roots within a unit disk. For example,

C0 = [1 0.5 0.5];
all(abs(roots(C0))<1)

1
Specifying as [ ], uses the default value of eps for the polynomial coefficients.

Note If the initial guesses are much smaller than the default InitialParameterCovariance, 10000, the initial guesses are given less importance during estimation. In that case, specify a smaller initial parameter covariance.

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name,Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Use Name, Value arguments to specify writable properties on page 1-1205 of recursiveARMA System object during object creation. For example, obj = recursiveARMA([2
1],'EstimationMethod', 'Gradient') creates a System object to estimate an ARMA model using the 'Gradient ' recursive estimation algorithm.

## Properties

recursiveARMA System object properties consist of read-only and writable properties. The writable properties are tunable and nontunable properties. The nontunable properties cannot be changed when the object is locked, that is, after you use the step command.

Use Name, Value arguments to specify writable properties of recursiveARMA objects during object creation. After object creation, use dot notation to modify the tunable properties.
obj = recursiveARMA;
obj.ForgettingFactor = 0.99;
A
Estimated coefficients of polynomial $A(q)$, returned as a row vector of real values specified in order of ascending powers of $q^{-1}$.

A is a read-only property and is initially empty after you create the object. It is populated after you use the step command for online parameter estimation.

## C

Estimated coefficients of polynomial $C(q)$, returned as a vector of real values specified in order of ascending powers of $q^{-1}$.

C is a read-only property and is initially empty after you create the object. It is populated after you use the step command for online parameter estimation.

## InitialA

Initial values for the coefficients of polynomial $A(q)$ of order na, specified as a row vector of length na +1 , with 1 as the first element. Specify the coefficients in order of ascending powers of $q^{-1}$.

If the initial guesses are much smaller than the default InitialParameterCovariance, 10000, the initial guesses are given less importance during estimation. In that case, specify a smaller initial parameter covariance.

InitialA is a tunable property. You can change it when the object is in a locked state.
Default: [1 eps]

## InitialC

Initial values for the coefficients of polynomial $C(q)$ of order nc , specified as a row vector of length nc +1 , with 1 as the first element. Specify the coefficients in order of ascending powers of $q^{-1}$.

The coefficients in InitialC must define a stable discrete-time polynomial with roots within a unit circle. For example,

```
InitialC = [1 0.5 0.5];
all(abs(roots(InitialC))<1)
ans =
1
```

If the initial guesses are much smaller than the default InitialParameterCovariance, 10000, the initial guesses are given less importance during estimation. In that case, specify a smaller initial parameter covariance.

InitialC is a tunable property. You can change it when the object is in a locked state.
Default: [1 eps]

## ParameterCovariance

Estimated covariance P of the parameters, returned as an $N$-by- $N$ symmetric positive-definite matrix. $N$ is the number of parameters to be estimated. The software computes P assuming that the residuals (difference between estimated and measured outputs) are white noise, and the variance of these residuals is 1. ParameterCovariance is applicable only when EstimationMethod is 'ForgettingFactor' or 'KalmanFilter'.

The interpretation of P depends on the estimation method:

- 'ForgettingFactor' $-R_{2} / 2 * \mathrm{P}$ is approximately equal to the covariance matrix of the estimated parameters, where $R_{2}$ is the true variance of the residuals.
- 'KalmanFilter' $-R_{2} * \mathrm{P}$ is the covariance matrix of the estimated parameters, and $R_{1} / R_{2}$ is the covariance matrix of the parameter changes. Where, $R_{1}$ is the covariance matrix that you specify in ProcessNoiseCovariance.

ParameterCovariance is a read-only property and is initially empty after you create the object. It is populated after you use the step command for online parameter estimation.

## InitialParameterCovariance

Covariance of the initial parameter estimates, specified as one of the following:

- Real positive scalar, $\alpha$ - Covariance matrix is an $N$-by- $N$ diagonal matrix, with $\alpha$ as the diagonal elements. $N$ is the number of parameters to be estimated.
- Vector of real positive scalars, $\left[\alpha_{1}, \ldots, \alpha_{N}\right]$ - Covariance matrix is an $N$-by- $N$ diagonal matrix, with [ $\alpha_{1}, \ldots, \alpha_{N}$ ] as the diagonal elements.
- $N$-by- $N$ symmetric positive-definite matrix.

InitialParameterCovariance represents the uncertainty in the initial parameter estimates. For large values of InitialParameterCovariance, less importance is placed on the initial parameter values and more on the measured data during beginning of estimation using step.

Use only when EstimationMethod is 'ForgettingFactor' or 'KalmanFilter'.
InitialParameterCovariance is a tunable property. You can change it when the object is in a locked state.

Default: 10000

## EstimationMethod

Recursive estimation algorithm used for online estimation of model parameters, specified as one of the following values:

- 'ForgettingFactor' - Algorithm used for parameter estimation
- 'KalmanFilter' - Algorithm used for parameter estimation
- 'NormalizedGradient' - Algorithm used for parameter estimation
- 'Gradient ' - Unnormalized gradient algorithm used for parameter estimation

Forgetting factor and Kalman filter algorithms are more computationally intensive than gradient and unnormalized gradient methods. However, they have better convergence properties. For information about these algorithms, see "Recursive Algorithms for Online Parameter Estimation".

EstimationMethod is a nontunable property. You cannot change it during execution, that is after the object is locked using the step command. If you want to deploy code using MATLAB Coder, EstimationMethod can only be assigned once.

## Default: 'ForgettingFactor'

## ForgettingFactor

Forgetting factor, $\lambda$, relevant for parameter estimation, specified as a scalar in the range $(0,1]$.
Suppose that the system remains approximately constant over $T_{0}$ samples. You can choose $\lambda$ such that:

$$
T_{0}=\frac{1}{1-\lambda}
$$

- Setting $\lambda=1$ corresponds to "no forgetting" and estimating constant coefficients.
- Setting $\lambda<1$ implies that past measurements are less significant for parameter estimation and can be "forgotten". Set $\lambda<1$ to estimate time-varying coefficients.

Typical choices of $\lambda$ are in the range [0.98 0.995].
Use only when EstimationMethod is 'ForgettingFactor'.
ForgettingFactor is a tunable property. You can change it when the object is in a locked state.

## Default: 1

## EnableAdapation

Enable or disable parameter estimation, specified as one of the following:

- true or 1 - The step command estimates the parameter values for that time step and updates the parameter values.
- false or 0 - The step command does not update the parameters for that time step and instead outputs the last estimated value. You can use this option when your system enters a mode where the parameter values do not vary with time.

Note If you set EnableAdapation to false, you must still execute the step command. Do not skip step to keep parameter values constant, because parameter estimation depends on current and past I/O measurements. step ensures past I/O data is stored, even when it does not update the parameters.

EnableAdapation is a tunable property. You can change it when the object is in a locked state.
Default: true

## DataType

Floating point precision of parameters, specified as one of the following values:

- 'double' - Double-precision floating point
- 'single' - Single-precision floating point

Setting DataType to 'single' saves memory, but leads to loss of precision. Specify DataType based on the precision required by the target processor where you will deploy generated code.

DataType is a nontunable property. It can only be set during object construction using Name, Value arguments and cannot be changed afterward.

Default: 'double'

## ProcessNoiseCovariance

Covariance matrix of parameter variations, specified as one of the following:

- Real nonnegative scalar, $\alpha$ - Covariance matrix is an $N$-by- $N$ diagonal matrix, with $\alpha$ as the diagonal elements.
- Vector of real nonnegative scalars, $\left[\alpha_{1}, \ldots, \alpha_{N}\right]$ - Covariance matrix is an $N$-by- $N$ diagonal matrix, with $\left[\alpha_{1}, \ldots, \alpha_{N}\right]$ as the diagonal elements.
- $N$-by- $N$ symmetric positive semidefinite matrix.
$N$ is the number of parameters to be estimated.
ProcessNoiseCovariance is applicable when EstimationMethod is 'KalmanFilter'.
Kalman filter algorithm treats the parameters as states of a dynamic system and estimates these parameters using a Kalman filter. ProcessNoiseCovariance is the covariance of the process noise acting on these parameters. Zero values in the noise covariance matrix correspond to estimating
constant coefficients. Values larger than 0 correspond to time-varying parameters. Use large values for rapidly changing parameters. However, the larger values result in noisier parameter estimates.

ProcessNoiseCovariance is a tunable property. You can change it when the object is in a locked state.

Default: 0.1
AdaptationGain
Adaptation gain, $\gamma$, used in gradient recursive estimation algorithms, specified as a positive scalar.
AdaptationGain is applicable when EstimationMethod is 'Gradient' or 'NormalizedGradient'.

Specify a large value for AdaptationGain when your measurements have a high signal-to-noise ratio.

AdaptationGain is a tunable property. You can change it when the object is in a locked state.

## Default: 1

## NormalizationBias

Bias in adaptation gain scaling used in the 'NormalizedGradient' method, specified as a nonnegative scalar.

NormalizationBias is applicable when EstimationMethod is 'NormalizedGradient'.
The normalized gradient algorithm divides the adaptation gain at each step by the square of the twonorm of the gradient vector. If the gradient is close to zero, this can cause jumps in the estimated parameters. NormalizationBias is the term introduced in the denominator to prevent these jumps. Increase NormalizationBias if you observe jumps in estimated parameters.

NormalizationBias is a tunable property. You can change it when the object is in a locked state.

## Default: eps

## Output Arguments

## obj - System object for online parameter estimation of ARMA model

recursiveARMA System object
System object for online parameter estimation of ARMA model, returned as a recursiveARMA System object. This object is created using the specified model orders and properties. Use step command to estimate the coefficients of the ARMA model polynomials. You can then access the estimated coefficients and parameter covariance using dot notation. For example, type obj. A to view the estimated $A$ polynomial coefficients.

## More About

ARMA Model Structure
The ARMA model structure is:

$$
A(q) y(t)=C(q) e(t)
$$

where,

$$
\begin{aligned}
& A(q)=1+a_{1} q^{-1}+\ldots+a_{n_{a}} q^{-n_{a}} \\
& C(q)=1+c_{1} q^{-1}+\ldots+c_{n_{c}} q^{-n_{C}}
\end{aligned}
$$

Here,

- $y(t)$ - Output at time $t$. Data is a time series that has no input channels and one output channel.
- na - Number of $A$ polynomial coefficients
- $n c$ - Number of $C$ polynomial coefficients
- $e(t)$ - White-noise disturbance value at time $t$
- $q^{-1}$ - Time-shift operator


## Tips

- Starting in R2016b, instead of using the step command to update model parameter estimates, you can call the System object with input arguments, as if it were a function. For example, [A,C,EstimatedOutput] = step(obj,y) and [A,C,EstimatedOutput] = obj(y) perform equivalent operations.


## Extended Capabilities

## C/C++ Code Generation

Generate C and C++ code using MATLAB® Coder $^{\text {TM }}$.
Usage notes and limitations:

- For Simulink-based workflows, use Recursive Polynomial Model Estimator.
- For limitations, see "Generate Code for Online Parameter Estimation in MATLAB".

Supports MATLAB Function block: No

## See Also

Recursive Polynomial Model Estimator | armax | clone | isLocked | recursiveAR | recursiveARMAX | recursiveARX|recursiveBJ|recursiveLS|recursive0E|release| reset|step

## Topics

"Perform Online Parameter Estimation at the Command Line"
"Validate Online Parameter Estimation at the Command Line"
"What Is Online Estimation?"
"Recursive Algorithms for Online Parameter Estimation"

## Introduced in R2015b

## recursiveARMAX

Create System object for online parameter estimation of ARMAX model

## Syntax

```
obj = recursiveARMAX
obj = recursiveARMAX(Orders)
obj = recursiveARMAX(Orders,A0,B0,C0)
obj = recursiveARMAX(
```

$\qquad$

``` ,Name, Value)
```


## Description

Use recursiveARMAX command for parameter estimation with real-time data. If all data necessary for estimation is available at once, and you are estimating a time-invariant model, use the offline estimation command, armax.
obj = recursiveARMAX creates a System object for online parameter estimation of default singleinput single-output (SISO) ARMAX model structure on page 1-1220. The default model structure has polynomials of order 1 and initial polynomial coefficient values eps.

After creating the object, use the step command to update model parameter estimates using recursive estimation algorithms and real-time data.
obj $=$ recursiveARMAX(Orders) specifies the polynomial orders of the ARMAX model to be estimated.
obj = recursiveARMAX(Orders,A0,B0,C0) specifies the polynomial orders and initial values of the polynomial coefficients. Specify initial values to potentially avoid local minima during estimation. If the initial values are small compared to the default InitialParameterCovariance property value, and you have confidence in your initial values, also specify a smaller
InitialParameterCovariance.
obj = recursiveARMAX( ___ ,Name, Value) specifies additional attributes of the ARMAX model structure and recursive estimation algorithm using one or more Name, Value pair arguments.

## Object Description

recursiveARMAX creates a System object for online parameter estimation of SISO ARMAX models using a recursive estimation algorithm.

A System object is a specialized MATLAB object designed specifically for implementing and simulating dynamic systems with inputs that change over time. System objects use internal states to store past behavior, which is used in the next computational step.

After you create a System object, you use commands to process data or obtain information from or about the object. System objects use a minimum of two commands to process data - a constructor to create the object and the step command to update object parameters using real-time data. This separation of declaration from execution lets you create multiple, persistent, reusable objects, each with different settings.

You can use the following commands with the online estimation System objects in System Identification Toolbox:

| Command | Description |
| :--- | :--- |
| step | Update model parameter estimates using <br> recursive estimation algorithms and real-time <br> data. <br> step puts the object into a locked state. In a <br> locked state, you cannot change any nontunable <br> properties or input specifications, such as model <br> order, data type, or estimation algorithm. During <br> execution, you can only change tunable <br> properties. |
| release | Unlock the System object. Use this command to <br> enable setting of nontunable parameters. |
| reset | Reset the internal states of a locked System <br> object to the initial values, and leave the object <br> locked. |
| clone | Create another System object with the same <br> object property values. |
|  | Do not create additional objects using syntax <br> obj2 $=$ obj. Any changes made to the <br> properties of the new object created this way <br> (obj2) also change the properties of the original <br> object (obj). |
| isLocked | Query locked status for input attributes and <br> nontunable properties of the System object. |

Use the recursiveARMAX command to create an online estimation System object. Then estimate the ARMAX model parameters ( $\mathrm{A}, \mathrm{B}$, and C ) and output using the step command with incoming input and output data, u , and y .

```
[A,B,C,EstimatedOutput] = step(obj,y,u)
```

For recursiveARMAX object properties, see "Properties" on page 1-1215.

## Examples

## Estimate an ARMAX Model Online

Create a System object for online parameter estimation of an ARMAX model.
obj = recursiveARMAX;
The ARMAX model has a default structure with polynomials of order 1 and initial polynomial coefficient values, eps.

Load the estimation data. In this example, use a static data set for illustration.

```
load iddatal z1;
output = z1.y;
input = z1.u;
```

Estimate ARMAX model parameters online using step.

```
for i = 1:numel(input)
[A,B,C,EstimatedOutput] = step(obj,output(i),input(i));
end
```

View the current estimated values of polynomial A coefficients.

```
obj.A
ans = 1\times2
    1.0000-0.8298
```

View the current covariance estimate of the parameters.

```
obj.ParameterCovariance
```

ans $=3 \times 3$

| 0.0001 | 0.0001 | 0.0001 |
| :--- | :--- | :--- |
| 0.0001 | 0.0032 | 0.0000 |
| 0.0001 | 0.0000 | 0.0001 |

View the current estimated output.
EstimatedOutput
EstimatedOutput = -4.5595

## Create System Object for ARMAX Model With Known Polynomial Orders

Specify ARMAX model orders and delays.

```
na = 1;
nb = 2;
nc = 1;
nk = 1;
```

Create a System object for online estimation of ARMAX model with the specified orders and delays.
obj = recursiveARMAX([na nb nc nk]);

## Create Online Estimation System Object for ARMAX Model With Known Initial Parameters

Specify ARMAX model orders and delays.

```
na = 1;
nb = 2;
```

```
nc = 1;
nk = 1;
```

Create a System object for online estimation of ARMAX model with known initial polynomial coefficients.

```
A0 = [1 0.5];
B0 = [l0 1 1];
C0 = [1 0.5];
obj = recursiveARMAX([na nb nc nk],A0,B0,C0);
```

Specify the initial parameter covariance.

```
obj.InitialParameterCovariance = 0.1;
```

InitialParameterCovariance represents the uncertainty in your guess for the initial parameters. Typically, the default InitialParameterCovariance (10000) is too large relative to the parameter values. This results in initial guesses being given less importance during estimation. If you have confidence in the initial parameter guesses, specify a smaller initial parameter covariance.

## Specify Estimation Method for Online Estimation of ARMAX Model

Create a System object that uses the Kalman filter algorithm for online parameter estimation of an ARMAX model.

```
obj = recursiveARMAX([1 2 1 1],'EstimationMethod','KalmanFilter');
obj.ProcessNoiseCovariance = 0.01;
```

The ProcessNoiseCovariance property of obj is applicable only when the Kalman filter algorithm is used for estimation.

## Input Arguments

## Orders - Model orders and delays

## 1-by-4 vector of integers

Model orders and delays of an ARMAX model on page 1-1220, specified as a 1-by-4 vector of integers, [na nb nc nk].

- na - Order of the polynomial $A(q)$, specified as a nonnegative integer. na represents the number of poles in your system.
- nb - Order of the polynomial $B(q)+1$, specified as a positive integer. nb represents the number of zeroes in your system plus 1 .
- nc - Order of the polynomial $C(q)$, specified as a nonnegative integer.
- nk - Input-output delay, specified as a nonnegative integer. nk is number of input samples that occur before the input affects the output. nk is expressed as fixed leading zeros of the $B$ polynomial.


## A0, B0, C0 - Initial value of polynomial coefficients

row vectors of real values | [ ]

Initial value of polynomial coefficients, specified as row vectors of real values with elements in order of ascending powers of $q^{-1}$.

- A0 - Initial guess for the coefficients of the polynomial $A(q)$, specified as a 1-by-(na+1) vector with 1 as the first element.
- B0 - Initial guess for the coefficients of the polynomial $B(q)$, specified as a 1-by-(nb+nk) vector with nk leading zeros.
- C0 - Initial guess for the coefficients of the polynomial $C(q)$, specified as a 1-by-(nc+1) vector with 1 as the first element.

The coefficients in C0 must define a stable discrete-time polynomial with roots within a unit disk. For example,

```
C0 = [1 0.5 0.5];
all(abs(roots(C0))<1)
ans =
    1
```

Specifying as [], uses the default value of eps for the polynomial coefficients.

Note If the initial guesses are much smaller than the default InitialParameterCovariance, 10000, the initial guesses are given less importance during estimation. In that case, specify a smaller initial parameter covariance.

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Use Name, Value arguments to specify writable properties on page 1-1215 of recursiveARMAX System object during object creation. For example, obj $=$ recursiveARMAX([2 21
1], 'EstimationMethod ', 'Gradient') creates a System object to estimate an ARMAX model using the 'Gradient ' recursive estimation algorithm.

## Properties

recursiveARMAX System object properties consist of read-only and writable properties. The writable properties are tunable and nontunable properties. The nontunable properties cannot be changed when the object is locked, that is, after you use the step command.

Use Name, Value arguments to specify writable properties of recursiveARMAX objects during object creation. After object creation, use dot notation to modify the tunable properties.
obj = recursiveARMAX;
obj.ForgettingFactor = 0.99;
A
Estimated coefficients of polynomial $A(q)$, returned as a row vector of real values specified in order of ascending powers of $q^{-1}$.

A is a read-only property and is initially empty after you create the object. It is populated after you use the step command for online parameter estimation.

B
Estimated coefficients of polynomial $B(q)$, returned as a vector of real values specified in order of ascending powers of $q^{-1}$.

B is a read-only property and is initially empty after you create the object. It is populated after you use the step command for online parameter estimation.

## C

Estimated coefficients of polynomial $C(q)$, returned as a vector of real values specified in order of ascending powers of $q^{-1}$.

C is a read-only property and is initially empty after you create the object. It is populated after you use the step command for online parameter estimation.

## InitialA

Initial values for the coefficients of polynomial $A(q)$ of order na, specified as a row vector of length na +1 , with 1 as the first element. Specify the coefficients in order of ascending powers of $q^{-1}$.

If the initial guesses are much smaller than the default InitialParameterCovariance, 10000, the initial guesses are given less importance during estimation. In that case, specify a smaller initial parameter covariance.

InitialA is a tunable property. You can change it when the object is in a locked state.
Default: [1 eps]

## InitialB

Initial values for the coefficients of polynomial $B(q)$ of order $\mathrm{nb}-1$, specified as a row vector of length $\mathrm{nb}+\mathrm{nk}$, with nk leading zeros. nk is the input-output delay. Specify the coefficients in order of ascending powers of $q^{-1}$.

If the initial guesses are much smaller than the default InitialParameterCovariance, 10000, the initial guesses are given less importance during estimation. In that case, specify a smaller initial parameter covariance.

InitialB is a tunable property. You can change it when the object is in a locked state.
Default: [0 eps]

## InitialC

Initial values for the coefficients of polynomial $C(q)$ of order nc, specified as a row vector of length nc +1 , with 1 as the first element. Specify the coefficients in order of ascending powers of $q^{-1}$.

The coefficients in InitialC must define a stable discrete-time polynomial with roots within a unit circle. For example,

```
InitialC = [1 0.5 0.5];
all(abs(roots(InitialC))<1)
```

If the initial guesses are much smaller than the default InitialParameterCovariance, 10000, the initial guesses are given less importance during estimation. In that case, specify a smaller initial parameter covariance.

InitialC is a tunable property. You can change it when the object is in a locked state.
Default: [1 eps]

## ParameterCovariance

Estimated covariance P of the parameters, returned as an $N$-by- $N$ symmetric positive-definite matrix. $N$ is the number of parameters to be estimated. The software computes P assuming that the residuals (difference between estimated and measured outputs) are white noise, and the variance of these residuals is 1. ParameterCovariance is applicable only when EstimationMethod is 'ForgettingFactor' or 'KalmanFilter'.

The interpretation of P depends on the estimation method:

- 'ForgettingFactor' $-R_{2} / 2 * \mathrm{P}$ is approximately equal to the covariance matrix of the estimated parameters, where $R_{2}$ is the true variance of the residuals.
- 'KalmanFilter' $-R_{2}{ }^{*} \mathrm{P}$ is the covariance matrix of the estimated parameters, and $R_{1} / R_{2}$ is the covariance matrix of the parameter changes. Where, $R_{1}$ is the covariance matrix that you specify in ProcessNoiseCovariance.

ParameterCovariance is a read-only property and is initially empty after you create the object. It is populated after you use the step command for online parameter estimation.

## InitialParameterCovariance

Covariance of the initial parameter estimates, specified as one of the following:

- Real positive scalar, $\alpha$ - Covariance matrix is an $N$-by- $N$ diagonal matrix, with $\alpha$ as the diagonal elements. $N$ is the number of parameters to be estimated.
- Vector of real positive scalars, $\left[\alpha_{1}, \ldots, \alpha_{N}\right]$ - Covariance matrix is an $N$-by- $N$ diagonal matrix, with [ $\alpha_{1}, \ldots, \alpha_{N}$ ] as the diagonal elements.
- $N$-by- $N$ symmetric positive-definite matrix.

InitialParameterCovariance represents the uncertainty in the initial parameter estimates. For large values of InitialParameterCovariance, less importance is placed on the initial parameter values and more on the measured data during beginning of estimation using step.

Use only when EstimationMethod is 'ForgettingFactor' or 'KalmanFilter'.
InitialParameterCovariance is a tunable property. You can change it when the object is in a locked state.

Default: 10000

## EstimationMethod

Recursive estimation algorithm used for online estimation of model parameters, specified as one of the following values:

- 'ForgettingFactor' - Algorithm used for parameter estimation
- 'KalmanFilter' - Algorithm used for parameter estimation
- 'NormalizedGradient' - Algorithm used for parameter estimation
- 'Gradient ' - Unnormalized gradient algorithm used for parameter estimation

Forgetting factor and Kalman filter algorithms are more computationally intensive than gradient and unnormalized gradient methods. However, they have better convergence properties. For information about these algorithms, see "Recursive Algorithms for Online Parameter Estimation".

EstimationMethod is a nontunable property. You cannot change it during execution, that is after the object is locked using the step command. If you want to deploy code using MATLAB Coder, EstimationMethod can only be assigned once.

Default: 'ForgettingFactor'

## ForgettingFactor

Forgetting factor, $\lambda$, relevant for parameter estimation, specified as a scalar in the range $(0,1]$.
Suppose that the system remains approximately constant over $T_{0}$ samples. You can choose $\lambda$ such that:

$$
T_{0}=\frac{1}{1-\lambda}
$$

- Setting $\lambda=1$ corresponds to "no forgetting" and estimating constant coefficients.
- Setting $\lambda<1$ implies that past measurements are less significant for parameter estimation and can be "forgotten". Set $\lambda<1$ to estimate time-varying coefficients.

Typical choices of $\lambda$ are in the range [0.98 0.995].
Use only when EstimationMethod is 'ForgettingFactor'.
ForgettingFactor is a tunable property. You can change it when the object is in a locked state.

## Default: 1

## EnableAdapation

Enable or disable parameter estimation, specified as one of the following:

- true or 1 - The step command estimates the parameter values for that time step and updates the parameter values.
- false or 0 - The step command does not update the parameters for that time step and instead outputs the last estimated value. You can use this option when your system enters a mode where the parameter values do not vary with time.

Note If you set EnableAdapation to false, you must still execute the step command. Do not skip step to keep parameter values constant, because parameter estimation depends on current
and past I/O measurements. step ensures past I/O data is stored, even when it does not update the parameters.

EnableAdapation is a tunable property. You can change it when the object is in a locked state.
Default: true

## DataType

Floating point precision of parameters, specified as one of the following values:

- 'double' - Double-precision floating point
- 'single' - Single-precision floating point

Setting DataType to 'single' saves memory, but leads to loss of precision. Specify DataType based on the precision required by the target processor where you will deploy generated code.

DataType is a nontunable property. It can only be set during object construction using Name, Value arguments and cannot be changed afterward.

Default: 'double'

## ProcessNoiseCovariance

Covariance matrix of parameter variations, specified as one of the following:

- Real nonnegative scalar, $\alpha$ - Covariance matrix is an $N$-by- $N$ diagonal matrix, with $\alpha$ as the diagonal elements.
- Vector of real nonnegative scalars, $\left[\alpha_{1}, \ldots, \alpha_{N}\right]$ - Covariance matrix is an $N$-by- $N$ diagonal matrix, with $\left[\alpha_{1}, \ldots, \alpha_{N}\right]$ as the diagonal elements.
- $N$-by- $N$ symmetric positive semidefinite matrix.
$N$ is the number of parameters to be estimated.
ProcessNoiseCovariance is applicable when EstimationMethod is 'KalmanFilter'.
Kalman filter algorithm treats the parameters as states of a dynamic system and estimates these parameters using a Kalman filter. ProcessNoiseCovariance is the covariance of the process noise acting on these parameters. Zero values in the noise covariance matrix correspond to estimating constant coefficients. Values larger than 0 correspond to time-varying parameters. Use large values for rapidly changing parameters. However, the larger values result in noisier parameter estimates.

ProcessNoiseCovariance is a tunable property. You can change it when the object is in a locked state.

Default: 0.1

## AdaptationGain

Adaptation gain, $\gamma$, used in gradient recursive estimation algorithms, specified as a positive scalar.
AdaptationGain is applicable when EstimationMethod is 'Gradient' or 'NormalizedGradient'.

Specify a large value for AdaptationGain when your measurements have a high signal-to-noise ratio.

AdaptationGain is a tunable property. You can change it when the object is in a locked state.
Default: 1

## NormalizationBias

Bias in adaptation gain scaling used in the 'NormalizedGradient' method, specified as a nonnegative scalar.

NormalizationBias is applicable when EstimationMethod is 'NormalizedGradient'.
The normalized gradient algorithm divides the adaptation gain at each step by the square of the twonorm of the gradient vector. If the gradient is close to zero, this can cause jumps in the estimated parameters. NormalizationBias is the term introduced in the denominator to prevent these jumps. Increase NormalizationBias if you observe jumps in estimated parameters.

NormalizationBias is a tunable property. You can change it when the object is in a locked state.

## Default: eps

## Output Arguments

## obj - System object for online parameter estimation of ARMAX model

recursiveARMAX System object
System object for online parameter estimation of ARMAX model, returned as a recursiveARMAX System object. This object is created using the specified model orders and properties. Use step command to estimate the coefficients of the ARMAX model polynomials. You can then access the estimated coefficients and parameter covariance using dot notation. For example, type obj. A to view the estimated $A$ polynomial coefficients.

## More About

## ARMAX Model Structure

The ARMAX (Autoregressive Moving Average with Extra Input) model structure is:

$$
\begin{aligned}
& y(t)+a_{1} y(t-1)+\ldots+a_{n_{a}} y\left(t-n_{a}\right)= \\
& b_{1} u\left(t-n_{k}\right)+\ldots+b_{n_{b}} u\left(t-n_{k}-n_{b}+1\right)+ \\
& \quad c_{1} e(t-1)+\ldots+c_{n_{c}} e\left(t-n_{c}\right)+e(t)
\end{aligned}
$$

A more compact way to write the difference equation is

$$
A(q) y(t)=B(q) u\left(t-n_{k}\right)+C(q) e(t)
$$

where

- $y(t)$ - Output at time $t$
- $n_{a}$ - Number of poles
- $n_{b}$ - Number of zeroes plus 1
- $n_{C}$ - Number of $C$ coefficients
- $n_{k}$ - Number of input samples that occur before the input affects the output, also called the dead time in the system
- $y(t-1) \ldots y\left(t-n_{a}\right)-$ Previous outputs on which the current output depends
- $u\left(t-n_{k}\right) \ldots u\left(t-n_{k}-n_{b}+1\right)-$ Previous and delayed inputs on which the current output depends
- $e(t-1) \ldots e\left(t-n_{C}\right)-$ White-noise disturbance value

The parameters na, $n b$, and $n c$ are the orders of the ARMAX model, and $n k$ is the delay. $q$ is the delay operator. Specifically,

$$
\begin{aligned}
& A(q)=1+a_{1} q^{-1}+\ldots+a_{n_{a}} q^{-n_{a}} \\
& B(q)=b_{1}+b_{2} q^{-1}+\ldots+b_{n_{b}} q^{-n_{b}+1} \\
& C(q)=1+c_{1} q^{-1}+\ldots+c_{n_{c}} q^{-n_{C}}
\end{aligned}
$$

## Tips

- Starting in R2016b, instead of using the step command to update model parameter estimates, you can call the System object with input arguments, as if it were a function. For example, $[A, B, C, E s t i m a t e d O u t p u t]=\operatorname{step}(o b j, y, u)$ and $[A, B, C, E s t i m a t e d O u t p u t]=$ $\mathrm{obj}(\mathrm{y}, \mathrm{u})$ perform equivalent operations.


## Extended Capabilities

## C/C++ Code Generation

Generate C and C++ code using MATLAB® Coder $^{\text {TM }}$.
Usage notes and limitations:

- For Simulink-based workflows, use Recursive Polynomial Model Estimator.
- For limitations, see "Generate Code for Online Parameter Estimation in MATLAB".

Supports MATLAB Function block: No

## See Also

Recursive Polynomial Model Estimator | armax |clone | isLocked | recursiveAR | recursiveARMA|recursiveARX|recursiveBJ|recursiveLS|recursive0E|release| reset|step

## Topics

"Perform Online Parameter Estimation at the Command Line"
"Validate Online Parameter Estimation at the Command Line"
"What Is Online Estimation?"
"Recursive Algorithms for Online Parameter Estimation"

Introduced in R2015b

## recursiveARX

Create System object for online parameter estimation of ARX model

## Syntax

```
obj = recursiveARX
obj = recursiveARX(Orders)
obj = recursiveARX(Orders,A0,B0)
obj = recursiveARX(
```

$\qquad$

``` ,Name, Value)
```


## Description

Use recursiveARX command for parameter estimation with real-time data. If all data necessary for estimation is available at once, and you are estimating a time-invariant model, use the offline estimation command, arx.
obj = recursiveARX creates a System object for online parameter estimation of a default ARX model structure on page 1-1234. The default model structure has polynomials of order 1 and initial polynomial coefficient values eps.

After creating the object, use the step command to update model parameter estimates using recursive estimation algorithms and real-time data.
obj = recursiveARX(Orders) specifies the polynomial orders of the ARX model to be estimated.
obj $=$ recursiveARX(Orders, $\mathrm{A} 0, \mathrm{~B} 0$ ) specifies the polynomial orders and initial values of the polynomial coefficients.
obj = recursiveARX( __ ,Name,Value) specifies additional attributes of the ARX model structure and recursive estimation algorithm using one or more Name, Value pair arguments.

## Object Description

recursiveARX creates a System object for online parameter estimation of single-input single-output (SISO) or multiple-input single-output (MISO) ARX models using a recursive estimation algorithm.

A System object is a specialized MATLAB object designed specifically for implementing and simulating dynamic systems with inputs that change over time. System objects use internal states to store past behavior, which is used in the next computational step.

After you create a System object, you use commands to process data or obtain information from or about the object. System objects use a minimum of two commands to process data - a constructor to create the object and the step command to update object parameters using real-time data. This separation of declaration from execution lets you create multiple, persistent, reusable objects, each with different settings.

You can use the following commands with the online estimation System objects in System Identification Toolbox:

| Command | Description |
| :--- | :--- |
| step | Update model parameter estimates using <br> recursive estimation algorithms and real-time <br> data. <br> step puts the object into a locked state. In a <br> locked state, you cannot change any nontunable <br> properties or input specifications, such as model <br> order, data type, or estimation algorithm. During <br> execution, you can only change tunable <br> properties. |
| release | Unlock the System object. Use this command to <br> enable setting of nontunable parameters. |
| reset | Reset the internal states of a locked System <br> object to the initial values, and leave the object <br> locked. |
| clone | Create another System object with the same <br> object property values. |
| Do not create additional objects using syntax |  |
| obj2 = obj. Any changes made to the |  |
| properties of the new object created this way |  |
| (obj2) also change the properties of the original |  |
| object (obj). |  |

Use the recursiveARX command to create an online estimation System object. Then estimate the ARX model parameters (A and B) and output using the step command with incoming input and output data, u and y .

```
[A,B,EstimatedOutput] = step(obj,y,u)
```

For recursiveARX object properties, see "Properties" on page 1-1227.

## Examples

## Estimate a SISO ARX Model Online

Create a System object for online parameter estimation of a SISO ARX model.
obj = recursiveARX;
The ARX model has a default structure with polynomials of order 1 and initial polynomial coefficient values, eps.

Load the estimation data. In this example, use a static data set for illustration.

```
load iddatal z1;
output = z1.y;
input = z1.u;
```

Estimate ARX model parameters online using step.

```
for i = 1:numel(input)
[A,B,EstimatedOutput] = step(obj,output(i),input(i));
end
```

View the current estimated values of polynomial B coefficients.

```
obj.B
ans = 1\times2
```

$$
0 \quad 0.7974
$$

View the current covariance estimate of the parameters.

```
obj.ParameterCovariance
ans = 2\times2
    0.0002 0.0001
    0.0001 0.0034
```

View the current estimated output.

```
EstimatedOutput
```

EstimatedOutput = -4.7766

## Create System Object for SISO ARX Model With Known Initial Parameters

Specify ARX model orders and delays.

```
na = 1;
nb = 2;
nk = 1;
```

Create a System object for online estimation of SISO ARX model with known initial polynomial coefficients.

```
A0 = [1 0.5];
B0 = [l0 1 1];
obj = recursiveARX([na nb nk],A0,B0);
```

Specify the initial parameter covariance.

```
obj.InitialParameterCovariance = 0.1;
```

InitialParameterCovariance represents the uncertainty in your guess for the initial parameters. Typically, the default InitialParameterCovariance (10000) is too large relative to the parameter values. This results in initial guesses being given less importance during estimation. If you have confidence in the initial parameter guesses, specify a smaller initial parameter covariance.

## Create System Object for MISO ARX Model With Known Initial Parameters

Specify orders and delays for ARX model with two inputs and one output.
na $=1$;
nb = [2 1];
nk = [1 3 3];
nb and nk are specified as row vectors of length equal to number of inputs, $N u$.
Specify initial polynomial coefficients.
$\mathrm{A} 0=\left[\begin{array}{ll}1 & 0.5\end{array}\right] ;$
$\mathrm{B} 0=\left[\begin{array}{lllllll}0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0.8\end{array}\right] ;$
B0 has $N u$ rows and max (nb+nk) columns. The $i$-th row corresponds to $i$-th input and is specified as having $n k(i)$ zeros, followed by $n b(i)$ initial values. Values after nb(i)+nk(i) are ignored.

Create a System object for online estimation of ARX model with known initial polynomial coefficients. obj = recursiveARX([na nb nk],A0,B0);

## Specify Estimation Method for Online Estimation of ARX Model

Create a System object that uses the normalized gradient algorithm for online parameter estimation of an ARX model.

```
obj = recursiveARX([1 2 1],'EstimationMethod','NormalizedGradient');
```


## Input Arguments

## Orders - Model orders and delays

1 -by-3 vector of integers | 1-by-3 vector of vectors
Model orders and delays of an ARX model on page 1-1234, specified as a 1-by-3 vector of integers or vectors, [na nb nk].

- na - Order of the polynomial $A(q)$, specified as a nonnegative integer.
- nb - Order of the polynomial $B(q)+1$, specified as 1 -by- $N u$ vector of positive integers. $N u$ is the number of inputs.

For MISO models, there are as many $B(q)$ polynomials as the number of inputs. $\mathrm{nb}(\mathrm{i})$ is the order of $i$ th polynomial $B_{i}(q)+1$ for the ith input.

- nk - Input-output delay, specified as a 1-by-Nu vector of nonnegative integers. $N u$ is the number of inputs.

For MISO models, there are as many $B(q)$ polynomials as the number of inputs. nk(i) is the input-output delay time corresponding to the ith input.

## A0, B0 - Initial value of polynomial coefficients

row vector and matrix of real values | [ ]

Initial value of coefficients of $A(q)$ and $B(q)$ polynomials, specified as row vector and matrix or real values, respectively. Specify the elements in order of ascending powers of $q^{-1}$.

- A0 - Initial value for the coefficients of the polynomial $A(q)$, specified as a 1-by-(na+1) row vector with 1 as the first element.
- B0 - Initial value for the coefficients of the polynomial $B(q)$, specified as $N u$-by-max (nb+nk) matrix. $N u$ is the number of inputs.

For MISO models, there are as many $B(q)$ polynomials as the number of inputs. The ith row of B0 corresponds to the ith input and must contain $\mathrm{nk}(\mathrm{i})$ leading zeros, followed by $\mathrm{nb}(\mathrm{i})$ initial parameter values. Entries beyond $\mathrm{nk}(\mathrm{i})+\mathrm{nb}(\mathrm{i})$ are ignored.
$\mathrm{na}, \mathrm{nb}$, and nk are the Orders of the model.
Specifying as [], uses the default value of eps for the polynomial coefficients.
If the initial parameter values are much smaller than InitialParameterCovariance, these initial values are given less importance during estimation. Specify a smaller initial parameter covariance if you have high confidence in the initial parameter values. This statement applies only for infinitehistory estimation. Finite-history estimation does not use InitialParameterCovariance.

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Use Name, Value arguments to specify writable properties on page 1-1227 of recursiveARX System object during object creation. For example, obj = recursiveARX([2 2
1],'EstimationMethod', 'Gradient') creates a System object to estimate an ARX model using the 'Gradient' recursive estimation algorithm.

## Properties

recursiveARX System object properties consist of read-only and writable properties. The writable properties are tunable and nontunable properties. The nontunable properties cannot be changed when the object is locked, that is, after you use the step command.

Use Name, Value arguments to specify writable properties of recursiveARX objects during object creation. After object creation, use dot notation to modify the tunable properties.

```
obj = recursiveARX;
obj.ForgettingFactor = 0.99;
```


## A

Estimated coefficients of polynomial $A(q)$, returned as a row vector of real values specified in order of ascending powers of $q^{-1}$.

A is a read-only property and is initially empty after you create the object. It is populated after you use the step command for online parameter estimation.

Estimated coefficients of polynomial $B(q)$, returned as a $N u$-by-max (nb+nk) matrix of real values. $N u$ is the number of inputs.

The ith row of B corresponds to the $i$ th input and contains nk (i) leading zeros, followed by $\mathrm{nb}(\mathrm{i})$ estimated parameters, specified in order of ascending powers of $q^{-1}$. Ignore zero entries beyond nk(i)+nb(i).
$B$ is a read-only property and is initially empty after you create the object. It is populated after you use the step command for online parameter estimation.

## InitialA

Initial values for the coefficients of polynomial $A(q)$ of order na, specified as a row vector of length na +1 , with 1 as the first element. Specify the coefficients in order of ascending powers of $q^{-1}$.

If the initial guesses are much smaller than the default InitialParameterCovariance, 10000, the initial guesses are given less importance during estimation. In that case, specify a smaller initial parameter covariance.

InitialA is a tunable property. You can change it when the object is in a locked state.
Default: [1 eps]

## InitialB

Initial values for the coefficients of polynomial $B(q)$, specified as an $N u$-by-max (nb+nk) matrix. $N u$ is the number of inputs.

For MISO models, there are as many $B(q)$ polynomials as the number of inputs. The ith row of B0 corresponds to the $i$ th input and must contain nk(i) zeros, followed by nb(i) initial parameter values. Entries beyond $\mathrm{nk}(\mathrm{i})+\mathrm{nb}(\mathrm{i})$ are ignored.

If the initial guesses are much smaller than the default InitialParameterCovariance, 10000, the initial guesses are given less importance during estimation. In that case, specify a smaller initial parameter covariance.

InitialB is a tunable property. You can change it when the object is in a locked state.
Default: [0 eps]

## InitialOutputs

Initial values of the measured outputs buffer in finite-history estimation, specified as 0 or as a ( $W$ $+n a)$-by- 1 vector, where $W$ is the window length and na is the order of the polynomial $A(q)$ that you specify when constructing the object.

The InitialOutputs property provides a means of controlling the initial behavior of the algorithm.
When Initial0utputs is set to 0, the object populates the buffer with zeros.
If the initial buffer is set to 0 or does not contain enough information, you see a warning message during the initial phase of your estimation. The warning should clear after a few cycles. The number of cycles it takes for sufficient information to be buffered depends upon the order of your polynomials and your input delays. If the warning persists, you should evaluate the content of your signals.

Specify Initial0utputs only when History is Finite.
Initialoutputs is a tunable property. You can change Initialoutputs when the object is in a locked state.

## Default: 0

## InitialInputs

Initial values of the inputs in the finite history window, specified as 0 or as a $(W-1+\max (n b)$ $+\max (n k)$ )-by-nu matrix, where $W$ is the window length and $n u$ is the number of inputs. $n b$ is the vector of $B(q)$ polynomial orders and $n k$ is vector of input delays that you specify when constructing the recursiveARX object.

The InitialInputs property provides a means of controlling the initial behavior of the algorithm.
When the InitialInputs is set to 0 , the object populates the buffer with zeros.
If the initial buffer is set to 0 or does not contain enough information, you see a warning message during the initial phase of your estimation. The warning should clear after a few cycles. The number of cycles it takes for sufficient information to be buffered depends upon the order of your polynomials and your input delays. If the warning persists, you should evaluate the content of your signals.

Specify InitialInputs only when History is Finite.
InitialInputs is a tunable property. You can change InitialInputs when the object is in a locked state.

## Default: 0

## ParameterCovariance

Estimated covariance P of the parameters, returned as an $N$-by- $N$ symmetric positive-definite matrix. $N$ is the number of parameters to be estimated. The software computes P assuming that the residuals (difference between estimated and measured outputs) are white noise, and the variance of these residuals is 1.

ParameterCovariance is applicable only when EstimationMethod is 'ForgettingFactor' or 'KalmanFilter' or when History is Finite.

The interpretation of P depends on your settings for the History and EstimationMethod properties.

- If History is Infinite, then your EstimationMethod selection results in one of the following:
- 'ForgettingFactor' $-\left(R_{2} / 2\right) \mathrm{P}$ is approximately equal to the covariance matrix of the estimated parameters, where $R_{2}$ is the true variance of the residuals.
- 'KalmanFilter' $-R_{2} \mathrm{P}$ is the covariance matrix of the estimated parameters, and $R_{1} / R_{2}$ is the covariance matrix of the parameter changes. Here, $R_{1}$ is the covariance matrix that you specify in ProcessNoiseCovariance.
- If History is Finite (sliding-window estimation) $-R_{2} \mathrm{P}$ is the covariance of the estimated parameters. The sliding-window algorithm does not use this covariance in the parameterestimation process. However, the algorithm does compute the covariance for output so that you can use it for statistical evaluation.

ParameterCovariance is a read-only property and is initially empty after you create the object. It is populated after you use the step command for online parameter estimation.

## InitialParameterCovariance

Covariance of the initial parameter estimates, specified as one of the following:

- Real positive scalar, $\alpha$ - Covariance matrix is an $N$-by- $N$ diagonal matrix, with $\alpha$ as the diagonal elements. $N$ is the number of parameters to be estimated.
- Vector of real positive scalars, $\left[\alpha_{1}, \ldots, \alpha_{N}\right]$ - Covariance matrix is an $N$-by- $N$ diagonal matrix, with [ $\alpha_{1}, \ldots, \alpha_{N}$ ] as the diagonal elements.
- $N$-by- $N$ symmetric positive-definite matrix.

InitialParameterCovariance represents the uncertainty in the initial parameter estimates. For large values of InitialParameterCovariance, less importance is placed on the initial parameter values and more on the measured data during beginning of estimation using step.

Use only when EstimationMethod is 'ForgettingFactor' or 'KalmanFilter'.
InitialParameterCovariance is a tunable property. You can change it when the object is in a locked state.

Default: 10000

## EstimationMethod

Recursive estimation algorithm used for online estimation of model parameters, specified as one of the following values:

- 'ForgettingFactor' - Algorithm used for parameter estimation
- 'KalmanFilter' - Algorithm used for parameter estimation
- 'NormalizedGradient' - Algorithm used for parameter estimation
- 'Gradient ' - Unnormalized gradient algorithm used for parameter estimation

Forgetting factor and Kalman filter algorithms are more computationally intensive than gradient and unnormalized gradient methods. However, they have better convergence properties. For information about these algorithms, see "Recursive Algorithms for Online Parameter Estimation".

These methods all use an infinite data history, and are available only when History is 'Infinite'.
EstimationMethod is a nontunable property. You cannot change it during execution, that is, after the object is locked using the step command.

## Default: Forgetting Factor

## ForgettingFactor

Forgetting factor, $\lambda$, relevant for parameter estimation, specified as a scalar in the range $(0,1]$.
Suppose that the system remains approximately constant over $T_{0}$ samples. You can choose $\lambda$ such that:

$$
T_{0}=\frac{1}{1-\lambda}
$$

- Setting $\lambda=1$ corresponds to "no forgetting" and estimating constant coefficients.
- Setting $\lambda<1$ implies that past measurements are less significant for parameter estimation and can be "forgotten". Set $\lambda<1$ to estimate time-varying coefficients.

Typical choices of $\lambda$ are in the range [0.98 0.995].
Use only when EstimationMethod is 'ForgettingFactor'.
ForgettingFactor is a tunable property. You can change it when the object is in a locked state.

## Default: 1

## EnableAdapation

Enable or disable parameter estimation, specified as one of the following:

- true or 1- The step command estimates the parameter values for that time step and updates the parameter values.
- false or 0 - The step command does not update the parameters for that time step and instead outputs the last estimated value. You can use this option when your system enters a mode where the parameter values do not vary with time.

Note If you set EnableAdapation to false, you must still execute the step command. Do not skip step to keep parameter values constant, because parameter estimation depends on current and past I/O measurements. step ensures past I/O data is stored, even when it does not update the parameters.

EnableAdapation is a tunable property. You can change it when the object is in a locked state.
Default: true

## DataType

Floating point precision of parameters, specified as one of the following values:

- 'double' - Double-precision floating point
- 'single' - Single-precision floating point

Setting DataType to 'single' saves memory, but leads to loss of precision. Specify DataType based on the precision required by the target processor where you will deploy generated code.

DataType is a nontunable property. It can only be set during object construction using Name, Value arguments and cannot be changed afterward.

## Default: 'double'

## ProcessNoiseCovariance

Covariance matrix of parameter variations, specified as one of the following:

- Real nonnegative scalar, $\alpha$ - Covariance matrix is an $N$-by- $N$ diagonal matrix, with $\alpha$ as the diagonal elements.
- Vector of real nonnegative scalars, $\left[\alpha_{1}, \ldots, \alpha_{N}\right]$ - Covariance matrix is an $N$-by- $N$ diagonal matrix, with $\left[\alpha_{1}, \ldots, \alpha_{N}\right]$ as the diagonal elements.
- $N$-by- $N$ symmetric positive semidefinite matrix.
$N$ is the number of parameters to be estimated.
ProcessNoiseCovariance is applicable when EstimationMethod is 'KalmanFilter'.
Kalman filter algorithm treats the parameters as states of a dynamic system and estimates these parameters using a Kalman filter. ProcessNoiseCovariance is the covariance of the process noise acting on these parameters. Zero values in the noise covariance matrix correspond to estimating constant coefficients. Values larger than 0 correspond to time-varying parameters. Use large values for rapidly changing parameters. However, the larger values result in noisier parameter estimates.

ProcessNoiseCovariance is a tunable property. You can change it when the object is in a locked state.

Default: 0.1

## AdaptationGain

Adaptation gain, $\gamma$, used in gradient recursive estimation algorithms, specified as a positive scalar.
AdaptationGain is applicable when EstimationMethod is 'Gradient' or
'NormalizedGradient'.
Specify a large value for AdaptationGain when your measurements have a high signal-to-noise ratio.

AdaptationGain is a tunable property. You can change it when the object is in a locked state.

## Default: 1

## NormalizationBias

Bias in adaptation gain scaling used in the 'NormalizedGradient' method, specified as a nonnegative scalar.

NormalizationBias is applicable when EstimationMethod is 'NormalizedGradient'.
The normalized gradient algorithm divides the adaptation gain at each step by the square of the twonorm of the gradient vector. If the gradient is close to zero, this can cause jumps in the estimated parameters. NormalizationBias is the term introduced in the denominator to prevent these jumps. Increase NormalizationBias if you observe jumps in estimated parameters.

NormalizationBias is a tunable property. You can change it when the object is in a locked state.

## Default: eps

## History

Data history type defining which type of recursive algorithm you use, specified as:

- 'Infinite' - Use an algorithm that aims to minimize the error between the observed and predicted outputs for all time steps from the beginning of the simulation.
- 'Finite ' - Use an algorithm that aims to minimize the error between the observed and predicted outputs for a finite number of past time steps.

Algorithms with infinite history aim to produce parameter estimates that explain all data since the start of the simulation. These algorithms still use a fixed amount of memory that does not grow over time. The object provides multiple algorithms of the 'Infinite' History type. Specifying this option activates the EstimationMethod property with which you specify an algorithm.

Algorithms with finite history aim to produce parameter estimates that explain only a finite number of past data samples. This method is also called sliding-window estimation. The object provides one algorithm of the 'Finite' type. Specifying this option activates the WindowLength property that sizes the window.

For more information on recursive estimation methods, see "Recursive Algorithms for Online Parameter Estimation".

History is a nontunable property. It can be set only during object construction using Name, Value arguments and cannot be changed afterward.

Default: 'Infinite'

## WindowLength

Window size determining the number of time samples to use for the sliding-window estimation method, specified as a positive integer. Specify WindowLength only when History is Finite.

Choose a window size that balances estimation performance with computational and memory burden. Sizing factors include the number and time variance of the parameters in your model. Always specify Window Length in samples, even if you are using frame-based input processing.

WindowLength must be greater than or equal to the number of estimated parameters.
Suitable window length is independent of whether you are using sample-based or frame-based input processing (see InputProcessing). However, when using frame-based processing, your window length must be greater than or equal to the number of samples (time steps) contained in the frame.

WindowLength is a nontunable property. It can be set only during object construction using Name, Value arguments and cannot be changed afterward.

Default: 200

## InputProcessing

Option for sample-based or frame-based input processing, specified as a character vector or string.

- Sample-based processing operates on signals streamed one sample at a time.
- Frame-based processing operates on signals containing samples from multiple time steps. Many machine sensor interfaces package multiple samples and transmit these samples together in frames. Frame-based processing allows you to input this data directly without having to first unpack it.

Your InputProcessing specification impacts the dimensions for the input and output signals when using the step command:

```
[theta,EstimatedOutput] = step(obj,y,u)
```

- Sample-based
- y and Estimated0utput are scalars.
- u is a 1-by-Nu vector, where $N u$ is the number of inputs.
-     - Frame-based with $M$ samples per frame
- y and EstimatedOutput are $M$-by-1 vectors.
- u is an $M$-by-Nu matrix.

InputProcessing is a nontunable property. It can be set only during object construction using Name, Value arguments and cannot be changed afterward.

Default: 'Sample-based'

## Output Arguments

## obj - System object for online parameter estimation of ARX model

## recursiveARX System object

System object for online parameter estimation of ARX model, returned as a recursiveARX System object. This object is created using the specified model orders and properties. Use step command to estimate the coefficients of the ARX model polynomials. You can then access the estimated coefficients and parameter covariance using dot notation. For example, type obj. A to view the estimated $A$ polynomial coefficients.

## More About

## ARX Model Structure

The ARX model structure is :

$$
\begin{aligned}
& y(t)+a_{1} y(t-1)+\ldots+a_{n a y} y(t-n a)= \\
& b_{1} u(t-n k)+\ldots+b_{n b} u(t-n b-n k+1)+e(t)
\end{aligned}
$$

The parameters na and nb are the orders of the ARX model, and nk is the delay.

- $y(t)-$ Output at time $t$.
- $n_{a}$ - Number of poles.
- $n_{b}$ - Number of zeroes plus 1 .
- $n_{k}$ - Number of input samples that occur before the input affects the output, also called the dead time in the system.
- $y(t-1) \ldots y\left(t-n_{a}\right)-$ Previous outputs on which the current output depends.
- $u\left(t-n_{k}\right) \ldots u\left(t-n_{k}-n_{b}+1\right)$ - Previous and delayed inputs on which the current output depends.
- $\quad e(t)$ - White-noise disturbance value.

A more compact way to write the difference equation is

$$
A(q) y(t)=B(q) u\left(t-n_{k}\right)+e(t)
$$

$q$ is the delay operator. Specifically,

$$
\begin{aligned}
& A(q)=1+a_{1} q^{-1}+\ldots+a_{n_{a}} q^{-n_{a}} \\
& B(q)=b_{1}+b_{2} q^{-1}+\ldots+b_{n_{b}} q^{-n_{b}+1}
\end{aligned}
$$

## Tips

- Starting in R2016b, instead of using the step command to update model parameter estimates, you can call the System object with input arguments, as if it were a function. For example, [A,B,EstimatedOutput] = step(obj,y,u) and [A,B,EstimatedOutput] = obj(y,u) perform equivalent operations.


## Extended Capabilities

## C/C++ Code Generation

Generate C and $\mathrm{C}++$ code using MATLAB® Coder $^{\mathrm{TM}}$.
Usage notes and limitations:

- For Simulink based workflows, use Recursive Polynomial Model Estimator.
- For limitations, see "Generate Code for Online Parameter Estimation in MATLAB".

Supports MATLAB Function block: No

## See Also

Recursive Polynomial Model Estimator \| arx |clone | isLocked| recursiveAR|recursiveARMA | recursiveARMAX |recursiveBJ|recursiveLS|recursiveOE|release|reset|step

## Topics

"Perform Online Parameter Estimation at the Command Line"
"Validate Online Parameter Estimation at the Command Line"
"Online ARX Parameter Estimation for Tracking Time-Varying System Dynamics"
"What Is Online Estimation?"
"Recursive Algorithms for Online Parameter Estimation"

## Introduced in R2015b

## recursiveBJ

Create System object for online parameter estimation of Box-Jenkins polynomial model

## Syntax

```
obj = recursiveBJ
obj = recursiveBJ(Orders)
obj = recursiveBJ(Orders,B0,C0,D0,F0)
obj = recursiveBJ(
```

$\qquad$

``` ,Name, Value)
```


## Description

Use recursiveBJ command for parameter estimation with real-time data. If all data necessary for estimation is available at once, and you are estimating a time-invariant model, use the offline estimation command, bj.
obj = recursiveBJ creates a System object for online parameter estimation of a default singleinput single-output (SISO) Box-Jenkins polynomial model structure on page 1-1246. The default model structure has polynomials of order 1 and initial polynomial coefficient values eps.

After creating the object, use the step command to update model parameter estimates using recursive estimation algorithms and real-time data.
obj $=$ recursiveBJ (Orders) specifies the polynomial orders of the Box-Jenkins model to be estimated.
obj = recursiveBJ (Orders,B0,C0,D0,F0) specifies the polynomial orders and initial values of the polynomial coefficients. Specify initial values to potentially avoid local minima during estimation. If the initial values are small compared to the default InitialParameterCovariance property value, and you have confidence in your initial values, also specify a smaller
InitialParameterCovariance.
obj = recursiveBJ( $\qquad$ ,Name, Value) specifies additional attributes of the Box-Jenkins model structure and recursive estimation algorithm using one or more Name, Value pair arguments.

## Object Description

recursiveBJ creates a System object for online parameter estimation of SISO Box-Jenkins polynomial models using a recursive estimation algorithm.

A System object is a specialized MATLAB object designed specifically for implementing and simulating dynamic systems with inputs that change over time. System objects use internal states to store past behavior, which is used in the next computational step.

After you create a System object, you use commands to process data or obtain information from or about the object. System objects use a minimum of two commands to process data - a constructor to create the object and the step command to update object parameters using real-time data. This separation of declaration from execution lets you create multiple, persistent, reusable objects, each with different settings.

You can use the following commands with the online estimation System objects in System Identification Toolbox:

| Command | Description |
| :--- | :--- |
| step | Update model parameter estimates using <br> recursive estimation algorithms and real-time <br> data. <br> step puts the object into a locked state. In a <br> locked state, you cannot change any nontunable <br> properties or input specifications, such as model <br> order, data type, or estimation algorithm. During <br> execution, you can only change tunable <br> properties. |
| release | Unlock the System object. Use this command to <br> enable setting of nontunable parameters. |
| reset | Reset the internal states of a locked System <br> object to the initial values, and leave the object <br> locked. |
| clone | Create another System object with the same <br> object property values. |
|  | Do not create additional objects using syntax <br> obj2 $=$ obj. Any changes made to the <br> properties of the new object created this way <br> (obj2) also change the properties of the original <br> object (obj). |
| isLocked | Query locked status for input attributes and <br> nontunable properties of the System object. |

Use the recursiveBJ command to create an online estimation System object. Then estimate the Box-Jenkins polynomial model parameters (B, C, D, and F) and output using the step command with incoming input and output data, $u$ and $y$.

```
[B,C,D,F,EstimatedOutput] = step(obj,y,u)
```

For recursiveBJ object properties, see "Properties" on page 1-1241.

## Examples

## Estimate Box-Jenkins Polynomial Model Online

Create a System object for online parameter estimation of a Box-Jenkins polynomial model.
obj = recursiveBJ;
The Box-Jenkins model has a default structure with polynomials of order 1 and initial polynomial coefficient values, eps.

Load the estimation data. In this example, use a static data set for illustration.

```
load iddatal z1;
output = z1.y;
input = z1.u;
```

Estimate Box-Jenkins model parameters online using step.

```
for i = 1:numel(input)
[B,C,D,F,EstimatedOutput] = step(obj,output(i),input(i));
end
```

View the current estimated values of polynomial D coefficients.

```
obj.D
```

```
ans = 1\times2
```

    \(1.0000-0.6876\)
    View the current covariance estimate of the parameters.

```
obj.ParameterCovariance
```

ans $=4 \times 4$

| 0.0020 | -0.0004 | -0.0001 | 0.0002 |
| ---: | ---: | ---: | ---: |
| -0.0004 | 0.0007 | 0.0006 | -0.0001 |
| -0.0001 | 0.0006 | 0.0007 | -0.0000 |
| 0.0002 | -0.0001 | -0.0000 | 0.0001 |

View the current estimated output.

```
EstimatedOutput
EstimatedOutput = -4.1905
```


## Create System Object for Box-Jenkins Model With Known Orders and Delays

Specify Box-Jenkins polynomial model orders and delays.

```
nb = 1;
nc = 1;
nd = 2;
nf = 1;
nk = 1;
```

Create a System object for online estimation of Box-Jenkins model with the specified orders and delays.

```
obj = recursiveBJ([nb nc nd nf nk]);
```


## Create System Object for Box-Jenkins Model With Known Initial Parameters

Specify Box-Jenkins polynomial model orders and delays.

```
nb = 1;
nc = 1;
nd = 1;
nf = 2;
nk = 1;
```

Create a System object for online estimation of Box-Jenkins model with known initial polynomial coefficients.

```
B0 = [0 1];
C0 = [1 0.5];
D0 = [1 0.9];
F0 = [1 0.7 0.8];
obj = recursiveBJ([nb nc nd nf nk],B0,C0,D0,F0);
```

Specify the initial parameter covariance.

```
obj.InitialParameterCovariance = 0.1;
```

InitialParameterCovariance represents the uncertainty in your guess for the initial parameters. Typically, the default InitialParameterCovariance (10000) is too large relative to the parameter values. This results in initial guesses being given less importance during estimation. If you have confidence in the initial parameter guesses, specify a smaller initial parameter covariance.

## Specify Estimation Method for Online Estimation of Box-Jenkins Model

Create a System object that uses the normalized gradient algorithm for online parameter estimation of a Box-Jenkins model.

```
obj = recursiveBJ([1 1 1 2 1],'EstimationMethod','NormalizedGradient');
```


## Input Arguments

## Orders - Model orders and delays

1 -by- 5 vector of integers
Model orders and delays of a Box-Jenkins polynomial model on page 1-1246, specified as a 1-by-5 vector of integers, [nb nc nd nf nk].

- nb - Order of the polynomial $B(q)+1$, specified as a positive integer.
- nc - Order of the polynomial $C(q)$, specified as a nonnegative integer.
- nd - Order of the polynomial $D(q)$, specified as a nonnegative integer.
- nf - Order of the polynomial $F(q)$, specified as a nonnegative integer.
- nk - Input-output delay, specified as a positive integer. nk is number of input samples that occur before the input affects the output. nk is expressed as fixed leading zeros of the $B$ polynomial.


## B0, C0, D0, F0 - Initial value of polynomial coefficients

row vectors of real values | [ ]
Initial value of polynomial coefficients, specified as row vectors of real values with elements in order of ascending powers of $q^{-1}$.

- B0 - Initial guess for the coefficients of the polynomial $B(q)$, specified as a 1-by-(nb+nk) vector with nk leading zeros.
- C0 - Initial guess for the coefficients of the polynomial $C(q)$, specified as a 1-by-(nc+1) vector with 1 as the first element.

The coefficients in C0 must define a stable discrete-time polynomial with roots within a unit disk. For example,

```
C0 = [1 0.5 0.5];
```

all(abs(roots(C0))<1)
ans $=$
1

- D0 - Initial guess for the coefficients of the polynomial $D(q)$, specified as a 1-by-(nd+1) vector with 1 as the first element.

The coefficients in D0 must define a stable discrete-time polynomial with roots within a unit disk. For example,

```
D0 = [1 0.9 0.8];
all(abs(roots(D0))<1)
ans =
    1
```

- F0 - Initial guess for the coefficients of the polynomial $F(q)$, specified as a 1-by-(nf+1) vector with 1 as the first element.

The coefficients in F0 must define a stable discrete-time polynomial with roots within a unit disk. For example,

```
F0 = [1 0.5 0.5];
all(abs(roots(F0))<1)
ans =
    1
```

Specifying as [], uses the default value of eps for the polynomial coefficients.

Note If the initial guesses are much smaller than the default InitialParameterCovariance, 10000, the initial guesses are given less importance during estimation. In that case, specify a smaller initial parameter covariance.

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Use Name, Value arguments to specify writable properties on page 1-1241 of recursiveBJ System object during object creation. For example, obj $=$ recursiveBJ ([ $\begin{array}{llll}1 & 1 & 1 & 2\end{array}$
1],'EstimationMethod', 'Gradient') creates a System object to estimate a Box-Jenkins polynomial model using the 'Gradient' recursive estimation algorithm.

## Properties

recursiveBJ System object properties consist of read-only and writable properties. The writable properties are tunable and nontunable properties. The nontunable properties cannot be changed when the object is locked, that is, after you use the step command.

Use Name, Value arguments to specify writable properties of recursiveBJ objects during object creation. After object creation, use dot notation to modify the tunable properties.

```
obj = recursiveBJ;
obj.ForgettingFactor = 0.99;
```


## B

Estimated coefficients of polynomial $B(q)$, returned as a vector of real values specified in order of ascending powers of $q^{-1}$.
$B$ is a read-only property and is initially empty after you create the object. It is populated after you use the step command for online parameter estimation.

## C

Estimated coefficients of polynomial $C(q)$, returned as a vector of real values specified in order of ascending powers of $q^{-1}$.

C is a read-only property and is initially empty after you create the object. It is populated after you use the step command for online parameter estimation.

## D

Estimated coefficients of polynomial $D(q)$, returned as a vector of real values specified in order of ascending powers of $q^{-1}$.

D is a read-only property and is initially empty after you create the object. It is populated after you use the step command for online parameter estimation.

## F

Estimated coefficients of polynomial $F(q)$, returned as a vector of real values specified in order of ascending powers of $q^{-1}$.

F is a read-only property and is initially empty after you create the object. It is populated after you use the step command for online parameter estimation.

## InitialB

Initial values for the coefficients of polynomial $B(q)$ of order nb-1, specified as a row vector of length $n b+n k$, with $n k$ leading zeros. $n k$ is the input-output delay. Specify the coefficients in order of ascending powers of $q^{-1}$.

If the initial guesses are much smaller than the default InitialParameterCovariance, 10000, the initial guesses are given less importance during estimation. In that case, specify a smaller initial parameter covariance.

InitialB is a tunable property. You can change it when the object is in a locked state.
Default: [0 eps]

## InitialC

Initial values for the coefficients of polynomial $C(q)$ of order nc, specified as a row vector of length nc +1 , with 1 as the first element. Specify the coefficients in order of ascending powers of $q^{-1}$.

The coefficients in InitialC must define a stable discrete-time polynomial with roots within a unit circle. For example,

```
InitialC = [1 0.5 0.5];
all(abs(roots(InitialC))<1)
ans =
    1
```

If the initial guesses are much smaller than the default InitialParameterCovariance, 10000, the initial guesses are given less importance during estimation. In that case, specify a smaller initial parameter covariance.

InitialC is a tunable property. You can change it when the object is in a locked state.
Default: [1 eps]

## InitialD

Initial values for the coefficients of polynomial $D(q)$ of order nd, specified as a row vector of length nd +1 , with 1 as the first element. Specify the coefficients in order of ascending powers of $q^{-1}$.

The coefficients in InitialD must define a stable discrete-time polynomial with roots within a unit circle. For example,

```
InitialD = [1 0.9 0.8];
all(abs(roots(InitialD))<1)
ans =
    1
```

If the initial guesses are much smaller than the default InitialParameterCovariance, 10000, the initial guesses are given less importance during estimation. In that case, specify a smaller initial parameter covariance.

InitialD is a tunable property. You can change it when the object is in a locked state.

Default: [1 eps]

## InitialF

Initial values for the coefficients of polynomial $F(q)$ of order nf , specified as a row vector of length nf +1 , with 1 as the first element. Specify the coefficients in order of ascending powers of $q^{-1}$.

The coefficients in InitialF must define a stable discrete-time polynomial with roots within a unit circle. For example,

```
InitialF = [1 0.9 0.8];
all(abs(roots(InitialF))<1)
ans =
    1
```

If the initial guesses are much smaller than the default InitialParameterCovariance, 10000, the initial guesses are given less importance during estimation. In that case, specify a smaller initial parameter covariance.

InitialF is a tunable property. You can change it when the object is in a locked state.
Default: [1 eps]

## ParameterCovariance

Estimated covariance P of the parameters, returned as an $N$-by- $N$ symmetric positive-definite matrix. $N$ is the number of parameters to be estimated. The software computes P assuming that the residuals (difference between estimated and measured outputs) are white noise, and the variance of these residuals is 1. ParameterCovariance is applicable only when EstimationMethod is 'ForgettingFactor' or 'KalmanFilter'.

The interpretation of P depends on the estimation method:

- 'ForgettingFactor' $-R_{2} / 2 * \mathrm{P}$ is approximately equal to the covariance matrix of the estimated parameters, where $R_{2}$ is the true variance of the residuals.
- 'KalmanFilter' $-R_{2}{ }^{*} \mathrm{P}$ is the covariance matrix of the estimated parameters, and $R_{1} / R_{2}$ is the covariance matrix of the parameter changes. Where, $R_{1}$ is the covariance matrix that you specify in ProcessNoiseCovariance.

ParameterCovariance is a read-only property and is initially empty after you create the object. It is populated after you use the step command for online parameter estimation.

## InitialParameterCovariance

Covariance of the initial parameter estimates, specified as one of the following:

- Real positive scalar, $\alpha$ - Covariance matrix is an $N$-by- $N$ diagonal matrix, with $\alpha$ as the diagonal elements. $N$ is the number of parameters to be estimated.
- Vector of real positive scalars, $\left[\alpha_{1}, \ldots, \alpha_{N}\right]$ - Covariance matrix is an $N$-by- $N$ diagonal matrix, with [ $\alpha_{1}, \ldots, \alpha_{N}$ ] as the diagonal elements.
- $N$-by- $N$ symmetric positive-definite matrix.

InitialParameterCovariance represents the uncertainty in the initial parameter estimates. For large values of InitialParameterCovariance, less importance is placed on the initial parameter values and more on the measured data during beginning of estimation using step.

Use only when EstimationMethod is 'ForgettingFactor' or 'KalmanFilter'.
InitialParameterCovariance is a tunable property. You can change it when the object is in a locked state.

Default: 10000

## EstimationMethod

Recursive estimation algorithm used for online estimation of model parameters, specified as one of the following values:

- 'ForgettingFactor' - Algorithm used for parameter estimation
- 'KalmanFilter' - Algorithm used for parameter estimation
- 'NormalizedGradient' - Algorithm used for parameter estimation
- 'Gradient ' - Unnormalized gradient algorithm used for parameter estimation

Forgetting factor and Kalman filter algorithms are more computationally intensive than gradient and unnormalized gradient methods. However, they have better convergence properties. For information about these algorithms, see "Recursive Algorithms for Online Parameter Estimation".

EstimationMethod is a nontunable property. You cannot change it during execution, that is after the object is locked using the step command. If you want to deploy code using MATLAB Coder, EstimationMethod can only be assigned once.

Default: 'ForgettingFactor'

## ForgettingFactor

Forgetting factor, $\lambda$, relevant for parameter estimation, specified as a scalar in the range $(0,1]$.
Suppose that the system remains approximately constant over $T_{0}$ samples. You can choose $\lambda$ such that:

$$
T_{0}=\frac{1}{1-\lambda}
$$

- Setting $\lambda=1$ corresponds to "no forgetting" and estimating constant coefficients.
- Setting $\lambda<1$ implies that past measurements are less significant for parameter estimation and can be "forgotten". Set $\lambda<1$ to estimate time-varying coefficients.

Typical choices of $\lambda$ are in the range [0.98 0.995].

Use only when EstimationMethod is 'ForgettingFactor'.
ForgettingFactor is a tunable property. You can change it when the object is in a locked state.
Default: 1

## EnableAdapation

Enable or disable parameter estimation, specified as one of the following:

- true or 1- The step command estimates the parameter values for that time step and updates the parameter values.
- false or 0 - The step command does not update the parameters for that time step and instead outputs the last estimated value. You can use this option when your system enters a mode where the parameter values do not vary with time.

Note If you set EnableAdapation to false, you must still execute the step command. Do not skip step to keep parameter values constant, because parameter estimation depends on current and past I/O measurements. step ensures past I/O data is stored, even when it does not update the parameters.

EnableAdapation is a tunable property. You can change it when the object is in a locked state.
Default: true

## DataType

Floating point precision of parameters, specified as one of the following values:

- 'double' - Double-precision floating point
- 'single' - Single-precision floating point

Setting DataType to 'single' saves memory, but leads to loss of precision. Specify DataType based on the precision required by the target processor where you will deploy generated code.

DataType is a nontunable property. It can only be set during object construction using Name, Value arguments and cannot be changed afterward.

## Default: 'double'

## ProcessNoiseCovariance

Covariance matrix of parameter variations, specified as one of the following:

- Real nonnegative scalar, $\alpha$ - Covariance matrix is an $N$-by- $N$ diagonal matrix, with $\alpha$ as the diagonal elements.
- Vector of real nonnegative scalars, $\left[\alpha_{1}, \ldots, \alpha_{N}\right]$ - Covariance matrix is an $N$-by- $N$ diagonal matrix, with $\left[\alpha_{1}, \ldots, \alpha_{N}\right]$ as the diagonal elements.
- $N$-by- $N$ symmetric positive semidefinite matrix.
$N$ is the number of parameters to be estimated.
ProcessNoiseCovariance is applicable when EstimationMethod is 'KalmanFilter'.
Kalman filter algorithm treats the parameters as states of a dynamic system and estimates these parameters using a Kalman filter. ProcessNoiseCovariance is the covariance of the process noise acting on these parameters. Zero values in the noise covariance matrix correspond to estimating constant coefficients. Values larger than 0 correspond to time-varying parameters. Use large values for rapidly changing parameters. However, the larger values result in noisier parameter estimates.

ProcessNoiseCovariance is a tunable property. You can change it when the object is in a locked state.

Default: 0.1
AdaptationGain
Adaptation gain, $\gamma$, used in gradient recursive estimation algorithms, specified as a positive scalar.
AdaptationGain is applicable when EstimationMethod is 'Gradient' or
'NormalizedGradient'.
Specify a large value for AdaptationGain when your measurements have a high signal-to-noise ratio.

AdaptationGain is a tunable property. You can change it when the object is in a locked state.

## Default: 1

## NormalizationBias

Bias in adaptation gain scaling used in the 'NormalizedGradient' method, specified as a nonnegative scalar.

NormalizationBias is applicable when EstimationMethod is 'NormalizedGradient'.
The normalized gradient algorithm divides the adaptation gain at each step by the square of the twonorm of the gradient vector. If the gradient is close to zero, this can cause jumps in the estimated parameters. NormalizationBias is the term introduced in the denominator to prevent these jumps. Increase NormalizationBias if you observe jumps in estimated parameters.

NormalizationBias is a tunable property. You can change it when the object is in a locked state.

## Default: eps

## Output Arguments

obj - System object for online parameter estimation of Box-Jenkins polynomial model recursiveBJ System object

System object for online parameter estimation of Box-Jenkins polynomial model, returned as a recursiveBJ System object. This object is created using the specified model orders and properties. Use step command to estimate the coefficients of the Box-Jenkins model polynomials. You can then access the estimated coefficients and parameter covariance using dot notation. For example, type obj. F to view the estimated $F$ polynomial coefficients.

## More About

## Box-Jenkins Polynomial Model Structure

The general Box-Jenkins model structure is:

$$
y(t)=\sum_{i=1}^{n u} \frac{B_{i}(q)}{F_{i}(q)} u_{i}\left(t-n k_{i}\right)+\frac{C(q)}{D(q)} e(t)
$$

where $n u$ is the number of input channels.
The orders of Box-Jenkins model are defined as follows:
$n b: \quad B(q)=b_{1}+b_{2} q^{-1}+\ldots+b_{n b} q^{-n b+1}$
$n c: \quad C(q)=1+c_{1} q^{-1}+\ldots+c_{n c} q^{-n c}$
$n d: \quad D(q)=1+d_{1} q^{-1}+\ldots+d_{n d} q^{-n d}$
$n f: \quad F(q)=1+f_{1} q^{-1}+\ldots+f_{n f} q^{-n f}$

## Tips

- Starting in R2016b, instead of using the step command to update model parameter estimates, you can call the System object with input arguments, as if it were a function. For example,
[B,C,D,F,EstimatedOutput] = step(obj,y,u) and [B,C,D,F,EstimatedOutput] = obj $(\mathrm{y}, \mathrm{u})$ perform equivalent operations.


## Extended Capabilities

## C/C++ Code Generation

Generate C and $\mathrm{C}++$ code using MATLAB® Coder $^{\mathrm{Tm}}$.
Usage notes and limitations:

- For Simulink-based workflows, use Recursive Polynomial Model Estimator.
- For limitations, see "Generate Code for Online Parameter Estimation in MATLAB".

Supports MATLAB Function block: No

## See Also

Recursive Polynomial Model Estimator | bj |clone |isLocked|recursiveAR|recursiveARMA | recursiveARMAX | recursiveARX|recursiveLS | recursiveOE|release|reset|step

## Topics

"Perform Online Parameter Estimation at the Command Line"
"Validate Online Parameter Estimation at the Command Line"
"What Is Online Estimation?"
"Recursive Algorithms for Online Parameter Estimation"

## Introduced in R2015b

## recursiveLS

Create System object for online parameter estimation using recursive least squares algorithm

## Syntax

```
obj = recursiveLS
obj = recursiveLS(Np)
obj = recursiveLS(Np,theta0)
obj = recursiveLS(
```

$\qquad$

``` ,Name, Value)
```


## Description

Use the recursiveLS command for parameter estimation with real-time data. If all data necessary for estimation is available at once and you are estimating a time-invariant model, use mldivide, $\backslash$.
obj = recursiveLS creates a System object for online parameter estimation of a default single output system that is linear in estimated parameters. Such a system can be represented as:
$y(t)=H(t) \theta(t)+e(t)$.
Here, $y$ is the output, $\theta$ are the parameters, $H$ are the regressors, and $e$ is the white-noise disturbance. The default system has one parameter with initial parameter value 1.

After creating the object, use the step command to update model parameter estimates using recursive least squares algorithms and real-time data. Alternatively, you can call the object directly. For more information, see "Tips" on page 1-1262.
obj $=$ recursiveLS(Np) also specifies the number of parameters to be estimated.
obj $=$ recursiveLS (Np,theta0) also specifies the number of parameters and initial values of the parameters.
obj = recursiveLS( __ , Name, Value) specifies additional attributes of the system and recursive estimation algorithm using one or more Name, Value pair arguments.

## Object Description

recursiveLS creates a System object for online parameter estimation of a single output system that is linear in its parameters.

A System object is a specialized MATLAB object designed specifically for implementing and simulating dynamic systems with inputs that change over time. System objects use internal states to store past behavior, which is used in the next computational step.

After you create a System object, you use commands to process data or obtain information from or about the object. System objects use a minimum of two commands to process data - a constructor to create the object and the step command to update object parameters using real-time data. This separation of declaration from execution lets you create multiple, persistent, reusable objects, each with different settings.

You can use the following commands with the online estimation System objects in System Identification Toolbox:
\(\left.$$
\begin{array}{|l|l|}\hline \text { Command } & \text { Description } \\
\hline \text { step } & \begin{array}{l}\text { Update model parameter estimates using } \\
\text { recursive estimation algorithms and real-time } \\
\text { data. } \\
\text { step puts the object into a locked state. In a } \\
\text { locked state, you cannot change any nontunable } \\
\text { properties or input specifications, such as model } \\
\text { order, data type, or estimation algorithm. During } \\
\text { execution, you can only change tunable } \\
\text { properties. }\end{array} \\
\hline \text { release } & \begin{array}{l}\text { Unlock the System object. Use this command to } \\
\text { enable setting of nontunable parameters. }\end{array} \\
\hline \text { reset } & \begin{array}{l}\text { Reset the internal states of a locked System } \\
\text { object to the initial values, and leave the object } \\
\text { locked. }\end{array} \\
\hline \text { clone } & \begin{array}{l}\text { Create another System object with the same } \\
\text { object property values. }\end{array}
$$ <br>
do not create additional objects using syntax <br>

obj2 = obj. Any changes made to the\end{array}\right\}\)| properties of the new object created this way |
| :--- |
| (obj2) also change the properties of the original |
| object (obj). |

Use the recursiveLS command to create an online estimation System object. Then estimate the system parameters (theta) and output using the step command with regressors and incoming output data, H and y .
[theta, EstimatedOutput] = step(obj, y, H)
For recursiveLS object properties, see "Properties" on page 1-1256.

## Examples

Create System Object for Online Estimation Using Recursive Least Squares Algorithm

```
obj = recursiveLS
```

obj =
recursiveLS with properties:
NumberOfParameters: 1
Parameters: []
InitialParameters: 1
ParameterCovariance: []

```
InitialParameterCovariance: 10000
    EstimationMethod: 'ForgettingFactor'
    ForgettingFactor: 1
    EnableAdaptation: true
            History: 'Infinite'
        InputProcessing: 'Sample-based'
            DataType: 'double'
```


## Estimate Parameters of System Using Recursive Least Squares Algorithm

The system has two parameters and is represented as:

$$
y(t)=a_{1} u(t)+a_{2} u(t-1)
$$

Here,

- $u$ and $y$ are the real-time input and output data, respectively.
- $u(t)$ and $u(t-1)$ are the regressors, H , of the system.
- $a_{1}$ and $a_{2}$ are the parameters, theta, of the system.

Create a System object for online estimation using the recursive least squares algorithm.
obj = recursiveLS(2);
Load the estimation data, which for this example is a static data set.

```
load iddata3
input = z3.u;
output = z3.y;
```

Create a variable to store $u(t-1)$. This variable is updated at each time step.
oldInput = 0;
Estimate the parameters and output using step and input-output data, maintaining the current regressor pair in H . Invoke the step function implicitly by calling the obj system object with input arguments.

```
for i = 1:numel(input)
    H = [input(i) oldInput];
    [theta, EstimatedOutput] = obj(output(i),H);
    estimatedOut(i)= EstimatedOutput;
    theta_est(i,:) = theta;
    oldInput = input(i);
end
```

Plot the measured and estimated output data.

```
numSample = 1:numel(input);
plot(numSample,output,'b',numSample,estimated0ut,'r--');
legend('Measured Output','Estimated Output');
```



Plot the parameters.
plot(numSample,theta_est(:,1), numSample,theta_est(:,2))
title('Parameter Estimates for Recursive Least Squares Estimation')
legend("theta1", "theta2")


View the final estimates.

```
theta_final = theta
theta_final = 2×1
    -1.5322
    -0.0235
```


## Use Frame-Based Data for Recursive Least Squares Estimation

Use frame-based signals with the recursiveLS command. Machine interfaces often provide sensor data in frames containing multiple samples, rather than in individual samples. The recursiveLS object accepts these frames directly when you set InputProcessing to Frame-based.

The object uses the same estimation algorithms for sample-based and frame-based input processing. The estimation results are identical. There are some special considerations, however, for working with frame-based inputs.

This example is the frame-based version of the sample-based recursiveLS example in "Estimate Parameters of System Using Recursive Least Squares Algorithm" on page 1-1250.

The system has two parameters and is represented as:

$$
y(t)=a_{1} u(t)+a_{2} u(t-1)
$$

Here,

- $u$ and $y$ are the real-time input and output data, respectively.
- $u(t)$ and $u(t-1)$ are the regressors, H , of the system.
- $a_{1}$ and $a_{2}$ are the parameters, $\theta$, of the system.

Create a System object for online estimation using the recursive least squares algorithm.

```
obj_f = recursiveLS(2,'InputProcessing','Frame-Based');
```

Load the data, which contains input and output time series signals. Each signal consists of 30 frames and each frame contains ten individual time samples.

```
load iddata3 frames input sig frame output sig frame
input = input_sig_frame.data;
output = output_sig_frame.data;
numframes = size(input,3)
numframes = 30
mframe = size(input,1)
mframe = 10
```

Initialize the regressor frame, which for a given frame, is of the form
$H_{f}=\left[\begin{array}{cc}u_{1} & u_{0} \\ u_{2} & u_{1} \\ \vdots & \vdots \\ u_{10} & u_{9}\end{array}\right]$,
where the most recent point in the frame is $u_{10}$.
Hframe $=$ zeros(10,2);
For this first-order example, the regressor frame includes one point from the previous frame. Initialize this point.
oldInput = 0;
Estimate the parameters and output using step and input-output data, maintaining the current regressor frame in Hframe.

- The input and output arrays have three dimensions. The third dimension is the frame index, and the first two dimensions represent the contents of individual frames.
- Use the circshift function to populate the second column of Hframe with the past input value for each regressor pair by shifting the input vector by one position.
- Populate the Hframe element holding the oldest value, $\operatorname{Hframe}(1,2)$, with the regressor value stored from the previous frame.
- Invoke the step function implicitly by calling the obj system object with input arguments. The step function is compatible with frames, so no loop function within the frame is necessary.
- Save the most recent input value to use for the next frame calculation.

```
EstimatedOutput = zeros(10,1,30);
theta = zeros(2,30);
for i = 1:numframes
    Hframe = [input(:,:,i) circshift(input(:,:,i),1)];
    Hframe(1,2) = oldInput;
    [theta(:,i), EstimatedOutput(:,:,i)] = obj_f(output(:,:,i),Hframe);
    oldInput = input(10,:,i);
end
```

Plot the parameters.

```
thetal = theta(1,:);
theta2 = theta(2,:);
iframe = 1:numframes;
plot(iframe,thetal,iframe,theta2)
title('Frame-Based Recursive Least Squares Estimation')
legend('theta1','theta2','location','best')
```



View the final estimates.

```
theta_final = theta(:,numframes)
theta final = 2×1
    -1.5322
    -0.0235
```

The final estimates are identical to the sample-based estimation.

## Specify Initial Parameters for Online Estimation Using Recursive Least Squares Algorithm

Create System object for online parameter estimation using recursive least squares algorithm of a system with two parameters and known initial parameter values.
obj $=$ recursiveLS(2,[0.8 1],'InitialParameterCovariance',0.1);
InitialParameterCovariance represents the uncertainty in your guess for the initial parameters. Typically, the default InitialParameterCovariance (10000) is too large relative to the parameter values. This results in initial guesses being given less importance during estimation. If you have confidence in the initial parameter guesses, specify a smaller initial parameter covariance.

## Input Arguments

## Np - Number of parameters

positive integer
Number of parameters in the system, specified as a positive integer.

## theta0 - Initial value of parameters <br> scalar | vector of real values

Initial value of parameters, specified as one of the following:

- Scalar - All the parameters have the same initial value.
- Vector of real values of length Np - The $i$ th parameter has initial value theta0(i).

The default initial value for all parameters is 1 .

Note If the initial parameter values are much smaller than InitialParameterCovariance, these initial values are given less importance during estimation. Specify a smaller initial parameter covariance if you have high confidence in the initial parameter values. This statement applies only for infinite-history estimation. Finite-history estimation does not use InitialParameterCovariance.

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Use Name, Value arguments to specify writable properties on page 1-1256 of recursiveLS System object during object creation. For example, obj $=$ recursiveLS(2,'EstimationMethod','Gradient') creates a System object to estimate the system parameters using the 'Gradient ' recursive estimation algorithm.

## Properties

recursiveLS System object properties consist of read-only and writable properties. The writable properties are tunable and nontunable properties. The nontunable properties cannot be changed when the object is locked, that is, after you use the step command.

Use Name, Value arguments to specify writable properties of recursiveLS objects during object creation. After object creation, use dot notation to modify the tunable properties.
obj = recursiveLS;
obj.ForgettingFactor $=0.99$;

## Number0fParameters

Number of parameters to be estimated, returned as a positive integer.
NumberOfParameters is a read-only property. If Np is specified during object construction,
NumberOfParameters takes the value assigned to Np .

## Default: 1

## Parameters

Estimated parameters, returned as a column vector of real values.
Parameters is a read-only property and is initially empty after you create the object. It is populated after you use the step command for online parameter estimation.

## InitialParameters

Initial values of parameters, specified as one of the following:

- Scalar - All the parameters have the same initial value.
- Vector of real values of length Np - The ith parameter has initial value InitialParameters(i).

If the initial parameter values are much smaller than InitialParameterCovariance, these initial values are given less importance during estimation. Specify a smaller initial parameter covariance if you have high confidence in initial parameter values. This statement applies only for infinite-history estimation. Finite-history estimation does not use InitialParameterCovariance.

InitialParameters is a tunable property. You can change InitialParameters when the object is in a locked state.

## Default: 1

## InitialOutputs

Initial values of the outputs buffer in finite-history estimation, specified as 0 or as a $W$-by- 1 vector, where $W$ is the window length.

The InitialOutputs property provides a means of controlling the initial behavior of the algorithm.
When InitialOutputs is set to 0 , the object populates the buffer with zeros.
If the initial buffer is set to 0 or does not contain enough information, you see a warning message during the initial phase of your estimation. The warning should clear after a few cycles. The number
of cycles it takes for sufficient information to be buffered depends upon the order of your polynomials and your input delays. If the warning persists, you should evaluate the content of your signals.

Specify InitialOutputs only when History is Finite.
InitialOutputs is a tunable property. You can change InitialOutputs when the object is in a locked state.

Default: 0

## InitialRegressors

Initial values of the regressors buffer in finite-history estimation, specified as 0 or as a $W$-by-Np matrix, where $W$ is the window length and $N p$ is the number of parameters.

The InitialRegressors property provides a means of controlling the initial behavior of the algorithm.

When the InitialRegressors is set to 0 , the object populates the buffer with zeros.
If the initial buffer is set to 0 or does not contain enough information, you see a warning message during the initial phase of your estimation. The warning should clear after a few cycles. The number of cycles it takes for sufficient information to be buffered depends upon the order of your polynomials and your input delays. If the warning persists, you should evaluate the content of your signals.

Specify InitialRegressors only when History is Finite.
InitialRegressors is a tunable property. You can change InitialRegressors when the object is in a locked state.

## Default: 0

## ParameterCovariance

Estimated covariance P of the parameters, returned as an N -by- N symmetric positive-definite matrix. $N$ is the number of parameters to be estimated. The software computes P assuming that the residuals (difference between estimated and measured outputs) are white noise, and the variance of these residuals is 1 .

ParameterCovariance is applicable only when EstimationMethod is 'ForgettingFactor' or 'KalmanFilter' or when History is Finite.

The interpretation of P depends on your settings for the History and EstimationMethod properties.

- If History is Infinite, then your EstimationMethod selection results in one of the following:
- 'ForgettingFactor' $-\left(R_{2} / 2\right) \mathrm{P}$ is approximately equal to the covariance matrix of the estimated parameters, where $R_{2}$ is the true variance of the residuals.
- 'KalmanFilter' $-R_{2} \mathrm{P}$ is the covariance matrix of the estimated parameters, and $R_{1} / R_{2}$ is the covariance matrix of the parameter changes. Here, $R_{1}$ is the covariance matrix that you specify in ProcessNoiseCovariance.
- If History is Finite (sliding-window estimation) $-R_{2} \mathrm{P}$ is the covariance of the estimated parameters. The sliding-window algorithm does not use this covariance in the parameter-
estimation process. However, the algorithm does compute the covariance for output so that you can use it for statistical evaluation.

ParameterCovariance is a read-only property and is initially empty after you create the object. It is populated after you use the step command for online parameter estimation.

## InitialParameterCovariance

Covariance of the initial parameter estimates, specified as one of the following:

- Real positive scalar, $\alpha$ - Covariance matrix is an $N$-by- $N$ diagonal matrix, with $\alpha$ as the diagonal elements. $N$ is the number of parameters to be estimated.
- Vector of real positive scalars, $\left[\alpha_{1}, \ldots, \alpha_{N}\right]$ - Covariance matrix is an $N$-by- $N$ diagonal matrix, with [ $\alpha_{1}, \ldots, \alpha_{N}$ ] as the diagonal elements.
- $N$-by- $N$ symmetric positive-definite matrix.

InitialParameterCovariance represents the uncertainty in the initial parameter estimates. For large values of InitialParameterCovariance, less importance is placed on the initial parameter values and more on the measured data during beginning of estimation using step.

Use only when EstimationMethod is 'ForgettingFactor' or 'KalmanFilter'.
InitialParameterCovariance is a tunable property. You can change it when the object is in a locked state.

## Default: 10000

## EstimationMethod

Recursive least squares estimation algorithm used for online estimation of model parameters, specified as one of the following values:

- 'ForgettingFactor' - Algorithm used for parameter estimation
- 'KalmanFilter' - Algorithm used for parameter estimation
- 'NormalizedGradient' - Algorithm used for parameter estimation
- 'Gradient' - Unnormalized gradient algorithm used for parameter estimation

Forgetting factor and Kalman filter algorithms are more computationally intensive than gradient and unnormalized gradient methods. However, they have better convergence properties. For information about these algorithms, see "Recursive Algorithms for Online Parameter Estimation".

These methods all use an infinite data history, and are available only when History is 'Infinite'.
EstimationMethod is a nontunable property. You cannot change it during execution, that is, after the object is locked using the step command.

Default: Forgetting Factor

## ForgettingFactor

Forgetting factor, $\lambda$, relevant for parameter estimation, specified as a scalar in the range $(0,1]$.
Suppose that the system remains approximately constant over $T_{0}$ samples. You can choose $\lambda$ such that:

$$
T_{0}=\frac{1}{1-\lambda}
$$

- Setting $\lambda=1$ corresponds to "no forgetting" and estimating constant coefficients.
- Setting $\lambda<1$ implies that past measurements are less significant for parameter estimation and can be "forgotten". Set $\lambda<1$ to estimate time-varying coefficients.

Typical choices of $\lambda$ are in the range [0.98 0.995].
Use only when EstimationMethod is 'ForgettingFactor'.
ForgettingFactor is a tunable property. You can change it when the object is in a locked state.

## Default: 1

## EnableAdapation

Enable or disable parameter estimation, specified as one of the following:

- true or 1 - The step command estimates the parameter values for that time step and updates the parameter values.
- false or 0 - The step command does not update the parameters for that time step and instead outputs the last estimated value. You can use this option when your system enters a mode where the parameter values do not vary with time.

Note If you set EnableAdapation to false, you must still execute the step command. Do not skip step to keep parameter values constant, because parameter estimation depends on current and past I/O measurements. step ensures past I/O data is stored, even when it does not update the parameters.

EnableAdapation is a tunable property. You can change it when the object is in a locked state.
Default: true

## DataType

Floating point precision of parameters, specified as one of the following values:

- 'double' - Double-precision floating point
- 'single' - Single-precision floating point

Setting DataType to 'single' saves memory, but leads to loss of precision. Specify DataType based on the precision required by the target processor where you will deploy generated code.

DataType is a nontunable property. It can only be set during object construction using Name, Value arguments and cannot be changed afterward.

Default: 'double'

## ProcessNoiseCovariance

Covariance matrix of parameter variations, specified as one of the following:

- Real nonnegative scalar, $\alpha$ - Covariance matrix is an $N$-by- $N$ diagonal matrix, with $\alpha$ as the diagonal elements.
- Vector of real nonnegative scalars, $\left[\alpha_{1}, \ldots, \alpha_{N}\right]$ - Covariance matrix is an $N$-by- $N$ diagonal matrix, with $\left[\alpha_{1}, \ldots, \alpha_{N}\right]$ as the diagonal elements.
- $N$-by- $N$ symmetric positive semidefinite matrix.
$N$ is the number of parameters to be estimated.
ProcessNoiseCovariance is applicable when EstimationMethod is 'KalmanFilter'.
Kalman filter algorithm treats the parameters as states of a dynamic system and estimates these parameters using a Kalman filter. ProcessNoiseCovariance is the covariance of the process noise acting on these parameters. Zero values in the noise covariance matrix correspond to estimating constant coefficients. Values larger than 0 correspond to time-varying parameters. Use large values for rapidly changing parameters. However, the larger values result in noisier parameter estimates.

ProcessNoiseCovariance is a tunable property. You can change it when the object is in a locked state.

Default: 0.1

## AdaptationGain

Adaptation gain, $\gamma$, used in gradient recursive estimation algorithms, specified as a positive scalar.
AdaptationGain is applicable when EstimationMethod is 'Gradient' or 'NormalizedGradient'.

Specify a large value for AdaptationGain when your measurements have a high signal-to-noise ratio.

AdaptationGain is a tunable property. You can change it when the object is in a locked state.

## Default: 1

## NormalizationBias

Bias in adaptation gain scaling used in the 'NormalizedGradient ' method, specified as a nonnegative scalar.

NormalizationBias is applicable when EstimationMethod is 'NormalizedGradient'.
The normalized gradient algorithm divides the adaptation gain at each step by the square of the twonorm of the gradient vector. If the gradient is close to zero, this can cause jumps in the estimated parameters. NormalizationBias is the term introduced in the denominator to prevent these jumps. Increase NormalizationBias if you observe jumps in estimated parameters.

NormalizationBias is a tunable property. You can change it when the object is in a locked state.

## Default: eps

## History

Data history type defining which type of recursive algorithm you use, specified as:

- 'Infinite' - Use an algorithm that aims to minimize the error between the observed and predicted outputs for all time steps from the beginning of the simulation.
- 'Finite' - Use an algorithm that aims to minimize the error between the observed and predicted outputs for a finite number of past time steps.

Algorithms with infinite history aim to produce parameter estimates that explain all data since the start of the simulation. These algorithms still use a fixed amount of memory that does not grow over time. The object provides multiple algorithms of the 'Infinite' History type. Specifying this option activates the EstimationMethod property with which you specify an algorithm.

Algorithms with finite history aim to produce parameter estimates that explain only a finite number of past data samples. This method is also called sliding-window estimation. The object provides one algorithm of the 'Finite' type. Specifying this option activates the WindowLength property that sizes the window.

For more information on recursive estimation methods, see "Recursive Algorithms for Online Parameter Estimation".

History is a nontunable property. It can be set only during object construction using Name, Value arguments and cannot be changed afterward.

## Default: 'Infinite'

## WindowLength

Window size determining the number of time samples to use for the sliding-window estimation method, specified as a positive integer. Specify WindowLength only when History is Finite.

Choose a window size that balances estimation performance with computational and memory burden. Sizing factors include the number and time variance of the parameters in your model. Always specify
Window Length in samples, even if you are using frame-based input processing.
WindowLength must be greater than or equal to the number of estimated parameters.
Suitable window length is independent of whether you are using sample-based or frame-based input processing (see InputProcessing). However, when using frame-based processing, your window length must be greater than or equal to the number of samples (time steps) contained in the frame.

WindowLength is a nontunable property. It can be set only during object construction using Name, Value arguments and cannot be changed afterward.

Default: 200

## InputProcessing

Option for sample-based or frame-based input processing, specified as a character vector or string.

- Sample-based processing operates on signals streamed one sample at a time.
- Frame-based processing operates on signals containing samples from multiple time steps. Many machine sensor interfaces package multiple samples and transmit these samples together in frames. Frame-based processing allows you to input this data directly without having to first unpack it.

Your InputProcessing specification impacts the dimensions for the input and output signals when using the step command:
[theta, EstimatedOutput] = step(obj,y,H)

- Sample-based
- y and EstimatedOutput are scalars.
- H is a 1 -by-Np vector, where Np is the number of parameters.
- Frame-based with $M$ samples per frame
- y and EstimatedOutput are M-by-1 vectors.
- $H$ is an $M$-by-Np matrix.

InputProcessing is a nontunable property. It can be set only during object construction using Name, Value arguments and cannot be changed afterward.

Default: 'Sample-based'

## Output Arguments

## obj - System object for online parameter estimation

recursiveLS System object
System object for online parameter estimation, returned as a recursiveLS System object. Use step command to estimate the parameters of the system. You can then access the estimated parameters and parameter covariance using dot notation. For example, type obj. Parameters to view the estimated parameters.

## Tips

- Starting in R2016b, instead of using the step command to update model parameter estimates, you can call the System object with input arguments, as if it were a function. For example, [theta,EstimatedOutput] = step(obj, y, H) and [theta,EstimatedOutput] = obj $(\mathrm{y}, \mathrm{H})$ perform equivalent operations.


## Extended Capabilities

## C/C++ Code Generation

Generate C and $\mathrm{C}++$ code using MATLAB® Coder $^{\mathrm{Tm}}$.
Usage notes and limitations:

- For Simulink based workflows, use Recursive Least Squares Estimator.
- For limitations, see "Generate Code for Online Parameter Estimation in MATLAB".

Supports MATLAB Function block: No

## See Also

Recursive Least Squares Estimator | clone \| isLocked |mldivide | recursiveAR| recursiveARMA | recursiveARMAX|recursiveARX|recursiveBJ | recursive0E | release | reset|step

## Topics

"Perform Online Parameter Estimation at the Command Line"
"Validate Online Parameter Estimation at the Command Line"
"Line Fitting with Online Recursive Least Squares Estimation"
"What Is Online Estimation?"
"Recursive Algorithms for Online Parameter Estimation"
Introduced in R2015b

## recursiveOE

Create System object for online parameter estimation of Output-Error polynomial model

## Syntax

```
obj = recursive0E
obj = recursiveOE(Orders)
obj = recursive0E(Orders,B0,F0)
obj = recursive0E(
```

$\qquad$

``` ,Name, Value)
```


## Description

Use recursive0E command for parameter estimation with real-time data. If all data necessary for estimation is available at once, and you are estimating a time-invariant model, use the offline estimation command, oe.
obj = recursive0E creates a System object for online parameter estimation of a default single-input-single output (SISO) Output-Error model structure on page 1-1275. The default model structure has polynomials of order 1 and initial polynomial coefficient values eps.

After creating the object, use the step command to update model parameter estimates using recursive estimation algorithms and real-time data.
obj $=$ recursiveOE(Orders) specifies the polynomial orders of the Output-Error model to be estimated.
obj = recursive0E(Orders,B0,F0) specifies the polynomial orders and initial values of the polynomial coefficients. Specify initial values to potentially avoid local minima during estimation. If the initial values are small compared to the default InitialParameterCovariance property value, and you have confidence in your initial values, also specify a smaller
InitialParameterCovariance.
obj = recursive0E( $\qquad$ ,Name, Value) specifies additional attributes of the Output-Error model structure and recursive estimation algorithm using one or more Name, Value pair arguments.

## Object Description

recursive0E creates a System object for online parameter estimation of SISO Output-Error polynomial models using a recursive estimation algorithm.

A System object is a specialized MATLAB object designed specifically for implementing and simulating dynamic systems with inputs that change over time. System objects use internal states to store past behavior, which is used in the next computational step.

After you create a System object, you use commands to process data or obtain information from or about the object. System objects use a minimum of two commands to process data - a constructor to create the object and the step command to update object parameters using real-time data. This separation of declaration from execution lets you create multiple, persistent, reusable objects, each with different settings.

You can use the following commands with the online estimation System objects in System Identification Toolbox:

| Command | Description |
| :--- | :--- |
| step | Update model parameter estimates using <br> recursive estimation algorithms and real-time <br> data. <br> step puts the object into a locked state. In a <br> locked state, you cannot change any nontunable <br> properties or input specifications, such as model <br> order, data type, or estimation algorithm. During <br> execution, you can only change tunable <br> properties. |
| release | Unlock the System object. Use this command to <br> enable setting of nontunable parameters. |
| reset | Reset the internal states of a locked System <br> object to the initial values, and leave the object <br> locked. |
| clone | Create another System object with the same <br> object property values. |
|  | Do not create additional objects using syntax <br> obj2 $=$ obj. Any changes made to the <br> properties of the new object created this way <br> (obj2) also change the properties of the original <br> object (obj). |
| isLocked | Query locked status for input attributes and <br> nontunable properties of the System object. |

Use the recursive0E command to create an online estimation System object. Then estimate the Output-Error polynomial model parameters ( B and F ) and output using the step command with incoming input and output data, u and y .
[B,F,EstimatedOutput] = step(obj, y, u)
For recursiveOE object properties, see "Properties" on page 1-1268.

## Examples

## Estimate Output-Error Polynomial Model Online

Create a System object for online parameter estimation of a Output-Error polynomial model using recursive estimation algorithms.
obj = recursive0E;
The Output-Error model has a default structure with polynomials of order 1 and initial polynomial coefficient values, eps.

Load the estimation data. In this example, use a static data set for illustration.

```
load iddatal z1;
output = z1.y;
input = z1.u;
```

Estimate Output-Error model parameters online using step.
for $i=1$ :numel(input)
[B, F,EstimatedOutput] = step(obj,output(i),input(i));
end

View the current estimated values of polynomial F coefficients.

```
obj.F
ans = 1\times2
    1.0000 -0.7618
```

View the current covariance estimate of the parameters.
obj. ParameterCovariance
ans $=2 \times 2$
$0.0024 \quad 0.0002$
$0.0002 \quad 0.0001$

View the current estimated output.
Estimated0utput
EstimatedOutput $=-4.1866$

## Create System Object for Output-Error Model With Known Orders and Delays

Specify Output-Error polynomial model orders and delays.

```
nb = 1;
nf = 2;
nk = 1;
```

Create a System object for online estimation of Output-Error polynomial model with the specified orders and delays.
obj = recursiveOE([nb nf nk]);

## Create System Object for Output-Error Model With Known Initial Parameters

Specify Output-Error polynomial model orders and delays.

```
nb = 1;
nf = 2;
nk = 1;
```

Create a System object for online estimation of Output-Error model with known initial polynomial coefficients.

```
B0 = [0 1];
F0 = [1 0.7 0.8];
obj = recursiveOE([nb nf nk],B0,F0);
```

Specify the initial parameter covariance.

```
obj.InitialParameterCovariance = 0.1;
```

InitialParameterCovariance represents the uncertainty in your guess for the initial parameters. Typically, the default InitialParameterCovariance (10000) is too large relative to the parameter values. This results in initial guesses being given less importance during estimation. If you have confidence in the initial parameter guesses, specify a smaller initial parameter covariance.

## Specify Estimation Method for Online Estimation of Output-Error Model

Create a System object that uses the unnormalized gradient algorithm for online parameter estimation of an Output-Error model.

```
obj = recursiveOE([1 2 1],'EstimationMethod','Gradient');
```


## Input Arguments

## Orders - Model orders and delays

1-by-3 vector of integers
Model orders and delays of a Output-Error polynomial model on page 1-1275, specified as a 1-by-3 vector of integers, [nb nf nk].

- nb - Order of the polynomial $B(q)+1$, specified as a positive integer.
- nf - Order of the polynomial $F(q)$, specified as a nonnegative integer.
- nk - Input-output delay, specified as a positive integer. nk is number of input samples that occur before the input affects the output. nk is expressed as fixed leading zeros of the $B$ polynomial.


## B0, F0 - Initial value of polynomial coefficients

row vectors of real values | [ ]
Initial value of polynomial coefficients, specified as row vectors of real values with elements in order of ascending powers of $q^{-1}$.

- B0 - Initial guess for the coefficients of the polynomial $B(q)$, specified as a 1-by-(nb+nk) vector with nk leading zeros.
- F0 - Initial guess for the coefficients of the polynomial $F(q)$, specified as a 1-by-(nf+1) vector with 1 as the first element.

The coefficients in F0 must define a stable discrete-time polynomial with roots within a unit disk. For example,

```
F0 = [1 0.5 0.5];
all(abs(roots(F0))<1)
ans =
    1
```

Specifying as [], uses the default value of eps for the polynomial coefficients.

Note If the initial parameter values are much smaller than InitialParameterCovariance, these initial values are given less importance during estimation. Specify a smaller initial parameter covariance if you have high confidence in the initial parameter values. This statement applies only for infinite-history estimation. Finite-history estimation does not use InitialParameterCovariance.

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

Use Name, Value arguments to specify writable properties on page 1-1268 of recursiveOE System object during object creation. For example, obj $=$ recursive0E ([1 2
1],'EstimationMethod','Gradient') creates a System object to estimate a Output-Error polynomial model using the 'Gradient' recursive estimation algorithm.

## Properties

recursive0E System object properties consist of read-only and writable properties. The writable properties are tunable and nontunable properties. The nontunable properties cannot be changed when the object is locked, that is, after you use the step command.

Use Name, Value arguments to specify writable properties of recursiveOE objects during object creation. After object creation, use dot notation to modify the tunable properties.

```
obj = recursive0E;
obj.ForgettingFactor = 0.99;
```


## B

Estimated coefficients of polynomial $B(q)$, returned as a vector of real values specified in order of ascending powers of $q^{-1}$.
$B$ is a read-only property and is initially empty after you create the object. It is populated after you use the step command for online parameter estimation.

## F

Estimated coefficients of polynomial $F(q)$, returned as a vector of real values specified in order of ascending powers of $q^{-1}$.

F is a read-only property and is initially empty after you create the object. It is populated after you use the step command for online parameter estimation.

## InitialB

Initial values for the coefficients of polynomial $B(q)$ of order $n b-1$, specified as a row vector of length $n b+n k$, with nk leading zeros. nk is the input-output delay. Specify the coefficients in order of ascending powers of $q^{-1}$.

If the initial guesses are much smaller than the default InitialParameterCovariance, 10000, the initial guesses are given less importance during estimation. In that case, specify a smaller initial parameter covariance.

InitialB is a tunable property. You can change it when the object is in a locked state.
Default: [0 eps]

## InitialF

Initial values for the coefficients of polynomial $F(q)$ of order $n f$, specified as a row vector of length nf +1 , with 1 as the first element. Specify the coefficients in order of ascending powers of $q^{-1}$.

The coefficients in InitialF must define a stable discrete-time polynomial with roots within a unit circle. For example,

```
InitialF = [1 0.9 0.8];
all(abs(roots(InitialF))<1)
ans =
    1
```

If the initial guesses are much smaller than the default InitialParameterCovariance, 10000, the initial guesses are given less importance during estimation. In that case, specify a smaller initial parameter covariance.

InitialF is a tunable property. You can change it when the object is in a locked state.
Default: [1 eps]

## Initial0utputs

Initial values of the measured outputs buffer in finite-history estimation, specified as 0 or as a ( $W+n f$ )-by- 1 vector, where $W$ is the window length and $n f$ is the order of the polynomial $F(q)$ that you specify when constructing the object.

The InitialOutputs property provides a means of controlling the initial behavior of the algorithm.
When Initialoutputs is set to 0 , the object populates the buffer with zeros.
If the initial buffer is set to 0 or does not contain enough information, you see a warning message during the initial phase of your estimation. The warning should clear after a few cycles. The number of cycles it takes for sufficient information to be buffered depends upon the order of your polynomials and your input delays. If the warning persists, you should evaluate the content of your signals.

Specify Initial0utputs only when History is Finite.

Initial0utputs is a tunable property. You can change Initial0utputs when the object is in a locked state.

## Default: 0

## InitialInputs

Initial values of the inputs in the finite history window, specified as 0 or as a $(W+n b+n k-1)$-by- 1 vector, where $W$ is the window length. $n b$ is the vector of $B(q)$ polynomial orders and $n k$ is vector of input delays that you specify when constructing the recursiveOE object.

The InitialInputs property provides a means of controlling the initial behavior of the algorithm.
When the Initial Inputs is set to 0 , the object populates the buffer with zeros.
If the initial buffer is set to 0 or does not contain enough information, you see a warning message during the initial phase of your estimation. The warning should clear after a few cycles. The number of cycles it takes for sufficient information to be buffered depends upon the order of your polynomials and your input delays. If the warning persists, you should evaluate the content of your signals.

Specify InitialInputs only when History is Finite.
InitialInputs is a tunable property. You can change InitialInputs when the object is in a locked state.

Default: 0

## ParameterCovariance

Estimated covariance P of the parameters, returned as an $N$-by- $N$ symmetric positive-definite matrix. $N$ is the number of parameters to be estimated. The software computes P assuming that the residuals (difference between estimated and measured outputs) are white noise, and the variance of these residuals is 1 .

ParameterCovariance is applicable only when EstimationMethod is 'ForgettingFactor' or 'KalmanFilter' or when History is Finite.

The interpretation of P depends on your settings for the History and EstimationMethod properties.

- If History is Infinite, then your EstimationMethod selection results in one of the following:
- 'ForgettingFactor' - $\left(R_{2} / 2\right) \mathrm{P}$ is approximately equal to the covariance matrix of the estimated parameters, where $R_{2}$ is the true variance of the residuals.
- 'KalmanFilter' $-R_{2} \mathrm{P}$ is the covariance matrix of the estimated parameters, and $R_{1} / R_{2}$ is the covariance matrix of the parameter changes. Here, $R_{1}$ is the covariance matrix that you specify in ProcessNoiseCovariance.
- If History is Finite (sliding-window estimation) $-R_{2} \mathrm{P}$ is the covariance of the estimated parameters. The sliding-window algorithm does not use this covariance in the parameterestimation process. However, the algorithm does compute the covariance for output so that you can use it for statistical evaluation.

ParameterCovariance is a read-only property and is initially empty after you create the object. It is populated after you use the step command for online parameter estimation.

## InitialParameterCovariance

Covariance of the initial parameter estimates, specified as one of the following:

- Real positive scalar, $\alpha$ - Covariance matrix is an $N$-by- $N$ diagonal matrix, with $\alpha$ as the diagonal elements. $N$ is the number of parameters to be estimated.
- Vector of real positive scalars, $\left[\alpha_{1}, \ldots, \alpha_{N}\right]$ - Covariance matrix is an $N$-by- $N$ diagonal matrix, with [ $\alpha_{1}, \ldots, \alpha_{N}$ ] as the diagonal elements.
- $N$-by- $N$ symmetric positive-definite matrix.

InitialParameterCovariance represents the uncertainty in the initial parameter estimates. For large values of InitialParameterCovariance, less importance is placed on the initial parameter values and more on the measured data during beginning of estimation using step.

Use only when EstimationMethod is 'ForgettingFactor' or 'KalmanFilter'.
InitialParameterCovariance is a tunable property. You can change it when the object is in a locked state.

Default: 10000

## EstimationMethod

Recursive estimation algorithm used for online estimation of model parameters, specified as one of the following values:

- 'ForgettingFactor' - Algorithm used for parameter estimation
- 'KalmanFilter' - Algorithm used for parameter estimation
- 'NormalizedGradient' - Algorithm used for parameter estimation
- 'Gradient ' - Unnormalized gradient algorithm used for parameter estimation

Forgetting factor and Kalman filter algorithms are more computationally intensive than gradient and unnormalized gradient methods. However, they have better convergence properties. For information about these algorithms, see "Recursive Algorithms for Online Parameter Estimation".

These methods all use an infinite data history, and are available only when History is 'Infinite'.
EstimationMethod is a nontunable property. You cannot change it during execution, that is, after the object is locked using the step command.

Default: Forgetting Factor

## ForgettingFactor

Forgetting factor, $\lambda$, relevant for parameter estimation, specified as a scalar in the range $(0,1]$.
Suppose that the system remains approximately constant over $T_{0}$ samples. You can choose $\lambda$ such that:

$$
T_{0}=\frac{1}{1-\lambda}
$$

- Setting $\lambda=1$ corresponds to "no forgetting" and estimating constant coefficients.
- Setting $\lambda<1$ implies that past measurements are less significant for parameter estimation and can be "forgotten". Set $\lambda<1$ to estimate time-varying coefficients.

Typical choices of $\lambda$ are in the range [0.98 0.995].
Use only when EstimationMethod is 'ForgettingFactor'.
ForgettingFactor is a tunable property. You can change it when the object is in a locked state.

## Default: 1

## EnableAdapation

Enable or disable parameter estimation, specified as one of the following:

- true or 1- The step command estimates the parameter values for that time step and updates the parameter values.
- false or 0 - The step command does not update the parameters for that time step and instead outputs the last estimated value. You can use this option when your system enters a mode where the parameter values do not vary with time.

Note If you set EnableAdapation to false, you must still execute the step command. Do not skip step to keep parameter values constant, because parameter estimation depends on current and past I/O measurements. step ensures past I/O data is stored, even when it does not update the parameters.

EnableAdapation is a tunable property. You can change it when the object is in a locked state.
Default: true

## DataType

Floating point precision of parameters, specified as one of the following values:

- 'double' - Double-precision floating point
- 'single' - Single-precision floating point

Setting DataType to 'single' saves memory, but leads to loss of precision. Specify DataType based on the precision required by the target processor where you will deploy generated code.

DataType is a nontunable property. It can only be set during object construction using Name, Value arguments and cannot be changed afterward.

Default: 'double'

## ProcessNoiseCovariance

Covariance matrix of parameter variations, specified as one of the following:

- Real nonnegative scalar, $\alpha$ - Covariance matrix is an $N$-by- $N$ diagonal matrix, with $\alpha$ as the diagonal elements.
- Vector of real nonnegative scalars, $\left[\alpha_{1}, \ldots, \alpha_{N}\right]$ - Covariance matrix is an $N$-by- $N$ diagonal matrix, with $\left[\alpha_{1}, \ldots, \alpha_{N}\right]$ as the diagonal elements.
- $N$-by- $N$ symmetric positive semidefinite matrix.
$N$ is the number of parameters to be estimated.
ProcessNoiseCovariance is applicable when EstimationMethod is 'KalmanFilter'.
Kalman filter algorithm treats the parameters as states of a dynamic system and estimates these parameters using a Kalman filter. ProcessNoiseCovariance is the covariance of the process noise acting on these parameters. Zero values in the noise covariance matrix correspond to estimating constant coefficients. Values larger than 0 correspond to time-varying parameters. Use large values for rapidly changing parameters. However, the larger values result in noisier parameter estimates.

ProcessNoiseCovariance is a tunable property. You can change it when the object is in a locked state.

## Default: 0.1

## AdaptationGain

Adaptation gain, $\gamma$, used in gradient recursive estimation algorithms, specified as a positive scalar.
AdaptationGain is applicable when EstimationMethod is 'Gradient ' or
'NormalizedGradient'.
Specify a large value for AdaptationGain when your measurements have a high signal-to-noise ratio.

AdaptationGain is a tunable property. You can change it when the object is in a locked state.

## Default: 1

## NormalizationBias

Bias in adaptation gain scaling used in the 'NormalizedGradient ' method, specified as a nonnegative scalar.

NormalizationBias is applicable when EstimationMethod is 'NormalizedGradient'.
The normalized gradient algorithm divides the adaptation gain at each step by the square of the twonorm of the gradient vector. If the gradient is close to zero, this can cause jumps in the estimated parameters. NormalizationBias is the term introduced in the denominator to prevent these jumps. Increase NormalizationBias if you observe jumps in estimated parameters.

NormalizationBias is a tunable property. You can change it when the object is in a locked state.

## Default: eps

## History

Data history type defining which type of recursive algorithm you use, specified as:

- 'Infinite' - Use an algorithm that aims to minimize the error between the observed and predicted outputs for all time steps from the beginning of the simulation.
- 'Finite' - Use an algorithm that aims to minimize the error between the observed and predicted outputs for a finite number of past time steps.

Algorithms with infinite history aim to produce parameter estimates that explain all data since the start of the simulation. These algorithms still use a fixed amount of memory that does not grow over time. The object provides multiple algorithms of the 'Infinite' History type. Specifying this option activates the EstimationMethod property with which you specify an algorithm.

Algorithms with finite history aim to produce parameter estimates that explain only a finite number of past data samples. This method is also called sliding-window estimation. The object provides one algorithm of the 'Finite' type. Specifying this option activates the WindowLength property that sizes the window.

For more information on recursive estimation methods, see "Recursive Algorithms for Online Parameter Estimation".

History is a nontunable property. It can be set only during object construction using Name, Value arguments and cannot be changed afterward.

Default: 'Infinite'

## WindowLength

Window size determining the number of time samples to use for the sliding-window estimation method, specified as a positive integer. Specify WindowLength only when History is Finite.

Choose a window size that balances estimation performance with computational and memory burden. Sizing factors include the number and time variance of the parameters in your model. Always specify Window Length in samples, even if you are using frame-based input processing.

WindowLength must be greater than or equal to the number of estimated parameters.
Suitable window length is independent of whether you are using sample-based or frame-based input processing (see InputProcessing). However, when using frame-based processing, your window length must be greater than or equal to the number of samples (time steps) contained in the frame.

WindowLength is a nontunable property. It can be set only during object construction using Name, Value arguments and cannot be changed afterward.

Default: 200

## InputProcessing

Option for sample-based or frame-based input processing, specified as a character vector or string.

- Sample-based processing operates on signals streamed one sample at a time.
- Frame-based processing operates on signals containing samples from multiple time steps. Many machine sensor interfaces package multiple samples and transmit these samples together in frames. Frame-based processing allows you to input this data directly without having to first unpack it.

Your InputProcessing specification impacts the dimensions for the input and output signals when using the step command:

```
[theta,EstimatedOutput] = step(obj,y,u)
```

- Sample-based
- y,u, and Estimated0utput are scalars.
- Frame-based with $M$ samples per frame
- y,u, and EstimatedOutput are $M$-by-1 vectors.

InputProcessing is a nontunable property. It can be set only during object construction using Name, Value arguments and cannot be changed afterward.

Default: 'Sample-based'

## Output Arguments

## obj - System object for online parameter estimation of Output-Error polynomial model recursive0E System object

System object for online parameter estimation of SISO Output-Error polynomial model, returned as a recursive0E System object. This object is created using the specified model orders and properties. Use step command to estimate the coefficients of the Output-Error model polynomials. You can then access the estimated coefficients and parameter covariance using dot notation. For example, type obj. B to view the estimated $B$ polynomial coefficients.

## More About

## Output-Error Model Structure

The general output-error model structure is:

$$
y(t)=\frac{B(q)}{F(q)} u\left(t-n_{k}\right)+e(t)
$$

The orders of the output-error model are:
$n b: \quad B(q)=b_{1}+b_{2} q^{-1}+\ldots+b_{n b} q^{-n b+1}$
$n f: \quad F(q)=1+f_{1} q^{-1}+\ldots+f_{n f} q^{-n f}$

## Tips

- Starting in R2016b, instead of using the step command to update model parameter estimates, you can call the System object with input arguments, as if it were a function. For example, $[B, F, E s t i m a t e d O u t p u t]=\operatorname{step}(o b j, y, u)$ and $[B, F, E s t i m a t e d O u t p u t]=o b j(y, u)$ perform equivalent operations.


## Extended Capabilities

## C/C++ Code Generation

Generate C and $\mathrm{C}++$ code using MATLAB® $\operatorname{Coder}^{\mathrm{TM}}$.
Usage notes and limitations:

- For Simulink-based workflows, use Recursive Polynomial Model Estimator.
- For limitations, see "Generate Code for Online Parameter Estimation in MATLAB".

Supports MATLAB Function block: No

## See Also

Recursive Polynomial Model Estimator |clone |isLocked | oe | recursiveAR|recursiveARMA | recursiveARMAX | recursiveARX|recursiveBJ|recursiveLS|release|reset|step

## Topics

"Perform Online Parameter Estimation at the Command Line"
"Validate Online Parameter Estimation at the Command Line"
"What Is Online Estimation?"
"Recursive Algorithms for Online Parameter Estimation"
Introduced in R2015b

## release

Unlock online parameter estimation System object

## Syntax

release(obj)

## Description

release(obj) unlocks the online parameter estimation System object, obj. Use release to change nontunable properties of the object.

Note You can use release on a System object in code generated using MATLAB Coder, but once you release its resources, you cannot use that System object again.

## Examples

## Unlock Online Estimation System Object

Create a System object ${ }^{\mathrm{TM}}$ for online estimation of an ARMAX model with default properties.
obj = recursiveARMAX;
Estimate model parameters online using step and input-output data.
[A, B, C, EstimatedOutput] = step(obj, 1, 1);
step puts the object in a locked state.
L = isLocked (obj)
L = logical
1

Unlock the object.
release(obj)
Check the locked status of the object.
L = isLocked(obj)
$\mathrm{L}=$ logical
0

## Input Arguments

```
obj - System object for online parameter estimation
recursiveAR object|recursiveARMA object | recursiveARX object | recursiveARMAX object | recursive0E object|recursiveBJ object|recursiveLS object
```

System object for online parameter estimation, created using one of the following commands:

- recursiveAR
- recursiveARMA
- recursiveARX
- recursiveARMAX
- recursive0E
- recursiveBJ
- recursiveLS


## See Also

clone|isLocked|recursiveAR|recursiveARMA|recursiveARMAX|recursiveARX| recursiveBJ|recursiveLS|recursive0E|reset|step

## Topics

"Perform Online Parameter Estimation at the Command Line"
"Validate Online Parameter Estimation at the Command Line"
"What Is Online Estimation?"
Introduced in R2015b

## repsys

Replicate and tile models

## Syntax

```
rsys = repsys(sys,[M N])
rsys = repsys(sys,N)
rsys = repsys(sys,[M N S1,...,Sk])
```


## Description

rsys $=$ repsys(sys,[M N]) replicates the model sys into an M-by-N tiling pattern. The resulting model rsys has size(sys,1)*M outputs and size(sys,2)*N inputs.
rsys = repsys(sys,N) creates an N -by-N tiling.
rsys = repsys(sys,[M N S1,..., Sk]) replicates and tiles sys along both I/O and array dimensions to produce a model array. The indices $S$ specify the array dimensions. The size of the array is [size(sys,1)*M, size(sys,2)*N, size(sys,3)*S1, ...].

## Input Arguments

## sys

Model to replicate.
M
Number of replications of sys along the output dimension.

## N

Number of replications of sys along the input dimension.

## S

Numbers of replications of sys along array dimensions.

## Output Arguments

## rsys

Model having size(sys,1)*M outputs and size(sys,2)*N inputs.
If you provide array dimensions $\mathrm{S} 1, \ldots, \mathrm{Sk}$, rsys is an array of dynamic systems which each have size(sys,1)*M outputs and size(sys,2)*N inputs. The size of rsys is [size(sys,1)*M, size(sys,2)*N, size(sys,3)*S1, ...].

## Examples

## Replicate SISO Transfer Function to Create MIMO Transfer Function

Create a single-input single-output (SISO) transfer function.
sys $=t f\left(2,\left[\begin{array}{ll}1 & 3\end{array}\right)\right.$
sys $=$
2
----
s + 3
Continuous-time transfer function.
Replicate the SISO transfer function to create a MIMO transfer function that has three inputs and two outputs.

```
rsys = repsys(sys,[2 3])
rsys =
    From input 1 to output...
            2
        1: -----
            s + 3
            2
        2: -----
        s + 3
    From input 2 to output...
            2
        1: -----
            s + 3
            2
        2: -----
        s + 3
    From input 3 to output...
            2
    1: -----
        s + 3
            2
    2: -----
        S + 3
Continuous-time transfer function.
```

Alternatively, you can obtain the MIMO transfer function as follows:
rsys = [sys sys sys; sys sys sys];

## Replicate SISO Transfer Function to Create Array of Transfer Functions

Create a SISO transfer function.
sys = tf(2,[1 3]);
Replicate the transfer function into a 3-by-4 array of two-input, one-output transfer functions.
rsys = repsys(sys,[11 2 3 4]);
Check the size of rsys.
size(rsys)
$3 \times 4$ array of transfer functions.
Each model has 1 outputs and 2 inputs.

## Tips

rsys $=$ repsys(sys,N) produces the same result as rsys $=$ repsys(sys,[N $N$ ]). To produce a diagonal tiling, use rsys = sys*eye(N).

## See Also

append

## Introduced in R2010b

## resample

Resample time-domain data by decimation or interpolation (requires Signal Processing Toolbox software)

## Syntax

```
resample(data,P,Q)
resample(data,P,Q,order)
```


## Description

resample (data, $\mathrm{P}, \mathrm{Q}$ ) resamples data such that the data is interpolated by a factor P and then decimated by a factor $Q$. resample ( $z, 1, Q$ ) results in decimation by a factor Q .
resample(data, $P, Q$, order) filters the data by applying a filter of specified order before interpolation and decimation.

## Input Arguments

## data

Name of time-domain iddata object. Can be input-output or time-series data.
Data must be sampled at equal time intervals.
P, Q
Integers that specify the resampling factor, such that the new sample time is $\mathrm{Q} / \mathrm{P}$ times the original one.
$(Q / P)>1$ results in decimation and $(Q / P)<1$ results in interpolation.
order
Order of the filters applied before interpolation and decimation.
Default: 10

## Examples

## Resample Time-Domain Data

Increase the sampling rate of data by a factor of 1.5 and compare the resampled and the original data signals.

```
u = idinput([20 1 2],'sine',[],[],[5 10 1]);
u = iddata([],u,1);
plot(u)
ur = resample(u,3,2);
plot(u,ur)
```


## Input-Output Data



## Algorithms

If you have installed the Signal Processing Toolbox software, resample calls the Signal Processing Toolbox resample function. The algorithm takes into account the intersample characteristics of the input signal, as described by data. InterSample.

## See Also

idresamp
Introduced before R2006a

## reset

Reset online parameter estimation System object

## Syntax

reset(obj)

## Description

reset ( obj ) resets the states of a locked online parameter estimation System object, obj, to initial values and leaves the object locked. The states of the object are the estimated parameters and parameter covariance. Use reset if you are not satisfied with the estimation or if your system changes modes.

## Examples

## Reset Online Estimation System Object

Create a System object for online estimation of an Output-Error model.

```
obj = recursive0E('InitialB',[0 0.5],'InitialF',[1 0.8],...
    'InitialParameterCovariance',0.1);
```

Load the estimation data. For this example, use a static data set for illustration.

```
load iddatal z1;
output = z1.y;
input = z1.u;
Estimate model parameters online using step.
```

```
for i = 1:numel(input)
```

for i = 1:numel(input)
[B,F,EstimatedOutput] = step(obj,output(i),input(i));
[B,F,EstimatedOutput] = step(obj,output(i),input(i));
end

```
end
```

View the object properties.

```
obj
obj =
    recursiveOE with properties:
                    B: [0 2.0014]
                    F: [1 -0.7639]
                            InitialB: [0 0.5000]
                            InitialF: [1 0.8000]
            ParameterCovariance: [2x2 double]
        InitialParameterCovariance: [2x2 double]
            EstimationMethod: 'ForgettingFactor'
            ForgettingFactor: 1
            EnableAdaptation: true
```

```
    History: 'Infinite'
InputProcessing: 'Sample-based'
    DataType: 'double'
```

Reset the System object.

```
reset(obj)
```

The estimated parameters, B and F, and parameter covariance, ParameterCovariance are reset to the initial values.

```
obj
```

```
obj =
    recursive0E with properties:
```

$\left[\begin{array}{ll}0 & 0.5000\end{array}\right]$
$\left[\begin{array}{ll}1 & 0.8000\end{array}\right]$
$\left[\begin{array}{ll}0 & 0.5000\end{array}\right]$
$\left[\begin{array}{l}1 \\ 0.8000\end{array}\right]$
$[2 \times 2$ double]
$[2 \times 2$ double]
'ForgettingFactor'
1
true
'Infinite'
'Sample-based'
'double'

## Input Arguments

## obj - System object for online parameter estimation

```
recursiveAR object|recursiveARMA object | recursiveARX object | recursiveARMAX object |
recursive0E object| recursiveBJ object| recursiveLS object
```

System object for online parameter estimation, created using one of the following commands:

| Online Estimation System Object | Estimated Parameters |
| :--- | :--- |
| recursiveAR | A - Reset to InitialA |
| recursiveARMA | A - Reset to InitialA <br> C - Reset to InitialC |
| recursiveARX | A - Reset to InitialA <br> B $~$ Reset to InitialB |
| recursiveARMAX | A - Reset to InitialA |
| B - Reset to InitialB |  |
| C - Reset to InitialC |  |


| Online Estimation System Object | Estimated Parameters |
| :--- | :--- |
| recursive0E | B - Reset to InitialB |
|  | F - Reset to InitialF |
| recursiveBJ | B - Reset to InitialB |
|  | C - Reset to InitialC |
|  | D - Reset to Initiald |
|  | F - Reset to InitialF |
| recursiveLS | Parameters - Reset to InitialParameters |

When EstimationMethod property of obj is 'ForgettingFactor' or 'KalmanFilter', the ParameterCovariance property of obj is reset to the value of InitialParameterCovariance.

## See Also

clone|isLocked|recursiveAR|recursiveARMA|recursiveARMAX|recursiveARX| recursiveBJ|recursiveLS|recursive0E|release|step

## Topics

"Perform Online Parameter Estimation at the Command Line"
"Validate Online Parameter Estimation at the Command Line"
"What Is Online Estimation?"

## Introduced in R2015b

## reshape

Change shape of model array

## Syntax

sys = reshape(sys,s1,s2,...,sk)
sys $=$ reshape(sys,[s1 s2 ... sk])

## Description

sys = reshape(sys,s1,s2,...,sk) (or, equivalently, sys = reshape(sys,[s1 s2 ...
sk])) reshapes the LTI array sys into an s1-by-s2-by-...-by-sk model array. With either syntax, there must be s1*s2*...*sk models in sys to begin with.

## Examples

## Change Shape of Model Array

Generate a 2-by-3 array of SISO models with four states each.

```
sys = rss(4,1,1,2,3);
size(sys)
2x3 array of state-space models.
Each model has 1 outputs, 1 inputs, and 4 states.
```

Change the shape of the array to create a 6-by-1 model array.
sys1 = reshape(sys,6,1);
size(sys1)
$6 \times 1$ array of state-space models.
Each model has 1 outputs, 1 inputs, and 4 states.

## See Also

ndims | size

Introduced before R2006a

## resid

Compute and test residuals

## Syntax

```
resid(Data,sys)
resid(Data,sys,Linespec)
resid(Data,sys1,...,sysn)
resid(Data,sys1,Linespec1,...,sysn,Linespecn)
resid(
```

$\qquad$

``` ,Options)
resid(
``` \(\qquad\)
``` ,Type)
[E,R] = resid(Data,sys)
```


## Description

resid(Data, sys) computes the 1-step-ahead prediction errors (residuals) for an identified model, sys, and plots residual-input dynamics as one of the following, depending on the data inData:

- For time-domain data, resid plots the autocorrelation of the residuals and the cross-correlation of the residuals with the input signals. The correlations are generated for lags -25 to 25 . To specify a different maximum lag value, use residOptions. The 99\% confidence region marking statistically insignificant correlations displays as a shaded region around the X-axis.
- For frequency-domain data, resid plots a bode plot of the frequency response from the input signals to the residuals. The $99 \%$ confidence region marking statistically insignificant response is shown as a region around the X -axis.

To change display options, right-click the plot to access the context menu. For more details about the menu, see "Tips" on page 1-1296.
resid(Data, sys, Linespec) sets the line style, marker symbol, and color.
resid(Data, sys1, ...,sysn) computes and plots the residual of multiple identified models sys1,...sysn.
resid(Data,sys1,Linespec1, ...,sysn,Linespecn) sets the line style, marker symbol, and color for each system.
resid(__ , Options) specifies additional residual calculation options. Use Options with any of the previous syntaxes.
resid( $\qquad$ ,Type) specifies the plot type. Use Type with any of the previous syntaxes.
$[E, R]=$ resid(Data,sys) returns the calculated residuals, $E$, and residual correlations, R. No plot is generated.

## Examples

Plot Model Residual Correlations
Load time-domain data.
load iddata1
data = z1;
Estimate an ARX model.
sys = arx(data,[11 0]);
Plot the autocorrelation of the residuals and cross-correlation between the residuals and the inputs.
resid(data,sys)


The correlations are calculated until the default maximum lag, 25. The 99\% confidence region marking statistically insignificant correlations displays as a shaded region around the X-axis.

Convert data to frequency domain.
data2 $=$ fft(data);
Compute the residuals for identified model, sys, and the frequency-domain data. Plot the residual response using red crosses.

```
resid(data2,sys,'rx')
```


## Residue Frequency Response



For frequency-domain data, resid plots the Bode plot showing frequency response from the input to the residuals.

## Compare the Residuals for Multiple Identified Models

Load time-domain data.
load iddatal
Estimate an ARX model.
sys1 = arx(z1,[11 0]);
Estimate a transfer function model.
sys2 = tfest(z1,2);
Plot the correlations of the residuals.
resid(z1,sys1,'b',sys2,'r')


The cross-correlation between residuals of sys2 and the inputs lie in the $99 \%$ confidence band for all lags.

## Specify Maximum Lag for Residual Impulse Response Calculations

Load time-domain data.
load iddatal
Estimate an ARX model.
sys = arx(z1,[11 1 0]);
Specify the maximum lag for residual correlation calculations.
opt = residOptions('MaxLag',35);
Plot the impulse response from the input to the residuals.
resid(z1,sys,opt,'ir')

Residue Impulse Response


## Calculate Residuals for a MIMO System

Load time-domain data.
load iddata7
The data is a two-input, single-output dataset.
Estimate an ARX model.
sys = tfest(z7,2);
Calculate the residuals and their autocorrelations and cross-correlations with inputs.
[E,R] = resid(z7,sys);
$R$ is a 26 -by- 3 -by- 3 matrix of correlations. For example,

- $R(:, 1,1)$ is the autocorrelation of the residuals until lag 25.
- $R(:, 1,2)$ is the cross-correlation of the residuals with the first input, until lag 25.

E is an iddata object with the residuals as output data and the inputs of validation data (z7) as input data. You can use E to identify error models and analyze the error dynamics.

Plot the error.
plot(E)


Estimate impulse response between inputs and residuals. Plot them with a 3 standard deviation confidence region.

I = impulseest(E);
showConfidence(impulseplot(I,20),3)

## Impulse Response



## Input Arguments

## Data - Validation data

iddata object
Validation input-output data, specified as an iddata object. Data can have multiple input-output channels. When sys is linear, Data is time-domain or frequency-domain. When sys is nonlinear, Data is time-domain.

## sys - System used for computing residuals

identified linear or nonlinear model
System used for computing residuals, specified as an identified linear or nonlinear model.
Example: idpoly

## Linespec - Line style, marker symbol, and color

character vector
Line style, marker symbol, and color, specified as a character vector. For more information, see plot. When Type is specified as 'corr', only the line style is used.

Example: 'Linespec', 'kx'

## Options - Residual analysis options

residOptions option set

Residual analysis options, specified as an residOptions option set.

## Type - Plot type <br> 'corr'|'ir'|'fr'

Plot type, specified as one of the following values:

- 'corr' - Plots the autocorrelation of the residuals, e, and the cross-correlation of the residuals with the input signals, $u$. The correlations are generated for lags -25 to 25 . Use residOptions to specify a different maximum lag value. The $99 \%$ confidence region marking statistically insignificant correlations is also shown as a shaded region around the X-axis. The computation of the confidence region is done assuming $e$ to be white and independent of $u$.
' corr' is default for time-domain data. This plot type is not available for frequency-domain data.
- 'ir' - Plots the impulse response up to lag 25 of a system from the input to the residuals. The impul seest command first estimates the impulse response model with e as output data and u as inputs. Then impul seest calculates the impulse response of the estimated model. The 99\% confidence region marking statistically insignificant response displays as a shaded region. A low magnitude indicates a reliable model.

This plot type is not available for frequency-domain data.

- ' fr ' - The frequency response from the input to the residuals (based on a high-order FIR model) is shown as a Bode plot. The $99 \%$ confidence region marking statistically insignificant response displays as a shaded region. A low magnitude in the frequency range of interest indicates a reliable model.
' fr ' is default for frequency-domain data.


## Output Arguments

## E - Model residuals

iddata object
Model residuals, returned as an iddata object. The residuals are stored in E.OutputData, and the inputs are stored in E . InputData. Use E to build models that describe the dynamics from the inputs to the residuals. The dynamics are negligible if sys is a reliable identified model.

## R - Correlations of the residuals

```
matrix of doubles | [ ]
```

Correlations of the residuals, returned as one of the following:

- Matrix of doubles - For time-domain-data

R is a matrix of size $M+1$-by- $(n y+n u)$-by-(ny+nu). Where, $M$ is the maximum lag specified in Options, $n y$ is the number of outputs, and $n u$ is the number of inputs. The default value of $M$ is 25.

At each lag $k(k=0: M), R(k, i, j)$ is the expectation value, $<Z(t, i) . Z(t+k-1, j)>$. Here, $Z=$ [E.OutputData, E.InputData].

For example, for a two-output, single-input model, $\mathrm{Z}=[\mathrm{e}, \mathrm{e} 2, \mathrm{u} 1]$. Where, $e 1$ is the residual of the first output, $e 2$ is the residual of the second output, and $u 1$ is the input. R is a 26 -by- 3 -by- 3 matrix, where:

- $R(5,1,2)=<e 1(t) . e 2(t+4)>$ is the cross-correlation at lag 4 between $e 1$ and $e 2$.
- $R(5,1,3)=<e 1(t) \cdot u 1(t+4)>$ is the cross-correlation at lag 4 between $e 1$ and $u 1$.
- $R(5,1,1), R(5,2,2), R(5,3,3)$ are the autocorrelations at lag 4 for $e 1, e 2$, and $u 1$, respectively.
- [] - For frequency-domain data


## Tips

- Right-clicking the plot opens the context menu, where you can access the following options:
- Systems - Select systems to view the residual correlation or response plots. By default, all systems are plotted.
- Show Confidence Region - View the 99\% confidence region marking statistically insignificant correlations. Applicable only for the correlation plots.
- Data Experiment - For multi-experiment data only. Toggle between data from different experiments.
- Characteristics - View data characteristics. Not applicable for correlation plots.
- Peak Response - View peak response of the data.
- Confidence Region - View the $99 \%$ confidence region marking statistically insignificant response.
- Show - Applicable only for frequency-response plots.
- Magnitude - View magnitude of frequency response.
- Phase - View phase of frequency response.
- I/O Grouping - For datasets containing more than one input or output channel. Select grouping of input and output channels on the plot. Not applicable for correlation plots.
- None - Plot input-output channels in their own separate axes.
- All - Group all input channels together and all output channels together.
- I/O Selector - For datasets containing more than one input or output channel. Select a subset of the input and output channels to plot. By default, all output channels are plotted.
- Grid - Add grids to the plot.
- Normalize - Normalize the y-scale of all data in the plot. Not applicable for frequencyresponse data.
- Full View - Return to full view. By default, the plot is scaled to full view.
- Initial Condition - Specify handling of initial conditions.

Specify as one of the following:

- Estimate - Treat the initial conditions as estimation parameters.
- Zero - Set all initial conditions to zero.
- Absorb delays and estimate - Absorb nonzero delays into the model coefficients and treat the initial conditions as estimation parameters. Use this option for discrete-time models only.
- Properties - Open the Property Editor dialog box to customize plot attributes.


## References

[1] Ljung, L. System Identification: Theory for the User. Upper Saddle River, NJ: Prentice-Hall PTR, 1999, Section 16.6.

## See Also

compare | predict | residOptions | sim | simsd

## Topics

"What Is Residual Analysis?"
Introduced before R2006a

## residOptions

Option set for resid

## Syntax

```
opt = residOptions
opt = residOptions(Name,Value)
```


## Description

opt $=$ residOptions creates the default option set for resid. Use dot notation to customize the option set, if needed.
opt $=$ residOptions(Name, Value) creates an option set with options specified by one or more Name, Value pair arguments. The options that you do not specify retain their default value.

## Examples

## Create and Modify Default Option Set for Residual Analysis

Create a default option set for resid.
opt = residOptions;
Specify the maximum lag for residual correlation calculations.

```
opt.MaxLag = 35;
```


## Specify Options for Residual Analysis

Create an option set for resid that specifies initial condition as zero.

```
opt = residOptions('InitialCondition','z');
```


## Input Arguments

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, ... , NameN, ValueN.
Example: residOptions('InitialCondition','e')

## MaxLag - Maximum positive lag

25 (default) | positive integer

Maximum positive lag for residual correlation and impulse response calculations, specified as the comma-separated pair consisting of 'MaxLag' and a positive integer.

## InitialCondition - Handling of initial conditions

'e' (default) | 'z' | 'd' | column vector | matrix \| initialCondition object | object array | structure | idpar object x00bj

Handling of initial conditions, specified as the comma-separated pair consisting of
'InitialCondition' and one of the following values:

- 'z' - Zero initial conditions.
- 'e' - Estimate initial conditions such that the prediction error for observed output is minimized.

For nonlinear grey-box models, only those initial states ithat are designated as free in the model (sys.InitialStates(i).Fixed = false) are estimated. To estimate all the states of the model, first specify all the Nx states of the idnlgrey model sys as free.

```
for i = 1:Nx
sys.InitialStates(i).Fixed = false;
end
```

Similarly, to fix all the initial states to values specified in sys. InitialStates, first specify all the states as fixed in the sys. InitialStates property of the nonlinear grey-box model.

- 'd' - Similar to 'e', but absorbs nonzero delays into the model coefficients. The delays are first converted to explicit model states, and the initial values of those states are also estimated and returned.

Use this option for linear models only.

- Vector or Matrix - Initial guess for state values, specified as a numerical column vector of length equal to the number of states. For multi-experiment data, specify a matrix with Ne columns, where $N e$ is the number of experiments. Otherwise, use a column vector to specify the same initial conditions for all experiments. Use this option for state-space (idss and idgrey) and nonlinear models (idnlarx, idnlhw, and idnlgrey) only.
- initialCondition object - initialCondition object that represents a model of the free response of the system to initial conditions. For multiexperiment data, specify a 1 -by- $N_{e}$ array of objects, where $N_{e}$ is the number of experiments.

Use this option for linear models only.

- Structure with the following fields, which contain the historical input and output values for a time interval immediately before the start time of the data used by resid:

| Field | Description |
| :--- | :--- |
| Input | Input history, specified as a matrix with $N u$ columns, where $N u$ is the number of <br> input channels. For time series models, use [ ]. The number of rows must be <br> greater than or equal to the model order. |
| Output | Output history, specified as a matrix with Ny columns, where Ny is the number of <br> output channels. The number of rows must be greater than or equal to the model <br> order. |

For multi-experiment data, configure the initial conditions separately for each experiment by specifying InitialCondition as a structure array with $N e$ elements. To specify the same initial conditions for all experiments, use a single structure.

The software uses data2state to map the historical data to states. If your model is not idss, idgrey, idnlgrey, or idnlarx, the software first converts the model to its state-space representation and then maps the data to states. If conversion of your model to idss is not possible, the estimated states are returned empty.

- x0obj - Specification object created using idpar. Use this object for discrete-time state-space (idss and idgrey) and nonlinear grey-box (idnlgrey) models only. Use x0obj to impose constraints on the initial states by fixing their value or specifying minimum or maximum bounds.


## InputOffset - Removal of offset from time-domain input data during estimation

[ ] (default) | vector of positive integers | matrix
Removal of offset from time-domain input data during estimation, specified as the comma-separated pair consisting of 'InputOffset' and one of the following:

- A column vector of positive integers of length $N u$, where $N u$ is the number of inputs.
- [] - Indicates no offset.
- $N u$-by-Ne matrix - For multi-experiment data, specify InputOffset as an $N u$-by-Ne matrix. $N u$ is the number of inputs, and $N e$ is the number of experiments.

Each entry specified by InputOffset is subtracted from the corresponding input data.

## OutputOffset - Removal of offset from time-domain output data during estimation <br> [] (default) | vector | matrix

Removal of offset from time-domain output data during estimation, specified as the comma-separated pair consisting of 'OutputOffset' and one of the following:

- A column vector of length $N y$, where $N y$ is the number of outputs.
- [ ] - Indicates no offset.
- $N y$-by-Ne matrix - For multi-experiment data, specify OutputOffset as a $N y$-by-Ne matrix. Ny is the number of outputs, and $N e$ is the number of experiments.

Each entry specified by OutputOffset is subtracted from the corresponding output data.

## OutputWeight - Weight of output for initial condition estimation

[] (default) | 'noise' | matrix
Weight of output for initial condition estimation, specified as the comma-separated pair consisting of 'OutputWeight ' and one of the following:

- [] - No weighting is used. This option is the same as using eye(Ny) for the output weight. Ny is the number of outputs.
- ' noise' - Inverse of the noise variance stored with the model.
- Matrix of doubles - A positive semidefinite matrix of dimension $N y$-by- Ny . Ny is the number of outputs.


## Output Arguments

opt - Option set for resid
residOptions option set
Option set for resid, returned as an residOptions option set.

## See Also

resid

Introduced in R2016a

## residual

Return measurement residual and residual covariance when using extended or unscented Kalman filter

## Syntax

[Residual,ResidualCovariance] = residual(obj,y)
[Residual,ResidualCovariance] = residual(obj,y,Um1,....,Umn)

## Description

The residual command returns the difference between the actual and predicted measurements for extendedKalmanFilter and unscentedKalmanFilter objects. Viewing the residual provides a way for you to validate the performance of the filter. Residuals, also known as innovations, quantify the prediction error and drive the correction step in the extended and unscented Kalman filter update sequence. When using correct and predict to update the estimated Kalman filter state, use the residual command immediately before using the correct command.
[Residual,ResidualCovariance] = residual(obj,y) returns the residual Residual between a measurement y and a predicted measurement produced by the Kalman filter obj. The function also returns the covariance of the residual ResidualCovariance.

You create obj using the extendedKalmanFilter or unscentedKalmanFilter commands. You specify the state transition function $f$ and measurement function $h$ of your nonlinear system in obj. The State property of the object stores the latest estimated state value. At each time step, you use correct and predict together to update the state $x$. The residual $s$ is the difference between the actual and predicted measurements for the time step, and is expressed as $s=y-h(x)$. The covariance of the residual $S$ is the sum $R+R_{P}$, where $R$ is the measurement noise matrix set by the MeasurementNoise property of the filter and $R_{P}$ is the state covariance matrix projected onto the measurement space.

Use this syntax if the measurement function $h$ that you specified in obj. MeasurementFcn has one of the following forms:

- $y(k)=h(x(k))$ for additive measurement noise
- $y(k)=h(x(k), v(k))$ for nonadditive measurement noise

Here, $y(k), x(k)$, and $v(k)$ are the measured output, states, and measurement noise of the system at time step $k$. The only inputs to $h$ are the states and measurement noise.
[Residual,ResidualCovariance] = residual(obj,y,Um1, $\ldots$, Umn) specifies additional input arguments if the measurement function of the system requires these inputs. You can specify multiple arguments.

Use this syntax if the measurement function $h$ has one of the following forms:

- $y(k)=h(x(k), U m 1, \ldots, U m n)$ for additive measurement noise
- $y(k)=h(x(k), v(k), U m 1, \ldots$, Umn $)$ for nonadditive measurement noise


## Examples

## Estimate States Online Using Extended Kalman Filter

Estimate the states of a van der Pol oscillator using an extended Kalman filter algorithm and measured output data. The oscillator has two states and one output.

Create an extended Kalman filter object for the oscillator. Use previously written and saved state transition and measurement functions, vdpStateFcn.m and vdpMeasurementFcn.m. These functions describe a discrete-approximation to a van der Pol oscillator with the nonlinearity parameter mu equal to 1 . The functions assume additive process and measurement noise in the system. Specify the initial state values for the two states as $[1 ; 0]$. This is the guess for the state value at initial time k , based on knowledge of system outputs until time $\mathrm{k}-1, \hat{x}[k \mid k-1]$.

```
obj = extendedKalmanFilter(@vdpStateFcn,@vdpMeasurementFcn,[1;0]);
```

Load the measured output data $y$ from the oscillator. In this example, use simulated static data for illustration. The data is stored in the vdp_data.mat file.
load vdp_data.mat y
Specify the process noise and measurement noise covariances of the oscillator.

```
obj.ProcessNoise = 0.01;
obj.MeasurementNoise = 0.16;
```

Initialize arrays to capture results of the estimation.

```
residBuf = [];
xcorBuf = [];
xpredBuf = [];
```

Implement the extended Kalman filter algorithm to estimate the states of the oscillator by using the correct and predict commands. You first correct $\hat{\chi}[k \mid k-1]$ using measurements at time $k$ to get $\hat{x}[k \mid k]$. Then, you predict the state value at the next time step $\hat{x}[k+1 \mid k]$ using $\hat{x}[k \mid k]$, the state estimate at time step $k$ that is estimated using measurements until time $k$.

To simulate real-time data measurements, use the measured data one time step at a time. Compute the residual between the predicted and actual measurement to assess how well the filter is performing and converging. Computing the residual is an optional step. When you use residual, place the command immediately before the correct command. If the prediction matches the measurement, the residual is zero.

After you perform the real-time commands for the time step, buffer the results so that you can plot them after the run is complete.

```
for k = 1:size(y)
    [Residual,ResidualCovariance] = residual(obj,y(k));
    [CorrectedState,CorrectedStateCovariance] = correct(obj,y(k));
    [PredictedState,PredictedStateCovariance] = predict(obj);
    residBuf(k,:) = Residual;
    xcorBuf(k,:) = CorrectedState';
    xpredBuf(k,:) = PredictedState';
end
```

When you use the correct command, obj. State and obj. StateCovariance are updated with the corrected state and state estimation error covariance values for time step k, CorrectedState and CorrectedStateCovariance. When you use the predict command, obj.State and obj. StateCovariance are updated with the predicted values for time step $\mathrm{k}+1$, PredictedState and PredictedStateCovariance. When you use the residual command, you do not modify any obj properties.

In this example, you used correct before predict because the initial state value was $\hat{x}[k \mid k-1]$, a guess for the state value at initial time $k$ based on system outputs until time $k$ - 1 . If your initial state value is $\hat{x}[k-1 \mid k-1]$, the value at previous time $k-1$ based on measurements until $k-1$, then use the predict command first. For more information about the order of using predict and correct, see "Using predict and correct Commands" on page 1-215.

Plot the estimated states, using the postcorrection values.
plot(xcorBuf(:,1), xcorBuf(:,2))
title('Estimated States')


Plot the actual measurement, the corrected estimated measurement, and the residual. For the measurement function in vdpMeasurementFcn, the measurement is the first state.

```
M = [y,xcorBuf(:,1),residBuf];
plot(M)
grid on
title('Actual and Estimated Measurements, Residual')
legend('Measured','Estimated','Residual')
```



The estimate tracks the measurement closely. After the initial transient, the residual remains relatively small throughout the run.

## Specify State Transition and Measurement Functions with Additional Inputs

Consider a nonlinear system with input $u$ whose state $x$ and measurement $y$ evolve according to the following state transition and measurement equations:

$$
\begin{aligned}
& x[k]=\sqrt{x[k-1]+u[k-1]}+w[k-1] \\
& y[k]=x[k]+2 * u[k]+v[k]^{2}
\end{aligned}
$$

The process noise $w$ of the system is additive while the measurement noise $v$ is nonadditive.
Create the state transition function and measurement function for the system. Specify the functions with an additional input u.
$f=@(x, u)(\operatorname{sqrt}(x+u)) ;$
$h=@(x, v, u)\left(x+2^{*} u+v^{\wedge} 2\right)$;
f and h are function handles to the anonymous functions that store the state transition and measurement functions, respectively. In the measurement function, because the measurement noise is nonadditive, $v$ is also specified as an input. Note that $v$ is specified as an input before the additional input $u$.

Create an extended Kalman filter object for estimating the state of the nonlinear system using the specified functions. Specify the initial value of the state as 1 and the measurement noise as nonadditive.
obj = extendedKalmanFilter(f,h,1,'HasAdditiveMeasurementNoise',false);
Specify the measurement noise covariance.
obj. MeasurementNoise = 0.01;
You can now estimate the state of the system using the predict and correct commands. You pass the values of $u$ to predict and correct, which in turn pass them to the state transition and measurement functions, respectively.

Correct the state estimate with measurement $\mathrm{y}[\mathrm{k}]=0.8$ and input $\mathrm{u}[\mathrm{k}]=0.2$ at time step k .
correct (obj, 0.8, 0.2)
Predict the state at the next time step, given $u[k]=0.2$.
predict(obj,0.2)
Retrieve the error, or residual, between the prediction and the measurement.
[Residual, ResidualCovariance] = residual(obj, 0.8,0.2);

## Input Arguments

obj - Extended or unscented Kalman filter
extendedKalmanFilter object | unscentedKalmanFilter object
Extended or unscented Kalman filter, created using one of the following commands:

- extendedKalmanFilter - Uses the extended Kalman filter algorithm
- unscentedKalmanFilter - Uses the unscented Kalman filter algorithm


## y - Measured system output

vector
Measured system output at the current time step, specified as an $N$-element vector, where $N$ is the number of measurements.

## Um1, . . . Umn - Additional input arguments to measurement function <br> input arguments of any type

Additional input arguments to the measurement function of the system, specified as input arguments of any type. The measurement function $h$ is specified in the MeasurementFcn or MeasurementLikelihoodFcn property of obj. If the function requires input arguments in addition to the state and measurement noise values, you specify these inputs in the residual command syntax. The residual command passes these inputs to the measurement or the measurement likelihood function to calculate estimated outputs. You can specify multiple arguments.

For instance, suppose that your measurement or measurement likelihood function calculates the estimated system output $y$ using system inputs $u$ and current time $k$, in addition to the state $x$. The $U_{m 1}$ and $U_{m 2}$ terms are therefore $\mathrm{u}(\mathrm{k})$ and k . These inputs result in the estimated output
$y(k)=h(x(k), u(k), k)$
Before you perform online state estimation correction at time step $k$, specify these additional inputs in the residual command syntax:
[Residual,ResidualCovariance] = residual(obj,y,u(k),k);
For an example showing how to use additional input arguments, see "Specify State Transition and Measurement Functions with Additional Inputs" on page 1-1305.

## Output Arguments

## Residual - Residual between current and predicted measurement

## scalar | vector

Residual between current and predicted measurement, returned as a:

- Scalar for a single-output system
- Vector of size $N$ for a multiple-output system, where $N$ is the number of measured outputs


## ResidualCovariance - Residual covariance

## matrix

Residual covariance, returned as an $N$-by- $N$ matrix where $N$ is the number of measured outputs.

## See Also

correct | extendedKalmanFilter | predict | unscentedKalmanFilter

## Topics

"Nonlinear State Estimation Using Unscented Kalman Filter and Particle Filter"
"Generate Code for Online State Estimation in MATLAB"
"What Is Online Estimation?"
"Extended and Unscented Kalman Filter Algorithms for Online State Estimation"
"Validate Online State Estimation at the Command Line"

## Introduced in R2019b

## retrend

Add offsets or trends to time-domain data signals stored in iddata objects

## Syntax

data $=$ retrend(data_d,T)

## Description

data $=$ retrend (data_d,T) returns a data object data by adding the trend information T to each signal in data_d. data_d is a time-domain iddata object. T is an TrendInfo object.

## Examples

## Retrend Simulated Model Output

Subtract means from input-output signals, estimate a linear model, and retrend the simulated output.
Load SISO data containing vectors u 2 and y 2 .
load dryer2
Create a data object with sample time of 0.08 seconds.
data $=$ iddata( $\mathrm{y} 2, \mathrm{u} 2,0.08$ );
Remove the mean from the data.
[data_d,T] = detrend(data,0);
Estimate a linear ARX model.
m = arx(data_d,[2 2 1]);
Simulate the model output with zero initial states.
y_sim = sim(m,data_d(:,[],:));
Retrend the simulated model output.
y_tot = retrend(y_sim,T);

## See Also

TrendInfo | detrend | getTrend

## Topics

"Handling Offsets and Trends in Data"

## Introduced in R2009a

## roe

(To be removed) Estimate recursively output-error models (IIR-filters)

Note roe will be removed in a future release. Use recursive0E instead.

## Syntax

```
thm = roe(z,nn,adm,adg)
```

[thm,yhat, P,phi,psi] = roe(z,nn,adm,adg,th0,P0,phi0,psi0)

## Description

The parameters of the output-error model structure

$$
y(t)=\frac{B(q)}{F(q)} u\left(t-n_{k}\right)+e(t)
$$

are estimated using a recursive prediction error method.
The input-output data are contained in $z$, which is either an iddata object or a matrix $z=[y \quad u]$ where $y$ and $u$ are column vectors. $n n$ is given as
$n n=[n b \mathrm{nf} n \mathrm{n}]$
where nb and nf are the orders of the output-error model, and nk is the delay. Specifically,

$$
\begin{aligned}
& n b: \quad B(q)=b_{1}+b_{2} q^{-1}+\ldots+b_{n b} q^{-n b+1} \\
& n f: \quad F(q)=1+f_{1} q^{-1}+\ldots+f_{n f} q^{-n f}
\end{aligned}
$$

See "What Are Polynomial Models?" for more information.
Only single-input, single-output models are handled by roe. Use rpem for the multiple-input case.
The estimated parameters are returned in the matrix thm. The kth row of thm contains the parameters associated with time k ; that is, they are based on the data in the rows up to and including row k in z .

Each row of thm contains the estimated parameters in the following order.
$\operatorname{thm}(k,:)=[b 1, \ldots, b n b, f 1, \ldots, f n f]$
yhat is the predicted value of the output, according to the current model; that is, row k of yhat contains the predicted value of $\mathrm{y}(\mathrm{k})$ based on all past data.

The actual algorithm is selected with the two arguments adg and adm. These are described under rarx.

The input argument th0 contains the initial value of the parameters, a row vector consistent with the rows of thm. The default value of th0 is all zeros.

The arguments P 0 and P are the initial and final values, respectively, of the scaled covariance matrix of the parameters. The default value of P0 is $10^{4}$ times the unit matrix. The arguments phi0, psi0, phi, and psi contain initial and final values of the data vector and the gradient vector, respectively. The sizes of these depend on the chosen model orders. The normal choice of phi0 and psi0 is to use the outputs from a previous call to roe with the same model orders. (This call could be a dummy call with default input arguments.) The default values of phi0 and psi0 are all zeros.

Note that the function requires that the delay nk be larger than 0 . If you want $n k=0$, shift the input sequence appropriately and use $\mathrm{nk}=1$.

## Algorithms

The general recursive prediction error algorithm (11.44) of Ljung (1999) is implemented. See also "Recursive Algorithms for Online Parameter Estimation".

## See Also

nkshift | recursive0E | rpem | rplr

## Topics

"Recursive Algorithms for Online Parameter Estimation"

Introduced before R2006a

## rpem

Estimate general input-output models using recursive prediction-error minimization method

## Syntax

```
thm = rpem(z,nn,adm,adg)
```

[thm,yhat, P, phi, psi] = rpem(z,nn,adm,adg,th0, P0, phi0, psi0)

## Description

rpem is not compatible with MATLAB Coder or MATLAB Compiler ${ }^{\text {TM }}$. For the special cases of ARX, AR, ARMA, ARMAX, Box-Jenkins, and Output-Error models, use recursiveARX, recursiveAR, recursiveARMA, recursiveARMAX, recursiveBJ, and recursiveOE, respectively.

The parameters of the general linear model structure

$$
A(q) y(t)=\frac{B_{1}(q)}{F_{1}(q)} u_{1}\left(t-n k_{1}\right)+\ldots+\frac{B_{n u}(q)}{F_{n u}(q)} u_{n u}\left(t-n k_{n u}\right)+\frac{C(q)}{D(q)} e(t)
$$

are estimated using a recursive prediction error method.
The input-output data is contained in $z$, which is either an iddata object or a matrix $z=[y \quad u]$ where $y$ and $u$ are column vectors. (In the multiple-input case, $u$ contains one column for each input.) $n n$ is given as
nn = [na nb nc nd nf nk]
where na, nb, nc, nd, and $n f$ are the orders of the model, and $n k$ is the delay. For multiple-input systems, nb, nf, and nk are row vectors giving the orders and delays of each input. See "What Are Polynomial Models?" for an exact definition of the orders.

The estimated parameters are returned in the matrix thm. The kth row of thm contains the parameters associated with time $k$; that is, they are based on the data in the rows up to and including row k in z . Each row of thm contains the estimated parameters in the following order.

```
thm(k,:) = [a1,a2,...,ana,b1,...,bnb,...
    c1,...,cnc,d1,...,dnd,f1,...,fnf]
```

For multiple-input systems, the $B$ part in the above expression is repeated for each input before the $C$ part begins, and the $F$ part is also repeated for each input. This is the same ordering as in m. par.
yhat is the predicted value of the output, according to the current model; that is, row $k$ of yhat contains the predicted value of $y(k)$ based on all past data.

The actual algorithm is selected with the two arguments adg and adm:

- $\operatorname{adm}=$ ' ff ' and adg = lam specify the forgetting factor algorithm with the forgetting factor $\lambda=$ lam. This algorithm is also known as recursive least squares (RLS). In this case, the matrix $P$ has the following interpretation: $R_{2} / 2 * \mathrm{P}$ is approximately equal to the covariance matrix of the estimated parameters. $R_{2}$ is the variance of the innovations (the true prediction errors $e(t)$ ).
adm ='ug' and adg = gam specify the unnormalized gradient algorithm with gain gamma = gam. This algorithm is also known as the normalized least mean squares (LMS).
adm $=$ ' $n g$ ' and adg $=$ gam specify the normalized gradient or normalized least mean squares (NLMS) algorithm. In these cases, P is not applicable.
adm $=$ ' kf ' and adg $=\mathrm{R} 1$ specify the Kalman filter based algorithm with $R_{2}=1$ and $R_{1}=\mathrm{R} 1$. If the variance of the innovations $e(t)$ is not unity but $R_{2}$; then $R_{2}{ }^{*} \mathrm{P}$ is the covariance matrix of the parameter estimates, while $R_{1}=\mathrm{R} 1 / R_{2}$ is the covariance matrix of the parameter changes.

The input argument th0 contains the initial value of the parameters, a row vector consistent with the rows of thm. The default value of th0 is all zeros.

The arguments $P 0$ and $P$ are the initial and final values, respectively, of the scaled covariance matrix of the parameters. The default value of P 0 is $10^{4}$ times the unit matrix. The arguments phi0, psi0, phi, and psi contain initial and final values of the data vector and the gradient vector, respectively. The sizes of these depend on the chosen model orders. The normal choice of phi0 and psi0 is to use the outputs from a previous call to rpem with the same model orders. (This call could be a dummy call with default input arguments.) The default values of phi0 and psi0 are all zeros.

Note that the function requires that the delay nk be larger than 0 . If you want $n k=0$, shift the input sequence appropriately and use $n k=1$.

## Examples

## Estimate Model Parameters Using Recursive Prediction-Error Minimization

Specify the order and delays of a polynomial model structure.

```
na = 2;
nb = 1;
nc = 1;
nd = 1;
nf = 0;
nk = 1;
```

Load the estimation data.
load iddata1 z1
Estimate the parameters using forgetting factor algorithm with forgetting factor 0.99.

```
EstimatedParameters = rpem(z1,[na nb nc nd nf nk],'ff',0.99);
```

Get the last set of estimated parameters.

```
p = EstimatedParameters(end,:);
```

Construct a polynomial model with the estimated parameters.

```
sys = idpoly([1 p(1:na)],... % A polynomial
    [zeros(1,nk) p(na+1:na+nb)],... % B polynomial
    [1 p(na+nb+1:na+nb+nc)],... % C polynomial
    [1 p(na+nb+nc+1:na+nb+nc+nd)]); % D polynomial
sys.Ts = z1.Ts;
```

Compare the estimated output with measured data.

```
compare(z1,sys);
```



## Algorithms

The general recursive prediction error algorithm (11.44) of Ljung (1999) is implemented. See also "Recursive Algorithms for Online Parameter Estimation".

## See Also

nkshift | recursiveAR | recursiveARMA | recursiveARMAX | recursiveARX|recursiveBJ | recursive0E|rplr

Topics
"What Is Online Estimation?"
"Recursive Algorithms for Online Parameter Estimation"

## Introduced before R2006a

## rplr

Estimate general input-output models using recursive pseudolinear regression method

## Syntax

thm $=r p l r(z, n n, a d m, a d g)$
[thm,yhat, P, phi] = rplr(z,nn,adm,adg,th0, P0, phi0)

## Description

$r p l r$ is not compatible with MATLAB Coder or MATLAB Compiler.
This is a direct alternative to rpem and has essentially the same syntax. See rpem for an explanation of the input and output arguments.
rplr differs from rpem in that it uses another gradient approximation. See Section 11.5 in Ljung (1999). Several of the special cases are well-known algorithms.

When applied to ARMAX models, ( $n n=\left[\begin{array}{lll}n a & n b \\ n c & 0 & n k\end{array}\right]$ ), $r p l r$ gives the extended least squares (ELS) method. When applied to the output-error structure (nn = [0 nb 00 nf nk ]), the method is known as HARF in the adm = ' ff ' case and SHARF in the adm $=$ ' ng ' case.
$r p l r$ can also be applied to the time-series case in which an ARMA model is estimated with:

```
z = y
nn = [na nc]
```


## Examples

## Estimate Output-Error Model Parameters Using Recursive Pseudolinear Regression

Specify the order and delays of an Output-Error model structure.

```
na = 0;
nb = 2;
nc = 0;
nd = 0;
nf = 2;
nk = 1;
```

Load the estimation data.
load iddata1 z1
Estimate the parameters using forgetting factor algorithm, with forgetting factor 0.99.
EstimatedParameters = rplr(z1,[na nb nc nd nf nk],'ff',0.99);

## References

For more information about HARF and SHARF, see Chapter 11 in Ljung (1999).

## See Also

nkshift | recursiveAR | recursiveARMA | recursiveARMAX | recursiveARX|recursiveBJ | recursive0E | rpem

## Topics

"What Is Online Estimation?"
"Recursive Algorithms for Online Parameter Estimation"
Introduced before R2006a

## rsample

Random sampling of linear identified systems

## Syntax

sys_array = rsample(sys,N)
sys_array = rsample(sys,N,sd)

## Description

sys_array $=$ rsample(sys, $N$ ) creates $N$ random samples of the identified linear system, sys. sys_array contains systems with the same structure as sys, whose parameters are perturbed about their nominal values, based on the parameter covariance.
sys_array $=$ rsample(sys, $N, s d$ ) specifies the standard deviation level, sd, for perturbing the parameters of sys.

## Input Arguments

## sys

Identifiable system.

## N

Number of samples to be generated.
Default: 10
sd
Standard deviation level for perturbing the identifiable parameters of sys.
Default: 1

## Output Arguments

```
sys_array
```

Array of random samples of sys.
If sys is an array of models, then the size of sys_array is equal to [size(sys) N ]. There are N randomized samples for each model in sys.

The parameters of the samples in sys_array vary from the original identifiable model within 1 standard deviation of their nominal values.

## Examples

## Create Random Samples of Estimated Model

Estimate a third-order, discrete-time, state-space model.
load iddata2 z2;
sys $=$ n4sid(z2,3);
Randomly sample the estimated model.
$\mathrm{N}=20$;
sys_array = rsample(sys,N);
Analyze the uncertainty in time (step) and frequency (Bode) responses.
opt = bodeoptions;
opt.PhaseMatching = 'on';
figure;
bodeplot(sys_array,'g',sys,'r.',opt)

figure;
stepplot(sys_array,'g',sys,'r.-')


## Specify Standard Deviation Level for Parameter Perturbation

Estimate a third-order, discrete-time, state-space model.
load iddata2 z2;
sys = n4sid(z2,3);
Randomly sample the estimated model. Specify the standard deviation level for perturbing the model parameters.
$\mathrm{N}=20$;
sd = 2;
sys_array = rsample(sys,N,sd);
Analyze the model uncertainty.
figure;
bode(sys_array);


## Compare Frequency Response Confidence Regions for Sampled ARMAX Model

Estimate an ARMAX model.
load iddatal z1
sys = armax(z1,[2 22 1]);
Randomly sample the ARMAX model. Perturb the model parameters up to 2 standard deviations.

```
N = 20;
sd = 2;
sys_array = rsample(sys,N,sd);
```

Compare the frequency response confidence region corresponding to 2 standard deviations (asymptotic estimate) with the model array response.

```
opt = bodeoptions; opt.PhaseMatching = 'on';
opt.ConfidenceRegionNumberSD = 2;
bodeplot(sys_array,'g',sys,'r',opt)
```



To view the confidence region, right click the plot, and choose Characteristics > Confidence Region.

## Tips

- For systems with large parameter uncertainties, the randomized systems may contain unstable elements. These unstable elements may make it difficult to analyze the properties of the identified system. Execution of analysis commands, such as step, bode, sim, etc., on such systems can produce unreliable results. Instead, use a dedicated Monte-Carlo analysis command, such as simsd.


## See Also

bode |init |iopzmap|noise2meas|noisecnv| simsd| step|translatecov
Introduced in R2012a

## saturation

Create a saturation nonlinearity estimator object

## Syntax

NL = saturation
NL = saturation('LinearInterval', [a, b])

## Description

$\mathrm{NL}=$ saturation creates a default saturation nonlinearity estimator object for estimating Hammerstein-Wiener models. The linear interval is set to [ $\mathrm{NaN} \mathrm{NaN]}$. interval is determined from the estimation data range during estimation using nlhw. Use dot notation to customize the object properties, if needed.

NL = saturation('LinearInterval', [a,b]) creates a saturation nonlinearity estimator object initialized with linear interval, [a,b].

Alternatively, use NL = saturation([a,b]).

## Object Description

saturation is an object that stores the saturation nonlinearity estimator for estimating Hammerstein-Wiener models.

Use saturation to define a nonlinear function $y=F(x, \theta)$, where $y$ and $x$ are scalars, and $\theta$ represents the parameters $a$ and $b$ that define the linear interval, $[a, b]$.

The saturation nonlinearity function has the following characteristics:

$$
\begin{array}{lr}
a \leq x<b & F(x)=x \\
a>x & F(x)=a \\
b \leq x & F(x)=b
\end{array}
$$

For example, in the following plot, the linear interval is [ $-4,3$ ].


The value $F(x)$ is computed by evaluate $(N L, x)$, where $N L$ is the saturation object.
For saturation object properties, see "Properties" on page 1-1325.

## Examples

## Create a Default Saturation Nonlinearity Estimator

NL = saturation;
Specify the linear interval.
NL. LinearInterval = [-4,5];

## Estimate a Hammerstein Model with Saturation

Load data.
load twotankdata;
z = iddata(y,u,0.2,'Name','Two tank system');
z1 = z(1:1000);
Create a saturation object with lower limit of 0 and upper limit of 5 .

InputNL = saturation('LinearInterval',[0 5]);
Estimate model with no output nonlinearity.

```
m = nlhw(z1,[2 3 0],InputNL,[]);
```


## Estimate MIMO Hammerstein-Wiener Model

Load the estimation data.
load motorizedcamera;
Create an iddata object.
z = iddata(y,u,0.02,'Name','Motorized Camera','TimeUnit','s');
$z$ is an iddata object with 6 inputs and 2 outputs.
Specify the model orders and delays.
Orders $=[\operatorname{ones}(2,6)$, ones $(2,6)$, ones $(2,6)]$;
Specify the same nonlinearity estimator for each input channel.
InputNL = saturation;
Specify different nonlinearity estimators for each output channel.

```
OutputNL = [deadzone,wavenet];
```

Estimate the Hammerstein-Wiener model.
sys $=$ nlhw(z,Orders,InputNL,OutputNL);
To see the shape of the estimated input and output nonlinearities, plot the nonlinearities.
plot(sys)


Click on the input and output nonlinearity blocks on the top of the plot to see the nonlinearities.

## Input Arguments

## [a,b] - Linear interval

[NaN NaN] (default)| 2 -element row vector
Linear interval of the saturation, specified as a 2 -element row vector of doubles.
The saturation nonlinearity is initialized at the interval [a,b]. The interval values are adjusted to the estimation data by nlhw. To remove the lower limit, set a to - Inf. The lower limit is not adjusted
during estimation. To remove the upper limit, set b to Inf. The upper limit is not adjusted during estimation.

When the interval is [ NaN NaN ], the initial value of the linear interval is determined from the estimation data range during estimation using nlhw.
Example: [-2 1]

## Properties

LinearInterval
Linear interval of the saturation, specified as a 2 -element row vector of doubles.
Default: [NaN NaN]

## Output Arguments

NL - Saturation nonlinearity estimator object
saturation object
Saturation nonlinearity estimator object, returned as a saturation object.

## See Also

nlhw

Introduced in R2007a

## segment

Segment data and estimate models for each segment

## Syntax

segm $=\operatorname{segment}(z, n n)$
[segm, V,thm, R2e] $=\operatorname{segment(z,nn,R2,q,R1,M,th0,P0,ll,mu)~}$

## Description

segment builds models of AR, ARX, or ARMAX/ARMA type,

$$
A(q) y(t)=B(q) u(t-n k)+C(q) e(t)
$$

assuming that the model parameters are piecewise constant over time. It results in a model that has split the data record into segments over which the model remains constant. The function models signals and systems that might undergo abrupt changes.

The input-output data is contained in $z$, which is either an iddata object or a matrix $z=[y \quad u]$ where $y$ and $u$ are column vectors. If the system has several inputs, $u$ has the corresponding number of columns.

The argument $n n$ defines the model order. For the ARMAX model
nn = [na nb nc nk];
where na, nb, and nc are the orders of the corresponding polynomials. See "What Are Polynomial Models?". Moreover, nk is the delay. If the model has several inputs, nb and nk are row vectors, giving the orders and delays for each input.

For an ARX model $(\mathrm{nc}=0)$ enter
nn = [na nb nk];
For an ARMA model of a time series

```
z = y;
```

nn = [na nc];
and for an AR model
nn = na;
The output argument segm is a matrix, where the kth row contains the parameters corresponding to time $k$. This is analogous to output estimates returned by the recursiveARX and recursiveARMAX estimators. The output argument thm of segment contains the corresponding model parameters that have not yet been segmented. Each row of thm contains the parameter estimates at the corresponding time instant. These estimates are formed by weighting together the parameters of M (default: 5) different time-varying models, with the participating models changing at every time step. Consider segment as an alternative to the online estimation commands when you are not interested in continuously tracking the changes in parameters of a single model, but need to detect abrupt changes in the system dynamics.

The output argument V contains the sum of the squared prediction errors of the segmented model. It is a measure of how successful the segmentation has been.

The input argument R2 is the assumed variance of the innovations $e(t)$ in the model. The default value of R2, R2 = [ ], is that it is estimated. Then the output argument R2e is a vector whose kth element contains the estimate of R 2 at time k .

The argument $q$ is the probability that the model exhibits an abrupt change at any given time. The default value is 0.01 .

R1 is the assumed covariance matrix of the parameter jumps when they occur. The default value is the identity matrix with dimension equal to the number of estimated parameters.
$M$ is the number of parallel models used in the algorithm (see below). Its default value is 5 .
th0 is the initial value of the parameters. Its default is zero. P0 is the initial covariance matrix of the parameters. The default is 10 times the identity matrix.
$l l$ is the guaranteed life of each of the models. That is, any created candidate model is not abolished until after at least $l l$ time steps. The default is $l l=1$. Mu is a forgetting parameter that is used in the scheme that estimates R2. The default is 0.97 .

The most critical parameter for you to choose is R2. It is usually more robust to have a reasonable guess of R2 than to estimate it. Typically, you need to try different values of R2 and evaluate the results. (See the example below.) sqrt (R2) corresponds to a change in the value $y(t)$ that is normal, giving no indication that the system or the input might have changed.

## Examples

## Divide Sinusoid into Segments with Constant Levels

Create a sinusoid for the simulated model output.
$y=\sin ([1: 50] / 3) ' ;$
Specify the input signal to be constant at 1.
u = ones(size(y));
Specify the estimated noise variance for the model.
R2 = 0.1;
Segment the signal and estimate an ARX model for each segment. Use the simple model $y(t)=b_{1} u(t-1)$, where $b 1$ is the model parameter describing the piecewise constant level of the estimated output, $y(t)$.
segm = segment([y,u],[011],R2);
Examine the result.
plot([segm,y])


Vary the value of R2 to change the estimated noise variance. Decreasing R2 increases the number of segments produced for this model.

## Model Abrupt Change In Time Delay Using Segmentation

Load and plot the estimation data.
load iddemo6m.mat z
z = iddata(z(:,1),z(:,2));
plot(z)

## Input-Output Data



This data contains a change in time delay from 2 to 1 , which is difficult to detect by examining the data.

Specify the model orders to estimate an ARX model of the form:

$$
\begin{aligned}
& \quad y(t)+a y(t-1)=b_{1} u(t-1)+b_{2} u(t-2) \\
& \mathrm{nn}=
\end{aligned}
$$

Segment the data and estimate ARX models for each segment. Specify an estimated noise variance of 0.1.
seg = segment(z,nn,0.1);
Examine the parameters of the segmented model.
plot(seg)
legend('a','b1','b2');


The data has been divided into two segments, as indicated by the change in model parameters around sample number 19. The increase in b1, along with a corresponding decrease in b2, shows the change in model delay.

## Limitations

segment is not compatible with MATLAB Coder or MATLAB Compiler.

## Algorithms

The algorithm is based on $M$ parallel models, each recursively estimated by an algorithm of Kalman filter type. Each model is updated independently, and its posterior probability is computed. The timevarying estimate thm is formed by weighting together the $M$ different models with weights equal to their posterior probability. At each time step the model (among those that have lived at least ll samples) that has the lowest posterior probability is abolished. A new model is started, assuming that the system parameters have changed, with probability q , a random jump from the most likely among the models. The covariance matrix of the parameter change is set to R1.

After all the data are examined, the surviving model with the highest posterior probability is tracked back and the time instances where it jumped are marked. This defines the different segments of the data. (If no models had been abolished in the algorithm, this would have been the maximum likelihood estimates of the jump instances.) The segmented model segm is then formed by smoothing the parameter estimate, assuming that the jump instances are correct. In other words, the last estimate over a segment is chosen to represent the whole segment.

## See Also

Topics
"Data Segmentation"
Introduced before R2006a

## selstruc

Select model order for single-output ARX models

## Syntax

nn = selstruc(v)
[nn,vmod] = selstruc(v,c)

## Description

Note Use selstruc for single-output systems only. selstruc supports both single-input and multiple-input systems.
selstruc is a function to help choose a model structure (order) from the information contained in the matrix $v$ obtained as the output from arxstruc or ivstruc.

The default value of c is 'plot'. The plot shows the percentage of the output variance that is not explained by the model as a function of the number of parameters used. Each value shows the best fit for that number of parameters. By clicking in the plot you can examine which orders are of interest. When you click Select, the variable nn is exported to the workspace as the optimal model structure for your choice of number of parameters. Several choices can be made.
$c=$ 'aic' gives no plots, but returns in nn the structure that minimizes

$$
\begin{aligned}
& V_{\mathrm{mod}}=\log \left(V\left(1+\frac{2 d}{N}\right)\right) \\
& =\log (V)+\frac{2 d}{N}, N \gg d
\end{aligned}
$$

where $V$ is the loss function, $d$ is the total number of parameters in the structure in question, and $N$ is the number of data points used for the estimation. $\log (V)+\frac{2 d}{N}$ is the Akaike's Information Criterion (AIC). See aic for more details.
$\mathrm{c}=$ ' mdl ' returns in nn the structure that minimizes Rissanen's Minimum Description Length (MDL) criterion.

$$
V_{\mathrm{mod}}=V\left(1+\frac{d \log (N)}{N}\right)
$$

When cequals a numerical value, the structure that minimizes $V_{\bmod }=V\left(1+\frac{c d}{N}\right)$
is selected.
The output argument vmod has the same format as v , but it contains the logarithms of the accordingly modified criteria.

## Examples

## Generate Model-Order Combinations and Estimate ARX Model Using IV Method

Create estimation and validation data sets

```
load iddata1;
```

ze = z1(1:150);
$z v=z 1(151: 300) ;$

Generate model-order combinations for estimation, specifying ranges for model orders and delays.
NN $=$ struc(1:3,1:2,2:4);
Estimate ARX models using the instrumental variable method, and compute the loss function for each model order combination.

V = ivstruc(ze,zv,NN);
Select the model order with the best fit to the validation data.
order $=\operatorname{selstruc}(\mathrm{V}, 0)$;
Estimate an ARX model of selected order.
M = iv4(ze,order);

## Generate Model-Order Combinations and Estimate Multi-Input ARX Model

Create estimation and validation data sets.
load co2data;
Ts $=0.5$; Sample time is 0.5 min
ze = iddata(Output_exp1,Input_exp1,Ts);
zv = iddata(Output_exp2,Input_exp2,Ts);
Generate model-order combinations for:

- $\mathrm{na}=2: 4$
- $n b=2: 5$ for the first input, and 1 or 4 for the second input.
- $n k=1: 4$ for the first input, and 0 for the second input.
$N N=\operatorname{struc}(2: 4,2: 5,[14], 1: 4,0)$;
Estimate an ARX model for each model order combination.
V = arxstruc(ze,zv,NN);
Select the model order with the best fit to the validation data.
order $=$ selstruc(V,0);
Estimate an ARX model of selected order.

```
M = arx(ze,order);
```

Introduced before R2006a

## set

Set or modify model properties

## Syntax

```
set(sys,'Property',Value)
set(sys,'Property1',Value1,'Property2',Value2,...)
sysnew = set(___)
set(sys,'Property')
```


## Description

set is used to set or modify the properties of a dynamic system model using property name/property value pairs.
set (sys,'Property', Value) assigns the value Value to the property of the model sys.
'Property' can be the full property name (for example, 'UserData') or any unambiguous caseinsensitive abbreviation (for example, 'user'). The specified property must be compatible with the model type. For example, if sys is a transfer function, Variable is a valid property but StateName is not. For a complete list of available system properties for any linear model type, see the reference page for that model type. This syntax is equivalent to sys.Property = Value.
set(sys,'Property1',Value1,'Property2', Value2,...) sets multiple property values with a single statement. Each property name/property value pair updates one particular property.
sysnew $=\operatorname{set}(\ldots \ldots)$ returns the modified dynamic system model, and can be used with any of the previous syntaxes.
set (sys,'Property') displays help for the property specified by 'Property'.

## Examples

## Specify Model Properties

Create a SISO state-space model with matrices $A, B, C$, and $D$ equal to $1,2,3$, and 4 , respectively.

```
sys = ss(1,2,3,4);
```

Modify the properties of the model. Add an input delay of 0.1 second, label the input as torque, and set the $D$ matrix to 0 .

```
set(sys,'InputDelay',0.1,'InputName','torque','D',0);
```

View the model properties, and verify the changes.

```
get(sys)
```

$$
\begin{array}{ll}
\text { A: } & 1 \\
\text { B: } & 2 \\
\text { C: } & 3
\end{array}
$$

```
                    D: 0
                    E: []
            Scaled: 0
        StateName: {''}
        StatePath: {''}
        StateUnit: {''}
InternalDelay: [0x1 double]
    InputDelay: 0.1000
    OutputDelay: 0
                    Ts: 0
            TimeUnit: 'seconds'
        InputName: {'torque'}
        InputUnit: {''}
    InputGroup: [1x1 struct]
    OutputName: {''}
    OutputUnit: {''}
    OutputGroup: [1x1 struct]
            Notes: [0x1 string]
        UserData: []
            Name: ''
SamplingGrid: [1x1 struct]
```


## Tips

For discrete-time transfer functions, the convention used to represent the numerator and denominator depends on the choice of variable (see $t f$ for details). Like $t f$, the syntax for set changes to remain consistent with the choice of variable. For example, if the Variable property is set to ' $z$ ' (the default),
set(h,'num',[1 2],'den',[lllll
produces the transfer function

$$
h(z)=\frac{z+2}{z^{2}+3 z+4}
$$

However, if you change the Variable to 'z^-1' by
set(h,'Variable','z^-1'),
the same command
set(h,'num',[1 2],'den',[1 3 4])
now interprets the row vectors [lll 12 2 and $\left[\begin{array}{lll}1 & 3 & 4\end{array}\right]$ as the polynomials $1+2 z^{-1}$ and $1+3 z^{-1}+4 z^{-2}$ and produces:

$$
\bar{h}\left(z^{-1}\right)=\frac{1+2 z^{-1}}{1+3 z^{-1}+4 z^{-2}}=z h(z)
$$

Note Because the resulting transfer functions are different, make sure to use the convention consistent with your choice of variable.

## See Also

frd|get|idfrd|idgrey|idnlarx|idnlgrey|idnlhw|idpoly|idproc|idss|idtf|ss | tf|zpk

Introduced before R2006a

## setcov

Set parameter covariance data in identified model

## Syntax

sys $=$ setcov(sys0,cov)

## Description

sys $=$ setcov(sys0,cov) sets the parameter covariance of identified model sys0 as cov.
The model parameter covariance is calculated and stored automatically when a model is estimated. Therefore, you do not need to set the parameter covariance explicitly for estimated models. Use this function for analysis, such as to study how the parameter covariance affects the response of a model obtained by explicit construction.

## Input Arguments

sys0
Identified model.
Identified model, specified as an idtf, idss, idgrey, idpoly, idproc, or idnlgrey model. You cannot set the covariance for nonlinear black-box models (idnlarx and idnlhw).
cov
Parameter covariance matrix.
cov is one of the following:

- an $n p$-by- $n p$ semi-positive definite symmetric matrix, where $n p$ is equal to the number of parameters of sys0.
- a structure with the following fields that describe the parameter covariance in a factored form:
- R - usually the Cholesky factor of inverse of covariance.
- T - transformation matrix.
- Free - logical vector of length $n p$ indicating if a parameter is free. Here $n p$ is equal to the number of parameters of sys0.

```
cov(Free,Free) = T*inv(R'*R)*T'.
```


## Default:

## Output Arguments

## sys

Identified model.

The values of all the properties of sys are the same as those in sys 0 , except for the parameter covariance values which are modified as specified by cov.

## Examples

## Set Raw Covariance Data for Identified Model

Create a transfer function model for the following system:

$$
\begin{aligned}
& \text { sys } 0=\frac{4}{s^{2}+2 s+1} \\
& \text { sys0 }=\operatorname{idtf}(4,[12 \quad 1]) ; \\
& \mathrm{np}=\text { nparams(sys0); } \\
& \text { sys0 contains np model parameters. }
\end{aligned}
$$

Specify the covariance values for the denominator parameters only.
cov $=$ zeros(np);
den_index = 2:3;
$\operatorname{cov}($ den_index, den_index) $=\operatorname{diag}([0.04$ 0.001]);
cov is a covariance matrix with nonzero entries for the denominator parameters.
Set the covariance for sys0.
sys = setcov(sys0,cov);

## See Also

getcov|rsample|setpvec|sim
Introduced in R2012a

## setinit

Set initial states of idnlgrey model object

## Syntax

model = setinit(model, Property, Values)

## Description

model = setinit(model,Property,Values) sets the values of the Property field of the InitialStates model property. Property can be 'Name', 'Unit', 'Value', 'Minimum', 'Maximum', and 'Fixed'.

## Input Arguments

model
Name of the idnlgrey model object.
Property
Name of the InitialStates model property field, such as 'Name', 'Unit', 'Value', 'Minimum', 'Maximum', and 'Fixed'.
Values
Values of the specified property Property. Values are an Nx -by-1 cell array of values, where Nx is the number of states.

See Also<br>getinit|getpar|idnlgrey|setpar<br>Introduced in R2007a

## setoptions

Set plot options for response plot

## Syntax

```
setoptions(h, PlotOpts)
setoptions(h, 'Property1', 'valuel', ...)
setoptions(h, PlotOpts, 'Property1', 'value1', ...)
```


## Description

setoptions( $\mathrm{h}, \mathrm{PlotOpts)} \mathrm{sets} \mathrm{preferences} \mathrm{for} \mathrm{response} \mathrm{plot} \mathrm{using} \mathrm{the} \mathrm{plot} \mathrm{handle}$.h is the plot handle, PlotOpts is a plot options handle containing information about plot options.

There are two ways to create a plot options handle:

- Use getoptions, which accepts a plot handle and returns a plot options handle.

```
p = getoptions(h)
```

- Create a default plot options handle using one of the following commands:
- bodeoptions - Bode plots
- hsvoptions - Hankel singular values plots
- nicholsoptions - Nichols plots
- nyquistoptions - Nyquist plots
- pzoptions - Pole/zero plots
- sigmaoptions - Sigma plots
- timeoptions - Time plots (step, initial, impulse, etc.)

For example,
$\mathrm{p}=$ bodeoptions
returns a plot options handle for Bode plots.
setoptions(h, 'Property1', 'value1', ...) assigns values to property pairs instead of using PlotOpts. To find out what properties and values are available for a particular plot, type help <function>options. For example, for Bode plots type
help bodeoptions
For a list of the properties and values available for each plot type, see "Properties and Values Reference" (Control System Toolbox).
setoptions(h, PlotOpts, 'Property1', 'value1', ...) first assigns plot properties as defined in @PlotOptions, and then overrides any properties governed by the specified property/ value pairs.

## Examples

## Change Frequency Units in Response Plot

Create the following continuous-time transfer function:

$$
H(s)=\frac{1}{s+1}
$$

sys $=t f\left(1,\left[\begin{array}{ll}1 & 1\end{array}\right)\right.$;
Create a Bode plot with plot handle h .
h = bodeplot(sys);


Create a plot options handle p.
p = getoptions(h);
Change frequency units of the plot to Hz .
p.FreqUnits = 'Hz';

Apply the plot options to the Bode plot.
setoptions(h, p);


Alternatively, use setoptions(h,'FrequencyUnits','Hz').

## See Also

getoptions
Introduced in R2012a

## setpar

Set attributes such as values and bounds of linear model parameters

## Syntax

```
sys1 = setpar(sys,'value',value)
sys1 = setpar(sys,'free',free)
sys1 = setpar(sys,'bounds',bounds)
sys1 = setpar(sys,'label',label)
```


## Description

sys1 = setpar(sys,'value', value) sets the parameter values of the model sys. For model arrays, use setpar separately on each model in the array.
sys1 = setpar(sys,'free',free) sets the free or fixed status of the parameters.
sys1 = setpar(sys,'bounds', bounds) sets the minimum and maximum bounds on the parameters.
sys1 = setpar(sys,'label',label) sets the labels for the parameters.

## Examples

## Assign Model Parameter Values

Estimate an ARMAX model.

```
load iddata8;
init_data = z8(1:100);
na = 1;
nb = [1 1 1];
nc = 1;
nk = [0 0 0];
sys = armax(init_data,[na nb nc nk]);
```

Set the parameter values.
sys = setpar(sys,'value',[0.5 0.1 0.3 0.02 0.5]');
To view the values, type val = getpar(sys, 'value').

## Fix or Free Model Parameters

Construct a process model.

```
m = idproc('P2DUZI');
m.Kp = 1;
```

```
m.Tw = 100;
m.Zeta = .3;
m.Tz = 10;
m.Td = 0.4;
```

Set the free status of the parameters.
m = setpar(m,'free',[ $\left.\begin{array}{lllll}1 & 1 & 1 & 1 & 0\end{array}\right]$ );
Here, you set Tz to be a fixed parameter.
To check the free status of Tz, type m. Structure.Tz.

## Set Minimum and Maximum Bounds on Parameters

Estimate an ARMAX model.

```
load iddata8;
init_data = z8(1:100);
na = 1;
nb = [1 1 1];
nc = 1;
nk = [0 0 0 0];
sys = armax(init_data,[na nb nc nk]);
```

Set the minimum and maximum bounds for the parameters. Each row represents the bounds for a single parameter. The first value in each row specifies the minimum bound and the second value specifies the maximum bound.
sys $=$ setpar(sys,'bounds',[0 1; 1 1.5; 0 2; 0.5 1; 0 1]);

## Assign Default Labels to Parameters

Estimate an ARMAX model.

```
load iddata8;
init_data = z8(1:100);
na = 1;
nb = [1 1 1];
nc = 1;
nk = [0 0 0];
sys = armax(init_data,[na nb nc nk]);
```

Assign default labels to model parameters.
sys = setpar(sys,'label','default');
View the default labels.

```
getpar(sys,'label')
ans = 5xl cell
    {'A1(1)'}
```

```
{'B0(1)'}
{'B0(2)'}
{'B0(3)'}
{'C1' }
```


## Input Arguments

## sys - Identified linear model

idss |idproc|idgrey|idtf|idpoly
Identified linear model, specified as an idss, idproc, idgrey, idtf, or idpoly model object.

## value - Parameter values

vector of doubles
Parameter values, specified as a double vector of length nparams(sys).

## free - Free or fixed status of parameters

vector of logical values
Free or fixed status of parameters, specified as a logical vector of length nparams(sys).
bounds - Minimum and maximum bounds on parameters
matrix of doubles
Minimum and maximum bounds on parameters, specified as a double matrix of size nparams(sys)-by-2. The first column specifies the minimum bound and the second column the maximum bound.

## label - Parameter labels

cell array of character vectors | 'default'
Parameter labels, specified as a cell array of character vectors. The cell array is of length nparams(sys). For example, \{'a1', 'a3'\}, if nparams(sys) is two.

Use 'default' to assign default labels, $\mathrm{A} 1, \mathrm{~A} 2 \ldots, \mathrm{~B} 1, \mathrm{~B} 2, \ldots$, to the parameters.

## Output Arguments

## sys1 - Model with specified values of parameter attributes <br> idss |idproc|idgrey|idtf|idpoly

Model with specified values of parameter attributes. The model sys you specify as the input to setpar gets updated with the specified parameter attribute values.

## See Also

getpar|setcov|setpvec

## Introduced in R2013b

## setpar

Set initial parameter values of idnlgrey model object

## Syntax

setpar(model, property, values)

## Input Arguments

model
Name of the idnlgrey model object.
property
Name of the Parameters model property field, such as 'Name', 'Unit', 'Value', 'Minimum', or 'Maximum'.

Default: 'Value'.
values
Values of the specified property Property. values are an Np-by-1 cell array of values, where Np is the number of parameters.

## Description

setpar(model, property, values) sets the model parameter values in the property field of the Parameters model property. property can be 'Name', 'Unit', 'Value', 'Minimum', and 'Maximum'.

## See Also

getinit|getpar|idnlgrey|setinit

Introduced in R2007a

## setPolyFormat

Specify format for $B$ and $F$ polynomials of multi-input polynomial model

## Syntax

```
modelOut = setPolyFormat(modelIn,'double')
modelOut = setPolyFormat(modelIn,'cell')
```


## Description

modelOut = setPolyFormat(modelIn,'double') converts the B and F polynomials of a multiinput polynomial model, modelIn, to double matrices.

By default, the B and F polynomials of an idpoly model are cell arrays. For MATLAB scripts written before R2012a, convert the cell arrays to double matrices for backward compatibility using this syntax. For example:

```
model = arx(data,[3 2 2 1 1]);
model = setPolyFormat(model,'double');
modelOut = setPolyFormat(modelIn,'cell') converts the B and F polynomials of modelIn to
cell arrays.
```

MATLAB data files saved before R2012a store idpoly models with their B and F polynomials represented as double matrices. If these models were previously set to operate in backwardcompatibility mode, they are not converted to use cell arrays when loaded. Convert these models to use cell arrays using this syntax. For example:

```
load polyData.mat model;
model = setPolyFormat(model,'cell');
```


## Examples

## Convert B and F Polynomials of a Multi-Input ARX Model to Double Matrices

Load estimation data.
load iddata8;
Estimate the model.
$\mathrm{ml}=\operatorname{arx}\left(z 8,\left[\begin{array}{lll}3 & {[2} & 2\end{array}\right]\left[\begin{array}{lll}1 & 1 & 1\end{array}\right]\right)$;
Convert the $b$ and $f$ polynomials to use double matrices.
m2 = setPolyFormat(m1,'double');
Extract pole and zero information from the model using matrix syntax.

```
Poles1 = roots(m2.F(1,:));
Zeros1 = roots(m2.B(1,:));
```


## Input Arguments

## modelIn - Polynomial model

idpoly object
Polynomial model, specified as an idpoly object. The B and F polynomials of model In are either:

- Cell arrays with $N_{u}$ elements, where $N_{u}$ is the number of model inputs, with each element containing a double vector. This configuration is the default.
- Double matrices with $N_{u}$ rows. This configuration applies to backward-compatible idpoly models stored in MATLAB data files before R2012a.

Note setPolyFormat only supports multi-input, single-output models. Specifying modelIn as a:

- Multi-output model generates an error.
- Single-input, single-output model has no effect. The B and F polynomials remain as double vectors.


## Output Arguments

## modelOut - Polynomial model

idpoly object
Polynomial model, returned as an idpoly object.
To access the $b$ and $f$ polynomials of modelOut, use:

- Matrix syntax after using modelOut = setPolyFormat(modelIn, 'double'). For example: modelOut.B(1,:);
- Cell array syntax after using modelOut = setPolyFormat(modelIn, 'cell' ). For example: modelOut.B\{1\};

After using modelOut = setPolyFormat(modelIn, 'cell'), you can resave the converted model in cell array format. For example:
save polyNew.mat modelOut;

## Tips

- To verify the current format of the B and F polynomials for a given idpoly model, enter:
class(model.B)
If the model uses double matrices, the displayed result is:
ans =
double
Otherwise, for cell arrays, the result is:

```
ans =
cell
```


## See Also

get |idpoly|polydata|set|tfdata
Topics
"Extracting Numerical Model Data"

Introduced in R2010a

## setpvec

Modify values of model parameters

## Syntax

```
sys = setpvec(sys0,par)
sys = setpvec(sys0,par,'free')
```


## Description

sys $=$ setpvec(sys0, par) modifies the value of the parameters of the identified model sys0 to the value specified by par.
par must be of length nparams(sys0). nparams(sys0) returns a count of all the parameters of sys0.
sys = setpvec(sys0, par,'free') modifies the value of all the free estimation parameters of sys0 to the value specified by par.
par must be of length nparams(sys0,'free'). nparams(sys0,'free') returns a count of all the free parameters of sys0. For idnlarx and idnlhw models, all parameters are treated as free.

## Input Arguments

## sys0

Identified model, specified as an idtf, idss, idgrey, idpoly, idproc, idnlarx, idnlhw, or idnlgrey object.
par
Replacement value for the parameters of the identified model sys0.
For the syntax sys = setpvec(sys0, par), par must be of length nparams(sys0). nparams (sys0) returns a count of all the parameters of sys0.

For the syntax sys = setpvec(sys0, par,'free'), par must be of length nparams(sys0,'free'). nparams(sys0,'free') returns a count of all the free parameters of sys0.

Use NaN to denote unknown parameter values.
If sys0 is an array of models, then specify par as a cell array with an entry corresponding to each model in sys 0 .

## Output Arguments

## sys

Identified model obtained from sys0 by updating the values of the specified parameters.

## Examples

## Modify Parameter Values of Transfer Function Model

Construct a transfer function model.
sys0 = idtf(1,[1 2]);
Define a parameter vector and use it to set the model parameters. The second parameter is set to NaN , indicating that its value is unknown.

```
par = [1;NaN;0];
sys = setpvec(sys0,par);
```

The constructed model, sys, can be used to initialize a model estimation.

## Modify Free Parameter Values of Transfer Function Model

Construct a transfer function model.
sys0 = idtf([10],[120]);
Set the first three parameters of sys0 as free parameters.

```
sys0 = setpar(sys0,'free',[1 1 1 0 0]);
```

Define a parameter vector and use it to set the free model parameters.

```
par = [1;2;1];
sys = setpvec(sys0,par,'free');
```


## See Also

getpvec|nparams | setcov

## Introduced in R2012a

## sgrid

Generate s-plane grid of constant damping factors and natural frequencies

## Syntax

sgrid
sgrid(zeta,wn)
sgrid(__,'new')
sgrid( $\overline{A X}$, $\qquad$ )

## Description

sgrid generates a grid of constant damping factors from 0 to 1 in steps of 0.1 and natural frequencies from 0 to $10 \mathrm{rad} / \mathrm{sec}$ in steps of one rad/sec for pole-zero and root locus plots. sgrid then plots the grid over the current axis. sgrid creates the grid over the plot if the current axis contains a continuous s-plane root locus diagram or pole-zero map.
sgrid(zeta,wn) plots a grid of constant damping factor and natural frequency lines for the damping factors and natural frequencies in the vectors zeta and wn, respectively. sgrid(zeta, wn) creates the grid over the plot if the current axis contains a continuous s-plane root locus diagram or pole-zero map.

Alternatively, you can select Grid from the context menu to generate the same s-plane grid.
sgrid( $\qquad$ , 'new' ) clears the current axes first and sets hold on.
sgrid(AX, $\qquad$ ) plots the s-plane grid on the Axes or UIAxes object in the current figure with the handle AX. Use this syntax when creating apps with sgrid in the App Designer.

## Examples

## Generate S-Plane Grid on Root Locus Plot

Create the following continuous-time transfer function:

$$
H(s)=\frac{2 s^{2}+5 s+1}{s^{2}+2 s+3}
$$

H = tf([2 5 1], $\left.\left.\begin{array}{lll}1 & 2 & 3\end{array}\right]\right)$;
Plot the root locus of the transfer function.
rlocus(H)


Plot $s$-plane grid lines on the root locus.
sgrid


## Input Arguments

zeta - Damping ratio
vector
Damping ratio, specified as a vector in the same order as wn.

## wn - Normalized natural frequency

vector
Normalized natural frequency, specified as a vector.

## AX - Object handle

Axes object | UIAxes object
Object handle, specified as an Axes or UIAxes object. Use AX to create apps with sgrid in the App Designer.

## See Also

pzmap|rlocus|zgrid
Introduced before R2006a

## showConfidence

Display confidence regions on response plots for identified models

## Syntax

showConfidence(plot_handle) showConfidence(plot_handle,sd)

## Description

showConfidence(plot_handle) displays the confidence region on the response plot, with handle plot_handle, for an identified model.
showConfidence(plot_handle,sd) displays the confidence region for sd standard deviations.

## Input Arguments

plot_handle
Response plot handle.
plot_handle is the handle for the response plot of an identified model on which the confidence region is displayed. It is obtained as an output of one of the following plot commands: bodeplot, stepplot, impulseplot, nyquistplot, or iopzplot.
sd
Standard deviation of the confidence region. A common choice is 3 standard deviations, which gives 99.7\% significance.

Default: getoptions(plot_handle,'ConfidenceRegionNumberSD')

## Examples

## View Confidence Region for Identified Model

Show the confidence bounds on the bode plot of an identified ARX model.
Obtain identified model and plot its bode response.

```
load iddatal z1
sys = arx(z1, [2 2 1]);
h = bodeplot(sys);
```


$z 1$ is an iddata object that contains time domain system response data. sys is an idpoly model containing the identified polynomial model. h is the plot handle for the bode response plot of sys.

Show the confidence bounds for sys.
showConfidence(h);


This plot depicts the confidence region for 1 standard deviation.

## Specify the Standard Deviation of the Confidence Region

Show the confidence bounds on the bode plot of an identified ARX model.
Obtain identified model and plot its bode response.
load iddatal z1
sys = arx(z1, [2 2 1]);
h = bodeplot(sys);

$z 1$ is an iddata object that contains time domain system response data. sys is an idpoly model containing the identified polynomial model. h is the plot handle for the bode response plot of sys.

Show the confidence bounds for sys using 2 standard deviations.
sd = 2;
showConfidence(h,sd);

sd specifies the number of standard deviations for the confidence region displayed on the plot.

## Alternatives

You can interactively turn on the confidence region display on a response plot. Right-click the response plot, and select Characteristics > Confidence Region.

## See Also

bodeplot | impulseplot | iopzplot | nyquistplot | stepplot
Introduced in R2012a

## sigmoidnet

Sigmoid network function for nonlinear ARX and Hammerstein-Wiener models

## Description

A sigmoidnet object implements a sigmoid network function, and is a nonlinear mapping function for estimating nonlinear ARX and Nonlinear Hammerstein-Weiner models. The mapping function, which is also referred to as a nonlinearity, uses a combination of linear weights, an offset and a nonlinear function to compute its output. The nonlinear function contains sigmoid unit functions that operate on a ridge combination (weighted linear sum) of inputs.


Mathematically, a sigmoidnet network is a function that maps $m$ inputs $X(t)=\left[\chi\left(t_{1}\right), x_{2}(t), \ldots, \chi_{m}(t)\right]^{T}$ to a scalar output $y(t)$ using the following relationship:

$$
y(t)=y_{0}+(X(t)-\bar{X})^{T} P L+S(X(t))
$$

Here:

- $X(t)$ is an $m$-by- 1 vector of inputs, or regressors, with mean $\bar{X}$.
- $y_{0}$ is the output offset, a scalar.
- $\quad P$ is an $m$-by- $p$ projection matrix, where $m$ is the number of regressors and is $p$ is the number of linear weights. $m$ must be greater than or equal to $p$.
- $L$ is a $p$-by- 1 vector of weights.
- $S(X)$ is a sum of dilated and translated sigmoid functions. The total number of sigmoid functions is referred to as the number of units $n$ of the network.

For the definition of the sigmoid function term $S(X)$, see "More About" on page 1-1367.
Use sigmoidnet as the value of the OutputFcn property of an idnlarx model or the InputNonlinearity and OutputLinearity properties of an idnlhw object. For example, specify sigmoidnet when you estimate an idnlarx model with the following command.

```
sys = nlarx(data,regressors,sigmoidnet)
```

When nlarx estimates the model, it essentially estimates the parameters of the sigmoidnet function.

You can configure the sigmoidnet function to disable components and fix parameters. To omit the linear component, set LinearFcn. Use to false. To omit the offset, set Offset. Use to false. To specify known values for the linear function and the offset, set their Value attributes directly and set the corresponding Free attributes to False. Use evaluate to compute the output of the function for a given vector of inputs.

## Creation

## Syntax

```
S = sigmoidnet
S = sigmoidnet(numUnits)
S = sigmoidnet(numUnits,UseLinearFcn)
S = sigmoidnet(numUnits,UseLinearFcn,UseOffset)
```


## Description

$S=$ sigmoidnet creates a sigmoidnet object $S$ that uses 10 units. The number of inputs is determined during model estimation and the number of outputs is 1.
$S=$ sigmoidnet (numUnits) specifies the number of sigmoid functions numUnits.
$S=$ sigmoidnet(numUnits,UseLinearFcn) specifies whether the function uses a linear function as a subcomponent.
$S=$ sigmoidnet (numUnits, UseLinearFcn,UseOffset) specifies whether the function uses an offset term $y_{0}$ parameter.

## Input Arguments

numUnits - Number of units
10 (default) | positive integer

Number of units, specified as a positive integer. numUnits determines the number of sigmoid functions.

This argument sets the S.NonlinearFcn. NumberOfUnits property.

## UseLinearFcn - Option to use linear function <br> true (default) | false

Option to use the linear function subcomponent, specified as true or false. This argument sets the value of the S.LinearFcn.Use property.

## UseOffset - Option to use offset term

true (default) | false
Option to use an offset term, specified as true or false. This argument sets the value of the S.Offset.Use property.

## Properties

## Input - Input signal information

signal property values
Input signal information for signals used for estimation, specified as vectors of $m$ property-specific values, where $m$ is the number of input signals. The Input properties for each input signal are as follows:

- Name - Names of the input signals, specified as a 1-by-m string or character array, where $m$ is the number of inputs
- Mean - Mean of the input signals, specified as a numeric scalar
- Range - Ranges of the input signals, specified as a 2-by-m numeric array that contains the minimum and maximum values


## Output - Output signal information

signal property values
Output signal information, specified as property-specific values. The Output properties are as follows:

- Name - Name of the output signal, specified as a string or a character array
- Mean - Mean of the output signal, specified as a numeric scalar
- Range - Range of the output signal, specified as a 2-by-1 numeric array that contains the minimum and maximum values


## LinearFcn - Parameters of linear function

linear function property values (default)
Parameters of the linear function, specified as follows:

- Use - Option to use the linear function in the sigmoid network, specified as a scalar logical. The default value is true.
- Value - Linear weights that compose $L^{\prime}$, specified as a 1-by-p vector.
- InputProjection - Input projection matrix $P$, specified as an $m$-by- $p$ matrix, that transforms the detrended input vector of length $m$ into a vector of length $p$. For Hammerstein-Wiener models, InputProjection is equal to 1.
- Free - Option to update entries of Value during estimation, specified as a 1-by-p logical vector. The software honors the Free specification only if the starting value of Value is finite. The default value is true.
- Minimum - Minimum bound on Value, specified as a 1-by-p vector. If Minimum is specified with a finite value and the starting value of Value is finite, then the software enforces that minimum bound during model estimation.
- Maximum - Maximum bound on Value, specified as a 1-by-p vector. If Maximum is specified with a finite value and the starting value of Value is finite, then the software enforces that maximum bound during model estimation.
- SelectedInputIndex - Indices of sigmoidnet inputs (see Input. Name) that are used as inputs to the linear function, specified as an 1-by- $n_{r}$ integer vector, where $n_{r}$ is the number of inputs. For nonlinear ARX models, the RegressorUsage property determines these indices. For Hammerstein-Wiener models, SelectedInputIndex is always 1.


## Offset - Parameters of offset term

offset property values
Parameters of the offset term, specified as follows:

- Use - Option to use the offset in the sigmoid network, specified as a scalar logical. The default value is true.
- Value - Offset value, specified as a scalar.
- Free - Option to update Value during estimation, specified as a scalar logical. The software honors the Free specification of false only if the value of Value is finite. The default value is true.
- Minimum - Minimum bound on Value, specified as a numeric scalar or -Inf. If Minimum is specified with a finite value and the value of Value is finite, then the software enforces that minimum bound during model estimation. The default value is - Inf.
- Maximum - Maximum bound on Value, specified as a numeric scalar or Inf. If Maximum is specified with a finite value and the starting value of Value is finite, then the software enforces that maximum bound during model estimation. The default value is Inf.

NonlinearFcn - Parameters of nonlinear function
nonlinear function property values
Parameters of the nonlinear function, specified as follows:

- NumberOfUnits - Number of units, specified as a positive integer. NumberOfUnits determines the number of sigmoid functions.
- Parameters - Parameters of sigmoidnet, specified as in the following table:

| Field Name | Description | Default |
| :--- | :--- | :--- |
| InputProject <br> ion | Projection matrix $Q$, specified as an $m$-by- $q$ matrix. $Q$ <br> transforms the detrended input vector $(X-\bar{X})$ of length $m$ <br> into a vector of length $q$. Typically, $Q$ has the same <br> dimensions as the linear projection matrix $P$. In this case, $q$ is <br> equal to $p$, which is the number of linear weights. | [] |
|  | For Hammerstein-Wiener models, InputProjection is <br> equal to 1. |  |
| OutputCoeffi <br> cient | Sigmoid function output coefficients $s_{i}$, specified as an $n$-by-1 <br> vector. | [ ] |
| Translation | Translation matrix, specified as an $n$-by- $q$ matrix of <br> translation row vectors $c_{i}$. | [ ] |
| Dilation | Dilation coefficients $b_{i,}$ specified as an $n$-by-1 vector. | [] |

- Free - Option to estimate parameters, specified as a logical scalar. If all the parameters have finite values, such as when the sigmoidnet object corresponds to a previously estimated model, then setting Free to false causes the parameters of the nonlinear function $S(X)$ to remain unchanged during estimation. The default value is true.
- SelectedInputIndex - Indices of sigmoidnet inputs (see Input. Name) that are used as inputs to the nonlinear function, specified as an 1-by- $n_{r}$ integer vector, where $n_{r}$ is the number of inputs. For nonlinear ARX models, the RegressorUsage property determines these indices. For Hammerstein-Wiener models, SelectedInputIndex is always 1.


## Examples

## Estimate Nonlinear ARX Model with sigmoidnet as Output Function

Load the data $\mathrm{z7}$ and create a subset to use as estimation data.

```
load iddata7 z7
ze = z7(1:300);
```

Create and configure a sigmoidnet mapping object. Fix the offset to 0.2 and the number of units to 15.

```
S = sigmoidnet;
S.Offset.Value = 0.2;
S.Offset.Free = false;
S.NonlinearFcn.NumberOfUnits = 15;
```

Create linear and polynomial model regressors. Use the input and output variable names from $\mathrm{z7}$ as the variable names for the regressors.

```
var_names = [z7.OutputName;z7.InputName]
var_names = 3\times1 cell
    {'y1'}
    {'u1'}
    {'u2'}
```

```
Reg1 = linearRegressor(var_names,{1:4,0:4,1});
Reg2 = polynomialRegressor(var_names,{1:2,0:2,0},2);
```

Set the estimation options.

```
opt = nlarxOptions('SearchMethod','fmincon');
```

opt.Search0ptions.MaxIterations = 40;

Estimate the nonlinear ARX model.

```
sys = nlarx(ze,[Reg1;Reg2],S,opt)
sys =
Nonlinear ARX model with 1 output and 2 inputs
    Inputs: u1, u2
    Outputs: y1
Regressors:
    1. Linear regressors in variables y1, u1, u2
    2. Order 2 regressors in variables y1, u1, u2
    List of all regressors
Output function: Sigmoid Network with 15 units
Sample time: 1 seconds
Status:
Estimated using NLARX on time domain data "ze".
Fit to estimation data: 72.23% (prediction focus)
FPE: 7.115, MSE: 0.7625
```


## Estimate Hammerstein-Wiener Model that Uses sigmoidnet

Estimate a Hammerstein-Wiener model that uses sigmoidnet as the output nonlinearity.
Load the data
load throttledata
Create a sigmoidnet mapping object that has 15 units and that has no input nonlinearity or offset.

```
S = sigmoidnet(15,false,false)
S =
Sigmoid Network
Nonlinear Function: Sigmoid network with 15 units.
Linear Function: not in use
Output Offset: not in use
                            Input: [1×1 idpack.Channel]
            Output: [1×1 idpack.Channel]
            LinearFcn: [1\times1 nlident.internal.UseProjectedLinearFcn]
        NonlinearFcn: [1×1 nlident.internal.RidgenetFcn]
            Offset: [1×1 nlident.internal.ChooseableOffset]
```

```
Estimate a Hammerstein-Wiener model.
sys = nlhw(ThrottleData,[4 4 1],[],S)
sys =
Hammerstein-Wiener model with 1 output and 1 input
    Linear transfer function corresponding to the orders nb = 4, nf = 4, nk = 1
    Input nonlinearity: absent
    Output nonlinearity: Sigmoid Network with 15 units
Sample time: 0.01 seconds
Status:
Estimated using NLHW on time domain data "ThrottleData".
Fit to estimation data: 72.91%
FPE: 114.9, MSE: 80.96
```


## More About

## Sigmoid Nonlinear Function S(X)

The sigmoid nonlinear function is a sum of the dilated and translated sigmoid functions, and is described by the following equation:

$$
S(X)=\sum_{i=1}^{n} s_{i} f\left((X-\bar{X})^{T} Q b_{i}+c_{i}\right)
$$

Here:

- $\quad Q$ is an $m$-by- $q$ projection matrix, where $m \geq q$.
- $s_{1}, s_{2}, \ldots, s_{n}$ are scalar weights called output coefficients.
- $b_{1}, b_{2}, \ldots, b_{n}$ are $q$-by- 1 vectors called dilation coefficients .
- $c_{1}, c_{2}, \ldots, c_{n}$ are scalars called translations.
- $f(z)=\frac{1}{e^{-z}+1}$. is the sigmoid function, also called a unit function of the sigmoid network. Here, $z$ is a scalar of the form $b_{i}(X-\bar{X})^{\mathrm{T}} Q+c_{i}$.


## Algorithms

sigmoidnet uses an iterative search technique for estimating parameters.

## Compatibility Considerations

## Use of previous nonlinearity estimator properties is not recommended

Not recommended starting in R2021a
Starting in R2021a, the properties of the mapping objects, previously known as nonlinearity estimators, have been reorganized. These objects are wavenet (W), sigmoidnet (S), treepartition (T), customnet (C), and linear (L). The property changes do not apply to neuralnet. The use of the pre-R2021a properties in the following table is discouraged. However, the software still accepts commands that set these properties. There are no plans to exclude such commands at this time.

| Pre-R2021a Property | R2021a Property | Applicable Mapping Objects |
| :--- | :--- | :--- |
| NumberOfUnits | NonlinearFcn.NumberOfUni <br> ts | W,S,T,C |
| LinearTerm | LinearFcn.Use, Offset.Use | W,S,C |
| Parameters | Split into three pieces: <br> - LinearFcn.Value | W,S,T,C,L <br> - <br> Offset.Value |
|  | - Nonlinear (L) excludes <br> s | NonlinearFcn.Parameters. |
| Options | NonlinearFcn.Structure | W,T |

## See Also

customnet | deadzone |evaluate |idnlarx|idnlhw|linear|neuralnet|nlarx|nlhw| polyld|pwlinear|saturation| treepartition|unitgain| wavenet

Introduced in R2007a

## sim

Simulate response of identified model

## Syntax

```
y = sim(sys,udata)
y = sim(sys,udata,opt)
```

[y,y_sd] $=\operatorname{sim}($
$\qquad$ )
$\left[y, y \_s d, x\right]=\operatorname{sim}(\ldots)$
[y,y_sd,x,x_sd] = sim( $\qquad$ )
sim( $\qquad$ )

## Description

$y=\operatorname{sim}(s y s, u d a t a)$ returns the simulated response of an identified model using the input data, udata. By default, zero initial conditions are used for all model types except idnlgrey, in which case the initial conditions stored internally in the model are used.
$y=\operatorname{sim}($ sys, udata, opt) uses the option set, opt, to configure the simulation option, including the specification of initial conditions.
[y,y_sd] = sim( $\qquad$ ) returns the estimated standard deviation, y_sd, of the simulated response.
$\left[y, y \_s d, x\right]=\operatorname{sim}($ $\qquad$ ) returns the state trajectory, $x$, for state-space models.
[y,y_sd,x,x_sd] = sim( $\qquad$ ) returns the standard deviation of the state trajectory, $x$ _sd, for state-space models.
sim (__ ) plots the simulated response of the identified model.

## Examples

## Simulate State-Space Model Using Input Data

Load the estimation data.
load iddata2 z2
Estimate a third-order state-space model.
sys $=$ ssest(z2,3);
Simulate the identified model using the input channels from the estimation data.
$y=\operatorname{sim}(s y s, z 2) ;$

## Add Noise to Simulated Model Response

Load the data, and obtain the identified model.

```
load iddata2 z2
```

sys $=n 4 s i d(z 2,3)$;
sys is a third-order state-space model estimated using a subspace method.
Create a simulation option set to add noise to the simulated model response.

```
opt1 = simOptions('AddNoise',true);
```

Simulate the model.
y $=$ sim(sys,z2,opt1);
Default Gaussian white noise is filtered by the noise transfer function of the model and added to the simulated model response.

You can also add your own noise signal, e, using the NoiseData option.

```
e = randn(length(z2.u),1);
opt2 = simOptions('AddNoise',true,'NoiseData',e);
```

Simulate the model.

```
y = sim(sys,z2,opt2);
```


## Simulate Model Using Initial Conditions Obtained During Estimation

## Load data.

load iddata1 z1
Specify the estimation option to estimate the initial state.

```
estimOpt = ssestOptions('InitialState','estimate');
```

Estimate a state-space model, and return the value of the estimated initial state.

```
[sys,x0] = ssest(z1,2,estimOpt);
```

Specify initial conditions for simulation

```
simOpt = simOptions('InitialCondition',x0);
```

Simulate the model, and obtain the model response and standard deviation.

```
[y,y_sd] = sim(sys,z1,sim0pt);
```


## Estimate Standard Deviation and State Trajectory for State-Space Models

Load estimation data, and estimate a state-space model.

```
load iddata1 z1
sys = ssest(z1,2);
```

Return the standard deviation and state trajectory.
$\left[y, y \_s d, x\right]=\operatorname{sim}(s y s, z 1) ;$

## Estimate State Trajectory and Standard Deviations of Simulated Response

Load estimation data, and estimate a state-space model.
load iddatal z1
sys = ssest(z1,2);
Create a simulation option set, and specify the initial states.
opt = simOptions('InitialCondition',[1;2]);
Specify the covariance of the initial states.
opt.X0Covariance = [0.1 0; 0 0.1];
Calculate the standard deviations of simulated response, $y$ _sd, and state trajectory, $x \_s d$.
[y,y_sd,x,x_sd] = sim(sys,z1,opt);

## Plot Simulated Model Response

Obtain the identified model.

```
load iddata2 z2
sys = tfest(z2,3);
```

sys is an idtf model that encapsulates the third-order transfer function estimated for the measured data $z 2$.

Simulate the model.

```
sim(sys,z2)
```

Simulated output \#1: y1


## Simulate Nonlinear ARX Model

Simulate a single-input single-output nonlinear ARX model around a known equilibrium point, with an input level of 1 and output level of 10 .

Load the sample data.
load iddata2
Estimate a nonlinear ARX model from the data.
M = nlarx(z2,[2 2 1],'treepartition');
Estimate current states of model based on past data. Specify as many past samples as there are lags in the input and output variables ( 2 here).
x0 = data2state(M,struct('Input',ones(2,1),'Output',10*ones(2,1)));
Simulate the model using the initial states returned by data2state.
opt = simOptions('InitialCondition',x0); sim(M,z2,opt)

Simulated output \#1: y1


## Continue from End of Previous Simulation

Continue the simulation of a nonlinear ARX model from the end of a previous simulation run.
Estimate a nonlinear ARX model from data.
load iddata2
M = nlarx(z2,[2 2 1],'treepartition');
Simulate the model using the first half of the input data z2. Start the simulation from zero initial states.
ul = z2(1:200,[]);
opt1 = simOptions('InitialCondition','zero');
ys1 = sim(M,u1,opt1);
Start another simulation using the second half of the input data $z 2$. Use the same states of the model from the end of the first simulation.
u2 = z2(201:end,[]);
To set the initial states for the second simulation correctly, package input $u 1$ and output ys1 from the first simulation into one iddata object. Pass this data as initial conditions for the next simulation.

```
firstSimData = [ys1,u1];
opt2 = simOptions('InitialCondition',firstSimData);
ys2 = sim(M,u2,opt2);
```

Verify the two simulations by comparing to a complete simulation using all the input data $\mathrm{z2}$. First, extract the whole set of input data.

```
uTotal = z2(:,[]);
opt3 = simOptions('InitialCondition','zero');
ysTotal = sim(M,uTotal,opt3);
```

Plot the three responses ys1, ys2 and ysTotal. ys1 should be equal to first half of ysTotal. ys2 should be equal to the second half of ysTotal.
plot(ys1,'b',ys2,'g',ysTotal,'k*')


The plot shows that the three responses ys1, ys2, and ysTotal overlap as expected.

## Match Model Response to Output Data

Estimate initial states of model M such that, the response best matches the output in data set z2.
Load the sample data.
load iddata2;

Estimate a nonlinear ARX model from the data.
$M=n l a r x\left(z 2,\left[\begin{array}{lll}4 & 3 & 2\end{array}\right]\right.$, wavenet('NumberOfUnits',20));
Estimate the initial states of $M$ to best fit $z 2 . y$ in the simulated response.
x0 = findstates(M,z2,Inf);
Simulate the model.

```
opt = simOptions('InitialCondition',x0);
```

ysim = sim(M,z2.u,opt);

Compare the simulated model output ysim with the output signal in z2.
time = z2.SamplingInstants;
plot(time,ysim,time,z2.y,'.')


## Simulate Model Near Steady State with Known Input and Unknown Output

Start simulation of a model near steady state, where the input is known to be 1 , but the output is unknown.

Load the sample data.
load iddata2

Estimate a nonlinear ARX model from the data.
M = nlarx(z2,[4 3 2],'wavenet');
Determine equilibrium state values for input 1 and unknown target output.
$x 0=$ findop(M,'steady',1, NaN);
Simulate the model using initial states $\times 0$.

```
opt = simOptions('InitialCondition',x0);
sim(M,z2.u,opt)
```



## Simulate Hammerstein-Wiener Model at Steady-State Operating Point

Load the sample data.
load iddata2
Create a Hammerstein-Wiener model.
M = nlhw(z2,[4 3 2],[],'pwlinear');
Compute steady-state operating point values corresponding to an input level of 1 and an unknown output level.
x0 $=$ findop (M,'steady', $1, N a N)$;
Simulate the model using the estimated initial states.

```
opt = simOptions('InitialCondition',x0);
```

sim(M,z2.u)

Simulated output \#1: y1


## Simulate Time Series Model

Load time series data, and estimate an AR model using the least-squares approach.
load iddata9 z9
sys = ar(z9,6,'ls');
For time series data, specify the desired simulation length, $N=200$ using an $N$-by- 0 input data set.
data $=$ iddata([],zeros(200, 0),z9.Ts);
Set the initial conditions to use the initial samples of the time series as historical output samples.
IC = struct('Input',[],'Output',z9.y(1:6));
opt = simOptions('InitialCondition',IC);
Simulate the model.
sim(sys,data,opt)

Simulated output \#1: y1


## Understand Use of Historical Data for Model Simulation

Use historical input-output data as a proxy for initial conditions when simulating your model. You first simulate using the sim command and specify the historical data using the simOptions option set. You then reproduce the simulated output by manually mapping the historical data to initial states.

Load a two-input, one-output data set.
load iddata7 z7
Identify a fifth-order state-space model using the data.
sys $=n 4 \operatorname{sid}(z 7,5)$;
Split the data set into two parts.
$z A=z 7(1: 15) ;$
zB = z7(16:end);
Simulate the model using the input signal in zB.
uSim = zB;

Simulation requires initial conditions. The signal values in zA are the historical data, that is, they are the input and output values for the time immediately preceding data in $z B$. Use $z A$ as a proxy for the required initial conditions.

```
IO = struct('Input',zA.InputData,'Output',zA.OutputData);
opt = simOptions('InitialCondition',IO);
```

Simulate the model.
ysim = sim(sys,uSim,opt);
Now reproduce the output by manually mapping the historical data to initial states of sys. To do so, use the data2state command.
xf = data2state(sys,zA);
xf contains the state values of sys at the time instant immediately after the most recent data sample in $z A$.

Simulate the system using $x f$ as the initial states.

```
opt2 = simOptions('InitialCondition',xf);
ysim2 = sim(sys,uSim,opt2);
```

Plot the output of the sim command ysim and the manually computed results ysim2.
plot(ysim,'b',ysim2,'--r')
Input-Output Data

ysim2 is the same as ysim.

## Input Arguments

sys - Identified model
identified linear model | identified nonlinear model
Identified model, specified as one of the following model objects:

|  | Model Type | Model Object |
| :--- | :--- | :--- |
| Identified <br> Linear <br> Model | Polynomial <br> model | idpoly |
|  | Process <br> model | idproc |
|  | State-space <br> model | idss |
|  | Transfer <br> function <br> model | idtf |
|  | Linear grey- <br> box model | idgrey |
| Identified <br> Nonlinear <br> Model | Nonlinear <br> ARX model | idnlarx |
|  | Nonlinear <br> Hammerstein- <br> Wiener model | idnlhw |
|  | Nonlinear <br> grey-box <br> model | idnlgrey |

## udata - Simulation input data

iddata object | matrix
Simulation input data, specified as an iddata object or a matrix. sim uses the input channels from this object as the simulation inputs. For time-domain simulation of discrete-time systems, you can also specify udata as a matrix with columns that correspond to each input channel.

If sys is a linear model, you can use either time-domain or frequency-domain data. If sys is a nonlinear model, you can only use time-domain data.

If sys is a time-series model, that is a model with no inputs, specify udata as an Ns-by-0 signal, where $N s$ is the wanted number of simulation output samples. For example, to simulate 100 output samples, specify udata as follows.
udata $=$ iddata([],zeros(100,0),Ts);
If you do not have data from an experiment, use idinput to generate signals with various characteristics.

## opt - Simulation options

simOptions option set
Simulation options, specified as a simOptions option set for setting the following options:

- Initial conditions
- Input/output offsets
- Additive noise


## Output Arguments

## y - Simulated response

iddata object | matrix
Simulated response for sys, returned as an iddata object or matrix, depending on how you specify udata. For example, if udata is an iddata object, then so is $y$.

If udata represents time-domain data, then y is the simulated response for the time vector corresponding to udata.

If udata represents frequency-domain data, $U(\omega)$, then $y$ contains the Fourier transform of the corresponding sampled time-domain output signal. This signal is the product of the frequency response of sys, $G(\omega)$, and $U(\omega)$.

For multi-experiment data, y is a corresponding multi-experiment iddata object.

## y_sd - Estimated standard deviation

double matrix
Estimated standard deviation of the simulated response for linear models or nonlinear grey-box models, returned as an Ns-by-Ny matrix, where $N s$ is the number of samples and $N y$ is the number of outputs. The software computes the standard deviation by taking into account the model parameter covariance, initial state covariance, and additive noise covariance. The additive noise covariance is stored in the NoiseVariance property of the model.
y_sd is derived using first order sensitivity considerations (Gauss approximation formula).
For nonlinear models, y _sd is [].
x - Estimated state trajectory for state-space models
matrix | []
Estimated state trajectory for state-space models, returned as an $N s$-by- $N x$ matrix, where $N s$ is the number of samples and $N x$ is the number of states.
$x$ is only relevant if sys is an idss, idgrey, or idnlgrey model. If sys is not a state-space model, $x$ is returned as [].

## x_sd - Estimated standard deviation of state trajectory <br> matrix | [ ]

Estimated standard deviation of state trajectory for state-space models, returned as an Ns -by- Nx matrix, where $N s$ is the number of samples and $N x$ is the number of states. The software computes the standard deviation by taking into account the model parameter covariance, initial state
covariance, and additive noise covariance. The additive noise covariance is stored in the NoiseVariance property of the model.
$x \_s d$ is only relevant if sys is an idss, idgrey, or idnlgrey model. If sys is not a state-space model, $x$ _sd is returned as [].

## Tips

- When the initial conditions of the estimated model and the system that measured the validation data set are different, the simulated and measured responses may also differ, especially at the beginning of the response. To minimize this difference, estimate the initial state values using findstates and use the estimated values to set the InitialCondition option using sim0ptions. For an example, see "Match Model Response to Output Data" on page 1-1374.


## Algorithms

Simulation means computing the model response using input data and initial conditions. sim simulates the following system:


Here,

- $u(t)$ is the simulation input data, udata.
- $y(t)$ is the simulated output response.
- $G$ is the transfer function from the input to the output and is defined in sys. The simulation initial conditions, as specified using sim0ptions, set the initial state of $G$.
- $\quad e(t)$ is an optional noise signal. Add noise to your simulation by creating a simOptions option set, and setting the AddNoise option to true. Additionally, you can change the default noise signal by specifying the NoiseData option.
- $H$ is the noise transfer function and is defined in sys.
- $\delta u$ is an optional input offset subtracted from the input signal, $u(t)$, before the input is used to simulate the model. Specify an input offset by setting the InputOffset option using sim0ptions.
- $\delta y$ is an optional output offset added to the output response, $y(t)$, after simulation. Specify an output offset by setting the Output0ffset option using simOptions.

For more information on specifying simulation initial conditions, input and output offsets, and noise signal data, see sim0ptions. For multiexperiment data, you can specify these options separately for each experiment.

## Alternatives

- Use simsd for a Monte-Carlo method of computing the standard deviation of the response.
- sim extends lsim to facilitate additional features relevant to identified models:
- Simulation of nonlinear models
- Simulation with additive noise
- Incorporation of signal offsets
- Computation of response standard deviation (linear models only)
- Frequency-domain simulation (linear models only)
- Simulations using different intersample behavior for different inputs

To obtain the simulated response without any of the preceding operations, use lsim.

## See Also

compare|findstates|forecast|idinput|lsim|predict|sim0ptions|simsd|step

## Topics

"Simulate and Predict Identified Model Output"
"Simulation and Prediction at the Command Line"

Introduced before R2006a

## simOptions

Option set for sim

## Syntax

```
opt = simOptions
opt = simOptions(Name,Value)
```


## Description

opt $=$ simOptions creates the default option set for sim.
opt $=$ simOptions(Name, Value) creates an option set with the options specified by one or more Name, Value pair arguments.

## Examples

## Create Default Option Set for Model Simulation

```
opt = simOptions;
```


## Specify Options for Model Simulation

Create an option set for sim specifying the following options.

- Zero initial conditions
- Input offset of 5 for the second input of a two-input model

```
opt = simOptions('InitialCondition','z','InputOffset',[0; 5]);
```


## Add Noise to Simulation Output

Create noise data for a simulation with 500 input data samples and two outputs.
noiseData $=$ randn $(500,2)$;
Create a default option set.
opt = simOptions;
Modify the option set to add the noise data.
opt.AddNoise = true;
opt.NoiseData = noiseData;

## Use Historical Data to Specify Initial Conditions for Model Simulation

Use historical input-output data as a proxy for initial conditions when simulating your model.
Load a two-input, one-output data set.
load iddata7 z7
Identify a fifth-order state-space model using the data.

```
sys = n4sid(z7, 5);
```

Split the data set into two parts.

```
zA = z7(1:15);
zB = z7(16:end);
```

Simulate the model using the input signal in $z B$.

```
uSim = zB;
```

Simulation requires initial conditions. The signal values in zA are the historical data, that is, they are the input and output values for the time immediately preceding data in $z B$. Use $z A$ as a proxy for the required initial conditions.

IO = struct('Input',zA.InputData,'Output',zA.OutputData);
opt = simOptions('InitialCondition',IO);
Simulate the model.
ysim = sim(sys,uSim,opt);
To understand how the past data is mapped to the initial states of the model, see "Understand Use of Historical Data for Model Simulation" on page 1-1378.

## Obtain and Apply Estimated Initial Conditions

Load and plot the data.

```
load iddatalic z1i
plot(zli)
```


## Input-Output Data



Examine the initial value of the output data $y(1)$.
ystart = z1i. $\mathrm{y}(1)$
ystart = -3.0491
The measured output does not start at 0 .
Estimate a second-order transfer function sys and return the estimated initial condition ic.

```
[sys,ic] = tfest(zli,2,1);
ic
ic =
    initialCondition with properties:
    A: [2x2 double]
    X0: [2x1 double]
    C: [0.2957 5.2441]
    Ts: 0
```

ic is an initialCondition object that encapsulates the free response of sys, in state-space form, to the initial state vector in X0.

Simulate sys using the estimation data but without incorporating the initial conditions. Plot the simulated output with the measured output.
y_no_ic = sim(sys,zli);
plot(y_no_ic,zli)
legend('Mōdel Response','Output Data')


The measured and simulated outputs do not agree at the beginning of the simulation.
Incorporate the initial condition into the simOptions option set.

```
opt = simOptions('InitialCondition',ic);
y_ic = sim(sys,zli,opt);
plot(y_ic,zli)
legend('Model Response','Output Data')
```


## Input-Output Data



The simulation combines the model response to the input signal with the free response to the initial condition. The measured and simulated outputs now have better agreement at the beginning of the simulation. This initial condition is valid only for the estimation data zli.

## Input Arguments

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.
Example: 'AddNoise',true', 'Input0ffset', [5;0] adds default Gaussian white noise to the response model and specifies an input offset of 5 for the first of two model inputs.

## InitialCondition - Simulation initial conditions

[ ] (default) | column vector | matrix | initialCondition object | object array | structure | structure array | 'model'

Simulation initial conditions, specified as one of the following:

- 'z' - Zero initial conditions.
- Numerical column vector of initial states with length equal to the model order.

For multiexperiment data, specify a matrix with $N e$ columns, where $N e$ is the number of experiments, to configure the initial conditions separately for each experiment. Otherwise, use a column vector to specify the same initial conditions for all experiments.

Use this option for state-space models (idss and idgrey) only.

- initialCondition object - initialCondition object that represents a model of the free response of the system to initial conditions. For multiexperiment data, specify a 1-by- $N_{e}$ array of objects, where $N_{e}$ is the number of experiments.

Use this option for linear models only. For an example, see "Obtain and Apply Estimated Initial Conditions" on page 1-1385.

- Structure with the following fields, which contain the historical input and output values for a time interval immediately before the start time of the data used in the simulation:

| Field | Description |
| :--- | :--- |
| Input | Input history, specified as a matrix with $N u$ columns, where $N u$ is the number of <br> input channels. For time-series models, use [ ]. The number of rows must be <br> greater than or equal to the model order. |
| Output | Output history, specified as a matrix with $N y$ columns, where $N y$ is the number of <br> output channels. The number of rows must be greater than or equal to the model <br> order. |

For an example, see "Use Historical Data to Specify Initial Conditions for Model Simulation" on page 1-1385.

For multiexperiment data, configure the initial conditions separately for each experiment by specifying InitialCondition as a structure array with $N e$ elements. To specify the same initial conditions for all experiments, use a single structure.

The software uses data2state to map the historical data to states. If your model is not idss, idgrey, idnlgrey, or idnlarx, the software first converts the model to its state-space representation and then maps the data to states. If conversion of your model to idss is not possible, the estimated states are returned empty.

- 'model' - Use this option for idnlgrey models only. The software sets the initial states to the values specified in the sys. InitialStates property of the model sys.
- [ ] - Corresponds to zero initial conditions for all models except idnlgrey. For idnlgrey models, the software treats [] as 'model' and specifies the initial states as sys.InitialStates.


## X0Covariance - Covariance of initial states vector

[] (default) | matrix
Covariance of initial states vector, specified as one of the following:

- Positive definite matrix of size $N x$-by- $N x$, where $N x$ is the model order.

For multiexperiment data, specify as an $N x$-by- $N x$-by- $N e$ matrix, where $N e$ is the number of experiments.

- [] - No uncertainty in the initial states.

Use this option only for state-space models (idss and idgrey) when 'InitialCondition' is specified as a column vector. Use this option to account for initial condition uncertainty when computing the standard deviation of the simulated response of a model.

## InputOffset - Input signal offset

[ ] (default) | column vector | matrix
Input signal offset, specified as a column vector of length $N u$. Use [ ] if there are no input offsets. Each element of InputOffset is subtracted from the corresponding input data before the input is used to simulate the model.

For multiexperiment data, specify Input0ffset as:

- An $N u$-by-Ne matrix to set offsets separately for each experiment.
- A column vector of length $N u$ to apply the same offset for all experiments.


## OutputOffset - Output signal offset

[ ] (default) | column vector | matrix
Output signal offset, specified as a column vector of length $N y$. Use [ ] if there are no output offsets. Each element of OutputOffset is added to the corresponding simulated output response of the model.

For multiexperiment data, specify Output0ffset as:

- An $N y$-by-Ne matrix to set offsets separately for each experiment.
- A column vector of length $N y$ to apply the same offset for all experiments.


## AddNoise - Noise addition toggle

false (default) | true
Noise addition toggle, specified as a logical value indicating whether to add noise to the response model.

## NoiseData - Noise signal data

[ ] (default) | matrix | cell array of matrices
Noise signal data specified as one of the following:

- [ ] — Default Gaussian white noise.
- Matrix with $N_{s}$ rows and $N_{y}$ columns, where $N_{s}$ is the number of input data samples, and $N_{y}$ is the number of outputs. Each matrix entry is scaled according to NoiseVariance property of the simulated model and added to the corresponding output data point. To set NoiseData at a level that is consistent with the model, use white noise with zero mean and a unit covariance matrix.
- Cell array of $N_{e}$ matrices, where $N_{e}$ is the number of experiments for multiexperiment data. Use a cell array to set the NoiseData separately for each experiment, otherwise set the same noise signal for all experiments using a matrix.

NoiseData is the noise signal, $e(t)$, for the model

$$
y(t)=G u(t)+H e(t) .
$$

Here, $G$ is the transfer function from the input, $u(t)$, to the output, $y(t)$, and $H$ is the noise transfer function.

NoiseData is used for simulation only when AddNoise is true.

## Output Arguments

opt - Option set for sim command
sim0ptions option set
Option set for sim command, returned as a sim0ptions option set.

## See Also

sim

Introduced in R2012a

## simsd

Simulate linear models with uncertainty using Monte Carlo method

## Syntax

```
simsd(sys,udata)
simsd(sys,udata,N)
simsd(sys,udata,N,opt)
y = simsd(
```

$\qquad$

```
[y,y_sd] = simsd(
```

$\qquad$

``` )
```


## Description

simsd simulates linear models using the Monte Carlo method. The command performs multiple simulations using different values of the uncertain parameters of the model, and different realizations of additive noise and simulation initial conditions. simsd uses Monte Carlo techniques to generate response uncertainty, whereas sim generates the uncertainty using the Gauss Approximation Formula.
simsd(sys, udata) simulates and plots the response of 10 perturbed realizations of the identified model sys. Simulation input data udata is used to compute the simulated response.

The parameters of the perturbed realizations of sys are consistent with the parameter covariance of the original model, sys. If sys does not contain parameter covariance information, the 10 simulated responses are identical. For information about how the parameter covariance information is used to generate the perturbed models, see "Generating Perturbations of Identified Model" on page 1-1399.
simsd(sys, udata, $N$ ) simulates and plots the response of $N$ perturbed realizations of the identified model sys.
simsd(sys, udata, N , opt) simulates the system response using the simulation behavior specified in the option set opt. Use opt to specify uncertainties in the initial conditions and include the effect of additive disturbances.

The simulated responses are all identical if sys does not contain parameter covariance information, and you do not specify additive noise or covariance values for initial states. You specify these values in the AddNoise and X0Covariance options of opt.
$\mathrm{y}=\operatorname{simsd}(\ldots \quad$ ) returns the N simulation results in y as a cell array. No simulated response plot is produced. Use with any of the input argument combinations in the previous syntaxes.
[y,y_sd] = simsd( ___ ) also returns the estimated standard deviation y_sd for the simulated response.

## Examples

## Simulate State-Space Model Using Monte Carlo Method

Load the estimation data.
load iddata1 z1
z1 is an iddata object that stores the input-output estimation data.
Estimate a third-order state-space model.
sys = ssest(z1,3);
Simulate the response of the estimated model using the Monte Carlo method and input estimation data, and plot the response.
simsd(sys,z1);


The blue line plots the simulated response of the original nominal model sys. The green lines plot the simulated response of 10 perturbed realizations of sys.

## Simulate Estimated Model Using Monte Carlo Method

Simulate an estimated model using the Monte Carlo method for a specified number of model perturbations.

Estimate a second-order state-space model using estimation data. Obtain sys in the observability canonical form.
load iddata3 z3

```
sys = ssest(z3,2,'Form','canonical');
```

Compute the simulated response of the estimated model using the Monte Carlo method, and plot the responses. Specify the number of random model perturbations as 20 .
$\mathrm{N}=20$;
simsd(sys,z3,N)

Simulated output\#1: y1


The blue line plots the simulated response of the original nominal model sys. The green lines plot the simulated response of the 20 perturbed realizations of sys.

You can also obtain the simulated response for each perturbation of sys. No plot is generated when you use this syntax.

```
y = simsd(sys,z3,N);
```

$y$ is the simulated response, returned as a cell array of $N+1$ elements. $y\{1\}$ contains the nominal response for sys. The remaining elements contain the simulated response for the N perturbed realizations.

## Simulate Time Series Model Using Monte Carlo Method

Load time series data.
load iddata9 z9
z9 is an iddata object with 200 output data samples and no inputs.
Estimate a sixth-order AR model using the least-squares algorithm.
sys $=\operatorname{ar}(z 9,6, ' l s ') ;$
For time series data, specify the desired simulation length, Ns = 200 using an Ns-by-0 input data set.
data $=$ iddata([],zeros(200,0),z9.Ts);
Set the initial conditions to use the initial samples of the time series as historical output samples. The past data is mapped to the initial states of each perturbed system individually.

```
IC = struct('Input',[],'Output',z9.y(1:6));
opt = simsdOptions('InitialCondition',IC);
```

Simulate the model using Monte Carlo method and specified initial conditions. Specify the number of random model perturbations as 20 .

```
simsd(sys,data,20,opt)
```

Simulated output\#1: y1


The blue line plots the simulated response of the original nominal model sys. The green lines plot the simulated response of the 20 perturbed realizations of sys.

## Study Effect of Initial Condition Uncertainty on Model Response

Load data, and split it into estimation and simulation data.

```
load iddata3
ze = z3(1:200);
zsim = z3(201:256);
```

Estimate a second-order state-space model sys using estimation data. Specify that no parameter covariance data is generated. Obtain sys in the observability canonical form.

```
opt = ssestOptions('EstimateCovariance',false);
sys = ssest(ze,2,'Form','canonical',opt);
```

Set the initial conditions for simulating the estimated model. Specify initial state values $x 0$ for the two states and also the covariance of initial state values x0Cov. The covariance is specified as a 2 -by-2 matrix because there are two states.

```
x0 = [1.2; -2.4];
x0Cov = [0.86 -0.39; -0.39 1.42];
opt = simsdOptions('InitialCondition',x0,'X0Covariance',x0Cov);
```

Simulate the model using Monte Carlo method and specified initial conditions. Specify the number of random model perturbations as 100 .

```
simsd(sys,zsim,100,opt)
```

Simulated output\#1: y1


The blue line plots the simulated response of the original nominal model sys. The green lines plot the simulated response of the 100 perturbed realizations of sys. The software uses a different realization of the initial states to simulate each perturbed model. Initial states are drawn from a Gaussian distribution with mean InitialCondition and covariance X0Covariance.

## Study Effect of Additive Disturbance on Response Uncertainty

Load the estimation data.
load iddatal z1
$z 1$ is an idddata object that stores 300 input-output estimation data samples.
Estimate a second-order state-space model using the estimation data.
sys $=\operatorname{ssest}(z 1,2)$;
Create a default option set for simsd, and modify the option set to add noise.
opt = simsdOptions;
opt.AddNoise = true;
Compute the simulated response of the estimated model using the Monte Carlo method. Specify the number of random model perturbations as 20, and simulate the model using the specified option set.
[y,y_sd] = simsd(sys,z1,20,opt);
$y$ is the simulated response, returned as a cell array of 21 elements. $y\{1\}$ contains the nominal, noise-free response for sys. The remaining elements contain the simulated response for the 20 perturbed realizations of sys with additive disturbances added to each response.
y _sd is the estimated standard deviation of simulated response, returned as an iddata object with no inputs. The standard deviations are computed from the 21 simulated outputs. To access the standard deviation, use y_sd.OutputData.

## Input Arguments

## sys - Model to be simulated

parametric linear identified model
Model to be simulated, specified as one of the following parametric linear identified models: idtf, idproc, idpoly, idss, or idgrey.

To generate the set of simulated responses, the software perturbs the parameters of sys in a way that is consistent with the parameter covariance information. Use getcov to examine the parameter uncertainty for sys. For information about how the perturbed models are generated from sys, see rsample.

The simulated responses are all identical if sys does not contain parameter covariance information and you do not specify additive noise or covariance values for initial states. You specify these values in the AddNoise and X0Covariance options of opt.

## udata - Simulation input data

iddata object | matrix
Simulation input data, specified as one of the following:

- iddata object - Input data can be either time-domain or frequency-domain. The software uses only the input channels of the iddata object.

If sys is a time series model, that is, a model with no inputs, specify udata as an $N s$-by- 0 signal, where $N s$ is the wanted number of simulation output samples for each of the $N$ perturbed realizations of sys. For example, to simulate 100 output samples, specify udata as follows.

```
udata = iddata([],zeros(100,0),Ts);
```

For an example, see "Simulate Time Series Model Using Monte Carlo Method" on page 1-1394.

- matrix - For simulation of discrete-time systems using time-domain data only. Columns of the matrix correspond to each input channel.

If you do not have data from an experiment, use idinput to generate signals with various characteristics.

## N - Number of perturbed realizations

10 (default) | positive integer
Number of perturbed realizations of sys to be simulated, specified as a positive integer.

## opt - Simulation options

simsdOptions option set
Simulation options for simulating models using Monte Carlo methods, specified as a simsdOptions option set. You can use this option set to specify:

- Input and output signal offsets - Specify an offset to remove from the input signal and an offset to add to the response of sys.
- Initial condition handling - Specify initial conditions for simulation and their covariance. For state-space and linear grey-box models (idss and idgrey), if you want to simulate the effect of uncertainty in initial states, set the InitialCondition option to a double vector, and specify its covariance using the X0Covariance option. For an example, see "Study Effect of Initial Condition Uncertainty on Model Response" on page 1-1396.
- Addition of noise to simulated data - If you want to include the influence of additive disturbances, specify the AddNoise option as true. For an example, see "Study Effect of Additive Disturbance on Response Uncertainty" on page 1-1397.


## Output Arguments

## y - Simulated response

cell array
Simulated response, returned as a cell array of $\mathrm{N}+1$ elements. y\{1\} contains the nominal response for sys. The remaining elements contain the simulated response for the $N$ perturbed realizations.

The command performs multiple simulations using different values of the uncertain parameters of the model, and different realizations of additive noise and simulation initial conditions. Thus, the simulated responses are all identical if sys does not contain parameter covariance information and you do not specify additive noise and covariance values for initial states in opt.

## y_sd - Estimated standard deviation of simulated response

## iddata object

Estimated standard deviation of simulated response, returned as an iddata object. The standard deviation is computed as the sample standard deviation of the $y$ ensemble:

$$
y_{-} s d=\sqrt{\frac{1}{N} \sum_{i=2}^{N+1}(y\{1\}-y\{i\})^{2}}
$$

Here $y\{1\}$ is the nominal response for sys, and $y\{i\}(i=2: N+1)$ are the simulated responses for the $N$ perturbed realizations of sys.

## More About

## Generating Perturbations of Identified Model

The software generates $N$ perturbations of the identified model sys and then simulates the response of each of these perturbations. The parameters of the perturbed realizations of sys are consistent with the parameter covariance of the original model sys. The parameter covariance of sys gives information about the distribution of the parameters. However, for some parameter values, the resulting perturbed systems can be unstable. To reduce the probability of generation of unrealistic systems, the software prescales the parameter covariance.

If $\Delta p$ is the parameter covariance for the parameters $p$ of sys, then the simulated output $f(p+\Delta p)$ of a perturbed model as a first-order approximation is:

$$
f(p+\Delta p)=f(p)+\frac{\partial f}{\partial p} \Delta p
$$

The simsd command first scales $\Delta p$ by a scaling factor $s$ (approximately $0.1 \%$ ) to generate perturbed systems with parameters $(p+s \Delta p)$. The command then computes $f(p+s \Delta p)$, the simulated response of these perturbed systems. Where,

$$
f(p+s \Delta p)=f(p)+s \frac{\partial f}{\partial p} \Delta p
$$

The command then computes the simulated response $f(p+\Delta p)$ as:

$$
f(p+\Delta p)=f(p)+\frac{1}{s}(f(p+s \Delta p)-f(p))
$$

Note This scaling is not applied to the free delays of idproc or idtf models.

If you specify the AddNoise option of simsdOptions as true, the software adds different realizations of the noise sequence to the noise-free responses of the perturbed system. The realizations of the noise sequence are consistent with the noise component of the model.

For state-space models, if you specify the covariance of initial state values in X0Covariance option of simsdOptions, different realizations of the initial states are used to simulate each perturbed model. Initial states are drawn from a Gaussian distribution with mean InitialCondition and covariance X0Covariance.

## See Also <br> getcov|rsample|showConfidence|sim|simsdOptions

Introduced before R2006a

## simsdOptions

Option set for simsd

## Syntax

```
opt = simsdOptions
opt = simsdOptions(Name,Value)
```


## Description

opt $=$ simsdOptions creates the default option set for simsd.
opt = simsdOptions(Name, Value) creates an option set with the options specified by one or more Name, Value pair arguments.

## Examples

## Create Default Option Set for Uncertain Model Simulation

opt = simsdOptions;

## Specify Options for Uncertain Model Simulation

Create an option set for simsd specifying the following options.

- Zero initial conditions
- Input offset of 5 for the second input of a two-input model

```
opt = simsdOptions('InitialCondition','z','InputOffset',[0; 5]);
```


## Add Noise to Uncertain Simulation Output

Create a default option set.
opt = simsdOptions;
Modify the option set to add noise to the data.
opt. AddNoise = true;
When you use this option set and simsd command to simulate the response of a model sys. The command returns the perturbed realizations of sys with additive disturbances added to each response.

## Input Arguments

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, ... , NameN, ValueN.
Example: opt = simsdOptions('AddNoise',true','Input0ffset', [5;0]) adds default Gaussian white noise to the response model, and specifies an input offset of 5 for the first of two model inputs.

## InitialCondition - Simulation initial conditions

'z' (default) | column vector | matrix | structure | structure array
Simulation initial conditions, specified as one of the following:

- 'z' - Zero initial conditions.
- Numerical column vector X 0 of initial states with length equal to the model order.

For multi-experiment data, specify a matrix with $N e$ columns, where $N e$ is the number of experiments, to configure the initial conditions separately for each experiment. Otherwise, use a column vector to specify the same initial conditions for all experiments.

Use this option for state-space models (idss and idgrey) only. You can also specify the covariance of the initial state vector in X0Covariance.

- Structure with the following fields, which contain the historical input and output values for a time interval immediately before the start time of the data used in the simulation:

| Field | Description |
| :--- | :--- |
| Input | Input history, specified as a matrix with $N u$ columns, where $N u$ is the number of <br> input channels. For time-series models, use [ ]. The number of rows must be <br> greater than or equal to the model order. |
| Output | Output history, specified as a matrix with Ny columns, where Ny is the number of <br> output channels. The number of rows must be greater than or equal to the model <br> order. |

For multi-experiment data, you can configure the initial conditions separately for each experiment by specifying InitialCondition as a structure array with $N e$ elements. Otherwise, use a single structure to specify the same initial conditions for all experiments.

The software uses data2state to map the historical data to states. If your model is not idss or idgrey, the software first converts the model to its state-space representation and then maps the data to states. If conversion of your model to idss is not possible, the estimated states are returned empty.

## X0Covariance - Covariance of initial states vector

[] (default) | matrix
Covariance of initial states vector, specified as one of the following:

- Positive definite matrix of size $N x$-by- $N x$, where $N x$ is the model order.

For multi-experiment data, specify as an $N x$-by- $N x$-by-Ne matrix, where $N e$ is the number of experiments. For the $k^{\text {th }}$ experiment, X0Covariance ( $\left., ~:, k\right)$ specifies the covariance of initial states X0(: , k).

- [ ] - No uncertainty in the initial states.

Use this option for state-space models (idss and idgrey) when 'InitialCondition' is specified as a numerical column vector X 0 . When you specify this option, the software uses a different realization of the initial states to simulate each perturbed model. Initial states are drawn from a Gaussian distribution with mean InitialCondition and covariance X0Covariance.

## InputOffset - Input signal offset <br> [ ] (default) | column vector | matrix

Input signal offset, specified as a column vector of length $N u$. Use [ ] if there are no input offsets. Each element of InputOffset is subtracted from the corresponding input data before the input is used to simulate the model.

For multiexperiment data, specify Input0ffset as:

- An $N u$-by-Ne matrix to set offsets separately for each experiment.
- A column vector of length $N u$ to apply the same offset for all experiments.


## OutputOffset - Output signal offset

[] (default) | column vector | matrix
Output signal offset, specified as a column vector of length $N y$. Use [ ] if there are no output offsets. Each element of OutputOffset is added to the corresponding simulated output response of the model.

For multiexperiment data, specify Output0ffset as:

- An Ny -by-Ne matrix to set offsets separately for each experiment.
- A column vector of length $N y$ to apply the same offset for all experiments.


## AddNoise - Noise addition toggle

false (default) | true
Noise addition toggle, specified as a logical value indicating whether to add noise to the response model. Set NoiseModel to true to study the effect of additive disturbances on the response. A different realization of the noise sequence, consistent with the noise component of the perturbed system, is added to the noise-free response of that system.

## Output Arguments

## opt - Option set for simsd command

simsdOptions option set
Option set for simsd command, returned as a simsdOptions option set.

## See Also

simsd

Introduced in R2012a

## size

Query output/input/array dimensions of input-output model and number of frequencies of FRD model

## Syntax

size(sys)
d = size(sys)
Ny = size(sys,1)
$\mathrm{Nu}=\operatorname{size}(\mathrm{sys}, 2)$
Sk = size(sys,2+k)
Nf = size(sys,'frequency')

## Description

When invoked without output arguments, size(sys) returns a description of type and the inputoutput dimensions of sys. If sys is a model array, the array size is also described. For identified models, the number of free parameters is also displayed. The lengths of the array dimensions are also included in the response to size when sys is a model array.
d = size(sys) returns:

- The row vector $\mathrm{d}=$ [ Ny Nu ] for a single dynamic model sys with Ny outputs and Nu inputs
- The row vector $d=[\mathrm{Ny}$ Nu S1 S2 ... Sp] for an S1-by-S2-by-...-by-Sp array of dynamic models with Ny outputs and Nu inputs

Ny = size(sys,1) returns the number of outputs of sys.
$N u=$ size (sys, 2 ) returns the number of inputs of sys.
Sk = size(sys, $2+\mathrm{k}$ ) returns the length of the $k$-th array dimension when sys is a model array.
Nf = size(sys,'frequency') returns the number of frequencies when sys is a frequency response data model. This is the same as the length of sys.frequency.

## Examples

## Query Dimensions of Model Array

Create a 3-by-1 model array of random state-space models with 3 outputs, 2 inputs, and 5 states.

```
sys = rss(5,3,2,3);
```

Verify the size of the model array.

```
size(sys)
3x1 array of state-space models.
Each model has 3 outputs, 2 inputs, and 5 states.
```


## Query Dimensions of Identified Model

Create a 2-input 2-output continuous-time process model with identifiable parameters.
type = \{'p1d','p2';'p3uz','p0'\};
sys = idproc(type);
Each element of the type cell array describes the model structure for the corresponding input-output pair.

Query the input-output dimensions and number of free parameters in the model.
size(sys)
Process model with 2 outputs, 2 inputs and 12 free parameters.

## See Also

isempty|issiso|ndims|nparams
Introduced before R2006a

## spa

Estimate frequency response with fixed frequency resolution using spectral analysis

## Syntax

G = spa(data)
$G=s p a(d a t a$, winSize,freq)
G = spa(data,winSize,freq,maxSize)

## Description

$\mathrm{G}=\mathrm{spa}$ (data) estimates the frequency response, along with uncertainty, and the noise spectrum from time- or frequency-domain data data. If data is a time series, spa(data) returns the output power spectrum along with uncertainty. spa computes the spectra at 128 equally spaced frequency values between 0 (excluded) and $\pi$, using a Hann window.

G = spa(data,winSize,freq) estimates the frequency response at the frequencies contained in freq, using a Hann window with size winSize.
$\mathrm{G}=\mathrm{spa}(\mathrm{data}$, winSize,freq, maxSize) splits the input-output data into segments, each segment containing fewer than maxSize elements. Use this syntax to improve computational performance.

## Examples

## Estimate Frequency Response

Estimate frequency response with fixed resolution at 128 equally spaced, logarithmic frequency values between 0 (excluded) and $\Pi$.

```
load iddata3;
g = spa(z3);
bode(g)
```



## Estimate Frequency Response at Specified Frequencies

Define the frequency vector.
w = logspace(-2,pi,128);
Compute the frequency response.
load iddata3;
g = spa(z3,[],w);
[] specifies the default lag window size.
Plot the Bode response and disturbance spectrum with confidence interval of 3 standard deviations.
h = bodeplot(g);
showConfidence(h,3)

figure
h = spectrumplot (g);
showConfidence(h,3)


## Input Arguments

## data - Input-output data

iddata object | idfrd object
Input-output data, specified as an iddata object or an idfrd object that can contain complex values. data can also contain time series data with only output.

## winSize - Window size

scalar integer
Hann window size, also known as lag size, specified as a scalar integer. By default, the function sets the window size to min(length (data)/10, 30).

## freq - Frequencies

row vector
Frequencies at which to estimate spectral response, specified as a row vector in units of rad/ timeUnit, where timeUnit refers to the timeUnit property of data. By default, the function sets freq to a vector of 128 values in the range ( $0, \Pi$ ], evenly spaced logarithmically. For discrete-time models, set freq within the Nyquist frequency bound.
maxSize - Maximum segment size
250 e 3 | positive integer

Maximum size of segments within data, specified as a positive integer. If you omit this argument, the function performs estimation using the full data set in data rather than segmenting the data.

## Output Arguments

## G - Frequency response and noise spectrum

## idfrd object

Frequency response with uncertainty and noise spectrum, specified as an idfrd object. For time series data, G is the estimated spectrum and standard deviation.

Information about the estimation results and options used is stored in the Report property of the model. Report has the following fields.

| Report Field | Description |  |
| :---: | :---: | :---: |
| Status | Summary of the model status, which indicates whether the model was created by construction or obtained by estimation. |  |
| Method | Estimation command used. |  |
| $\begin{aligned} & \text { windowSiz } \\ & e \end{aligned}$ | Size of the Hann window. |  |
| DataUsed | Attributes of the data used for estimation. Structure with the following fields: |  |
|  | Field | Description |
|  | Name | Name of the data set. |
|  | Type | Data type. |
|  | Length | Number of data samples. |
|  | Ts | Sample time. This is equivalent to Data. Ts. |
|  | InterSam ple | Input intersample behavior. One of the following values: <br> - 'zoh' - Zero-order hold maintains a piecewise-constant input signal between samples. <br> - ' foh ' - First-order hold maintains a piecewise-linear input signal between samples. <br> - 'bl' - Band-limited behavior specifies that the continuous-time input signal has zero power above the Nyquist frequency. <br> The value of Intersample has no effect on estimation results for discrete-time models. |
|  | Input0ff set | Offset removed from time-domain input data during estimation. |
|  | Output0f fset | Offset removed from time-domain output data during estimation. |

## More About

## Frequency Response Function

A Frequency response function describes the steady-state response of a system to sinusoidal inputs. For a linear system, a sinusoidal input of a specific frequency results in an output that is also a sinusoid with the same frequency, but with a different amplitude and phase. The frequency response function describes the amplitude change and phase shift as a function of frequency.

To better understand the frequency response function, consider the following description of a linear dynamic system:

$$
y(t)=G(q) u(t)+v(t)
$$

Here $u(t)$ and $y(t)$ are the input and output signals, respectively. $G(q)$ is called the transfer function of the system-it captures the system dynamics that take the input to the output. The notation $G(q) u(t)$ represents the following operation:

$$
G(q) u(t)=\sum_{k=1}^{\infty} g(k) u(t-k)
$$

$q$ is the shift operator, defined by the following equation:

$$
G(q)=\sum_{k=1}^{\infty} g(k) q^{-k} \quad q^{-1} u(t)=u(t-1)
$$

$G(q)$ is the frequency-response function, which is evaluated on the unit circle, $G\left(q=e^{i \omega}\right)$.
Together, $G\left(q=e^{i \omega}\right)$ and the output noise spectrum $\widehat{\Phi}_{v}(\omega)$ are the frequency-domain description of the system.

The frequency-response function estimated using the Blackman-Tukey approach is given by the following equation:

$$
\widehat{G}_{N}\left(e^{i \omega}\right)=\frac{\widehat{\Phi}_{y u}(\omega)}{\widehat{\Phi}_{u}(\omega)}
$$

In this case, ^ represents approximate quantities. For a derivation of this equation, see the chapter on nonparametric time- and frequency-domain methods in [1].

## Output Noise Spectrum

The output noise spectrum (spectrum of $v(t)$ ) is given by the following equation:

$$
\widehat{\Phi}_{v}(\omega)=\widehat{\Phi}_{y}(\omega)-\frac{\left|\widehat{\Phi}_{y u}(\omega)\right|^{2}}{\widehat{\Phi}_{u}(\omega)}
$$

This equation for the noise spectrum is derived by assuming that the linear relationship $y(t)=G(q) u(t)+v(t)$, that $u(t)$ is independent of $v(t)$, and that the following relationships between the spectra:

$$
\Phi_{y}(\omega)=\left|G\left(e^{i \omega}\right)\right|^{2} \Phi_{u}(\omega)+\Phi_{v}(\omega)
$$

$$
\Phi_{y u}(\omega)=G\left(e^{i \omega}\right) \Phi_{u}(\omega)
$$

Here, the noise spectrum is given by the following equation:

$$
\Phi_{v}(\omega) \equiv \sum_{\tau=-\infty}^{\infty} R_{\nu}(\tau) e^{-i w \tau}
$$

$\widehat{\Phi}_{y u}(\omega)$ is the output-input cross-spectrum and $\widehat{\Phi}_{u}(\omega)$ is the input spectrum.
Alternatively, the disturbance $v(t)$ can be described as filtered white noise:

$$
v(t)=H(q) e(t)
$$

Here, $e(t)$ is the white noise with variance $\lambda$ and the noise power spectrum is given by the following equation:

$$
\Phi_{v}(\omega)=\lambda\left|H\left(e^{i \omega}\right)\right|^{2}
$$

## Algorithms

spa applies the Blackman-Tukey spectral analysis method by following these steps:
1 Compute the covariances and cross-covariance from $u(t)$ and $y(t)$ :

$$
\begin{aligned}
& \widehat{R}_{y}(\tau)=\frac{1}{N} \sum_{t=1}^{N} y(t+\tau) y(t) \\
& \widehat{R}_{u}(\tau)=\frac{1}{N} \sum_{t=1}^{N} u(t+\tau) u(t) \\
& \widehat{R}_{y u}(\tau)=\frac{1}{N} \sum_{t=1}^{N} y(t+\tau) u(t)
\end{aligned}
$$

2 Compute the Fourier transforms of the covariances and the cross-covariance:

$$
\begin{aligned}
& \widehat{\Phi}_{y}(\omega)=\sum_{\tau=-M}^{M} \widehat{R}_{y}(\tau) W_{M}(\tau) e^{-i \omega \tau} \\
& \widehat{\Phi}_{u}(\omega)=\sum_{\tau=-M}^{M} \widehat{R}_{u}(\tau) W_{M}(\tau) e^{-i \omega \tau} \\
& \widehat{\Phi}_{y u}(\omega)=\sum_{\tau=-M}^{M} \widehat{R}_{y u}(\tau) W_{M}(\tau) e^{-i \omega \tau}
\end{aligned}
$$

where $W_{M}(\tau)$ is the Hann window with a width (lag size) of $M$. You can specify $M$ to control the frequency resolution of the estimate, which is approximately equal $2 \pi / M \mathrm{rad} /$ sample time.

By default, this operation uses 128 equally spaced frequency values between 0 (excluded) and $n$, where $w=[1: 128] / 128^{*} \mathrm{pi} / \mathrm{Ts}$ and Ts is the sample time of that data set. The default lag size of the Hann window is $M=\min$ (length (data) $/ 10,30$ ). For default frequencies, the operation uses fast Fourier transforms (FFT), which is more efficient than for user-defined frequencies.

Note $M=\gamma$ is in Table 6.1 of [1]. Standard deviations are on pages 184 and 188 in [1].
3 Compute the frequency-response function $\widehat{G}_{N}\left(e^{i \omega}\right)$ and the output noise spectrum $\widehat{\Phi}_{v}(\omega)$.

$$
\begin{aligned}
& \widehat{G}_{N}\left(e^{i \omega}\right)=\frac{\widehat{\Phi}_{y u}(\omega)}{\widehat{\Phi}_{u}(\omega)} \\
& \Phi_{v}(\omega) \equiv \sum_{\tau=-\infty}^{\infty} R_{v}(\tau) e^{-i \omega \tau}
\end{aligned}
$$

spectrum is the spectrum matrix for both the output and the input channels. That is, if $z=$ [data.OutputData, data.InputData], spectrum contains as spectrum data the matrix-valued power spectrum of $z$.

$$
S=\sum_{m=-M}^{M} E z(t+m) z(t)^{\prime} W_{M}\left(T_{S}\right) \exp (-i \omega m)
$$

' is a complex-conjugate transpose.

## References

[1] Ljung, Lennart. System Identification: Theory for the User, Second Ed., Prentice Hall PTR, 1999.

## See Also

bode |etfe|freqresp|idfrd|spafdr|spectrum

## Topics

"Estimate Impulse-Response Models at the Command Line"
"Spectrum Normalization"
"What is a Frequency-Response Model?"
Introduced before R2006a

## spafdr

Estimate frequency response and spectrum using spectral analysis with frequency-dependent resolution

## Syntax

g = spafdr(data)
g = spafdr(data,Resol,w)

## Description

$\mathrm{g}=\operatorname{spafdr}($ data $)$ estimates the input-to-output frequency response $\mathrm{G}(\omega)$ and noise spectrum $\Phi_{v}$ of the general linear model

$$
y(t)=G(q) u(t)+v(t)
$$

where $\Phi_{v}(\omega)$ is the spectrum of $v(t)$. data contains the output-input data as an iddata object. The data can be complex valued, and either time or frequency domain. It can also be an idfrd object containing frequency-response data. g is an idfrd object with the estimate of $G\left(e^{i \omega}\right)$ at the frequencies $\omega$ specified by row vector $w$. $g$ also includes information about the spectrum estimate of $\Phi_{v}(\omega)$ at the same frequencies. Both results are returned with estimated covariances, included in g . The normalization of the spectrum is the same as described in spa.

Information about the estimation results and options used is stored in the model's Report property. Report has the following fields:

- Status - Summary of the model status, which indicates whether the model was created by construction or obtained by estimation.
- Method - Estimation command used.
- WindowSize - Frequency resolution.
- DataUsed - Attributes of the data used for estimation. Structure with the following fields:
- Name - Name of the data set.
- Type - Data type.
- Length - Number of data samples.
- Ts - Sample time.
- InterSample - Input intersample behavior.
- InputOffset - Offset removed from time-domain input data during estimation.
- OutputOffset - Offset removed from time-domain output data during estimation.
$g=s p a f d r(d a t a, R e s o l, w)$ specifies frequencies and frequency resolution.


## Frequencies

The frequency variable $w$ is either specified as a row vector of frequencies in rad/TimeUnit, where TimeUnit refers to the TimeUnit property of data, or as a cell array $\{w m i n, w m a x\}$. In the latter
case the covered frequencies will be 50 logarithmically spaced points from wmin to wmax. You can change the number of points to NP by entering \{wmin, wmax, NP\}.

Omitting w or entering it as an empty matrix gives the default value, which is 100 logarithmically spaced frequencies between the smallest and largest frequency in data. For time-domain data, the default range goes from $\frac{2 \Pi}{N T_{s}}$ to $\frac{\Pi}{T_{s}}$, where $T s$ is the sample time of data and $N$ is the number of data points.

## Resolution

The argument Resol defines the frequency resolution of the estimates. The resolution (measured in $\mathrm{rad} /$ TimeUnit) is the size of the smallest detail in the frequency function and the spectrum that is resolved by the estimate. The resolution is a tradeoff between obtaining estimates with fine, reliable details, and suffering from spurious, random effects: The finer the resolution, the higher the variance in the estimate. Resol can be entered as a scalar (measured in rad/TimeUnit), which defines the resolution over the whole frequency interval. It can also be entered as a row vector of the same length as $w$. Then Resol ( $k$ ) is the local, frequency-dependent resolution around frequency $w(k)$.

The default value of Resol, obtained by omitting it or entering it as the empty matrix, is Resol(k) = $2(w(k+1)-w(k))$, adjusted upwards, so that a reasonable estimate is guaranteed. In all cases, the resolution is returned in the variable g. Report.WindowSize.

## Algorithms

If the data is given in the time domain, it is first converted to the frequency domain. Then averages of $Y(w) \operatorname{Conj}(U(w))$ and $U(w) \operatorname{Conj}(U(w))$ are formed over the frequency ranges $w$, corresponding to the desired resolution around the frequency in question. The ratio of these averages is then formed for the frequency-function estimate, and corresponding expressions define the noise spectrum estimate.

## See Also

bode |etfe|freqresp|idfrd|nyquist|spa|spectrum

## Introduced before R2006a

## spectrum

Plot or return output power spectrum of time series model or disturbance spectrum of linear inputoutput model

## Syntax

```
spectrum(sys)
spectrum(sys,{wmin, wmax})
spectrum(sys,w)
spectrum(sys1,...,sysN,w)
ps = spectrum(sys,w)
ps = spectrum(sys,w)
[ps,wout] = spectrum(sys)
[ps,wout,sdps] = spectrum(sys)
```


## Description

## Plot Results

spectrum(sys) plots the output power spectrum of an identified time series model sys or the disturbance spectrum of an identified input-output model sys. The function chooses the frequency range and number of points automatically.

- If sys is a time series model, then sys represents the system:

$$
y(t)=H e(t)
$$

Here, $e(t)$ is a Gaussian white noise and $y(t)$ is the observed output.
spect rum plots $\left|H^{\prime} H\right|$, scaled by the variance of $e(t)$ and the sample time.

- If sys is an input-output model, sys represents the system:

$$
y(t)=G u(t)+H e(t)
$$

Here, $u(t)$ is the measured input, $e(t)$ is Gaussian white noise and $y(t)$ is the observed output.
In this case, spectrum plots the spectrum of the disturbance component $\mathrm{He}(t)$.
For discrete-time models with sample time $T_{s}$, spectrum uses the transformation $z=e^{j \omega T_{s}}$ to map the unit circle to the real frequency axis. The function plots the spectrum only for frequencies smaller than the Nyquist frequency $\pi / T_{s}$, and uses the default value of 1 time unit when $T s$ is unspecified.
spectrum(sys, \{wmin, wmax\}) creates a spectrum plot for frequencies ranging from win to wmax.
spectrum(sys,w) creates a spectrum plot using the frequencies specified in the vector w .
spectrum(sys1, ..., sysN, w) creates a spectrum plot of several identified models on a single plot. The w argument is optional.

You can specify a color, line style and marker for each model. For example, spectrum(sys1,'r',sys2,'y--',sys3,'gx') uses red for sys1, yellow dash markers for sys2, and green x markers for sys3.

Return Results
ps = spectrum(sys,w) returns the power spectrum amplitude of sys for the specified frequencies, $w$. There is no plot display.
ps = spectrum(sys,w) returns the power spectrum amplitude of sys for the specified frequencies, $w$.
[ps, wout] = spectrum(sys) returns the frequency vector, wout, for which the output power spectrum is computed.
[ps,wout,sdps] = spectrum(sys) returns the estimated standard deviations of the power spectrum.

## Examples

Plot Output Spectrum of Time-Series Model
Load the time-series estimation data.
load iddata9 z9
Estimate a fourth-order AR model using a least-squares approach.
sys = ar(z9,4,'ls');
Plot the output spectrum of the model.

```
spectrum(sys);
```



Plot Noise Spectrum of SISO Linear Identified Model
Load the estimation data.
load iddatal $\mathrm{z1}$;
Estimate a single-input single-output state-space model.
sys = n4sid(z1,2);
Plot the noise spectrum for the model. Specify a frequency range from 0.1 to $50 \mathrm{rad} / \mathrm{sec}$.
spectrum(sys,\{0.1,50\});

## Power Spectrum



The function plots the spectrum, but limits the frequency range to the Nyquist frequency of approximately $31.4 \mathrm{rad} / \mathrm{sec}$.

## Compare Spectrum of Sinusoid Signal with Square

Create an input consisting of the sum of five sinusoids, each spread over the full frequency range. Compare the spectrum of this signal with that of its square.

Create a sum-of-sinusoids input that extends for 20 periods, with each period containing 100 samples. Specify that the signal combine 5 sinusoids of random phase, using 10 trials to find the set with the lowest signal spread. For more information on this step, see idinput.
u = idinput([100 1 20],'sine',[],[],[5 10 1]);
Create an input-only iddata object $u$ that contains the input $u$ and has a period of 100 .
u = iddata([],u,1,'per',100);
Square the input values and store them in new iddata object u2.

```
u2 = u.u.^2;
u2 = iddata([],u2,1,'per',100);
```

Use etfe to estimate empirical transfer function models from $u$ and $u 2$. Plot the power spectra of these models together. Use different marker colors and types to distinguish the spectrum sources.
spectrum(etfe(u),'r*',etfe(u2),'+')

## Power Spectrum



The plot shows some frequency splitting where the u2-based spectrum does not line up with the u1based spectrum, but instead, contains two spectral points that flank certain u1-based points. This indicates the nonlinearity of the squared system.

## Input Arguments

## sys - Identified model

idpoly object | idproc object | idss object | idtf object
Identified model, specified as an idpoly object, an idproc object, an idss object, or an idtf object.

- If sys is a time series model, then sys represents the system:

$$
y(t)=H e(t)
$$

Here, $\mathrm{e}(\mathrm{t})$ is a Gaussian white noise and $\mathrm{y}(\mathrm{t})$ is the observed output.

- If sys is an input-output model, then sys represents the system:

$$
y(t)=G u(t)+H e(t)
$$

Here, $u(t)$ is the measured input, $e(t)$ is a Gaussian white noise and $y(t)$ is the observed output.

## wmin - Minimum frequency

positive number
Minimum frequency of the frequency range for which to plot the spectrum, specified as a positive number.

Specify wmin in rad/TimeUnit, where TimeUnit is sys.TimeUnit.
Example: "Plot Noise Spectrum of SISO Linear Identified Model" on page 1-1419

## wmax - Maximum frequency

positive number
Maximum frequency of the frequency range for which to plot the spectrum, specified as a positive number. By default, the function uses the Nyquist frequency of sys as wmax.

Specify wmax in rad/TimeUnit, where TimeUnit is sys.TimeUnit. If you specify wmax to be greater than the Nyquist frequency, then spectrum will use the Nyquist frequency instead.

## Example: "Plot Noise Spectrum of SISO Linear Identified Model" on page 1-1419

## w- Frequencies

positive numeric vector
Frequencies for which to plot the spectrum, specified as a vector of positive numbers.
Specify w in rad/TimeUnit, where TimeUnit is sys.TimeUnit.

## Output Arguments

## ps - Power spectrum amplitude

## numeric array

Power spectrum amplitude returned as a numeric vector or a numeric array.

- For single-output models, ps is a 1-by-1- $N_{w}$ array, where $N_{w}$ is the length of the frequency vector.
- For multiple-output models, ps is an $N_{y}$-by $N_{y}$-by $N_{w}$ array, where $N_{y}$ is the number of outputs. ps (:,$:, k)$ corresponds to the power spectrum for the frequency at $w(k)$.

For amplitude values in dB , type $\mathrm{psdb}=10 * \log 10(\mathrm{ps})$.

## wout - Frequencies

numeric vector
Frequencies for which the spectrum is plotted, returned as a numeric vector. If you supply w as an input argument, the function returns the identical vector in wout.

## sdps - Standard deviation

numeric array
Estimated standard deviation of the power spectrum, returned as an array with the same dimensions as ps.

## See Also

ar | armax | arx|bode |etfe|forecast | freqresp|idinput

## Topics

"Frequency Response Plots"
"Noise Spectrum Plots"
"Plot the Noise Spectrum Using the System Identification App"
"Plot the Noise Spectrum at the Command Line"
"Plot Bode Plots Using the System Identification App"
"Plot Bode and Nyquist Plots at the Command Line"

Introduced in R2012a

## spectrumoptions

Option set for spectrumplot

## Syntax

```
opt = spectrumoptions
opt = spectrumoptions('identpref')
```


## Description

opt $=$ spectrumoptions creates the default option set for spectrumplot. Use dot notation to customize the option set, if needed.
opt $=$ spectrumoptions('identpref') initializes the plot options with the System Identification Toolbox preferences. Use this syntax to change a few plot options but otherwise use your toolbox preferences.

## Examples

## Specify Options for Spectrum Plot

Specify the plot options.

```
plot_options = spectrumoptions;
plot_options.FreqUnits = 'Hz';
plot_options.FreqScale = 'linear';
plot_options.Xlim = {[0 20]};
plot_options.MagUnits = 'abs';
Estimate an AR model.
```

```
load iddata9 z9
sys = ar(z9,4);
```

Plot the output spectrum for the model.

```
spectrumplot(sys,plot_options);
```



## Initialize Plot Options Using Toolbox Preferences

opt = spectrumoptions('identpref');

## Output Arguments

opt - Option set for spectrumplot
spectrumpoptions option set
Option set containing the specified options for spectrumplot.

| Field | Description |
| :---: | :---: |
| Title, XLabel, YLabel | Text and style for axes labels and plot title, specified as a structure array with the following fields: <br> - String - Title and axes label text, specified as a character vector. <br> Default Title: 'Power Spectrum' <br> Default XLabel: ' $F$ requency' <br> Default YLabel: 'Power' <br> - FontSize - Font size, specified as data type scalar. <br> Default: 8 <br> - FontWeight - Thickness of text, specified as one of the following values: 'Normal ' \| 'Bold' <br> Default: 'Normal' <br> - Font Angle - Text character angle, specified as one of the following values: 'Normal'\|'Italic' <br> Default: 'Normal' <br> - Color - Color of text, specified as vector of RGB values between 0 to 1 . <br> Default: [0, 0, 0] <br> - Interpreter - Interpretation of text characters, specified as one of the following values: 'tex'\|'latex'|'none' Default: 'tex' |


| Field | Description |
| :---: | :---: |
| TickLabel | Tick label style, specified as a structure array with the following fields: <br> - FontSize - Font size, specified as data type scalar. <br> Default: 8 <br> - FontWeight - Thickness of text, specified as one of the following values: 'Normal ' \| 'Bold' <br> Default: 'Normal' <br> - Font Angle - Text character angle, specified as one of the following values: 'Normal'\|'Italic' Default: 'Normal' <br> - Color - Color of text, specified as vector of RGB values between 0 to 1 \| character vector of color name | ' none '. For example, for yellow color, specify as one of the following: [1 1 0 0], 'yellow', or 'y'. Default: [0, 0, 0] |
| Grid | Show or hide the grid, specified as one of the following values: 'off'\|'on' <br> Default: ' off' |
| GridColor | Color of the grid lines, specified as one of the following: vector of RGB values in the range $[0,1] \mid$ character vector of color name \| 'none ' For example, for yellow color, specify as one of the following: $\left.\begin{array}{lll}1 & 1 & 0\end{array}\right]$, 'yellow', or 'y'. <br> Default: [0.15, 0. 15, 0.15] |
| XLimMode, YLimMode | Axes limit modes, specified as one of the following values: <br> - 'auto' - The axes limits are based on the data plotted <br> - 'manual' - The values are explicitly set with Xlim, Ylim <br> Default: 'auto' |
| XLim, YLim | Axes limits, specified as an array of the form [min, max] |
| IOGrouping | Grouping of input-output pairs in the plot, specified as one of the following values: ' none' \| 'inputs'|'outputs'|'all' <br> Default: ' $n o n e$ ' |


| Field | Description |
| :--- | :--- |
| InputLabels, OutputLabels | Input and output label styles, specified as a <br> structure array with the following fields: |
|  | FontSize - Font size, specified as data type <br> scalar. <br> Default: 8 <br> Fontweight - Thickness of text, specified as <br> one of the following values: 'Normal ' \| |
| ' Bold' |  |
| Default: 'Normal' |  |


| Field | Description |
| :---: | :---: |
| FreqUnits | Frequency units, specified as one of the following values: <br> - 'Hz' <br> - 'rad/second' <br> - 'rpm' <br> - 'kHz' <br> - 'MHz' <br> - 'GHz' <br> - 'rad/nanosecond' <br> - 'rad/microsecond' <br> - 'rad/millisecond' <br> - 'rad/minute' <br> - 'rad/hour' <br> - 'rad/day' <br> - 'rad/week' <br> - 'rad/month' <br> - 'rad/year' <br> - 'cycles/nanosecond' <br> - 'cycles/microsecond' <br> - 'cycles/millisecond' <br> - 'cycles/hour' <br> - 'cycles/day' <br> - 'cycles/week' <br> - 'cycles/month' <br> - 'cycles/year' <br> Default: 'rad/s' <br> You can also specify 'auto' , which uses frequency units rad/TimeUnit relative to system time units specified in the TimeUnit property. For multiple systems with different time units, the units of the first system are used. |
| FreqScale | Frequency scale, specified as one of the following values: 'linear'\|'log' <br> Default: ' $\log$ ' |
| MagUnits | Magnitude units, specified as one of the following values: 'dB' \| 'abs' <br> Default: ' dB ' |


| Field | Description |
| :--- | :--- |
| MagScale | Magnitude scale, specified as one of the following <br> values: ' linear' \| ' log' <br> Default: ' linear' ' |
| MagLowerLimMode | Enables a lower magnitude limit, specified as one <br> of the following values: 'auto ' \| 'manual ' <br> Default: ' auto ' |
| MagLowerLim | Lower magnitude limit, specified as data type <br> double. |

## See Also

getoptions |identpref|setoptions | spectrumplot
Introduced in R2012a

## spectrumplot

Plot disturbance spectrum of linear identified models

## Syntax

```
spectrumplot(sys)
spectrumplot(sys,line_spec)
spectrumplot(sys1,line_spec1,...,sysN,line_specN)
spectrumplot(ax,
```

$\qquad$

```
spectrumplot(___
        ,plot_options)
spectrumplot(sys,w)
h = spectrumplot(
```

$\qquad$

``` )
```


## Description

spectrumplot(sys) plots the disturbance spectrum of the model, sys. The software chooses the number of points on the plot and the plot frequency range.

If sys is a time-series model, its disturbance spectrum is the same as the model output spectrum. You generally use this function with time-series models.
spectrumplot(sys,line_spec) uses line_spec to specify the line type, marker symbol, and color.
spectrumplot(sys1,line_spec1,...,sysN,line_specN) plots the disturbance spectrum for one or more models on the same axes.

You can mix sys,line_spec pairs with sys models as in spectrumplot (sys1, sys2,line_spec2, sys3). spectrumplot automatically chooses colors and line styles in the order specified by the ColorOrder and LineStyleOrder properties of the current axes.
spectrumplot(ax, $\qquad$ ) plots into the axes with handle ax. All input arguments described for the previous syntaxes also apply here.
spectrumplot ( __ , plot_options) uses plot_options to specify options such as plot title, frequency units, etc. All input arguments described for the previous syntaxes also apply here.
spect rumplot (sys,w) uses w to specify the plot frequencies.

- If $w$ is specified as a 2 -element cell array, $\{w m i n, ~ w m a x\}$, the plot spans the frequency range \{wmin, wmax\}.
- If $w$ is specified as vector, the spectrum is plotted for the specified frequencies.

Specify w as radians/time_unit, where time_unit must equal sys.TimeUnit.
h = spectrumplot (__ ) returns the handle to the spectrum plot. You use the handle to customize the plot. All input arguments described for the previous syntaxes also apply here.

## Input Arguments

sys
Identified linear model.

## Default:

line_spec
Line style, marker, and color of both the line and marker, specified as a character vector. For example, 'b','b+:'.

For more information, see Chart Line .
ax
Plot axes handle.
Specify as a double-precision value.
You can obtain the current axes handle by using the function, gca.

```
plot_options
```

Plot customization options.
Specify as a plot options object.
You use the command, spectrumoptions, to create plot_options. For more information, type help spectrumoptions.
w
Frequency range.
Specify in radians/time_unit, where time_unit must equal sys.TimeUnit.

## Output Arguments

## h

Plot handle for spectrum plot, returned as a double-precision value.

## Examples

Plot Model Output Spectrum for Identified Model
Obtain the identified model.
load iddata9 z9
sys $=\operatorname{ar}(z 9,4)$;
Plot the output spectrum for the model.
spectrumplot(sys);


## Specify Line Width and Marker Style on Spectrum Plot

Obtain the identified model.
load iddata9 z9
sys $=\operatorname{ar}(z 9,4)$;
Specify the line width and marker style for the spectrum plot.
spectrumplot(sys,'k*--');

' $\mathrm{k}^{*}-\mathrm{-}^{\text {' }}$, specifies a dashed line (--) that is black (k), with star markers (*).

## Plot Multiple Models on the Same Axes

Obtain multiple identified models.

```
load iddata9 z9
sys1 = ar(z9,4);
sys2 = ar(z9,2);
```

Plot the output spectrum for both models.

```
spectrumplot(sys1,'b*-',sys2,'g^:');
```

legend('sys1','sys2');


## Specify Plot Axes for Spectrum Plot

Obtain the axes handle for a plot.
load iddata9 z9
sys1 = ar(z9,4);
spectrumplot(sys1);

ax = gca;
ax is the handle for the spectrum plot axes.
Plot the output spectrum for another model on the specified axes.
sys2 $=\operatorname{ar}(z 9,2)$;
hold on;
spectrumplot(ax,sys2,'r*--');
legend('sys1','sys2');


## Specify Options for Spectrum Plot

Specify the plot options.

```
plot_options = spectrumoptions;
plot-options.FreqUnits = 'Hz';
plot_options.FreqScale = 'linear';
plot_options.Xlim = {[0 20]};
plot_options.MagUnits = 'abs';
```

Estimate an AR model.
load iddata9
sys $=\operatorname{ar}(z 9,4)$;
sys = ar(z9,4);
Plot the output spectrum for the model.
spectrumplot(sys,plot_options);


## Specify Spectrum Plot Frequency Range

Obtain the identified model.
load iddata9 z9
sys = ar(z9,4);
Specify the frequency range for the output spectrum plot for the model.
spectrumplot(sys,\{1,1000\});


The 2-element cell array $\{1,1000\}$ specifies the frequency range from $1 \mathrm{rad} / \mathrm{s}$ to $1000 \mathrm{rad} / \mathrm{s}$.

## Get Plot Handle for Spectrum Plot Customization

Obtain the identified model.
load iddata9 z9
sys $=\operatorname{ar}(z 9,4)$;
Get the plot handle for the model spectrum plot.
h = spectrumplot(sys);

(Optional) Specify the plot options, using the plot handle.

```
setoptions(h,'FreqUnits','Hz','FreqScale','linear','Xlim',{[0 20]},'MagUnits','abs');
```



## See Also

Axes | Chart Line \| getoptions \| setoptions \| showConfidence \| spectrum | spectrumoptions Introduced in R2012b

## ss2ss

State coordinate transformation for state-space model

## Syntax

sysT $=$ ss2ss (sys,T)

## Description

Given a state-space model sys with equations

$$
\begin{aligned}
& \dot{x}=A x+B u \\
& y=C x+D u
\end{aligned}
$$

or the innovations form used by the identified state-space (IDSS) models:

$$
\begin{aligned}
\frac{d x}{d t} & =A x+B u+K e \\
y & =C x+D u+e
\end{aligned}
$$

(or their discrete-time counterpart), ss2ss performs the similarity transformation $\bar{x}=T x$ on the state vector $x$ and produces the equivalent state-space model sysT with equations.

$$
\begin{aligned}
& \dot{\bar{x}}=T A T^{-1} \bar{x}+T B u \\
& y=C T^{-1} \overline{\bar{x}}+D u
\end{aligned}
$$

or, in the case of an IDSS model:

$$
\begin{aligned}
& \dot{\bar{x}}=T A T^{-} 1 \bar{x}+T B u+T K e \\
& y=C T^{-} 1 \bar{\chi}+D u+e
\end{aligned}
$$

sysT $=$ ss2ss(sys,T) returns the transformed state-space model sysT given sys and the state coordinate transformation T. The model sys must be in state-space form and the matrix T must be invertible. ss2ss is applicable to both continuous- and discrete-time models.

## Examples

Perform a similarity transform to improve the conditioning of the $A$ matrix.

```
[T,~] = balance(sys.A)
sysb = ss2ss(sys,inv(T))
```


## See Also

balreal|canon

## Introduced before R2006a

## ssdata

Access state-space model data

## Syntax

[a,b,c,d] = ssdata(sys)
[a,b,c,d,Ts] = ssdata(sys)

## Description

$[a, b, c, d]=$ ssdata(sys) extracts the matrix (or multidimensional array) data $A, B, C, D$ from the state-space model (LTI array) sys. If sys is a transfer function or zero-pole-gain model (LTI array), it is first converted to state space. See ss for more information on the format of state-space model data.

If sys appears in descriptor form (nonempty E matrix), an equivalent explicit form is first derived.
If sys has internal delays, $A, B, C, D$ are obtained by first setting all internal delays to zero (creating a zero-order Padé approximation). For some systems, setting delays to zero creates singular algebraic loops, which result in either improper or ill-defined, zero-delay approximations. For these systems, ssdata cannot display the matrices and returns an error. This error does not imply a problem with the model sys itself.

For generalized state-space (genss) models, ssdata returns the state-space models evaluated at the current, nominal value of all control design blocks. To access the dependency of a genss model on its static control design blocks, use the $A, B, C$, and $D$ properties of the model.
$[a, b, c, d, T s]=$ ssdata(sys) also returns the sample time Ts.
You can access the remaining LTI properties of sys with get or by direct referencing. For example:

```
sys.statename
```

For arrays of state-space models with variable numbers of states, use the syntax:
[a,b,c,d] = ssdata(sys,'cell')
to extract the state-space matrices of each model as separate cells in the cell arrays $a, b, c$, and $d$.

See Also<br>dssdata| get|getDelayModel|idssdata| set|ss|tfdata|zpkdata

Introduced before R2006a

## ssest

Estimate state-space model using time-domain or frequency-domain data

## Syntax

```
sys = ssest(data,nx)
sys = ssest(data,nx,Name,Value)
sys = ssest(
```

$\qquad$

``` , opt)
sys = ssest(data,init_sys)
sys = ssest(data,init_sys,opt)
[sys,x0] = ssest(
```

$\qquad$

## Description

## Estimate a State-Space Model

sys = ssest(data, nx ) estimates a continuous-time state-space model sys of order $n \mathrm{x}$, using data data that can be in the time domain or the frequency domain. sys is a model of the following form:

$$
\begin{aligned}
& \dot{x}(t)=A x(t)+B u(t)+K e(t) \\
& y(t)=C x(t)+D u(t)+e(t)
\end{aligned}
$$

$A, B, C, D$, and $K$ are state-space matrices. $u(t)$ is the input, $y(t)$ is the output, $e(t)$ is the disturbance, and $x(t)$ is the vector of $n x$ states.

All entries of $A, B, C$, and $K$ are free estimable parameters by default. $D$ is fixed to zero by default, meaning that there is no feedthrough, except for static systems ( $n x=0$ ).
sys = ssest(data, nx, Name, Value) incorporates additional options specified by one or more name-value pair arguments. For example, estimate a discrete-time model by specifying the sample time 'Ts' name-value pair argument. Use the 'Form', 'Feedthrough', and
' DisturbanceModel ' name-value pair arguments to modify the default behavior of the $A, B, C, D$, and $K$ matrices.
sys = ssest(__, opt) incorporates an option set opt that specifies options such as estimation objective, handling of initial conditions, regularization, and numerical search method used for estimation. You can specify opt after any of the previous input-argument combinations.

## Configure Initial Parameters

sys = ssest(data,init_sys) uses the state-space model init_sys to configure the initial parameterization.
sys = ssest(data,init_sys,opt) estimates the model using an option set opt.

## Return Estimated Initial States

[sys,x0] = ssest( ___ ) returns the value of initial states computed during estimation. You can use this syntax with any of the previous input-argument combinations.

## Examples

## State-Space Model

Estimate a state-space model and compare its response with the measured output.
Load the input-output data, which is stored in an iddata object.
load iddatal z1
Estimate a fourth-order state-space model.
$\mathrm{nx}=4$;
sys = ssest(z1,nx);
Compare the simulated model response with the measured output.
compare(z1,sys)


The plot shows that the fit percentage between the simulated model and the estimation data is greater than $70 \%$.

You can view more information about the estimation by exploring the idss property sys.Report. sys.Report

```
ans =
Status: 'Estimated using SSEST with prediction focus'
            Method: 'SSEST'
InitialState: 'zero'
            N4Weight: 'CVA'
        N4Horizon: [6 10 10]
            Fit: [1x1 struct]
        Parameters: [1x1 struct]
    OptionsUsed: [1x1 idoptions.ssest]
        RandState: []
        DataUsed: [1x1 struct]
    Termination: [1x1 struct]
```

For example, find out more information about the termination conditions.

```
sys.Report.Termination
ans = struct with fields:
    WhyStop: 'No improvement along the search direction with line search.'
    Iterations: 7
    FirstOrderOptimality: 85.9759
            FcnCount: 123
        UpdateNorm: 9.3748
        LastImprovement: 0
```

The report includes information on the number of iterations and the reason the estimation stopped iterating.

## Determine Optimal Estimated Model Order

Load the input-output data $z 1$, which is stored in an iddata object. This is the same data used to estimate a fourth-order model in "State-Space Model" on page 1-1445.
load iddatal z1
Determine the optimal model order by specifying argument nx as a range from 1:10.
$n \mathrm{n}=1$ : 10 ;
sys = ssest(z1,nx);
An automatically generated plot shows the Hankel singular values for models of the orders specified by $n$.


States with relatively small Hankel singular values can be safely discarded. The suggested default order choice is 2 .

Select the model order in the Chosen Order list and click Apply.

## Identify State-Space Model with Input Delay

Load time-domain system response data.
load iddata7 z7;
Identify a fourth-order state-space model of the data. Specify a known delay of 2 seconds for the first input and 0 seconds for the second input.

```
nx = 4;
sys = ssest(z7(1:300),nx,'InputDelay',[2;0]);
```


## Modify Form, Feedthrough, and Disturbance-Model Matrices

Modify the canonical form of the A, B, and C matrices, include a feedthrough term in the D matrix, and eliminate disturbance-model estimation in the K matrix.

Load input-output data and estimate a fourth-order system using the ssest default options.

```
load iddatal z1
sys1 = ssest(z1,4);
```

Specify the companion form and compare the A matrix with the default A matrix.

```
sys2 = ssest(z1,4,'Form','companion');
A1 = sys1.A
A1 = 4×4
\begin{tabular}{rrrr}
-0.5155 & -3.8483 & 0.6657 & -0.2666 \\
5.8665 & -2.7285 & 1.0649 & -1.4694 \\
-0.4487 & 0.9308 & -0.6235 & 18.8148 \\
-0.4192 & 0.5595 & -16.0688 & 0.5399
\end{tabular}
A2 = sys2.A
A2 = 4\times4
103 x
\begin{tabular}{rrrr}
0 & 0 & 0 & -7.1122 \\
0.0010 & 0 & 0 & -0.9547 \\
0 & 0.0010 & 0 & -0.3263 \\
0 & 0 & 0.0010 & -0.0033
\end{tabular}
```

Include a feedthrough term and compare D matrices.

```
sys3 = ssest(z1,4,'Feedthrough',1);
D1 = sys1.D
D1 = 0
D3 = sys3.D
D3 = 0.0339
```

Eliminate disturbance modeling and compare K matrices.

```
sys4 = ssest(zl,4,'DisturbanceModel','none');
K1 = sys1.K
K1 = 4×1
    0.0520
    0.0973
    0.0151
    0.0270
K4 = sys4.K
K4 = 4×1
    0
    0
    0
    0
```


## Estimate Initial States as Independent Parameters

Specify ssest estimate initial states as independent estimation parameters.
ssest can handle initial states using one of several methods. By default, ssest chooses the method automatically based on your estimation data. You can choose the method yourself by modifying the option set using ssestOptions.

Load the input-output data z1 and estimate a second-order state-space model sys using the default options. Use the syntax that returns initial states $x 0$.

```
load iddata1 z1
[sys,x0] = ssest(z1,2);
x0
x0 = 2x1
    0
    0
```

By default, the estimation is performed using the 'auto' setting for InitialState. Find out which method ssest applied by reviewing the value of InitialState in sys.Report.

```
sys.Report.InitialState
ans =
'zero'
```

The software applied the 'zero ' method, meaning that the software set the initial states to zero instead of estimating them. This selection is consistent with the 0 values returned for x 0 .

Specify that ssest estimate the initial states instead as independent parameters using the 'estimate' setting. Use ssestOptions to create a modified option set and specify that option set to estimate a new model.

```
opt = ssestOptions('InitialState','estimate');
[sys1,x0] = ssest(z1,2,opt);
x0
x0 = 2x1
    0.0068
    0.0052
```

$x 0$ now has estimated parameters with nonzero values.

## Estimate State-Space Model Using Regularization

Obtain a regularized fifth-order state-space model for a second-order system from a narrow bandwidth signal.

Load estimation data.
load regularizationExampleData eData;
Create the transfer function model used for generating the estimation data (true system).

```
trueSys = idtf([0.02008 0.04017 0.02008],[1 -1.561 0.6414],1);
```

Estimate an unregularized state-space model.
opt = ssestOptions('SearchMethod','lm');

```
m = ssest(eData,5,'form','modal','DisturbanceModel','none','Ts',eData.Ts,opt);
```

Estimate a regularized state-space model.

```
opt.Regularization.Lambda = 10;
```

mr = ssest(eData, $5, '$ form','modal','DisturbanceModel', 'none','Ts',eData.Ts,opt);

Compare the model outputs with the estimation data.
compare(eData,m,mr);

Simulated Response Comparison


Compare the model impulse responses.

```
impulse(trueSys,m,mr,50);
legend('trueSys','m','mr');
```



## Estimate Partially Known State-Space Model Using Structured Estimation

Estimate a state-space model of measured input-output data. Configure the parameter constraints and initial values for estimation using a state-space model.

Create an idss model to specify the initial parameterization for estimation.

```
A = blkdiag([-0.1 0.4; -0.4 -0.1],[-1 5; -5 -1]);
B = [1; zeros(3,1)];
C = [1 1 1 1];
D = 0;
K = zeros(4,1);
x0 = [0.1 0.1 0.1 0.1];
Ts = 0;
init_sys = idss(A,B,C,D,K,x0,Ts);
```

Setting all entries of K to 0 creates an idss model with no state disturbance element.
Use the Structure property to fix the values of some of the model parameters. Configure the model so that B and K are fixed, and only the nonzero entries of A are estimable.

```
init_sys.Structure.A.Free = (A~=0);
init_sys.Structure.B.Free = false;
init_sys.Structure.K.Free = false;
```

The entries in init_sys.Structure.A.Free determine whether the corresponding entries in init_sys.A are free (true) or fixed (false).

Load the measured data and estimate a state-space model using the parameter constraints and initial values specified by init_sys.

```
load iddata2 z2;
```

sys = ssest(z2,init_sys);

The estimated parameters of sys satisfy the constraints specified by init_sys.

## Input Arguments

## data - Estimation data

iddata object | frd object | idfrd object
Estimation data, specified as an iddata object, an frd object, or an idfrd object.
For time-domain estimation, data must be an iddata object containing the input and output signal values.

For frequency-domain estimation, data can be one of the following:

- Recorded frequency response data (frd or idfrd)
- iddata object with properties specified as follows.
- InputData - Fourier transform of the input signal
- OutputData - Fourier transform of the output signal
- Domain - 'Frequency'

Estimation data must be uniformly sampled. By default, the software sets the sample time of the model to the sample time of the estimation data.

For multiexperiment data, the sample times and intersample behavior of all the experiments must match.

The domain of your data determines the type of model you can estimate.

- Time-domain or discrete-time frequency-domain data - Continuous-time and discrete-time models
- Continuous-time frequency-domain data - Continuous-time models only


## nx - Order of estimated model

1:10 (default) | positive integer scalar | positive integer vector | 0
Order of the estimated model, specified as a nonnegative integer or as a vector containing a range of positive integers.

- If you already know what order you want your estimated model to have, specify nx as a scalar.
- If you want to compare a range of potential orders to choose the most effective order for your estimated model, specify the range in $n x$. ssest creates a Hankel singular-value plot that shows the relative energy contributions of each state in the system. States with relatively small Hankel singular values contribute little to the accuracy of the model and can be discarded with little
impact. The index of the highest state you retain is the model order. The plot window includes a suggestion for the order to use. You can accept this suggestion or enter a different order. For an example, see "Determine Optimal Estimated Model Order" on page 1-1446.

If you do not specify $n x$, or if you specify $n x$ as best, the software automatically chooses $n x$ from the range 1:10.

- If you are identifying a static system, set nx to 0 .


## opt - Estimation options

ssestOptions option set
Estimation options, specified as an ssestOptions option set. Options specified by opt include:

- Estimation objective
- Handling of initial conditions
- Regularization
- Numerical search method used for estimation

For examples showing how to use opt, see "Estimate Initial States as Independent Parameters" on page 1-1448 and "Estimate State-Space Model Using Regularization" on page 1-1449.

## init_sys - Linear system that configures initial parameterization of sys

idss model | linear model | structure
Linear system that configures the initial parameterization of sys, specified as an idss model or as a structure. You obtain init_sys by either performing an estimation using measured data or by direct construction.

If init_sys is an idss model, ssest uses the parameter values of init_sys as the initial guess for estimating sys. For information on how to specify idss, see "Estimate State-Space Models with Structured Parameterization". ssest honors constraints on the parameters of init_sys, such as fixed coefficients and minimum/maximum bounds.

Use the Structure property of init_sys to configure initial parameter values and constraints for the $A, B, C, D$, and $K$ matrices. For example:

- To specify an initial guess for the $A$ matrix of init_sys, set init_sys.Structure.A.Value as the initial guess.
- To specify constraints for the $B$ matrix of init_sys:
- Set init_sys.Structure.B.Minimum to the minimum $B$ matrix value
- Set init_sys.Structure.B.Maximum to the maximum $B$ matrix value
- Set init_sys.Structure.B.Free to indicate if entries of the $B$ matrix are free parameters for estimation

To set more complex constraints, such as interdependence of coefficients, use grey-box estimation using greyest and idgrey.

You must assign finite initial values for all matrix parameters.
If init_sys is not a state-space (idss) model, the software first converts init_sys to an idss model. ssest uses the parameters of the resulting model as the initial guess for estimation.

If you do not specify opt and init_sys was obtained by estimation, then the software uses estimation options from init_sys.Report.OptionsUsed.

For an example, see "Estimate Partially Known State-Space Model Using Structured Estimation" on page 1-1451.

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.
Example: sys $=$ ssest(data, nx,'Ts',0.1)

## Ts - Sample time of estimated model

0 (continuous time) (default) | sample time of data (data.Ts) | positive scalar
Sample time of the estimated model, specified as the comma-separated pair consisting of ' Ts ' and either 0 or a positive scalar.

- For continuous-time models, specify 'Ts' as 0.
- For discrete-time models, specify 'Ts' as the data sample time in the units stored in the TimeUnit property.


## InputDelay - Input delays

0 (default) | scalar | vector
Input delay for each input channel, specified as the comma-separated pair consisting of ' InputDelay' and a numeric vector.

- For continuous-time models, specify 'InputDelay' in the time units stored in the TimeUnit property.
- For discrete-time models, specify 'InputDelay' in integer multiples of the sample time Ts. For example, setting 'InputDelay' to 3 specifies a delay of three sampling periods.

For a system with $N_{u}$ inputs, set InputDelay to an $N_{u}$-by-1 vector. Each entry of this vector is a numerical value that represents the input delay for the corresponding input channel. For an example, see "Identify State-Space Model with Input Delay" on page 1-1447.

To apply the same delay to all channels, specify InputDelay as a scalar.

## Form - Type of canonical form

'free' (default)|'modal' | 'companion' | 'canonical'
Type of canonical form of sys, specified as the comma-separated pair consisting of 'Form' and one of the following values:

- 'free' - All entries of the matrices $A, B, C, D$, and $K$ are treated as free.
- 'modal' - Obtain sys in modal form.
- 'companion' - Obtain sys in companion form.
- 'canonical' - Obtain sys in the observability canonical form.

For definitions of the canonical forms, see "Canonical State-Space Realizations".

For more information, see "Estimate State-Space Models with Canonical Parameterization". For an example, see "Modify Form, Feedthrough, and Disturbance-Model Matrices" on page 1-1447.

## Feedthrough - Direct feedthrough from input to output

0 (default) | 1 | logical vector
Direct feedthrough from input to output, specified as the comma-separated pair consisting of 'Feedthrough ' and a logical vector of length $N_{u}$, where $N_{u}$ is the number of inputs. If you specify Feedthrough as a logical scalar, that value is applied to all the inputs. For static systems, the software always assumes 'Feedthrough' is 1.

For an example, see "Modify Form, Feedthrough, and Disturbance-Model Matrices" on page 1-1447.

## DisturbanceModel - Option to estimate time-domain noise component parameters

'estimate' (default)|'none'
Option to estimate time-domain noise component parameters in the K matrix, specified as the commaseparated pair consisting of 'DisturbanceModel' and one of the following values:

- 'estimate' - Estimate the noise component. The $K$ matrix is treated as a free parameter.
- 'none' - Do not estimate the noise component. The elements of the $K$ matrix are fixed at zero.

For frequency-domain data, the software assumes that 'DisturbanceModel' is 'none '.
For an example, see "Modify Form, Feedthrough, and Disturbance-Model Matrices" on page 1-1447.

## Output Arguments

## sys - Identified state-space model

idss model
Identified state-space model, returned as an idss model. This model is created using the specified model orders, delays, and estimation options.

Information about the estimation results and options used is stored in the Report property of the model. Report has the following fields.

| Report <br> Field | Description |
| :--- | :--- |
| Status | Summary of the model status, which indicates whether the model was created by <br> construction or obtained by estimation. |
| Method | Estimation command used. |


| Report Field | Description |  |
| :---: | :---: | :---: |
| InitialSt ate | How initial states were handled during estimation, returned as one of the following values: <br> - 'zero' - The initial state is set to zero. <br> - 'estimate' - The initial state is treated as an independent estimation parameter. <br> - 'backcast' - The initial state is estimated using the best least squares fit. <br> - Column vector of length $N x$, where $N x$ is the number of states. For multi-experiment data, a matrix with $N e$ columns, where $N e$ is the number of experiments. <br> - Parametric initial condition object (x0obj) created using idpar. Only for discretetime state-space models. <br> This field is especially useful when the InitialState option in the estimation option set is 'auto'. |  |
| N4Weight | Weighting scheme used for singular-value decomposition by the N4SID algorithm, returned as one of the following values: <br> - 'MOESP' - Uses the MOESP algorithm. <br> - 'CVA' - Uses the Canonical Variate Algorithm. <br> - 'SSARX' - A subspace identification method that uses an ARX estimation-based algorithm to compute the weighting. <br> This option is especially useful when the N4Weight option in the estimation option set is 'auto'. |  |
| N4Horizon | Forward and backward prediction horizons used by the N4SID algorithm, returned as a row vector with three elements - [r sy su], where $r$ is the maximum forward prediction horizon. sy is the number of past outputs, and su is the number of past inputs that are used for the predictions. |  |
| Fit | Quantitative assessment of the estimation, returned as a structure. See "Loss Function and Model Quality Metrics" for more information on these quality metrics. The structure has the following fields: |  |
|  | Field | Description |
|  | FitPerce <br> nt | Normalized root mean squared error (NRMSE) measure of how well the response of the model fits the estimation data, expressed as the percentage fit $=100(1-\mathrm{NRMSE})$. |
|  | LossFen | Value of the loss function when the estimation completes. |
|  | MSE | Mean squared error (MSE) measure of how well the response of the model fits the estimation data. |
|  | FPE | Final prediction error for the model. |
|  | AIC | Raw Akaike Information Criteria (AIC) measure of model quality. |
|  | AICc | Small sample-size corrected AIC. |
|  | nAIC | Normalized AIC. |
|  | BIC | Bayesian Information Criteria (BIC). |


| Report Field | Description |  |
| :---: | :---: | :---: |
| Parameter <br> s | Estimated values of model parameters. |  |
| OptionsUs ed | Option set used for estimation. If no custom options were configured, this is a set of default options. See ssestOptions for more information. |  |
| RandState | State of the random number stream at the start of estimation. Empty, [ ], if randomization was not used during estimation. For more information, see rng. |  |
| DataUsed | Attributes of the data used for estimation. Structure with the following fields: |  |
|  | Field | Description |
|  | Name | Name of the data set. |
|  | Type | Data type. |
|  | Length | Number of data samples. |
|  | Ts | Sample time. This is equivalent to Data.Ts. |
|  | InterSam ple | Input intersample behavior. One of the following values: <br> - 'zoh ' - Zero-order hold maintains a piecewise-constant input signal between samples. <br> - ' foh ' - First-order hold maintains a piecewise-linear input signal between samples. <br> - 'bl' - Band-limited behavior specifies that the continuous-time input signal has zero power above the Nyquist frequency. <br> The value of Intersample has no effect on estimation results for discrete-time models. |
|  | Input0ff set | Offset removed from time-domain input data during estimation. |
|  | $\begin{aligned} & \text { Output0f } \\ & \text { fset } \end{aligned}$ | Offset removed from time-domain output data during estimation. |


| Report Field | Description |  |
| :---: | :---: | :---: |
| Terminati on | Termination conditions for the iterative search used for prediction error minimization, returned as a structure with the following fields: |  |
|  | Field | Description |
|  | WhyStop | Reason for terminating the numerical search. |
|  | Iteratio ns | Number of search iterations performed by the estimation algorithm. |
|  | First0rd erOptima lity | $\infty$-norm of the gradient search vector when the search algorithm terminates. |
|  | FcnCount | Number of times the objective function was called. |
|  | UpdateNo rm | Norm of the gradient search vector in the last iteration. Omitted when the search method is 'lsqnonlin' or 'fmincon'. |
|  | LastImpr ovement | Criterion improvement in the last iteration, expressed as a percentage. Omitted when the search method is 'lsqnonlin' or 'fmincon'. |
|  | $\begin{aligned} & \text { Algorith } \\ & \mathrm{m} \end{aligned}$ | Algorithm used by ' lsqnonlin' or 'fmincon' search method. Omitted when other search methods are used. |
|  | For estimation methods that do not require numerical search optimization, the Termination field is omitted. |  |

For more information on using Report, see "Estimation Report".
x0 - Initial states computed during estimation
column vector $\mid$ matrix
Initial states computed during the estimation, returned as an array containing a column vector corresponding to each experiment.

This array is also stored in the Parameters field of the model Report property.
For an example, see "Estimate Initial States as Independent Parameters" on page 1-1448.

## Algorithms

ssest initializes the parameter estimates using either a noniterative subspace approach or an iterative rational function estimation approach. It then refines the parameter values using the prediction error minimization approach. For more information, see pem and ssest0ptions.

## References

[1] Ljung, L. System Identification: Theory for the User, Second Edition. Upper Saddle River, NJ: Prentice Hall PTR, 1999.

## Extended Capabilities

## Automatic Parallel Support

Accelerate code by automatically running computation in parallel using Parallel Computing Toolbox ${ }^{\mathrm{TM}}$.
Parallel computing support is available for estimation using the lsqnonlin search method (requires
Optimization Toolbox). To enable parallel computing, use ssest0ptions, set SearchMethod to 'lsqnonlin', and set Search0ptions.Advanced.UseParallel to true.

For example:
opt = ssestOptions;
opt.SearchMethod = 'lsqnonlin';
opt.Search0ptions.Advanced.UseParallel = true;

## See Also

## Functions

canon |iddata|idfrd|idgrey|idss|n4sid|pem|polyest|procest|ssest0ptions| ssregest|tfest

## Live Editor Tasks

## Estimate State Space Model

## Topics

"Estimate State-Space Models at the Command Line"
"Estimate State-Space Models with Free-Parameterization"
"Estimate State-Space Models with Canonical Parameterization"
"Estimate State-Space Models with Structured Parameterization"
"Use State-Space Estimation to Reduce Model Order"
"What Are State-Space Models?"
"Supported State-Space Parameterizations"
"State-Space Model Estimation Methods"
"Regularized Estimates of Model Parameters"
"Estimating Models Using Frequency-Domain Data"

## Introduced in R2012a

## ssestOptions

Option set for ssest

## Syntax

```
opt = ssestOptions
```

opt = ssestOptions(Name,Value)

## Description

opt $=$ ssestOptions creates the default option set for ssest.
opt $=$ ssestOptions(Name, Value) creates an option set with the options specified by one or more Name, Value pair arguments.

## Input Arguments

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.

## InitializeMethod - Algorithm used to initialize the state-space parameters

'auto' (default) | 'n4sid'|'lsrf'
Algorithm used to initialize the state-space parameter values for ssest, specified as one of the following values:

- 'auto' - ssest selects automatically:
- lsrf, if the system is non-MIMO, the data is frequency-domain, and the state-space parameters are real-valued.
- n4sid otherwise (time-domain, MIMO, or with complex-valued state-space parameters).
- 'n4sid ' - Subspace state-space estimation approach - can be used with all systems (see n4sid).
- 'lsrf' - Least-squares rational function estimation-based approach [7] (see "Continuous-Time Transfer Function Estimation Using Continuous-Time Frequency-Domain Data" on page 1-1574) can provide higher-accuracy results for non-MIMO frequency-domain systems with real-valued state-space parameters, but cannot be used for any other systems (time-domain, MIMO, or with complex-valued state-space parameters).

InitialState - Handling of initial states
'auto' (default) | 'zero' | 'estimate' | 'backcast' | vector | parametric initial condition object (x0obj)

Handling of initial states during estimation, specified as one of the following values:

- 'zero' - The initial state is set to zero.
- 'estimate' - The initial state is treated as an independent estimation parameter.
- 'backcast ' - The initial state is estimated using the best least squares fit.
- 'auto' - ssest chooses the initial state handling method, based on the estimation data. The possible initial state handling methods are 'zero', 'estimate' and 'backcast'.
- Vector of doubles - Specify a column vector of length $N x$, where $N x$ is the number of states. For multi-experiment data, specify a matrix with $N e$ columns, where $N e$ is the number of experiments. The specified values are treated as fixed values during the estimation process.
- Parametric initial condition object (x0obj) - Specify initial conditions by using idpar to create a parametric initial condition object. You can specify minimum/maximum bounds and fix the values of specific states using the parametric initial condition object. The free entries of x0obj are estimated together with the idss model parameters.

Use this option only for discrete-time state-space models.

## N4Weight - Weighting scheme used for singular-value decomposition by the N4SID algorithm <br> 'auto' (default)|'MOESP' | 'CVA'|'SSARX'

Weighting scheme used for singular-value decomposition by the N4SID algorithm, specified as one of the following values:

- 'MOESP' - Uses the MOESP algorithm by Verhaegen [2].
- 'CVA' - Uses the Canonical Variate Algorithm by Larimore [1].
- 'SSARX' - A subspace identification method that uses an ARX estimation based algorithm to compute the weighting.

Specifying this option allows unbiased estimates when using data that is collected in closed-loop operation. For more information about the algorithm, see [6].

- 'auto' - The estimating function chooses between the MOESP and CVA algorithms.


## N4Horizon - Forward- and backward-prediction horizons used by the N4SID algorithm 'auto' (default) | vector [r sy su] |k-by-3 matrix

Forward and backward prediction horizons used by the N4SID algorithm, specified as one of the following values:

- A row vector with three elements - [r sy su], where $r$ is the maximum forward prediction horizon. The algorithm uses up to $r$ step-ahead predictors. sy is the number of past outputs, and $s u$ is the number of past inputs that are used for the predictions. See pages 209 and 210 in [4] for more information. These numbers can have a substantial influence on the quality of the resulting model, and there are no simple rules for choosing them. Making 'N4Horizon' a k-by-3 matrix means that each row of 'N4Horizon' is tried, and the value that gives the best (prediction) fit to data is selected. k is the number of guesses of [ $r$ sy su] combinations. If you specify N4Horizon as a single column, $r=s y=$ su is used.
- 'auto' - The software uses an Akaike Information Criterion (AIC) for the selection of sy and su.


## Focus - Error to be minimized

'prediction' (default)|'simulation'
Error to be minimized in the loss function during estimation, specified as the comma-separated pair consisting of 'Focus' and one of the following values:

- 'prediction ' - The one-step ahead prediction error between measured and predicted outputs is minimized during estimation. As a result, the estimation focuses on producing a good predictor model.
- 'simulation' - The simulation error between measured and simulated outputs is minimized during estimation. As a result, the estimation focuses on making a good fit for simulation of model response with the current inputs.

The Focus option can be interpreted as a weighting filter in the loss function. For more information, see "Loss Function and Model Quality Metrics".

## WeightingFilter - Weighting prefilter

[] (default) | vector | matrix | cell array | linear system | 'inv ' 'invsqrt '
Weighting prefilter applied to the loss function to be minimized during estimation. To understand the effect of WeightingFilter on the loss function, see "Loss Function and Model Quality Metrics".

Specify WeightingFilter as one of the following values:

- [ ] - No weighting prefilter is used.
- Passbands - Specify a row vector or matrix containing frequency values that define desired passbands. You select a frequency band where the fit between estimated model and estimation data is optimized. For example, [ $\mathrm{wl} \mathrm{l}, \mathrm{wh}$ ] where wl and wh represent lower and upper limits of a passband. For a matrix with several rows defining frequency passbands,
$[w 1 l, w 1 h ; w 2 l, w 2 h ; w 3 l, w 3 h ; \ldots]$, the estimation algorithm uses the union of the frequency ranges to define the estimation passband.

Passbands are expressed in rad/TimeUnit for time-domain data and in FrequencyUnit for frequency-domain data, where TimeUnit and FrequencyUnit are the time and frequency units of the estimation data.

- SISO filter - Specify a single-input-single-output (SISO) linear filter in one of the following ways:
- A SISO LTI model
- $\{A, B, C, D\}$ format, which specifies the state-space matrices of a filter with the same sample time as estimation data.
- \{numerator, denominator\} format, which specifies the numerator and denominator of the filter as a transfer function with same sample time as estimation data.

This option calculates the weighting function as a product of the filter and the input spectrum to estimate the transfer function.

- Weighting vector - Applicable for frequency-domain data only. Specify a column vector of weights. This vector must have the same length as the frequency vector of the data set, Data. Frequency. Each input and output response in the data is multiplied by the corresponding weight at that frequency.
- 'invsqrt ' - Applicable for frequency-domain data only, with InitializeMethod set to ' lsrf' only. Uses $1 / \sqrt{|G(\omega)|}$ as the weighting filter, where $G(\omega)$ is the complex frequencyresponse data. Use this option for capturing relatively low amplitude dynamics in data.
- 'inv' - Applicable for frequency-domain data only, with InitializeMethod set to 'lsrf' only. Uses $1 /|G(\omega)|$ as the weighting filter. Similarly to 'invsqrt', this option captures relatively low-amplitude dynamics in data. Use it when 'invsqrt' weighting produces an estimate that is missing dynamics in the low-amplitude regions. 'inv' is more sensitive to noise than 'invsqrt'.


## EnforceStability - Control whether to enforce stability of model <br> false (default)|true

Control whether to enforce stability of estimated model, specified as the comma-separated pair consisting of 'EnforceStability' and either true or false.

Data Types: logical

## EstimateCovariance - Control whether to generate parameter covariance data true (default) | false

Controls whether parameter covariance data is generated, specified as true or false.
If EstimateCovariance is true, then use getcov to fetch the covariance matrix from the estimated model.

## Display - Specify whether to display the estimation progress

'off' (default)|'on'
Specify whether to display the estimation progress, specified as one of the following values:

- 'on ' - Information on model structure and estimation results are displayed in a progress-viewer window.
- 'off' - No progress or results information is displayed.


## InputOffset - Removal of offset from time-domain input data during estimation

[ ] (default) | vector of positive integers | matrix
Removal of offset from time-domain input data during estimation, specified as the comma-separated pair consisting of 'InputOffset' and one of the following:

- A column vector of positive integers of length $N u$, where $N u$ is the number of inputs.
- [ ] - Indicates no offset.
- Nu-by-Ne matrix - For multi-experiment data, specify InputOffset as an Nu -by-Ne matrix. Nu is the number of inputs, and $N e$ is the number of experiments.

Each entry specified by Input0ffset is subtracted from the corresponding input data.

## OutputOffset - Removal of offset from time-domain output data during estimation

[] (default) | vector | matrix
Removal of offset from time-domain output data during estimation, specified as the comma-separated pair consisting of 'OutputOffset' and one of the following:

- A column vector of length $N y$, where $N y$ is the number of outputs.
- [] - Indicates no offset.
- $N y$-by-Ne matrix - For multi-experiment data, specify OutputOffset as a $N y$-by-Ne matrix. $N y$ is the number of outputs, and $N e$ is the number of experiments.

Each entry specified by OutputOffset is subtracted from the corresponding output data.

## OutputWeight - Weighting of prediction errors in multi-output estimations

[] (default)|'noise' | positive semidefinite symmetric matrix
Weighting of prediction errors in multi-output estimations, specified as one of the following values:

- 'noise ' - Minimize $\operatorname{det}\left(E^{\prime *} E / N\right)$, where $E$ represents the prediction error and $N$ is the number of data samples. This choice is optimal in a statistical sense and leads to maximum likelihood estimates if nothing is known about the variance of the noise. It uses the inverse of the estimated noise variance as the weighting function.

Note OutputWeight must not be 'noise' if SearchMethod is 'lsqnonlin'.

- Positive semidefinite symmetric matrix ( $W$ ) - Minimize the trace of the weighted prediction error matrix trace (E'*E*W/N) where:
- $E$ is the matrix of prediction errors, with one column for each output, and $W$ is the positive semidefinite symmetric matrix of size equal to the number of outputs. Use $W$ to specify the relative importance of outputs in multiple-output models, or the reliability of corresponding data.
- $N$ is the number of data samples.
- [] - The software chooses between the 'noise' or using the identity matrix for W.

This option is relevant for only multi-output models.

## Regularization - Options for regularized estimation of model parameters

## structure

Options for regularized estimation of model parameters. For more information on regularization, see "Regularized Estimates of Model Parameters".

Regularization is a structure with the following fields:

- Lambda - Constant that determines the bias versus variance tradeoff.

Specify a positive scalar to add the regularization term to the estimation cost.
The default value of zero implies no regularization.
Default: 0

- R - Weighting matrix.

Specify a vector of nonnegative numbers or a square positive semi-definite matrix. The length must be equal to the number of free parameters of the model.

For black-box models, using the default value is recommended. For structured and grey-box models, you can also specify a vector of np positive numbers such that each entry denotes the confidence in the value of the associated parameter.

The default value of 1 implies a value of eye(npfree), where npfree is the number of free parameters.

## Default: 1

- Nominal - The nominal value towards which the free parameters are pulled during estimation.

The default value of zero implies that the parameter values are pulled towards zero. If you are refining a model, you can set the value to 'model' to pull the parameters towards the parameter values of the initial model. The initial parameter values must be finite for this setting to work.

Default: 0

## SearchMethod - Numerical search method used for iterative parameter estimation <br> 'auto' (default)|'gn'|'gna'|'lm'|'grad'|'lsqnonlin' |'fmincon'

Numerical search method used for iterative parameter estimation, specified as the comma-separated pair consisting of 'SearchMethod' and one of the following:

- 'auto' - A combination of the line search algorithms, 'gn', 'lm', 'gna', and 'grad' methods is tried in sequence at each iteration. The first descent direction leading to a reduction in estimation cost is used.
- 'gn' - Subspace Gauss-Newton least squares search. Singular values of the Jacobian matrix less than GnPinvConstant*eps*max(size(J))*norm(J) are discarded when computing the search direction. $J$ is the Jacobian matrix. The Hessian matrix is approximated as $J^{T} J$. If there is no improvement in this direction, the function tries the gradient direction.
- 'gna' - Adaptive subspace Gauss-Newton search. Eigenvalues less than gamma*max(sv) of the Hessian are ignored, where $s v$ contains the singular values of the Hessian. The Gauss-Newton direction is computed in the remaining subspace. gamma has the initial value
InitialGnaTolerance (see Advanced in 'Search0ptions' for more information). This value is increased by the factor LMStep each time the search fails to find a lower value of the criterion in fewer than five bisections. This value is decreased by the factor $2 *$ LMStep each time a search is successful without any bisections.
- 'lm' - Levenberg-Marquardt least squares search, where the next parameter value is - pinv (H $\left.+d^{*} \mathrm{I}\right) *$ grad from the previous one. $H$ is the Hessian, $I$ is the identity matrix, and grad is the gradient. $d$ is a number that is increased until a lower value of the criterion is found.
- 'grad ' - Steepest descent least squares search.
- 'lsqnonlin' - Trust-region-reflective algorithm of lsqnonlin. Requires Optimization Toolbox software.
- 'fmincon' - Constrained nonlinear solvers. You can use the sequential quadratic programming (SQP) and trust-region-reflective algorithms of the fmincon solver. If you have Optimization Toolbox software, you can also use the interior-point and active-set algorithms of the fmincon solver. Specify the algorithm in the SearchOptions.Algorithm option. The fmincon algorithms may result in improved estimation results in the following scenarios:
- Constrained minimization problems when there are bounds imposed on the model parameters.
- Model structures where the loss function is a nonlinear or non smooth function of the parameters.
- Multi-output model estimation. A determinant loss function is minimized by default for multioutput model estimation. fmincon algorithms are able to minimize such loss functions directly. The other search methods such as ' lm ' and 'gn' minimize the determinant loss function by alternately estimating the noise variance and reducing the loss value for a given noise variance value. Hence, the fmincon algorithms can offer better efficiency and accuracy for multi-output model estimations.


## SearchOptions - Option set for the search algorithm

search option set
Option set for the search algorithm, specified as the comma-separated pair consisting of
'SearchOptions ' and a search option set with fields that depend on the value of SearchMethod.

| SearchOptions Structure When SearchMethod is Specified as 'gn ', 'gna', 'lm', 'grad', or |
| :--- |
| 'auto' |
| Field <br> Name Description Default <br> Toleran <br> ce Minimum percentage difference between the current value of the loss <br> function and its expected improvement after the next iteration, specified as <br> a positive scalar. When the percentage of expected improvement is less <br> than Tolerance, the iterations stop. The estimate of the expected loss- <br> function improvement at the next iteration is based on the Gauss-Newton <br> vector computed for the current parameter value. 0.01MaxIter <br> ationsMaximum number of iterations during loss-function minimization, specified <br> as a positive integer. The iterations stop when MaxIterations is reached <br> or another stopping criterion is satisfied, such as Tolerance. |
| Setting MaxIterations = 0 returns the result of the start-up procedure. <br> Use sys. Report. Termination. Iterations to get the actual number <br> of iterations during an estimation, where sys is an idtf model. |


| Field Name | Description |  | Default |
| :---: | :---: | :---: | :---: |
| Advance <br> d | Advanced search settings, specified as a structure with the following fields: |  |  |
|  | Field Name | Description | Default |
|  | GnPinvCons tant | Jacobian matrix singular value threshold, specified as a positive scalar. Singular values of the Jacobian matrix that are smaller than GnPinvConstant*max(size(J)*norm(J)*eps) are discarded when computing the search direction. Applicable when SearchMethod is 'gn'. | 10000 |
|  | InitialGna Tolerance | Initial value of gamma, specified as a positive scalar. Applicable when SearchMethod is 'gna'. | 0.0001 |
|  | LMStartVal ue | Starting value of search-direction length $d$ in the Levenberg-Marquardt method, specified as a positive scalar. Applicable when SearchMethod is ' lm'. | 0.001 |
|  | LMStep | Size of the Levenberg-Marquardt step, specified as a positive integer. The next value of the search-direction length $d$ in the Levenberg-Marquardt method is LMStep times the previous one. Applicable when SearchMethod is ' lm '. | 2 |
|  | MaxBisecti ons | Maximum number of bisections used for line search along the search direction, specified as a positive integer. | 25 |
|  | MaxFunctio nEvaluatio ns | Maximum number of calls to the model file, specified as a positive integer. Iterations stop if the number of calls to the model file exceeds this value. | Inf |
|  | MinParamet erChange | Smallest parameter update allowed per iteration, specified as a nonnegative scalar. | 0 |
|  | RelativeIm provement | Relative improvement threshold, specified as a nonnegative scalar. Iterations stop if the relative improvement of the criterion function is less than this value. | 0 |
|  | StepReduct ion | Step reduction factor, specified as a positive scalar that is greater than 1. The suggested parameter update is reduced by the factor StepReduction after each try. This reduction continues until MaxBisections tries are completed or a lower value of the criterion function is obtained. <br> StepReduction is not applicable for SearchMethod <br> ' lm' (Levenberg-Marquardt method). | 2 |

SearchOptions Structure When SearchMethod is Specified as ' Isqnonlin'

| Field Name | Description | Default |
| :---: | :---: | :---: |
| Function Toleranc e | Termination tolerance on the loss function that the software minimizes to determine the estimated parameter values, specified as a positive scalar. <br> The value of FunctionTolerance is the same as that of opt.SearchOptions.Advanced.TolFun. | 1e-5 |
| StepTole rance | Termination tolerance on the estimated parameter values, specified as a positive scalar. <br> The value of StepTolerance is the same as that of opt.SearchOptions.Advanced.TolX. | 1e-6 |
| MaxItera tions | Maximum number of iterations during loss-function minimization, specified as a positive integer. The iterations stop when MaxIterations is reached or another stopping criterion is satisfied, such as FunctionTolerance. <br> The value of MaxIterations is the same as that of opt.SearchOptions.Advanced.MaxIter. | 20 |
| Advanced | Advanced search settings, specified as an option set for lsqnonlin. <br> For more information, see the Optimization Options table in "Optimization Options" (Optimization Toolbox). | Use optimset('lsqnonl in') to create a default option set. |

SearchOptions Structure When SearchMethod is Specified as 'fmincon'

| Field Name | Description | Default |
| :---: | :---: | :---: |
| Algorithm | fmincon optimization algorithm, specified as one of the following: <br> 'sqp ' - Sequential quadratic programming algorithm. The algorithm satisfies bounds at all iterations, and it can recover from NaN or Inf results. It is not a large-scale algorithm. For more information, see "Large-Scale vs. MediumScale Algorithms" (Optimization Toolbox). 'trust-regionreflective' - Subspace trust-region method based on the interior-reflective Newton method. It is a largescale algorithm. <br> - 'interior-point' -Large-scale algorithm that requires Optimization Toolbox software. The algorithm satisfies bounds at all iterations, and it can recover from NaN or Inf results. <br> - 'active-set' - Requires Optimization Toolbox software. The algorithm can take large steps, which adds speed. It is not a large-scale algorithm. <br> For more information about the algorithms, see "Constrained Nonlinear Optimization Algorithms" (Optimization Toolbox) and "Choosing the Algorithm" (Optimization Toolbox). | 'sqp ' |


| Field Name | Description | Default |
| :--- | :--- | :--- |
| FunctionTolerance | Termination tolerance on the <br> loss function that the software <br> minimizes to determine the <br> estimated parameter values, <br> specified as a positive scalar. | $1 \mathrm{e}-6$ |
| StepTolerance | Termination tolerance on the <br> estimated parameter values, <br> specified as a positive scalar. | $1 \mathrm{e}-6$ |
| MaxIterations | Maximum number of iterations <br> during loss function <br> minimization, specified as a <br> positive integer. The iterations <br> stop when MaxIterations is <br> reached or another stopping <br> criterion is satisfied, such as <br> FunctionTolerance. | 100 |

## Advanced - Additional advanced options

structure
Additional advanced options, specified as a structure with the following fields:

- ErrorThreshold - Specifies when to adjust the weight of large errors from quadratic to linear.

Errors larger than ErrorThreshold times the estimated standard deviation have a linear weight in the loss function. The standard deviation is estimated robustly as the median of the absolute deviations from the median of the prediction errors, divided by 0.7. For more information on robust norm choices, see section 15.2 of [4].

ErrorThreshold $=0$ disables robustification and leads to a purely quadratic loss function. When estimating with frequency-domain data, the software sets ErrorThreshold to zero. For time-domain data that contains outliers, try setting ErrorThreshold to 1.6.

Default: 0

- MaxSize - Specifies the maximum number of elements in a segment when input-output data is split into segments.

MaxSize must be a positive integer.
Default: 250000

- StabilityThreshold - Specifies thresholds for stability tests.

StabilityThreshold is a structure with the following fields:

- s-Specifies the location of the right-most pole to test the stability of continuous-time models. A model is considered stable when its right-most pole is to the left of $s$.

Default: 0

- $z$ - Specifies the maximum distance of all poles from the origin to test stability of discretetime models. A model is considered stable if all poles are within the distance $z$ from the origin.

Default: $1+$ sqrt(eps)

- AutoInitThreshold - Specifies when to automatically estimate the initial conditions.

The initial condition is estimated when

$$
\frac{\left\|y_{p, z}-y_{\text {meas }}\right\|}{\left\|y_{p, e}-y_{\text {meas }}\right\|}>\text { AutoInitThreshold }
$$

- $y_{\text {meas }}$ is the measured output.
- $y_{p, z}$ is the predicted output of a model estimated using zero initial states.
- $y_{p, e}$ is the predicted output of a model estimated using estimated initial states.

Applicable when InitialState is 'auto'.
Default: 1.05

- DDC - Specifies if the Data Driven Coordinates algorithm [5] is used to estimate freely parameterized state-space models.

Specify DDC as one of the following values:

- ' on ' - The free parameters are projected to a reduced space of identifiable parameters using the Data Driven Coordinates algorithm.
- 'off' - All the entries of $A, B$, and $C$ updated directly using the chosen SearchMethod.

Default: ' on '

## Output Arguments

## opt - Option set for ssest

ssest0ptions option set
Option set for ssest, returned as an ssestOptions option set.

## Examples

## Create Default Option Set for State Space Estimation

opt = ssestOptions;

## Specify Options for State Space Estimation

Create an option set for ssest using the 'backcast' algorithm to initialize the state and set the Display to 'on'.
opt = ssestOptions('InitialState','backcast','Display','on');
Alternatively, use dot notation to set the values of opt.

```
opt = ssestOptions;
opt.InitialState = 'backcast';
opt.Display = 'on';
```


## Compatibility Considerations

## Renaming of Estimation and Analysis Options

The names of some estimation and analysis options were changed in R2018a. Prior names still work. For details, see the R2018a release note "Renaming of Estimation and Analysis Options".

## References

[1] Larimore, W.E. "Canonical variate analysis in identification, filtering and adaptive control." Proceedings of the 29th IEEE Conference on Decision and Control, pp. 596-604, 1990.
[2] Verhaegen, M. "Identification of the deterministic part of MIMO state space models." Automatica, Vol. 30, No. 1, 1994, pp. 61-74.
[3] Wills, Adrian, B. Ninness, and S. Gibson. "On Gradient-Based Search for Multivariable System Estimates." Proceedings of the 16th IFAC World Congress, Prague, Czech Republic, July 3-8, 2005. Oxford, UK: Elsevier Ltd., 2005.
[4] Ljung, L. System Identification: Theory for the User. Upper Saddle River, NJ: Prentice-Hall PTR, 1999.
[5] McKelvey, T., A. Helmersson,, and T. Ribarits. "Data driven local coordinates for multivariable linear systems and their application to system identification." Automatica, Volume 40, No. 9, 2004, pp. 1629-1635.
[6] Jansson, M. "Subspace identification and ARX modeling." 13th IFAC Symposium on System Identification, Rotterdam, The Netherlands, 2003.
[7] Ozdemir, A. A., and S. Gumossoy. "Transfer Function Estimation in System identification Toolbox via Vector Fitting." Proceedings of the 20th World Congress of the International Federation of Automatic Control. Toulouse, France, July 2017.

## See Also

ssest
Topics
"Loss Function and Model Quality Metrics"

Introduced in R2012a

## ssform

Quick configuration of state-space model structure

## Syntax

sys1 = ssform(sys,Name,Value)

## Description

sys1 = ssform(sys,Name,Value) specifies the type of parameterization and whether feedthrough and disturbance dynamics are present for the state-space model sys using one or more Name, Value pair arguments.

## Input Arguments

## sys

State-space model

## Default:

## Name-Value Pair Arguments

Specify comma-separated pairs of Name, Value arguments, where Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify several name and value pair arguments in any order as Name1, Value1, ... , NameN, ValueN.

## Form

Specify structure of A, B and C matrices as one of the following values:

- 'free'

All entries of A, B, C are set free

- 'companion'

Companion form of the model where the characteristic polynomial appears in the far-right column of the state matrix A

- 'modal'

Modal decomposition form, where the state matrix A is block diagonal. Each block corresponds to a real or complex-conjugate pair of poles.

You cannot use this value for models with repeated poles.

- 'canonical'

Observability canonical form of A, B, and C matrices, as described in [1].

## Default:

## Feedthrough

Specify whether the model has direct feedthrough from the input $u(t)$ to the output $y(t)$, (whether the elements of the matrix D are nonzero).

Must be a logical vector (true or false) of length equal to the number of inputs ( Nu ).
Feedthrough(i) = false sets sys.Structure.D.Value(:,i) to zero and sys.Structure.D.Free(: i) to false.

Feedthrough(i) = true sets sys.Structure.D.Free(:,i) to true.

Note Specifying this option for a previously estimated model causes the model parameter covariance information to be lost. Use translatecov to recompute the covariance.

## Default:

## DisturbanceModel

Specify whether to estimate the noise component of the model, specified as one of the following values:

- 'none'

The value of the K matrix is fixed to zero.

- 'estimate'

The K matrix is treated as a free parameter

Note Specifying this option for a previously estimated model causes the model parameter covariance information to be lost. Use translatecov to recompute the covariance.

## Default:

## Output Arguments

## sys1

State-space model with configured parameterization, feedthrough, and disturbance dynamics

## Examples

## Convert State-Space Model to Canonical Form

Create a state-space model.
rng('default');
A = randn(2) - 2*eye(2);
$\mathrm{B}=\operatorname{randn}(2,1)$;
C = randn(1,2);

```
D = 0;
K = randn(2,1);
model \(=\) idss (A, B, C, D, K,'Ts', 0);
```

The state-space model has free parameterization and no feedthrough.
Convert the model to observability canonical form.

```
model1 = ssform(model,'Form','canonical');
```


## Estimate State-Space Model Parameters in Canonical Form with Feedthrough

Load the estimation data.

```
load iddatal z1;
```

Create a state-space model.

```
rng('default');
```

A = randn(2) - 2*eye(2);
$B=\operatorname{randn}(2,1)$;
C $=$ randn(1,2);
D $=0$;
$\mathrm{K}=\operatorname{randn}(2,1)$;
model = idss(A,B,C,D,K,'Ts',0);

The state-space model has free parameterization and no feedthrough.
Convert the model to observability canonical form and specify to estimate its feedthrough behavior.

```
model1 = ssform(model,'Form','canonical','Feedthrough', true);
```

Estimate the parameters of the model.

```
model2 = ssest(z1,model1);
```


## Alternatives

Use the Structure property of an idss model to specify the parameterization, feedthrough, and disturbance dynamics by modifying the Value and Free attributes of the A, B, C, D and K parameters.

## References

[1] Ljung, L. System Identification: Theory For the User, Second Edition, Appendix 4A, pp 132-134, Upper Saddle River, N.J: Prentice Hall, 1999.

## See Also

idss|n4sid|ssest

## Topics

"Estimate State-Space Models at the Command Line"
"Supported State-Space Parameterizations"
Introduced in R2012b

## ssregest

Estimate state-space model by reduction of regularized ARX model

## Syntax

```
sys = ssregest(data,nx)
sys = ssregest(data,nx,Name,Value)
sys = ssregest(
```

$\qquad$

``` ,opt)
[sys,x0] = ssregest(
``` \(\qquad\)
``` )
```


## Description

sys = ssregest(data, nx ) estimates a state-space model by reduction of a regularized ARX model.
sys = ssregest(data, nx , Name, Value) specifies additional options using one or more Name, Value pair arguments.
sys = ssregest(__, opt) specifies estimation options that configure the estimation objective, ARX orders, and order reduction options. This syntax can include any of the input argument combinations in the previous syntaxes.
[sys,x0] = ssregest (__ ) returns the value of initial states computed during estimation. This syntax can include any of the input argument combinations in the previous syntaxes.

## Examples

## Estimate State-Space Model by Reduction of Regularized ARX Model

Load estimation data.
load iddata2 z2;
$z 2$ is an iddata object that contains time-domain system response data.
Identify a third-order state-space model.
sys = ssregest(z2,3);

## Estimate State-Space Model With Input Delay

Load estimation data.
load iddata2 z2
Estimate a third-order state-space model with input delay.

```
sys = ssregest(z2,3,'InputDelay',2);
```


## Configure the ARX Orders and Estimation Focus

Load estimation data.
load iddata2 z2;
Specify the order of the regularized ARX model used by the software during estimation. Also, set the estimation focus to simulation.

```
opt = ssregestOptions('ARXOrder',[100 100 1],'Focus','simulation');
```

Identify a third-order state-space model.

```
sys = ssregest(z2,3,opt);
```


## Return Initial State Values Computed During Estimation

Load estimation data.
load iddata2 z2;
Obtain the initial state values when identifying a third-order state-space model.

```
[sys,x0] = ssregest(z2,3);
```


## Compare Regularized State-Space Models Estimated Using Impulse Response and Reduction of ARX Models

Load data.

```
load regularizationExampleData eData;
```

Create a transfer function model used for generating the estimation data (true system).
trueSys $=$ idtf([0.02008 0.04017 0.02008],[1-1.561 0.6414],1);
Obtain regularized impulse response (FIR) model.

```
opt = impulseestOptions('RegularizationKernel','DC');
```

m0 = impulseest(eData,70,opt);

Convert the model into a state-space model and reduce the model order.
m1 = balred(idss(m0),15);
Obtain a second state-space model using regularized reduction of an ARX model.
m2 = ssregest(eData, 15);

Compare the impulse responses of the true system and the estimated models.
impulse(trueSys,m1,m2,50);
legend('trueSys','m1','m2');


## Input Arguments

## data - Estimation data

iddata|idfrd|frd
Estimation data, specified as an iddata, idfrd or frd object.
For time-domain estimation, data must be an iddata object containing the input and output signal values.

For frequency-domain estimation, data can be one of the following:

- Recorded frequency response data (frd or idfrd)
- iddata object with its properties specified as follows:
- InputData - Fourier transform of the input signal
- OutputData - Fourier transform of the output signal
- Domain - 'Frequency'

The sample time Ts of the iddata object must be nonzero.

## nx - Order of estimated model

positive scalar | positive vector | 'best'
Order of the estimated model, specified as a positive scalar or vector.
If nx is a vector, then ssregest creates a plot which you can use to choose a suitable model order. The plot shows the Hankel singular values for models of chosen values in the vector. States with relatively small Hankel singular values can be safely discarded. A default choice is suggested in the plot.

You can also specify $\mathrm{nx}=$ 'best', as in ssregest(data, 'best'), in which case the optimal order is chosen automatically in the 1:10 range.

## opt - Options set for ssregest

ssregest0ptions options set
Estimation options for ssregest, specified as an options set you create using ssregest0ptions.

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, ... , NameN, ValueN.
Example: sys $=$ ssregest(z2,3,'InputDelay',2) specifies a delay of 2 sampling periods.

## Ts - Sample time

sample time of data (data. Ts) (default) | positive scalar | 0
Sample time of the model, specified as 0 or equal to the sample time of data.
For continuous-time models, use Ts $=0$. For discrete-time models, specify Ts as a positive scalar whose value is equal to the data sample time.

## InputDelay - Input delays

## 0 (default) | scalar | vector

Input delay for each input channel, specified as a numeric vector. For continuous-time systems, specify input delays in the time unit stored in the TimeUnit property. For discrete-time systems, specify input delays in integer multiples of the sample time Ts. For example, InputDelay $=3$ means a delay of three sampling periods.

For a system with Nu inputs, set InputDelay to an Nu-by-1 vector. Each entry of this vector is a numerical value that represents the input delay for the corresponding input channel.

You can also set InputDelay to a scalar value to apply the same delay to all channels.

## Form - Type of canonical form

'free' (default) | 'modal' | 'companion' | 'canonical'
Type of canonical form of sys, specified as one of the following values:

- 'modal' - Obtain sys in modal form on page 1-1483.
- 'companion' - Obtain sys in companion form on page 1-1484.
- 'free' - All entries of the $A, B$ and $C$ matrices are treated as free.
- 'canonical' - Obtain sys in the observability canonical form [1].

Use the Form, Feedthrough and DisturbanceModel name-value pair arguments to modify the default behavior of the $A, B, C, D$, and $K$ matrices.

## Feedthrough - Direct feedthrough from input to output

0 (default) | 1 |logical vector
Direct feedthrough from input to output, specified as a logical vector of length $N u$, where $N u$ is the number of inputs. If Feedthrough is specified as a logical scalar, it is applied to all the inputs.

Use the Form, Feedthrough and DisturbanceModel name-value pair arguments to modify the default behavior of the $A, B, C, D$, and $K$ matrices.

## DisturbanceModel - Specify whether to estimate the $K$ matrix <br> 'estimate' (default)|'none'

Specify whether to estimate the $K$ matrix which specifies the noise component, specified as one of the following values:

- 'none ' - Noise component is not estimated. The value of the $K$ matrix is fixed to zero value.
- 'estimate ' - The K matrix is treated as a free parameter.

DisturbanceModel must be 'none' when using frequency-domain data.
Use the Form, Feedthrough and DisturbanceModel name-value pair arguments to modify the default behavior of the $A, B, C, D$, and $K$ matrices.

## Output Arguments

## sys - Estimated state-space model

idss
Estimated state-space model of order $n x$, returned as an idss model object. The model represents:

$$
\begin{aligned}
& \dot{x}(t)=A x(t)+B u(t)+K e(t) \\
& y(t)=C x(t)+D u(t)+e(t)
\end{aligned}
$$

$A, B, C, D$, and $K$ are state-space matrices. $u(t)$ is the input, $y(t)$ is the output, $e(t)$ is the disturbance and $x(t)$ is the vector of $n x$ states.

All the entries of $A, B, C$, and $K$ are free estimable parameters by default. $D$ is fixed to zero by default, meaning that there is no feedthrough, except for static systems ( $\mathrm{nx}=0$ ).

Information about the estimation results and options used is stored in the Report property of the model. Report has the following fields:

| Report <br> Field | Description |
| :--- | :--- |
| Status | Summary of the model status, which indicates whether the model was created by <br> construction or obtained by estimation. |


| Report Field | Description |  |
| :---: | :---: | :---: |
| Method | Estimation command used. |  |
| InitialSt ate | Handling of initial states during estimation, returned as one of the following values: <br> - 'zero' - The initial state was set to zero. <br> - 'estimate ' - The initial state was treated as an independent estimation parameter. <br> This field is especially useful when the InitialState option in the estimation option set is 'auto'. |  |
| ARXOrder | ARX model orders, returned as a matrix of nonnegative integers [na nb nk]. |  |
| Fit | Quantitative assessment of the estimation, returned as a structure. See "Loss Function and Model Quality Metrics" for more information on these quality metrics. The structure has the following fields: |  |
|  | Field | Description |
|  | FitPerce nt | Normalized root mean squared error (NRMSE) measure of how well the response of the model fits the estimation data, expressed as the percentage fit $=100(1-$ NRMSE $)$. |
|  | LossFcn | Value of the loss function when the estimation completes. |
|  | MSE | Mean squared error (MSE) measure of how well the response of the model fits the estimation data. |
|  | FPE | Final prediction error for the model. |
|  | AIC | Raw Akaike Information Criteria (AIC) measure of model quality. |
|  | AICc | Small sample-size corrected AIC. |
|  | nAIC | Normalized AIC. |
|  | BIC | Bayesian Information Criteria (BIC). |
| Parameter s | Estimated values of model parameters. |  |
| OptionsUs ed | Option set used for estimation. If no custom options were configured, this is a set of default options. See ssregestOptions for more information. |  |
| RandState | State of the random number stream at the start of estimation. Empty, [ ], if randomization was not used during estimation. For more information, see rng. |  |


| Report Field | Description |  |
| :---: | :---: | :---: |
| DataUsed | Attributes of the data used for estimation, returned as a structure with the following fields: |  |
|  | Field | Description |
|  | Name | Name of the data set. |
|  | Type | Data type. |
|  | Length | Number of data samples. |
|  | Ts | Sample time. |
|  | InterSam ple | Input intersample behavior, returned as one of the following values: <br> - 'zoh ' - Zero-order hold maintains a piecewise-constant input signal between samples. <br> - ' foh ' - First-order hold maintains a piecewise-linear input signal between samples. <br> - 'bl' - Band-limited behavior specifies that the continuous-time input signal has zero power above the Nyquist frequency. |
|  | Input0ff set | Offset removed from time-domain input data during estimation. For nonlinear models, it is [ ]. |
|  | Output0f fset | Offset removed from time-domain output data during estimation. For nonlinear models, it is []. |

For more information on using Report, see "Estimation Report".

## x0 - Initial states computed during estimation

scalar | matrix
Initial states computed during estimation, returned as a scalar. If data contains multiple experiments, then x 0 is a matrix with each column corresponding to an experiment.

This value is also stored in the Parameters field of the model's Report property.

## More About

## Modal Form

In modal form, $A$ is a block-diagonal matrix. The block size is typically 1 -by- 1 for real eigenvalues and 2-by-2 for complex eigenvalues. However, if there are repeated eigenvalues or clusters of nearby eigenvalues, the block size can be larger.

For example, for a system with eigenvalues ( $\lambda_{1}, \sigma \pm j \omega, \lambda_{2}$ ), the modal $A$ matrix is of the form

$$
\left[\begin{array}{cccc}
\lambda_{1} & 0 & 0 & 0 \\
0 & \sigma & \omega & 0 \\
0 & -\omega & \sigma & 0 \\
0 & 0 & 0 & \lambda_{2}
\end{array}\right]
$$

## Companion Form

In the companion realization, the characteristic polynomial of the system appears explicitly in the rightmost column of the $A$ matrix.

For a system with characteristic polynomial

$$
P(s)=s^{n}+\alpha_{1} s^{n-1}+\ldots+\alpha_{n-1} s+\alpha_{n}
$$

the corresponding companion $A$ matrix is

$$
A=\left[\begin{array}{cccccc}
0 & 0 & 0 & \ldots & 0 & -\alpha_{n} \\
1 & 0 & 0 & \ldots & 0 & -\alpha_{n-1} \\
0 & 1 & 0 & \ldots & 0 & -\alpha_{n-2} \\
0 & 0 & 1 & \ldots & 0 & -\alpha_{n-3} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & \ldots & 1 & -\alpha_{1}
\end{array}\right]
$$

The companion transformation requires that the system be controllable from the first input. The companion form is poorly conditioned for most state-space computations; avoid using it when possible.

## Tips

- ssregest function provides improved accuracy than n4sid for short, noisy data sets.
- For some problems, the quality of fit using n4sid is sensitive to options, such as N4Horizon, whose values can be difficult to determine. In comparison, the quality of fit with ssregest is less sensitive to its options, which makes ssregest simpler to use.


## Algorithms

ss regest estimates a regularized ARX model and converts the ARX model to a state-space model. The software then uses balanced model reduction techniques to reduce the state-space model to the specified order.

## References

[1] Ljung, L. System Identification: Theory For the User, Second Edition, Appendix 4A, pp 132-134, Upper Saddle River, N.J: Prentice Hall, 1999.

## See Also

arx|arxRegul|balred|n4sid|ssest|ssregest0ptions

## Topics

"Regularized Estimates of Model Parameters"

## Introduced in R2014a

## ssregestOptions

Option set for ssregest

## Syntax

options = ssregest0ptions
options $=$ ssregestOptions(Name, Value)

## Description

options $=$ ssregestOptions creates a default option set for ssregest.
options = ssregestOptions(Name, Value) specifies additional options using one or more Name, Value pair arguments.

## Examples

## Create Default Option Set for State-Space Estimation Using Reduction of Regularized ARX Model <br> ```options = ssregestOptions;```

## Specify Options for State-Space Estimation Using Reduction of Regularized ARX Model

Create an option set for ssregest that fixes the value of the initial states to 'zero'. Also, set the Display to 'on'.

```
opt = ssregestOptions('InitialState','zero','Display','on');
```

Alternatively, use dot notation to set the values of opt.

```
opt = ssregestOptions;
opt.InitialState = 'zero';
opt.Display = 'on';
```


## Input Arguments

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.
Example: opt = ssregestOptions('InitialState','zero') fixes the value of the initial states to zero.

## InitialState - Handling of initial states <br> 'estimate' (default)|'zero'

Handling of initial states during estimation, specified as one of the following values:

- 'zero' - The initial state is set to zero.
- 'estimate ' - The initial state is treated as an independent estimation parameter.


## ARXOrder - ARX model orders

'auto ' (default) | matrix of nonnegative integers
ARX model orders, specified as a matrix of nonnegative integers [na nb nk]. The max (ARXOrder) +1 must be greater than the desired state-space model order (number of states). If you specify a value, it is recommended that you use a large value for nb order. To learn more about ARX model orders, see arx.

## RegularizationKernel - Regularizing kernel <br> 'TC' (default) |'SE'|'SS'|'HF'|'DI'|'DC'

Regularizing kernel used for regularized estimates of the underlying ARX model, specified as one of the following values:

- 'TC' - Tuned and correlated kernel
- 'SE' - Squared exponential kernel
- 'SS' - Stable spline kernel
- 'HF' - High frequency stable spline kernel
- 'DI ' - Diagonal kernel
- 'DC' - Diagonal and correlated kernel

For more information, see [1].

## Reduction - Options for model order reduction

structure
Options for model order reduction, specified as a structure with the following fields:

- StateElimMethod

State elimination method. Specifies how to eliminate the weakly coupled states (states with smallest Hankel singular values). Specified as one of the following values:
'MatchDC' Discards the specified states and alters the remaining states to preserve the DC gain.
'Truncate ' Discards the specified states without altering the remaining states. This method tends to product a better approximation in the frequency domain, but the DC gains are not guaranteed to match.

Default: 'Truncate'

- AbsTol, RelTol

Absolute and relative error tolerance for stable/unstable decomposition. Positive scalar values. For an input model $G$ with unstable poles, the reduction algorithm of ssregest first extracts the
stable dynamics by computing the stable/unstable decomposition $G \rightarrow G S+G U$. The AbsTol and RelTol tolerances control the accuracy of this decomposition by ensuring that the frequency responses of $G$ and $G S+G U$ differ by no more than AbsTol + RelTol*abs( $G$ ). Increasing these tolerances helps separate nearby stable and unstable modes at the expense of accuracy. See stabsep for more information.

Default: AbsTol = 0; RelTol = 1e-8

- Offset

Offset for the stable/unstable boundary. Positive scalar value. In the stable/unstable decomposition, the stable term includes only poles satisfying

- $\operatorname{Re}(s)<-0 f f s e t * \max (1,|\operatorname{Im}(s)|)$ (Continuous time)
- $|z|<1$ - Offset (Discrete time)

Increase the value of Offset to treat poles close to the stability boundary as unstable.
Default: 1e-8
Focus - Error to be minimized
'prediction' (default)|'simulation'
Error to be minimized in the loss function during estimation, specified as the comma-separated pair consisting of 'Focus' and one of the following values:

- 'prediction' - The one-step ahead prediction error between measured and predicted outputs is minimized during estimation. As a result, the estimation focuses on producing a good predictor model.
- 'simulation' - The simulation error between measured and simulated outputs is minimized during estimation. As a result, the estimation focuses on making a good fit for simulation of model response with the current inputs.

The Focus option can be interpreted as a weighting filter in the loss function. For more information, see "Loss Function and Model Quality Metrics".

## WeightingFilter - Weighting prefilter

[ ] (default) | vector | matrix | cell array | linear system
Weighting prefilter applied to the loss function to be minimized during estimation. To understand the effect of WeightingFilter on the loss function, see "Loss Function and Model Quality Metrics".

Specify WeightingFilter as one of the following values:

- [] - No weighting prefilter is used.
- Passbands - Specify a row vector or matrix containing frequency values that define desired passbands. You select a frequency band where the fit between estimated model and estimation data is optimized. For example, [ $\mathrm{wl} \mathrm{l}, \mathrm{wh}$ ] where wl and wh represent lower and upper limits of a passband. For a matrix with several rows defining frequency passbands,
[w1l,w1h;w2l,w2h;w3l,w3h;...], the estimation algorithm uses the union of the frequency ranges to define the estimation passband.

Passbands are expressed in rad/TimeUnit for time-domain data and in FrequencyUnit for frequency-domain data, where TimeUnit and FrequencyUnit are the time and frequency units of the estimation data.

- SISO filter - Specify a single-input-single-output (SISO) linear filter in one of the following ways:
- A SISO LTI model
- $\{A, B, C, D\}$ format, which specifies the state-space matrices of a filter with the same sample time as estimation data.
- \{numerator, denominator\} format, which specifies the numerator and denominator of the filter as a transfer function with same sample time as estimation data.

This option calculates the weighting function as a product of the filter and the input spectrum to estimate the transfer function.

- Weighting vector - Applicable for frequency-domain data only. Specify a column vector of weights. This vector must have the same length as the frequency vector of the data set, Data. Frequency. Each input and output response in the data is multiplied by the corresponding weight at that frequency.


## EstimateCovariance - Control whether to generate parameter covariance data true (default) | false

Controls whether parameter covariance data is generated, specified as true or false.
If EstimateCovariance is true, then use getcov to fetch the covariance matrix from the estimated model.

## Display - Specify whether to display the estimation progress

'off' (default)|'on'
Specify whether to display the estimation progress, specified as one of the following values:

- 'on ' - Information on model structure and estimation results are displayed in a progress-viewer window.
- 'off' - No progress or results information is displayed.


## InputOffset - Removal of offset from time-domain input data during estimation

[ ] (default) | vector of positive integers | matrix
Removal of offset from time-domain input data during estimation, specified as the comma-separated pair consisting of 'Input0ffset' and one of the following:

- A column vector of positive integers of length $N u$, where $N u$ is the number of inputs.
- [ ] - Indicates no offset.
- $N u$-by-Ne matrix - For multi-experiment data, specify Input0ffset as an $N u$-by- $N e$ matrix. $N u$ is the number of inputs, and $N e$ is the number of experiments.

Each entry specified by Input0ffset is subtracted from the corresponding input data.

## OutputOffset - Removal of offset from time-domain output data during estimation [] (default) | vector | matrix

Removal of offset from time-domain output data during estimation, specified as the comma-separated pair consisting of 'OutputOffset' and one of the following:

- A column vector of length $N y$, where $N y$ is the number of outputs.
- [] - Indicates no offset.
- $N y$-by-Ne matrix - For multi-experiment data, specify OutputOffset as a $N y$-by- $N e$ matrix. $N y$ is the number of outputs, and $N e$ is the number of experiments.

Each entry specified by Output0ffset is subtracted from the corresponding output data.

## OutputWeight - Weight of prediction errors in multi-output estimation

[ ] (default) | positive semidefinite, symmetric matrix
Weight of prediction errors in multi-output estimation, specified as one of the following values:

- Positive semidefinite, symmetric matrix (W). The software minimizes the trace of the weighted prediction error matrix trace ( $E^{\prime *} E^{* W} / N$ ) where:
- $E$ is the matrix of prediction errors, with one column for each output, and $W$ is the positive semidefinite, symmetric matrix of size equal to the number of outputs. Use $W$ to specify the relative importance of outputs in multiple-output models, or the reliability of corresponding data.
- $N$ is the number of data samples.
- [ ] - No weighting is used. Specifying as [] is the same as eye (Ny), where Ny is the number of outputs.

This option is relevant only for multi-output models.

## Advanced - Advanced estimation options

structure
Advanced options for regularized estimation, specified as a structure with the following fields:

- MaxSize - Maximum allowable size of Jacobian matrices formed during estimation, specified as a large positive number.

Default: 250e3

- SearchMethod - Search method for estimating regularization parameters, specified as one of the following values:
- 'gn': Quasi-Newton line search.
- 'fmincon ': Trust-region-reflective constrained minimizer. In general, 'fmincon ' is better than 'gn' for handling bounds on regularization parameters that are imposed automatically during estimation.

Default: 'fmincon'

## Output Arguments

## options - Option set for ssregest

ssregest0ptions options set
Estimation options for ss regest, returned as an ssregestoptions option set.

## Compatibility Considerations

## Renaming of Estimation and Analysis Options

The names of some estimation and analysis options were changed in R2018a. Prior names still work. For details, see the R2018a release note "Renaming of Estimation and Analysis Options".

## References

[1] T. Chen, H. Ohlsson, and L. Ljung. "On the Estimation of Transfer Functions, Regularizations and Gaussian Processes - Revisited", Automatica, Volume 48, August 2012.

## See Also

ssregest

## Topics

"Loss Function and Model Quality Metrics"
Introduced in R2014a

## stack

Build model array by stacking models or model arrays along array dimensions

## Syntax

sys = stack(arraydim,sys1,sys2,...)

## Description

sys $=$ stack(arraydim,sys1,sys2,...) produces an array of dynamic system models sys by stacking (concatenating) the models (or arrays) sys1,sys2,... along the array dimension arraydim. All models must have the same number of inputs and outputs (the same I/O dimensions), but the number of states can vary. The I/O dimensions are not counted in the array dimensions. For more information about model arrays and array dimensions, see "Model Arrays" (Control System Toolbox).

For arrays of state-space models with variable order, you cannot use the dot operator (e.g., sys.A) to access arrays. Use the syntax

```
[A,B,C,D] = ssdata(sys,'cell')
```

to extract the state-space matrices of each model as separate cells in the cell arrays A, B, C, and D.

## Examples

## Example 1

If sys1 and sys2 are two models:

- stack(1,sys1,sys2) produces a 2-by-1 model array.
- stack(2,sys1,sys2) produces a 1-by-2 model array.
- stack(3,sys1,sys2) produces a 1-by-1-by-2 model array.


## Example 2

Stack identified state-space models derived from the same estimation data and compare their bode responses.

```
load iddatal z1
sysc = cell(1,5);
opt = ssestOptions('Focus','simulation');
for i = 1:5
sysc{i} = ssest(z1,i-1,opt);
end
sysArray = stack(1, sysc{:});
bode(sysArray);
```


## Introduced in R2012a

## step

Step response plot of dynamic system; step response data

## Syntax

```
step(sys)
step(sys,tFinal)
step(sys,t)
step(sys1,sys2,...,sysN, __)
step(sys1,LineSpec1,...,sysN,LineSpecN,___)
step(___,opts)
y = step(sys,t)
[y,tOut] = step(sys)
[y,tOut] = step(sys,tFinal)
[y,t,x] = step(sys)
[y,t,x,ysd] = step(sys)
[___] = step(___,opts)
```


## Description

## Step Response Plots

step (sys) plots the response of a dynamic system model to a step input of unit amplitude. The model sys can be continuous- or discrete-time, and SISO or MIMO. For MIMO systems, the plot displays the step responses for each I/O channel. step automatically determines the time steps and duration of the simulation based on the system dynamics.
step(sys, tFinal ) simulates the step response from $\mathrm{t}=0$ to the final time $\mathrm{t}=\mathrm{tFinal}$. The function uses system dynamics to determine the intervening time steps.
step (sys,t) plots the step response at the times that you specify in the vector t .
step(sys1, sys $2, \ldots$, sysN,__) plots the step response of multiple dynamic systems on the same plot. All systems must have the same number of inputs and outputs. You can use multiple dynamic systems with any of the previous input-argument combinations.
step (sys1,LineSpec1,..., sysN, LineSpecN,___) specifies a color, line style, and marker for each system in the plot. You can use LineSpec with any of the previous input-argument combinations. When you need additional plot customization options, use stepplot instead.
step (__ , opts) specifies additional options for computing the step response, such as the step amplitude or input offset. Use stepData0ptions to create the option set opts. You can use opts with any of the previous input-argument and output-argument combinations.

## Step Response Data

$y=s t e p(s y s, t)$ returns the step response of a dynamic system model sys at the times specified in the vector $t$. This syntax does not draw a plot.
[y,tOut] = step(sys) also returns a vector of times tOut corresponding to the responses in $y$. If you do not provide an input vector $t$ of times, step chooses the length and time step of $t$ Out based on the system dynamics.
[y,tOut] = step(sys,tFinal) computes the step response up to the end time tFinal. step chooses the time step of tOut based on the system dynamics.
$[y, t, x]=$ step(sys) also returns the state trajectories $x$, when sys is a state-space model such as an ss or idss model.
[ $y, t, x, y s d]=$ step(sys) also computes the standard deviation ysd of the step response $y$, when sys is an identified model such as an idss, idtf, or idnlarx model.
[___ ] = step (__ , opts) specifies additional options for computing the step response, such as the step amplitude or input offset. Use stepDataOptions to create the option set opts. You can use opts with any of the previous input-argument and output-argument combinations.

## Examples

## Step Response of Dynamic System

Plot the step response of a continuous-time system represented by the following transfer function.

$$
\operatorname{sys}(s)=\frac{4}{s^{2}+2 s+10} .
$$

For this example, create a tf model that represents the transfer function. You can similarly plot the step response of other dynamic system model types, such as zero-pole gain (zpk) or state-space (ss) models.

```
sys = tf(4,[1 2 10]);
```

Plot the step response.

```
step(sys)
```



The step plot automatically includes a dotted horizontal line indicating the steady-state response. In a MATLAB® figure window, you can right-click on the plot to view other step-response characteristics such as peak response and settling time. For more information about these characteristics, see stepinfo (Control System Toolbox).

## Step Response of Discrete-Time System

Plot the step response of a discrete-time system. The system has a sample time of 0.2 s and is represented by the following state-space matrices.
$A=\left[\begin{array}{ll}1.6 & -0.7 ;\end{array}\right.$
1 0];
$B=[0.5 ; 0] ;$
$C=[0.10 .1] ;$
D = 0;
Create the state-space model and plot its step response.
sys = ss(A,B,C,D,0.2);
step(sys)


The step response reflects the discretization of the model, showing the response computed every 0.2 seconds.

## Step Response at Specified Times

Examine the step response of the following transfer function.

```
sys = zpk(-1,[-0.2+3j,-0.2-3j],1) * tf([1 1],[1 0.05])
sys =
```

|  | $(s+1)^{\wedge} 2$ |
| :---: | :---: |
| ( $\mathrm{s}+0.05$ ) | $\left(s^{\wedge} 2+0.4 s+9.04\right)$ |

Continuous-time zero/pole/gain model.
step(sys)


By default, step chooses an end time that shows the steady state that the response is trending toward. This system has fast transients, however, which are obscured on this time scale. To get a closer look at the transient response, limit the step plot to $t=15 \mathrm{~s}$.
step(sys,15)


Alternatively, you can specify the exact times at which you want to examine the step response, provided they are separated by a constant interval. For instance, examine the response from the end of the transient until the system reaches steady state.
$\mathrm{t}=20: 0.2: 120$;
step(sys,t)


Even though this plot begins at $\mathrm{t}=20$, step always applies the step input at $\mathrm{t}=0$.

## Step Response Plot of MIMO Systems

Consider the following second-order state-space model:

$$
\begin{aligned}
& {\left[\begin{array}{l}
\dot{x}_{1} \\
\dot{x}_{2}
\end{array}\right]=\left[\begin{array}{cc}
-0.5572 & -0.7814 \\
0.7814 & 0
\end{array}\right]\left[\begin{array}{l}
x_{1} \\
x_{2}
\end{array}\right]+\left[\begin{array}{cc}
1 & -1 \\
0 & 2
\end{array}\right]\left[\begin{array}{l}
u_{1} \\
u_{2}
\end{array}\right]} \\
& y=\left[\begin{array}{lll}
1.9691 & 6.4493
\end{array}\right]\left[\begin{array}{l}
x_{1} \\
x_{2}
\end{array}\right]
\end{aligned}
$$

$A=[-0.5572,-0.7814 ; 0.7814,0] ;$
$B=[1,-1 ; 0,2]$;
$C=[1.9691,6.4493]$;
sys $=s s(A, B, C, 0)$;
This model has two inputs and one output, so it has two channels: from the first input to the output, and from the second input to the output. Each channel has its own step response.

When you use step, it computes the responses of all channels.
step(sys)


The left plot shows the step response of the first input channel, and the right plot shows the step response of the second input channel. Whenever you use step to plot the responses of a MIMO model, it generates an array of plots representing all the I/O channels of the model. For instance, create a random state-space model with five states, three inputs, and two outputs, and plot its step response.

```
sys = rss(5,2,3);
step(sys)
```


## Step Response




In a MATLAB figure window, you can restrict the plot to a subset of channels by right-clicking on the plot and selecting I/O Selector.

## Compare Responses of Multiple Systems

step allows you to plot the responses of multiple dynamic systems on the same axis. For instance, compare the closed-loop response of a system with a PI controller and a PID controller. Create a transfer function of the system and tune the controllers.

```
H = tf(4,[1 2 10]);
C1 = pidtune(H,'PI');
C2 = pidtune(H,'PID');
```

Form the closed-loop systems and plot their step responses.

```
sys1 = feedback(H*C1,1);
sys2 = feedback(H*C2,1);
step(sys1,sys2)
legend('PI','PID','Location','SouthEast')
```



By default, step chooses distinct colors for each system that you plot. You can specify colors and line styles using the LineSpec input argument.

```
step(sys1,'r--',sys2,'b')
legend('PI','PID','Location','SouthEast')
```


## Step Response



The first LineSpec 'r-- ' specifies a dashed red line for the response with the PI controller. The second LineSpec 'b' specifies a solid blue line for the response with the PID controller. The legend reflects the specified colors and linestyles. For more plot customization options, use stepplot.

## Step Response of Systems in a Model Array

The example Compare Responses of Multiple Systems shows how to plot responses of several individual systems on a single axis. When you have multiple dynamic systems arranged in a model array, step plots all their responses at once.

Create a model array. For this example, use a one-dimensional array of second-order transfer functions having different natural frequencies. First, preallocate memory for the model array. The following command creates a 1-by-5 row of zero-gain SISO transfer functions. The first two dimensions represent the model outputs and inputs. The remaining dimensions are the array dimensions.

```
sys = tf(zeros(1,1,1,5));
```

Populate the array.

```
w0 = 1.5:1:5.5; % natural frequencies
zeta = 0.5; % damping constant
for i = 1:length(w0)
```

```
    sys(:,:,1,i) = tf(w0(i)^2,[1 2*zeta*w0(i) w0(i)^2]);
end
```

(For more information about model arrays and how to create them, see "Model Arrays" (Control System Toolbox).) Plot the step responses of all models in the array.
step(sys)

step uses the same linestyle for the responses of all entries in the array. One way to distinguish among entries is to use the SamplingGrid property of dynamic system models to associate each entry in the array with the corresponding $w 0$ value.

```
sys.SamplingGrid = struct('frequency',w0);
```

Now, when you plot the responses in a MATLAB figure window, you can click a trace to see which frequency value it corresponds to.

## Step Response Data

When you give it an output argument, step returns an array of response data. For a SISO system, the response data is returned as a column vector of length equal to the number of time points at which the response is sampled. You can provide the vector t of time points, or allow step to select time points for you based on system dynamics. For instance, extract the step response of a SISO system at 101 time points between $t=0$ and $t=5 \mathrm{~s}$.

```
sys = tf(4,[1 2 10]);
t = 0:0.05:5;
y = step(sys,t);
size(y)
ans = 1\times2
    101 1
```

For a MIMO system, the response data is returned in an array of dimensions $N$-by- $N y$-by- Nu , where $N y$ and $N u$ are the number of outputs and inputs of the dynamic system. For instance, consider the following state-space model, representing a two-input, one-output system.

```
A = [-0.5572,-0.7814;0.7814,0];
B = [1,-1;0,2];
C = [1.9691,6.4493];
sys = ss(A,B,C,0);
```

Extract the step response of this system at 200 time points between $t=0$ and $t=20 \mathrm{~s}$.

```
t = linspace(0,20,200);
y = step(sys,t);
size(y)
ans = 1\times3
    200 1 2
```

$y(:, i, j)$ is a column vector containing the step response from the $j$ th input to the ith output at the times $t$. For instance, extract the step response from the second input to the output.

```
y12 = y(:,1,2);
plot(t,y12)
```



## Step Response Plot of Feedback Loop with Delay

Create a feedback loop with delay and plot its step response.

```
s = tf('s');
G = exp(-s) * (0.8* s^2+s+2)/(s^2+s);
sys = feedback(ss(G),1);
step(sys)
```



The system step response displayed is chaotic. The step response of systems with internal delays may exhibit odd behavior, such as recurring jumps. Such behavior is a feature of the system and not software anomalies.

## Response to Custom Step Input

By default, step applies an input signal that changes from 0 to 1 at $t=0$. To customize the amplitude and offset, use stepDataOptions. For instance, compute the response of a SISO state-space model to a signal that changes from 1 to -1 to at $t=0$.

```
A = [1.6 -0.7;
    1 0];
B = [0.5; 0];
C = [0.1 0.1];
D = 0;
sys = ss(A,B,C,D,0.2);
opt = stepDataOptions;
opt.InputOffset = 1;
opt.StepAmplitude = - 2;
step(sys,opt)
```



For responses to arbitrary input signals, use lsim (Control System Toolbox).

## Step Responses of Identified Models with Confidence Regions

Compare the step response of a parametric identified model to a non-parametric (empirical) model. Also view their $3 \sigma$ confidence regions.

Load the data.
load iddatal $z 1$
Estimate a parametric model.
sys1 = ssest(z1,4);
Estimate a non-parametric model.
sys2 = impulseest(z1);
Plot the step responses for comparison.
$\mathrm{t}=(0: 0.1: 10) \mathrm{C}$
[y1, ~, ~, ysd1] = step(sys1,t);
[y2, ~, ~, ysd2] = step(sys2,t);
plot(t, y1, 'b', t, y1+3*ysd1, 'b:', t, y1-3*ysd1, 'b:')
hold on
plot(t, y2, 'g', t, y2+3*ysd2, 'g:', t, y2-3*ysd2, 'g:')


## Step Response of Identified Time-Series Model

Compute the step response of an identified time-series model.
A time-series model, also called a signal model, is one without measured input signals. The step plot of this model uses its (unmeasured) noise channel as the input channel to which the step signal is applied.

Load the data.
load iddata9;
Estimate a time-series model.
sys = ar(z9, 4);
ys is a model of the form A $y(t)=e(t)$, where $e(t)$ represents the noise channel. For computation of step response, $e(t)$ is treated as an input channel, and is named e@y1.

Plot the step response.
step(sys)


## Validate Linearization of Identified Nonlinear ARX Model

Validate the linearization of a nonlinear ARX model by comparing the small amplitude step responses of the linear and nonlinear models.

Load the data.
load iddata2 z2;
Estimate a nonlinear ARX model.
nlsys = nlarx(z2,[4 3 10],'tree','custom', \{'sin(y1(t-2)*ul(t))+y1(t-2)*ul(t)+ul(t).*ul(t-13)','
Determine an equilibrium operating point for $n l$ sys corresponding to a steady-state input value of 1.
$u 0=1$;
[ $\mathrm{X}, \sim, r]=$ findop(nlsys, 'steady', 1 );
y0 = r.SignalLevels.Output;
Obtain a linear approximation of nlsys at this operating point.
sys = linearize(nlsys,u0,X);
Validate the usefulness of sys by comparing its small-amplitude step response to that of nlsys.

The nonlinear system $n l$ sys is operating at an equilibrium level dictated by (u0, y0). Introduce a step perturbation of size 0.1 about this steady-state and compute the corresponding response.

```
opt = stepDataOptions;
opt.InputOffset = u0;
opt.StepAmplitude = 0.1;
t = (0:0.1:10)';
ynl = step(nlsys, t, opt);
```

The linear system sys expresses the relationship between the perturbations in input to the corresponding perturbation in output. It is unaware of the nonlinear system's equilibrium values.

Plot the step response of the linear system.

```
opt = stepDataOptions;
opt.StepAmplitude = 0.1;
yl = step(sys, t, opt);
```

Add the steady-state offset, y 0 , to the response of the linear system and plot the responses.
plot(t, ynl, t, yl+y0)
legend('Nonlinear', 'Linear with offset')


## Input Arguments

## sys - Dynamic system

dynamic system model | model array
Dynamic system, specified as a SISO or MIMO dynamic system model or array of dynamic system models. Dynamic systems that you can use include:

- Continuous-time or discrete-time numeric LTI models, such as tf, zpk, or ss models.
- Generalized or uncertain LTI models such as genss or uss models. (Using uncertain models requires Robust Control Toolbox software.)
- For tunable control design blocks, the function evaluates the model at its current value for both plotting and returning response data.
- For uncertain control design blocks, the function plots the nominal value and random samples of the model. When you use output arguments, the function returns response data for the nominal model only.
- Sparse state-space models such as sparss and mechss models.
- Identified LTI models, such as idtf, idss, or idproc models. For such models, the function can also plot confidence intervals and return standard deviations of the frequency response. See "Step Responses of Identified Models with Confidence Regions" on page 1-1507.
step does not support frequency-response data models such as frd , genfrd, or idfrd models.
If sys is an array of models, the function plots the responses of all models in the array on the same axes. See "Step Response of Systems in a Model Array" on page 1-1502.


## tFinal - End time for step response

positive scalar
End time for the step response, specified as a positive scalar value. step simulates the step response from $\mathrm{t}=0$ to $\mathrm{t}=\mathrm{tFinal}$.

- For continuous-time systems, the function determines the step size and number of points automatically from system dynamics. Express tFinal in the system time units, specified in the TimeUnit property of sys.
- For discrete-time systems, the function uses the sample time of sys as the step size. Express tFinal in the system time units, specified in the TimeUnit property of sys.
- For discrete-time systems with unspecified sample time (Ts = -1), step interprets tFinal as the number of sampling periods to simulate.


## t - Time vector

vector
Time vector at which to compute the step response, specified as a vector of positive scalar values. Express $t$ in the system time units, specified in the TimeUnit property of sys.

- For continuous-time models, specify t in the form Ti:dt:Tf. To obtain the response at each time step, the function uses dt as the sample time of a discrete approximation to the continuous system (see "Algorithms" on page 1-1513).
- For discrete-time models, specify t in the form Ti:Ts:Tf, where Ts is the sample time of sys.
step always applies the step input at $t=0$, regardless of Ti.


## LineSpec - Line style, marker, and color

character vector | string
Line style, marker, and color, specified as a string or vector of one, two, or three characters. The characters can appear in any order. You do not need to specify all three characteristics (line style, marker, and color). For example, if you omit the line style and specify the marker, then the plot shows only the marker and no line. For more information about configuring this argument, see the LineSpec input argument of the plot function.
Example: 'r--' specifies a red dashed line
Example: '*b' specifies blue asterisk markers
Example: 'y' specifies a yellow line

## opts - Input offset and amplitude

stepData0ptions options set
Input offset and amplitude of the applied step signal, specified as a stepDataOptions option set. By default, step applies an input that goes from 0 to 1 at time $t=0$. Use this input argument to change the initial and final values of the step input. See "Response to Custom Step Input" on page 11506 for an example.

## Output Arguments

## y - Step response data

array
Step response data, returned as an array.

- For SISO systems, $y$ is a column vector of the same length as $t$ (if provided) or tOut (if you do not provide t).
- For single-input, multi-output systems, y is a matrix with as many rows as there are time samples and as many columns as there are outputs. Thus, the $j$ th column of $y$, or $y(:, j)$, contains the step response of from the input to the $j$ th output.
- For MIMO systems, the step responses of each input channel are stacked up along the third dimension of y . The dimensions of y are then N -by- Ny -by- Nu , where:
- $N$ is the number of time samples.
- $N y$ is the number of system outputs.
- $N u$ is the number of system inputs.

Thus, $y(:, i, j)$ is a column vector containing the step response from the $j$ th input to the $i$ th output at the times specified in $t$ or tOut.

## tOut - Times at which step response is computed

vector
Times at which step response is computed, returned as a vector. When you do not provide a specific vector $t$ of times, step chooses this time vector based on the system dynamics. The times are expressed in the time units of sys.

## x - State trajectories

array

State trajectories, returned as an array. When sys is a state-space model, $x$ contains the evolution of the states of sys at each time in $t$ or tOut. The dimensions of $x$ are $N$-by- $N x$-by- $N u$, where:

- $N$ is the number of time samples.
- $N x$ is the number of states.
- $N u$ is the number of system inputs.

Thus, the evolution of the states in response to a step injected at the $k$ th input is given by the array $x(:,:, k)$. The row vector $x(i,:, k)$ contains the state values at the $i$ th time step.
ysd - Standard deviation of step response
array
Standard deviation of the step response of an identified model, returned as an array of the same dimensions as $y$. If sys does not contain parameter covariance information, then ysd is empty.

## Tips

- When you need additional plot customization options, use stepplot instead.
- To simulate system responses to arbitrary input signals, use lsim.


## Algorithms

To obtain samples of continuous-time models without internal delays, step converts such models to state-space models and discretizes them using a zero-order hold on the inputs. step chooses the sampling time for this discretization automatically based on the system dynamics, except when you supply the input time vector $t$ in the form $t=0: d t: T f$. In that case, step uses $d t$ as the sampling time. The resulting simulation time steps t0ut are equisampled with spacing dt .

For systems with internal delays, Control System Toolbox software uses variable step solvers. As a result, the time steps tOut are not equisampled.

## References

[1] L.F. Shampine and P. Gahinet, "Delay-differential-algebraic equations in control theory," Applied Numerical Mathematics, Vol. 56, Issues 3-4, pp. 574-588.

## See Also

## Functions

impulse|initial|lsim|stepData0ptions|stepplot

## Apps

Linear System Analyzer

Introduced before R2006a

## step

Update model parameters and output online using recursive estimation algorithm

## Syntax

[EstimatedParameters,EstimatedOutput] = step(obj,y,InputData)

## Description

[EstimatedParameters,EstimatedOutput] = step(obj,y,InputData) updates parameters and output of the model specified in System object, obj, using measured output, y, and input data.
step puts the object into a locked state. In a locked state you cannot change any nontunable properties of the object, such as model order, data type, or estimation algorithm.

The EstimatedParameters and InputData depend on the online estimation System object:

- recursiveAR - step returns the estimated polynomial $A(q)$ coefficients of a single output AR model using time-series output data.
[A,EstimatedOutput] = step(obj,y)
- recursiveARMA - step returns the estimated polynomial $A(q)$ and $C(q)$ coefficients of a single output ARMA model using time-series output data, $y$.
[A,C,EstimatedOutput] = step(obj,y)
- recursiveARX - step returns the estimated polynomial $A(q)$ and $B(q)$ coefficients of a SISO or MISO ARX model using measured input and output data, $u$ and $y$, respectively. [A,B,EstimatedOutput] = step(obj,y,u).
- recursiveARMAX - step returns the estimated polynomial $A(q), B(q)$, and $C(q)$ coefficients of a SISO ARMAX model using measured input and output data, $u$ and $y$, respectively. [A,B,C,EstimatedOutput] = step(obj,y,u).
- recursive0E - step returns the estimated polynomial $B(q)$, and $F(q)$ coefficients of a SISO Output-Error polynomial model using measured input and output data, $u$ and $y$, respectively. [B,F,EstimatedOutput] = step(obj,y,u).
- recursiveBJ - step returns the estimated polynomial $B(q), C(q), D(q)$, and $F(q)$ coefficients of a SISO Box-Jenkins polynomial model using measured input and output data, $u$ and $y$, respectively. [B,C,D,F,EstimatedOutput] = step(obj,y,u).
- recursiveLS - step returns the estimated system parameters, $\theta$, of a single output system that is linear in estimated parameters, using regressors $H$ and output data $y$.
[theta, EstimatedOutput] = step(obj,y,H).


## Examples

## Estimate an ARMAX Model Online

Create a System object for online parameter estimation of an ARMAX model.
obj = recursiveARMAX;

The ARMAX model has a default structure with polynomials of order 1 and initial polynomial coefficient values, eps.

Load the estimation data. In this example, use a static data set for illustration.

```
load iddatal z1;
output = z1.y;
input = z1.u;
Estimate ARMAX model parameters online using step.
```

```
for i = 1:numel(input)
```

for i = 1:numel(input)
[A,B,C,EstimatedOutput] = step(obj,output(i),input(i));
[A,B,C,EstimatedOutput] = step(obj,output(i),input(i));
end

```
end
```

View the current estimated values of polynomial A coefficients.

```
obj.A
ans = 1\times2
    1.0000-0.8298
```

View the current covariance estimate of the parameters.

```
obj.ParameterCovariance
ans = 3\times3
    0.0001 0.0001 0.0001
    0.0001 0.0032 0.0000
    0.0001 0.0000 0.0001
```

View the current estimated output.
Estimated0utput
EstimatedOutput $=-4.5595$

## Tune Recursive Estimation Algorithm Properties During Online Parameter Estimation

Create a System object for online parameter estimation of an ARMAX model.
obj = recursiveARMAX;
The ARMAX model has a default structure with polynomials of order 1 and initial polynomial coefficient values, eps.

Load the estimation data. In this example, use a static data set for illustration.

```
load iddatal z1;
output = z1.y;
input = zl.u;
dataSize = numel(input);
```

Estimate ARMAX model parameters online using the default recursive estimation algorithm, Forgetting Factor. Change the ForgettingFactor property during online parameter estimation.

```
for i = 1:dataSize
    if i == dataSize/2
        obj.ForgettingFactor = 0.98;
    end
[A,B,C,EstimatedOutput] = step(obj,output(i),input(i));
end
```


## Estimate Parameters of System Using Recursive Least Squares Algorithm

The system has two parameters and is represented as:

$$
y(t)=a_{1} u(t)+a_{2} u(t-1)
$$

Here,

- $u$ and $y$ are the real-time input and output data, respectively.
- $u(t)$ and $u(t-1)$ are the regressors, H , of the system.
- $a_{1}$ and $a_{2}$ are the parameters, theta, of the system.

Create a System object for online estimation using the recursive least squares algorithm.

```
obj = recursiveLS(2);
```

Load the estimation data, which for this example is a static data set.

```
load iddata3
input = z3.u;
output = z3.y;
```

Create a variable to store $u(t-1)$. This variable is updated at each time step.
oldInput = 0;
Estimate the parameters and output using step and input-output data, maintaining the current regressor pair in H . Invoke the step function implicitly by calling the obj system object with input arguments.

```
for i = 1:numel(input)
    H = [input(i) oldInput];
    [theta, EstimatedOutput] = obj(output(i),H);
    estimatedOut(i)= EstimatedOutput;
    theta_est(i,:) = theta;
    oldInput = input(i);
end
```

Plot the measured and estimated output data.

```
numSample = 1:numel(input);
plot(numSample,output,'b',numSample,estimated0ut,'r--');
legend('Measured Output','Estimated Output');
```



Plot the parameters.
plot(numSample,theta_est(:,1), numSample,theta_est(:,2))
title('Parameter Estimates for Recursive Least Squares Estimation')
legend("theta1", "theta2")


View the final estimates.
theta_final = theta
theta_final $=2 \times 1$
-1.5322
-0.0235

## Input Arguments

## obj - System object for online parameter estimation

```
recursiveAR object | recursiveARMA object | recursiveARX object | recursiveARMAX object | recursive0E object | recursiveBJ object|recursiveLS object
```

System object for online parameter estimation, created using one of the following commands:

- recursiveAR
- recursiveARMA
- recursiveARX
- recursiveARMAX
- recursive0E
- recursiveBJ
- recursiveLS

The step command updates parameters of the model using the recursive estimation algorithm specified in obj and the incoming input-output data.

## y - Output data

## real scalar

Output data acquired in real time, specified as a real scalar.

## InputData - Input data

scalar | vector of real values
Input data acquired in real time, specified as a scalar or vector of real values depending on the type of System object.

| System object | Model Type | InputData |
| :--- | :--- | :--- |
| recursiveAR | Time-series | Not Applicable |
| recursiveARMA | Time-series | Not Applicable |
| recursiveARX | SISO ARX | Real scalar |
|  | MISO ARX with $N$ inputs | Column vector of length $N$, <br> specified as real values |
| recursiveARMAX | SISO | Real scalar |
| recursive0E | SISO | Real scalar |
| recursiveBJ | SISO | Real scalar |
| recursiveLS | Single output system with $N p$ <br> system parameters | Regressors, $H$, specified as a <br> vector of real values of length <br> $N p$ |

## Output Arguments

## EstimatedParameters - Estimated model parameters

vector of real values for each parameter
Estimated model parameters, returned as vectors of real values. The number of estimated parameters, and so the step syntax, depend on the type of System object:

| Online Estimation System Object | Estimated Parameters |
| :--- | :--- |
| recursiveAR | Polynomial $A(q)$ coefficients |
| recursiveARMA | Polynomials $A(q)$ and $C(q)$ coefficients |
| recursiveARX | Polynomials $A(q)$ and $B(q)$ coefficients |
| recursiveARMAX | Polynomials $A(q), B(q)$, and $C(q)$ coefficients |
| recursive0E | Polynomials $B(q)$ and $F(q)$ |
| recursiveBJ | Polynomials $B(q), C(q), D(q)$, and $F(q)$ coefficients |
| recursiveLS | System parameters, $\theta$ |

## EstimatedOutput - Estimated output <br> real scalar

Estimated output, returned as a real scalar. The output is estimated using input-output estimation data, current parameter values, and recursive estimation algorithm specified in obj.

## Tips

- Starting in R2016b, instead of using the step command to update model parameter estimates, you can call the System object with input arguments, as if it were a function. For example, [A, EstimatedOutput] = step(obj,y) and [A,EstimatedOutput] =obj(y) perform equivalent operations.


## See Also

clone|isLocked|recursiveAR|recursiveARMA|recursiveARMAX|recursiveARX| recursiveBJ|recursiveLS|recursive0E|release|reset

## Topics

"Perform Online Parameter Estimation at the Command Line"
"Validate Online Parameter Estimation at the Command Line"
"Online ARX Parameter Estimation for Tracking Time-Varying System Dynamics"
"Line Fitting with Online Recursive Least Squares Estimation"
"What Is Online Estimation?"
Introduced in R2015b

## stepDataOptions

Options for the step command

## Syntax

```
opt = stepDataOptions
opt = stepDataOptions(Name,Value)
```


## Description

opt $=$ stepDataOptions creates the default options for step.
opt = stepDataOptions(Name,Value) creates an options set with the options specified by one or more Name, Value pair arguments.

## Examples

## Specify Input Offset and Step Amplitude Level for Step Response

Create a transfer function model.
sys = tf(1,[1,1]);
Create an option set for step to specify input offset and step amplitude level.

```
opt = stepDataOptions('InputOffset',-1,'StepAmplitude',2);
```

Calculate the step response using the specified options.
[y,t] = step(sys,opt);

## Input Arguments

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.
Example: ...,'Input0ffset',2
InputOffset - Input signal offset level
0 (default) | scalar
Input signal offset level, specified as the comma-separated pair consisting of 'InputOffset' and a scalar, for all time $\mathrm{t}<0$, as shown in the following figure.


## StepAmplitude - Change of input signal offset

1 (default) | scalar
Change of input signal offset level, specified as the comma-separated pair consisting of 'StepAmplitude' and a scalar, which occurs at time $t=0$, as shown in the previous figure.

## Output Arguments

## opt - Options for the step command

step options set
Options for the step command, returned as a step options set.

See Also<br>step<br>Introduced in R2012a

## stepinfo

Rise time, settling time, and other step-response characteristics

## Syntax

```
S = stepinfo(sys)
S = stepinfo(y,t)
S = stepinfo(y,t,yfinal)
S = stepinfo(
,'SettlingTimeThreshold',ST)
S = stepinfo(
,'RiseTimeLimits',RT)
```


## Description

S = stepinfo(sys) computes the step-response characteristics for a dynamic system model sys. The function returns the characteristics in a structure containing the fields:

- RiseTime - Time it takes for the response to rise from $10 \%$ to $90 \%$ of the steady-state response.
- SettlingTime - Time it takes for the error $e(t)=\left|y(t)-y_{f n a l}\right|$ between the response $y(t)$ and the steady-state response $y_{\text {fnal }}$ to fall below $2 \%$ of the peak value of $e(t)$.
- SettlingMin - Minimum value of $y(t)$ once the response has risen.
- SettlingMax - Maximum value of $y(t)$ once the response has risen.
- Overshoot - Percentage overshoot, relative to $y_{\text {fnal }}$.
- Undershoot - Percentage undershoot.
- Peak - Peak absolute value of $y(t)$
- PeakTime - Time at which the peak value occurs.

The following figure illustrates some of these quantities on a typical second-order response.

Step Response


Using this syntax requires a Control System Toolbox license.
S = stepinfo(y,t) computes step-response characteristics from an array of step-response data y and corresponding time vector $t$. For SISO system responses, $y$ is a vector with the same number of entries as $t$. For MIMO response data, $y$ is an array containing the responses of each I/O channel. This syntax uses the last value in $y$ (or the last value in each channel's corresponding response data) as the steady-state value for computing characteristics that depend on that value.
$\mathrm{S}=$ stepinfo( $\mathrm{y}, \mathrm{t}, \mathrm{yfinal})$ computes step-response characteristics relative to the steady-state value yfinal. This syntax is useful when you know that the expected steady-state system response differs from the last value in $y$ for reasons such as measurement noise.

For SISO responses, t and y are vectors with the same length NS. For systems with NU inputs and NY outputs, you can specify y as an NS-by-NY-by-NU array (see step) and yfinal as an NY-by-NU array. stepinfo then returns a NY-by-NU structure array S of response characteristics corresponding to each I/O pair.

S = stepinfo( $\qquad$ , 'SettlingTimeThreshold ', ST) lets you specify the threshold ST used in the definition of settling time. The response has settled when the error $e(t)=\left|y(t)-y_{\text {fnal }}\right|$ becomes smaller than a fraction ST of its peak value. The default value is ST $=0.02(2 \%)$. You can use this syntax with any of the previous input-argument combinations.

S = stepinfo(__, 'RiseTimeLimits', RT) lets you specify the lower and upper thresholds used in the definition of rise time. By default, the rise time is defined as the time the response takes to rise from 10 to $90 \%$ of the steady-state value ( $\mathrm{RT}=$ [0.1 0.9]). The upper threshold RT(2) is also used to calculate SettlingMin and SettlingMax. These values are the minimum and maximum values of the response occurring after the response has reached the upper threshold. You can use this syntax with any of the previous input-argument combinations.

## Examples

## Step-Response Characteristics of Dynamic System

Compute step-response characteristics such as rise time, settling time, and overshoot for a dynamic system model. For this example, use the continuous-time transfer function:

$$
\text { sys }=\frac{s^{2}+5 s+5}{s^{4}+1.65 s^{3}+5 s^{2}+6.5 s+2}
$$

Create the transfer function and examine its step response.

```
sys = tf([1 5 5],[1 1.65 5 6.5 2]);
```

step(sys)


The plot shows that the response rises in a few seconds, and then rings down to a steady-state value of about 2.5. Compute the characteristics of this response using stepinfo.

```
S = stepinfo(sys)
S = struct with fields:
            RiseTime: 3.8456
    SettlingTime: 27.9762
        SettlingMin: 2.0689
        SettlingMax: 2.6873
            Overshoot: 7.4915
        Undershoot: 0
            Peak: 2.6873
            PeakTime: 8.0530
```

By default, the settling time is the time it takes for $\left|y(t)-y_{\text {final }}\right|$ to fall below $2 \%$ of its peak value, where $y(t)$ is the system response at time $t$ and $y_{\text {final }}$ is the steady-state response. The result S. SettlingTime shows that for sys, this condition occurs after about 28 seconds. The default definition of rise time is the time it takes for the response to go from $10 \%$ of its steady-state value to $90 \%$ of that value. S. RiseTime shows that for sys, this rise occurs in less than 4 seconds. The maximum overshoot is returned in S.Overshoot. For this system, the peak value S. Peak, which occurs at the time S.PeakTime, overshoots the steady-state value by about $7.5 \%$ of the steady-state value.

## Step-Response Characteristics of MIMO System

For a MIMO system, stepinfo returns a structure array in which each entry contains the response characteristics of the corresponding I/O channel of the system. For this example, use a two-output, two-input discrete-time system. Compute the step-response characteristics.

```
A = [0.68 -0.34; 0.34 0.68];
B = [0.18 -0.05; 0.04 0.11];
C = [0 -1.53; -1.12 -1.10];
D = [0 0; 0.06 -0.37];
sys = ss(A,B,C,D,0.2);
S = stepinfo(sys)
S=2\times2 struct array with fields:
    RiseTime
    SettlingTime
    SettlingMin
    SettlingMax
    Overshoot
    Undershoot
    Peak
    PeakTime
```

Access the response characteristics for a particular I/0 channel by indexing into S. For instance, examine the response characteristics for the response from the first input to the second output of sys, corresponding to $S(2,1)$.

S(2,1)
ans = struct with fields:
RiseTime: 0.4000

```
SettlingTime: 2.8000
    SettlingMin: -0.6724
    SettlingMax: -0.5188
    Overshoot: 24.6476
    Undershoot: 11.1224
            Peak: 0.6724
        PeakTime: 1
```

To access a particular value, use dot notation. For instance, extract the rise time of the $(2,1)$ channel.

```
rt21 = S(2,1).RiseTime
rt21 = 0.4000
```


## Specify Definition of Settling Time or Rise Time

By default, stepinfo defines settling time as the time it takes for the error $e(t)=\left|y(t)-y_{\text {final }}\right|$ between the response $y(t)$ and the steady-state response $y_{\text {final }}$ to fall below $2 \%$ of the peak value of $e(t)$. Also, stepinfo defines the rise time as the time it takes for the response to rise from $10 \%$ of $y_{\text {final }}$ to $90 \%$ of $y_{\text {final }}$. You can change these definitions using SettlingTimeThreshold and RiseTimeThreshold. For this example, use the system given by:

$$
\text { sys }=\frac{s^{2}+5 s+5}{s^{4}+1.65 s^{3}+6.5 s+2} .
$$

Create the transfer function.
sys $=t f\left(\left[\begin{array}{lll}1 & 5 & 5\end{array}\right],\left[\begin{array}{llll}1 & 1.65 & 5 & 6.5\end{array}\right]\right) ;$
Compute the time it takes for the error in the response of sys to to reach $0.5 \%$ of the peak error. To do so, set SettlingTimeThreshold to $0.5 \%$, or 0.005 .

```
S1 = stepinfo(sys,'SettlingTimeThreshold',0.005);
st1 = S1.SettlingTime
st1 = 46.1325
```

Compute the time it takes the response of sys to rise from $5 \%$ to $95 \%$ of the steady-state value. To do so, set RiseTimeThreshold to a vector containing those bounds.

```
S2 = stepinfo(sys,'RiseTimeThreshold',[0.05 0.95]);
rt2 = S2.RiseTime
rt2 = 4.1690
```

You can define both settling time and rise time in the same computation.

```
S3 = stepinfo(sys,'SettlingTimeThreshold',0.005,'RiseTimeThreshold',[0.05 0.95])
S3 = struct with fields:
        RiseTime: 4.1690
    SettlingTime: 46.1325
    SettlingMin: 2.0689
    SettlingMax: 2.6873
```

Overshoot: 7.4915
Undershoot: 0
Peak: 2.6873
PeakTime: 8.0530

## Step-Response Characteristics from Response Data

You can extract step-response characteristics from step-response data even if you do not have a model of your system. For instance, suppose you have measured the response of your system to a step input, and saved the resulting response data in a vector $y$ of response values at the times stored in another vector, t . Load the response data and examine it.
load StepInfoData t y
plot(t,y)


Compute step-response characteristics from this response data using stepinfo. If you do not specify the steady-state response value yfinal, then stepinfo assumes that the last value in the response vector y is the steady-state response. Because there is some noise in the data, the last value in y is likely not the true steady-state response value. When you know what the steady-state value should be, you can provide it to stepinfo. For this example, suppose that the steady-state response is 2.4.

S1 = stepinfo(y, t, 2.4)

```
S1 = struct with fields:
            RiseTime: 1.2713
    SettlingTime: 19.6478
    SettlingMin: 2.0219
    SettlingMax: 3.3302
        Overshoot: 38.7575
        Undershoot: 0
            Peak: 3.3302
            PeakTime: 3.4000
```

Because of the noise in the data, the default definition of the settling time is too stringent, resulting in an arbitrary value of almost 20 seconds. To allow for the noise, increase the settling-time threshold from the default $2 \%$ to $5 \%$.

```
S2 = stepinfo(y,t,2.4,'SettlingTimeThreshold',0.05)
S2 = struct with fields:
    RiseTime: 1.2713
    SettlingTime: 10.4201
    SettlingMin: 2.0219
    SettlingMax: 3.3302
        Overshoot: 38.7575
        Undershoot: 0
            Peak: 3.3302
            PeakTime: 3.4000
```


## Input Arguments

## sys - Dynamic system

dynamic system model
Dynamic system, specified as a SISO or MIMO dynamic system model. Dynamic systems that you can use include:

- Continuous-time or discrete-time numeric LTI models, such as $t f, z p k$, or ss models.
- Generalized or uncertain LTI models such as genss or uss models. (Using uncertain models requires Robust Control Toolbox software.) For generalized models, stepinfo computes the stepresponse characteristics using the current value of tunable blocks and the nominal value of uncertain blocks.
- Identified LTI models, such as idtf, idss, or idproc models.


## y - Step-response data

vector | array
Step-response data, specified as:

- For SISO response data, a vector of length Ns, where Ns is the number of samples in the response data.
- For MIMO response data, an Ns-by-Ny-by-Nu array, where Ny is the number of system outputs, and Nu is the number of system inputs.


## t-Time vector

vector
Time vector corresponding to the response data in y , specified as a vector of length Ns .

## yfinal - Steady-state response

scalar | array
Steady-state response, specified as:

- For SISO response data, a scalar value.
- For MIMO response data, an Ny-by-Nu array, where each entry provides the steady-state response value for the corresponding system channel.

If you do not provide yfinal, then stepinfo uses the last value in the corresponding channel of $y$ as the steady-state response value.

## ST - Threshold for defining settling time

0.02 (default) | scalar between 0 and 1

Threshold for defining settling time, specified as a scalar value between 0 and 1. By default, stepinfo defines settling time as the time it takes for the error $e(t)=\left|y(t)-y_{\text {fnal }}\right|$ to fall below $2 \%$ of the peak value of $e(t)$. To change this definition, set ST to a different value. For instance, to measure when the error falls below $5 \%$ of the peak error, set ST to 0.05 .

## RT - Thresholds for defining rise time

[0.1 0.9] (default)| 2 -element row vector
Threshold for defining rise time, specified as a 2 -element row vector of nondescending values between 0 and 1. By default, stepinfo defines rise time as the time it takes for the response to rise from $10 \%$ to $90 \%$ of the steady-state value $y_{\text {final }}$. To change this definition, set RT to a different value. For instance, to define the rise time as the time it takes for the response to rise from $5 \%$ to $95 \%$ of the steady-state value, set RT to [0.05 0.95].

## Output Arguments

## S - Step-response characteristics

structure
Step-response characteristics of sys, returned as a structure containing the fields:

- RiseTime - Time it takes for the response to rise from $10 \%$ to $90 \%$ of the steady-state response.
- SettlingTime - Time it takes for the error $e(t)=\left|y(t)-y_{\text {final }}\right|$ between the response $y(t)$ and the steady-state response $y_{\text {fnal }}$ to fall below $2 \%$ of the peak value of $e(t)$.
- SettlingMin - Minimum value of $y(t)$ once the response has risen.
- SettlingMax - Maximum value of $y(t)$ once the response has risen.
- Overshoot - Percentage overshoot, relative to $y_{\text {final }}$ ).
- Undershoot - Percentage undershoot.
- Peak - Peak absolute value of $y(t)$
- PeakTime - Time at which the peak value occurs.

For MIMO models or responses data, S is a structure array in which each entry contains the stepresponse characteristics of the corresponding I/O channel. For instance, if you provide a 3 -input, 3output model or array of response data, then $S(2,3)$ contains the characteristics of the response from the third input to the second output. For an example, see "Step-Response Characteristics of MIMO System" (Control System Toolbox).

If sys is unstable, then all step-response characteristics are NaN, except for Peak and PeakTime, which are Inf.

## See Also

lsiminfo|step
Introduced in R2006a

## stepplot

Plot step response and return plot handle

## Syntax

```
h = stepplot(sys)
stepplot(sys,Tfinal)
stepplot(sys,t)
stepplot(sys1,sys2,...,sysN)
stepplot(sys1,sys2,...,sysN,Tfinal)
stepplot(sys1,sys2,...,sysN,t)
stepplot(AX,...)
stepplot(..., plotoptions)
stepplot(..., dataoptions)
```


## Description

$\mathrm{h}=$ stepplot (sys) plots the step response of the dynamic system model sys. It also returns the plot handle $h$. You can use this handle to customize the plot with the getoptions and setoptions commands. Type
help timeoptions
for a list of available plot options.
For multiinput models, independent step commands are applied to each input channel. The time range and number of points are chosen automatically.
stepplot(sys,Tfinal) simulates the step response from $t=0$ to the final time $t=T f i n a l$. Express Tfinal in the system time units, specified in the TimeUnit property of sys. For discretetime systems with unspecified sample time ( $\mathrm{Ts}=-1$ ), stepplot interprets Tfinal as the number of sampling intervals to simulate.
stepplot (sys, t ) uses the user-supplied time vector t for simulation. Express t in the system time units, specified in the TimeUnit property of sys. For discrete-time models, $t$ should be of the form Ti:Ts:Tf, where Ts is the sample time. For continuous-time models, $t$ should be of the form Ti:dt:Tf, where dt becomes the sample time of a discrete approximation to the continuous system (see step). The stepplot command always applies the step input at $\mathrm{t}=0$, regardless of Ti .

To plot the step responses of multiple models sys1,sys2,... on a single plot, use:

```
stepplot(sys1,sys2,...,sysN)
stepplot(sys1,sys2,...,sysN,Tfinal)
stepplot(sys1,sys2,...,sysN,t)
```

You can also specify a color, line style, and marker for each system, as in

```
stepplot(sys1,'r',sys2,'y--',sys3,'gx')
stepplot(AX,...) plots into the axes with handle AX.
```

stepplot(..., plotoptions) customizes the plot appearance using the options set, plotoptions. Use timeoptions to create the options set.
stepplot(..., dataoptions) specifies options such as the step amplitude and input offset using the options set, dataoptions. Use stepData0ptions to create the options set.

## Examples

## Display Normalized Response on Step Plot

Generate a step response plot for two dynamic systems.

```
sys1 = rss(3);
sys2 = rss(3);
h = stepplot(sys1,sys2);
```


## Step Response



Each step response settles at a different steady-state value. Use the plot handle to normalize the plotted response.

```
setoptions(h,'Normalize','on')
```

Step Response


Now, the responses settle at the same value expressed in arbitrary units.

## Plot Step Responses of Identified Models with Confidence Region

Compare the step response of a parametric identified model to a nonparametric (empirical) model, and view their $3-\sigma$ confidence regions. (Identified models require System Identification Toolbox ${ }^{\mathrm{TM}}$ software.)

Identify a parametric and a nonparametric model from sample data.

```
load iddatal z1
sys1 = ssest(z1,4);
sys2 = impulseest(z1);
```

Plot the step responses of both identified models. Use the plot handle to display the 3- $\sigma$ confidence regions.

```
t = -1:0.1:5;
h = stepplot(sys1,sys2,t);
showConfidence(h,3)
legend('parametric','nonparametric')
```



The nonparametric model sys2 shows higher uncertainty.

## Plot Step Response of Nonlinear Identified Model

Load data for estimating a nonlinear Hammerstein-Wiener model.
load(fullfile(matlabroot,'toolbox','ident','iddemos','data','twotankdata'));
z = iddata(y,u,0.2,'Name','Two tank system');
$z$ is an iddata object that stores the input-output estimation data.
Estimate a Hammerstein-Wiener Model of order [15 3] using the estimation data. Specify the input nonlinearity as piecewise linear and output nonlinearity as one-dimensional polynomial.
sys = nlhw(z,[1 5 3],pwlinear,poly1d);
Create an option set to specify input offset and step amplitude level.
opt $=$ stepData0ptions('Input0ffset',2,'StepAmplitude',0.5);
Plot the step response until 60 seconds using the specified options.
stepplot(sys,60,opt);


## Tips

You can change the properties of your plot, for example the units. For information on the ways to change properties of your plots, see "Ways to Customize Plots" (Control System Toolbox).

## See Also <br> getoptions| setoptions| showConfidence | step <br> Introduced in R2012a

## strseq

Create sequence of indexed character vectors

## Syntax

txtarray $=$ strseq(TXT,INDICES)

## Description

txtarray = strseq(TXT,INDICES) creates a sequence of indexed character vectors in the cell array txtarray by appending the integer values INDICES to the character vector TXT.

Note You can use strvec to aid in system interconnection. For an example, see the sumblk reference page.

## Examples

## Create a Cell Array of Indexed Text

Index the text 'e' with the numbers 1,2 , and 4.

```
txtarray = strseq('e',[1 2 4])
txtarray = 3x1 cell
    {'e1'}
    {'e2'}
    {'e4'}
```


## See Also

connect | strcat
Introduced in R2012a

## struc

Generate model-order combinations for single-output ARX model estimation

## Syntax

$n n=\operatorname{struc}(n a, n b, n k)$
$n n=\operatorname{struc}\left(n a, n b \_1, \ldots, n b \_n u, n k \_1, \ldots, n k \_n u\right)$

## Description

$n n=$ struc (na, $n b, n k$ ) generates model-order combinations for single-input, single-output ARX model estimation. na and $n b$ are row vectors that specify ranges of model orders. $n k$ is a row vector that specifies a range of model delays. $n n$ is a matrix that contains all combinations of the orders and delays.
$n n=\operatorname{struc}\left(n a, n b \_1, \ldots, n b \_n u, n k \_1, \ldots, n k \_n u\right)$ generates model-order combinations for an ARX model with $n u$ input channels.

## Examples

## Generate Model-Order Combinations and Estimate ARX Model Using IV Method

Create estimation and validation data sets

```
load iddata1;
ze = z1(1:150);
zv = z1(151:300);
```

Generate model-order combinations for estimation, specifying ranges for model orders and delays.
NN $=$ struc ( $1: 3,1: 2,2: 4$ );
Estimate ARX models using the instrumental variable method, and compute the loss function for each model order combination.
$\mathrm{V}=$ ivstruc(ze,zv,NN);
Select the model order with the best fit to the validation data.
order $=$ selstruc(V,0);
Estimate an ARX model of selected order.
M = iv4(ze,order);

## Generate Model-Order Combinations and Estimate Multi-Input ARX Model

Create estimation and validation data sets.

```
load co2data;
Ts = 0.5; % Sample time is 0.5 min
ze = iddata(Output_exp1,Input_exp1,Ts);
zv = iddata(Output_exp2,Input_exp2,Ts);
```

Generate model-order combinations for:

- $\mathrm{na}=2: 4$
- $\mathrm{nb}=2: 5$ for the first input, and 1 or 4 for the second input.
- $n k=1: 4$ for the first input, and 0 for the second input.
$N N=\operatorname{struc}(2: 4,2: 5,[14], 1: 4,0) ;$
Estimate an ARX model for each model order combination.
V = arxstruc(ze,zv,NN);
Select the model order with the best fit to the validation data.
order $=\operatorname{selstruc}(\mathrm{V}, 0)$;
Estimate an ARX model of selected order.
$M=\operatorname{arx}(z e, o r d e r) ;$


## Tips

- Use with arxstruc or ivstruc to compute loss functions for ARX models, one for each model order combination returned by struc.


## See Also

arxstruc|ivstruc|selstruc

## Topics

"Estimating Model Orders Using an ARX Model Structure"
"Preliminary Step - Estimating Model Orders and Input Delays"
Introduced before R2006a

## System Identification

Identify models of dynamic systems from measured data

## Description

The System Identification app enables you to identify models of dynamic systems from measured input-output data. You can estimate both linear and nonlinear models and compare responses of different models.

Using this app you can:

- Import, plot, and preprocess measured input-output data.
- Estimate linear models such as transfer functions, process models, polynomial models, and statespace models using time-domain, time series, or frequency-domain data.
- Estimate nonlinear ARX and Hammerstein-Wiener models using time-domain data.
- Validate estimated models using independent data sets.
- Export estimated models for further analysis to MATLAB workspace or to the Linear System Analyzer app in Control System Toolbox.

For more information, in the System Identification app, select Help > System Identification App Help.


## Open the System Identification App

- MATLAB Toolstrip: On the Apps tab, under Control System Design and Analysis, click the app icon.
- MATLAB command prompt: Enter systemIdentification.


## Examples

- "Working with System Identification App"
- "Identify Linear Models Using System Identification App"
- "Identify Nonlinear Black-Box Models Using System Identification App"


## Programmatic Use

systemIdentification opens the System Identification app. If the app is already open, the command brings the app into focus.
systemIdentification(sessionFile) opens the app and loads a previously saved session file, sessionFile, on the MATLAB path. A session includes data sets and models in the app at the time of saving. If the app is already open, the command merges the contents of the session file with those already present in the app.

For example, systemIdentification('mySession') opens the app and loads the previously saved app session mySession. sid.

To save a session, in the System Identification app, select File > Save session. The session is saved to a file with a . sid extension.
systemIdentification(sessionFile, path) specifies the path to the session file. Use this syntax if the file is not on the MATLAB path.

For example, systemIdentification('mySession', 'C:\matlab\work') opens the app and loads the previously saved app session mySession. sid located at C: \matlab\work.

## Limitations

MATLAB Online ${ }^{\text {TM }}$ does not support the System Identification app.

## See Also

## Functions

arx|iddata|midprefs|nlarx|nlhw|polyest| procest| ssest|tfest

## Topics

"Working with System Identification App"
"Identify Linear Models Using System Identification App"
"Identify Nonlinear Black-Box Models Using System Identification App"

Introduced before R2006a

## tfdata

Access transfer function data

## Syntax

```
[num,den] = tfdata(sys)
[num,den,ts] = tfdata(sys)
[num,den,ts,sdnum,sdden] = tfdata(sys)
    = tfdata(sys,J1,...,JN)
[num,den] = tfdata(sys,'v')
```


## Description

[num,den] = tfdata(sys) returns the numerator and denominator coefficients of the transfer function for the tf , ss and zpk model objects or the array of model objects represented by sys.

The outputs num and den are two-dimensional cell arrays if sys contains a single LTI model. When sys is an array of models, num and den are returned as multidimensional cell arrays.
[num,den,ts] = tfdata(sys) also returns the sample time ts.
[num,den,ts,sdnum,sdden] = tfdata(sys) also returns the uncertainties in the numerator and denominator coefficients of identified system sys. $\operatorname{sdnum\{ i}, \mathrm{j}\}(\mathrm{k})$ is the 1 standard uncertainty in the value num $\{\mathrm{i}, \mathrm{j}\}(\mathrm{k})$ and $\operatorname{sdden}\{\mathrm{i}, \mathrm{j}\}(\mathrm{k})$ is the 1 standard uncertainty in the value den $\{\mathrm{i}, \mathrm{j}\}$ $(k)$. If sys does not contain uncertainty information, sdnum and sdden are empty [].
$\ldots \quad$ tfdata (sys $, \mathrm{J} 1, \ldots, \mathrm{JN}$ ) extracts the data for the $\mathrm{J} 1, \ldots, \mathrm{JN}$ entry in the model array sys.
[num,den] = tfdata(sys, 'v') returns the numerator and denominator coefficients as row vectors rather than cell arrays for a SISO transfer function represented by sys.

## Examples

## Extract Numerator and Denominator Coefficients from Transfer Function

For this example, consider tfData.mat which contains a continuous-time SISO transfer function sys1.

Load the data and use tfdata to extract the numerator and denominator coefficients.

```
load('tfData.mat','sys1');
[num,den] = tfdata(sys1);
```

num and den are returned as cell arrays. To display data, use celldisp.

```
celldisp(num)
```

num $\{1\}=$

```
        0
celldisp(den)
den{1} =
    7 4 2 1
```

You can also extract the numerator and denominator coefficients as row vectors with the following syntax.
[num,den] = tfdata(sys1,'v');

## Extract Discrete-Time Transfer Function Data

For this example, consider tfData.mat which contains a discrete-time SISO transfer function sys2.
Load the data and use tfdata to extract the numerator and denominator coefficients along with the sample time.

```
load('tfData.mat','sys2');
[num,den,ts] = tfdata(sys2)
num = 1xl cell array
    {[0 0 2 0]}}
den = 1x1 cell array
    {[4 0 3 -1]}
```

$\mathrm{ts}=0.1000$
num and den are returned as cell arrays. To display data, use celldisp.
celldisp(num)
num $\{1\}=$
$0 \quad 0 \quad 20$
celldisp(den)
$\operatorname{den}\{1\}=$
$\begin{array}{llll}4 & 0 & 3 & -1\end{array}$

## Extract Identified Transfer Function Data

For this example, estimate a transfer function with 2 poles and 1 zero from identified data contained in iddata7.mat with an input delay value.

Load the identified data and estimate the transfer function.

```
load('iddata7.mat');
sys = tfest(z7,2,1,'InputDelay',[1 0]);
```

Extract the numerator, denominator and their standard deviations for the 2 -input, 1 output identified transfer function.

```
[num,den,~,sdnum,sdden] = tfdata(sys)
num=1\times2 cell array
    {[0 -0.5212 1.1886]} {[0 0.0552 -0.0013]}
den=1\times2 cell array
    {[1 0.3390 0.2353]} {[1 0.0360 0.0314]}
sdnum=1\times2 cell array
    {[0 0.1311 0.0494]} {[0 0.0246 0.0033]}
sdden=1\times2 cell array
    {[0 0.0183 0.0085]} {[0 0.0278 0.0048]}
```


## Extract Data from Specific Model in Transfer Function Array

For this example, extract numerator and denominator coefficients for a specific transfer function contained in the $3 x 1$ array of continuous-time transfer functions sys.

Load the data and extract the numerator and denominator coefficients of the second model in the array.

```
load('tfArray.mat','sys');
[num,den] = tfdata(sys,2);
```

Use celldisp to visualize the data in the cell array num and den.

```
celldisp(num)
```

num $\{1\}=$
$0 \quad 0 \quad 2$

```
celldisp(den)
den{1} =
    1 1 2
```


## Input Arguments

## sys - Dynamic system

dynamic system model | model array
Dynamic system, specified as a SISO or MIMO dynamic system model, or an array of SISO or MIMO dynamic system models. Dynamic systems that you can use include continuous-time or discrete-time numeric LTI models such as tf, ss and zpk models.

If sys is a state-space or zero-pole-gain model, it is first converted to transfer function form using tf. For more information on the format of transfer function model data, see the $t f$ reference page.

For SISO transfer functions, use the following syntax to return the numerator and denominator coefficients directly as row vectors rather than as cell arrays:
[num,den] = tfdata(sys,'v')
J1, ..., JN - Indices of models in array whose data you want to access positive integer

Indices of models in array whose data you want to access, specified as a positive integer. You can provide as many indices as there are array dimensions in sys. For example, if sys is a 4 -by- 5 array of transfer functions, the following command accesses the data for entry $(2,3)$ in the array.
[num,den] = tfdata(sys,2,3);

## Output Arguments

## num - Coefficients of the numerator

cell array | row vector
Coefficients of the numerator of the transfer function, returned as a cell array or row vector.
When sys contains a single LTI model, the output num is returned as a cell array with the following characteristics:

- num has as many rows as outputs and as many columns as inputs of sys.
- The ( $i, j$ ) entries in num $\{i, j\}$ are row vectors specifying the numerator coefficients of the transfer function from input $j$ to output $i$. tfdata orders these coefficients in descending powers of $s$ or $z$.

When sys contains an array of LTI models, num is returned as a multidimensional cell array of the same size as sys.

## den - Coefficients of the denominator

cell array | row vector
Coefficients of the denominator of the transfer function, returned as a cell array or row vector.
When sys contains a single LTI model, the output den is returned as a cell array with the following characteristics:

- den has as many rows as outputs and as many columns as inputs of sys.
- The ( $\mathrm{i}, \mathrm{j}$ ) entries in den $\{\mathrm{i}, \mathrm{j}\}$ are row vectors specifying the denominator coefficients of the transfer function from input $j$ to output i. tfdata orders these coefficients in descending powers of $s$ or $z$.

When sys contains an array of LTI models, den is returned as a multidimensional cell array of the same size as sys.

## ts - Sample time

non-negative scalar
Sample time, returned as a non-negative scalar.

## sdnum - Standard uncertainty of the numerator coefficients

cell array
Standard uncertainty of the numerator coefficients of the identified system sys, returned as a cell array of the same size as num. $\operatorname{sdnum}\{i, j\}(k)$ is the 1 standard uncertainty in the value num $\{i, j\}$ $(k)$. If sys does not contain uncertainty information, sdnum is empty [ ].

## sdden - Standard uncertainty the denominator coefficients

cell array
Standard uncertainty of the denominator coefficients of the identified system sys, returned as a cell array of the same size as den. $\operatorname{sdden}\{\mathrm{i}, \mathrm{j}\}(\mathrm{k})$ is the 1 standard uncertainty in the value den $\{\mathrm{i}, \mathrm{j}\}$ $(k)$. If sys does not contain uncertainty information, sdden is empty [ ].

See Also<br>get|ss|ssdata|tf|zpk|zpkdata

## Introduced before R2006a

## tfest

Estimate transfer function

## Syntax

```
sys = tfest(data,np)
sys = tfest(data,np,nz)
sys = tfest(data,np,nz,iodelay)
sys = tfest(
```

$\qquad$

``` , Name, Value)
sys = tfest(data,init_sys)
sys = tfest(
```

$\qquad$

``` , opt)
[sys,ic] = tfest(
``` \(\qquad\)
``` )
```


## Description

## Estimate a Transfer Function Model

sys $=$ tfest (data, np) estimates a continuous-time transfer function sys using the time-domain or frequency-domain data data and containing np poles. The number of zeros in sys is $\max (n p-1,0)$.
sys $=$ tfest(data, $n p, n z)$ estimates a transfer function containing $n z$ zeros.
sys = tfest(data, np,nz,iodelay) estimates a transfer function with transport delay for the input-output pairs in iodelay.
sys $=$ tfest( $\qquad$ , Name, Value) uses additional options specified by one or more name-value pair arguments. You can use this syntax with any of the previous input-argument combinations.

## Configure Initial Parameters

sys = tfest(data,init_sys) uses the linear system init_sys to configure the initial parameterization of sys.

## Specify Additional Options

sys = tfest( $\qquad$ , opt) specifies the estimation behavior using the option set opt. You can use this syntax with any of the previous input-argument combinations.

## Return Estimated Initial Conditions

[sys,ic] = tfest( $\qquad$ ) returns the estimated initial conditions as an initialCondition object. Use this syntax if you plan to simulate or predict the model response using the same estimation input data and then compare the response with the same estimation output data. Incorporating the initial conditions yields a better match during the first part of the simulation.

## Examples

## Estimate Transfer Function Model by Specifying Number of Poles

Load the time-domain system-response data $z 1$.
load iddatal z1;
Set the number of poles $n p$ to 2 and estimate a transfer function.

```
np = 2;
```

sys $=$ tfest(z1,np);
sys is an idtf model containing the estimated two-pole transfer function.
View the numerator and denominator coefficients of the resulting estimated model sys.

```
sys.Numerator
ans = 1\times2
    2.4554 176.9856
sys.Denominator
ans = 1\times3
    1.0000 3.1625 23.1631
```

To view the uncertainty in the estimates of the numerator and denominator and other information, use tfdata.

## Specify Number of Poles and Zeros in Estimated Transfer Function

Load time-domain system response data $z 2$ and use it to estimate a transfer function that contains two poles and one zero.

```
load iddata2 z2;
np = 2;
nz = 1;
sys = tfest(z2,np,nz);
```

sys is an idtf model containing the estimated transfer function.

## Estimate Transfer Function Containing Known Transport Delay

Load the data $z 2$, which is an iddata object that contains time-domain system response data.
load iddata2 z2;
Estimate a transfer function model sys that contains two poles and one zero, and which includes a known transport delay iodelay.

```
np = 2;
nz = 1;
iodelay = 0.2;
sys = tfest(z2,np,nz,iodelay);
```

sys is an idtf model containing the estimated transfer function, with the IODelay property set to 0.2 seconds.

## Estimate Transfer Function Containing Unknown Transport Delay

Load time-domain system response data z2 and use it to estimate a two-pole one-zero transfer function for the system. Specify an unknown transport delay for the transfer function by setting the value of iodelay to NaN.

```
load iddata2 z2;
np = 2;
nz = 1;
iodelay = NaN;
sys = tfest(z2,np,nz,iodelay);
```

sys is an idtf model containing the estimated transfer function, whose IODelay property is estimated using the data.

## Estimate Discrete-Time Transfer Function

Load time-domain system response data z2.

```
load iddata2 z2
```

Estimate a discrete-time transfer function with two poles and one zero. Specify the sample time Ts as 0.1 seconds and the transport delay iodelay as 2 seconds.

```
np = 2;
nz = 1;
iodelay = 2;
Ts = 0.1;
sysd = tfest(z2,np,nz,iodelay,'Ts',Ts)
sysd =
    From input "u1" to output "y1":
        1.8 z^-1
    z^(-2) * ---------------------------
            1 - 1.418 z^-1 + 0.6613 z^-2
Sample time: 0.1 seconds
Discrete-time identified transfer function.
Parameterization:
    Number of poles: 2 Number of zeros: 1
    Number of free coefficients: 3
    Use "tfdata", "getpvec", "getcov" for parameters and their uncertainties.
```


## Status:

Estimated using TFEST on time domain data "z2".
Fit to estimation data: 80.26\%
FPE: 2.095, MSE: 2.063
By default, the model has no feedthrough, and the numerator polynomial of the estimated transfer function has a zero leading coefficient b0. To estimate b0, specify the Feedthrough property during estimation.

## Estimate Discrete-Time Transfer Function with Feedthrough

Load the estimation data z5.
load iddata5 z5
First, estimate a discrete-time transfer function model with two poles, one zero, and no feedthrough. Get the sample time from the Ts property of z5.

```
np = 2;
nz = 1;
sys = tfest(z5,np,nz,'Ts',z5.Ts);
```

The estimated transfer function has the following form:

$$
H\left(z^{-1}\right)=\frac{b 1 z^{-1}+b 2 z^{-2}}{1+a 1 z^{-1}+a 2 z^{-2}}
$$

By default, the model has no feedthrough, and the numerator polynomial of the estimated transfer function has a zero leading coefficient b0. To estimate b0, specify the Feedthrough property during estimation.

```
sys = tfest(z5,np,nz,'Ts',z5.Ts,'Feedthrough',true);
```

The numerator polynomial of the estimated transfer function now has a nonzero leading coefficient:

$$
H\left(z^{-1}\right)=\frac{b 0+b 1 z^{-1}+b 2 z^{-2}}{1+a 1 z^{-1}+a 2 z^{-2}}
$$

## Analyze Origin of Delay in Measured Data

Compare two discrete-time models with and without feedthrough and transport delay.
If there is a delay from the measured input to output, it can be attributed either to a lack of feedthrough or to an actual transport delay. For discrete-time models, absence of feedthrough corresponds to a lag of one sample between the input and output. Estimating a model using Feedthrough $=$ false and iodelay $=0$ thus produces a discrete-time system that is equivalent to a system estimated using Feedthrough $=$ true and iodelay $=1$. Both systems show the same time- and frequency-domain responses, for example, on step and Bode plots. However, you get different results if you reduce these models using bal red or convert them to their continuous-time
representations. Therefore, a best practice is to check if the observed delay can be attributed to a transport delay or to a lack of feedthrough.

Estimate a discrete-time model with no feedthrough.
load iddatal z1
$\mathrm{np}=2$;
nz = 2;
sys1 = tfest(z1,np,nz,'Ts',z1.Ts);
Because sys 1 has no feedthrough and therefore has a numerator polynomial that beings with $z^{-1}$, sys1 has a lag of one sample. The IODelay property is 0 .

Estimate another discrete-time model with feedthrough and with a reduction from two zeros to one, incurring a one-sample input-output delay.
sys2 = tfest(z1,np,nz-1,1,'Ts',z1.Ts,'Feedthrough',true);
Compare the Bode responses of the models.
bode(sys1,sys2);


The discrete equations that underlie sys1 and sys2 are equivalent, and so are the Bode responses.
Convert the models to continuous time and compare the Bode responses for these models.
sys1c = d2c(sys1);
sys2c = d2c(sys2);
bode(sys1c,sys2c);
legend


As the plot shows, the Bode responses of the two models do not match when you convert them to continuous time. When there is no feedthrough, as with sys1c, there must be some lag. When there is feedthrough, as with sys2c, there can be no lag. Continuous-time feedthrough maps to discretetime feedthrough. Continuous-time lag maps to discrete-time delays.

## Estimate MISO Discrete-Time Transfer Function with Feedthrough and Delay Specifications for Individual Channels

Estimate a two-input, one-output discrete-time transfer function with a delay of two 2 samples on the first input and zero samples on the second input. Both inputs have no feedthrough.

Load the data and split the data into estimation and validation data sets.

```
load iddata7 z7
ze = z7(1:300);
zv = z7(200:400);
```

Estimate a two-input, one-output transfer function with two poles and one zero for each input-tooutput transfer function.

```
Lag = [2;0];
Ft = [false,false];
model = tfest(ze,2,1,'Ts',z7.Ts,'Feedthrough',Ft,'InputDelay',Lag);
```

The Feedthrough value you choose dictates whether the leading numerator coefficient is zero (no feedthrough) or not (nonzero feedthrough). Delays are generally expressed separately using the InputDelay or IODelay property. This example uses InputDelay only to express the delays.

Validate the estimated model. Exclude the data outliers for validation.

```
I = 1:201;
I(114:118) = [];
opt = compareOptions('Samples',I);
compare(zv,model,opt)
```


## Simulated Response Comparison



## Estimate Transfer Function Model Using Regularized Impulse Response Model

Identify a 15th order transfer function model by using regularized impulse response estimation.
Load the data.
load regularizationExampleData m0simdata;
Obtain a regularized impulse response (FIR) model.

```
opt = impulseestOptions('RegularizationKernel','DC');
m0 = impulseest(m0simdata,70,opt);
```

Convert the model into a transfer function model after reducing the order to 15.
m = idtf(balred(idss(m0), 15));
Compare the model output with the data.
compare(m0simdata,m);

Simulated Response Comparison


## Estimate Transfer Function Using Estimation Option Set

Create an option set for tfest that specifies the initialization and search methods. Also set the display option, which specifies that the loss-function values for each iteration be shown.

```
opt = tfestOptions('InitializeMethod','n4sid','Display','on','SearchMethod','lsqnonlin');
```

Load time-domain system response data $z 2$ and use it to estimate a transfer function with two poles and one zero. Specify opt for the estimation options.

```
load iddata2 z2;
np = 2;
nz = 1;
iodelay = 0.2;
sys = tfest(z2,np,nz,iodelay,opt);
```

sys is an idtf model containing the estimated transfer function.

## Specify Model Properties of Estimated Transfer Function

Load the time-domain system response data z2, and use it to estimate a two-pole, one-zero transfer function. Specify an input delay.

```
load iddata2 z2;
np = 2;
nz = 1;
input_delay = 0.2;
sys = tfest(z2,np,nz,'InputDelay',input_delay);
```

sys is an idtf model containing the estimated transfer function with an input delay of 0.2 seconds.

## Convert Frequency-Response Data into Transfer Function

Use bode to obtain the magnitude and phase response for the following system:

$$
H(s)=\frac{s+0.2}{s^{3}+2 s^{2}+s+1}
$$

Use 100 frequency points, ranging from $0.1 \mathrm{rad} / \mathrm{s}$ to $10 \mathrm{rad} / \mathrm{s}$, to obtain the frequency-response data. Use frd to create a frequency-response data object.

```
freq = logspace(-1,1,100);
[mag,phase] = bode(tf([1 0.2],[1 2 1 1]),freq);
data = frd(mag.*exp(1j*phase*pi/180),freq);
```

Estimate a three-pole, one-zero transfer function using data.
$n p=3 ;$
nz = 1;
sys = tfest(data,np,nz);
sys is an idtf model containing the estimated transfer function.

## Estimate Transfer Function with Known Transport Delays for Multiple Inputs

Load the time-domain system response data co2data, which contains the data from two experiments, each with two inputs and one output. Convert the data from the first experiment into an iddata object data with a sample time of 0.5 seconds.

```
load co2data;
Ts = 0.5;
data = iddata(Output_exp1,Input_exp1,Ts);
```

Specify estimation options for the search method and the input and output offsets. Also specify the maximum number of search iterations.

```
opt = tfestOptions('SearchMethod','gna');
opt.InputOffset = [170;50];
opt.Output0ffset = mean(data.y(1:75));
opt.SearchOptions.MaxIterations = 50;
```

Estimate a transfer function using the measured data and the estimation option set opt. Specify the transport delays from the inputs to the output.

```
np = 3;
nz = 1;
iodelay = [2 5];
sys = tfest(data,np,nz,iodelay,opt);
```

iodelay specifies the input-to-output delay from the first and second inputs to the output as 2 seconds and 5 seconds, respectively.
sys is an idtf model containing the estimated transfer function.

## Estimate Transfer Function with Known and Unknown Transport Delays

Load time-domain system response data and use it to estimate a transfer function for the system. Specify the known and unknown transport delays.

```
load co2data;
Ts = 0.5;
data = iddata(Output_exp1,Input_exp1,Ts);
```

data is an iddata object with two input channels and one output channels, and which has a sample rate of 0.5 seconds.

Create an option set opt. Specify estimation options for the search method and the input and output offsets. Also specify the maximum number of search iterations.

```
opt = tfestOptions('Display','on','SearchMethod','gna');
opt.InputOffset = [170; 50];
opt.OutputOffset = mean(data.y(1:75));
opt.Search0ptions.MaxIterations = 50;
```

Specify the unknown and known transport delays in iodelay, using 2 for a known delay of 2 seconds and nan for the unknown delay. Estimate the transfer function using iodelay and opt.

```
np = 3;
nz = 1;
iodelay = [2 nan];
sys = tfest(data,np,nz,iodelay,opt);
sys is an idtf model containing the estimated transfer function.
```


## Estimate Transfer Function with Unknown, Constrained Transport Delays

Create a transfer function model with the expected numerator and denominator structure and delay constraints.

In this example, the experiment data consists of two inputs and one output. Both transport delays are unknown and have an identical upper bound. Additionally, the transfer functions from both inputs to the output are identical in structure.

```
init_sys = idtf(NaN(1,2),[1,NaN(1,3)],'IODelay',NaN);
init_sys.Structure(1).IODelay.Free = true;
init_sys.Structure(1).IODelay.Maximum = 7;
```

init_sys is an idtf model describing the structure of the transfer function from one input to the output. The transfer function consists of one zero, three poles, and a transport delay. The use of NaN indicates unknown coefficients.
init_sys.Structure(1).IODelay.Free = true indicates that the transport delay is not fixed.
init_sys.Structure(1).IODelay.Maximum = 7 sets the upper bound for the transport delay to 7 seconds.

Specify the transfer function from both inputs to the output.

```
init_sys = [init_sys,init_sys];
```

Load time-domain system response data and use it to estimate a transfer function. Specify options in the tfestOptions option set opt.

```
load co2data;
Ts = 0.5;
data = iddata(Output_exp1,Input_exp1,Ts);
opt = tfestOptions('Display','on','SearchMethod','gna');
opt.InputOffset = [170;50];
opt.OutputOffset = mean(data.y(1:75));
opt.SearchOptions.MaxIterations = 50;
sys = tfest(data,init_sys,opt);
```

sys is an idtf model containing the estimated transfer function.
Analyze the estimation result by comparison. Create a compare0ptions option set opt2 and specify input and output offsets, and then use compare.

```
opt2 = compareOptions;
opt2.InputOffset = opt.InputOffset;
opt2.OutputOffset = opt.OutputOffset;
compare(data,sys,opt2)
```



## Estimate Transfer Function Containing Different Numbers of Poles for Input-Output Pairs

Estimate a multiple-input, single-output transfer function containing different numbers of poles for input-output pairs for given data.

Obtain frequency-response data.
For example, use frd to create a frequency-response data model for the following system:

$$
G=\left[\begin{array}{c}
e^{-4 s} \frac{s+2}{s^{3}+2 s^{2}+4 s+5} \\
e^{-0.6 s} \frac{5}{s^{4}+2 s^{3}+s^{2}+s}
\end{array}\right]
$$

Use 100 frequency points, ranging from $0.01 \mathrm{rad} / \mathrm{s}$ to $100 \mathrm{rad} / \mathrm{s}$, to obtain the frequency-response data.

```
G = tf({[1 2],[5]},{[1 2 4 5],[1 2 1 1 0]},0,'IODelay',[4 0.6]);
```

data $=\mathrm{frd}(\mathrm{G}$, logspace $(-2,2,100))$;
data is an frd object containing the continuous-time frequency response for $G$.
Estimate a transfer function for data.

```
np = [3 4];
nz = [1 0];
iodelay = [4 0.6];
sys = tfest(data,np,nz,iodelay);
```

np specifies the number of poles in the estimated transfer function. The first element of $n \mathrm{p}$ indicates that the transfer function from the first input to the output contains three poles. Similarly, the second element of np indicates that the transfer function from the second input to the output contains four poles.
$n z$ specifies the number of zeros in the estimated transfer function. The first element of $n z$ indicates that the transfer function from the first input to the output contains one zero. Similarly, the second element of $n p$ indicates that the transfer function from the second input to the output does not contain any zeros.
iodelay specifies the transport delay from the first input to the output as 4 seconds. The transport delay from the second input to the output is specified as 0.6 seconds.
sys is an idtf model containing the estimated transfer function.

## Estimate Transfer Function for Unstable System

Estimate a transfer function describing an unstable system using frequency-response data.
Use idtf to construct a transfer function model $G$ of the following system:

$$
G=\left[\begin{array}{c}
\frac{s+2}{s^{3}+2 s^{2}+4 s+5} \\
\frac{5}{s^{4}+2 s^{3}+s^{2}+s+1}
\end{array}\right]
$$

$G=\operatorname{idtf}\left(\left\{\left[\begin{array}{ll}1 & 2\end{array}\right], 5\right\},\left\{\left[\begin{array}{lll}1 & 2 & 4\end{array}\right],\left[\begin{array}{llll}1 & 2 & 1 & 1\end{array}\right]\right\}\right)$;
Use idfrd to obtain a frequency-response data model data for G. Specify 100 frequency points ranging from $0.01 \mathrm{rad} / \mathrm{s}$ to $100 \mathrm{rad} / \mathrm{s}$.
data $=$ idfrd(G,logspace(-2,2,100));
data is an idfrd object.
Estimate a transfer function for data.
np = [3 4];
nz = [10];
sys = tfest(data,np,nz);
np specifies the number of poles in the estimated transfer function. The first element of np indicates that the transfer function from the first input to the output contains three poles. Similarly, the second element of np indicates that the transfer function from the second input to the output contains four poles.
$n z$ specifies the number of zeros in the estimated transfer function. The first element of $n z$ indicates that the transfer function from the first input to the output contains one zero. Similarly, the second
element of $n z$ indicates that the transfer function from the second input to the output does not contain any zeros.
sys is an idtf model containing the estimated transfer function.

```
pole(sys)
ans = 7x1 complex
    -1.5260 + 0.0000i
    -0.2370 + 1.7946i
    -0.2370 - 1.7946i
    -1.4656 + 0.0000i
    -1.0000 + 0.0000i
    0.2328 + 0.7926i
    0.2328 - 0.7926i
```

sys is an unstable system, as the pole display indicates.

## Estimate Transfer Function using High Modal Density Frequency Response Data

Load the high-density frequency-response measurement data. The data corresponds to an unstable process maintained at equilibrium using feedback control.

```
load HighModalDensityData FRF f
```

Package the data as an idfrd object for identification and find the Bode magnitude response.

```
G = idfrd(permute(FRF,[2 3 1]),f,0,'FrequencyUnit','Hz');
bodemag(G)
```



Estimate a transfer function with 32 poles and 32 zeros, and compare the Bode magnitude response.
sys = tfest(G,32,32);
bodemag(G, sys)
$x \lim ([0.01,2 e 3])$
legend


Obtain and Apply Estimated Initial Conditions
Load and plot the data.
load iddatalic zli
plot(z1i)

## Input-Output Data



Examine the initial value of the output data $y(1)$.
ystart = z1i. $\mathrm{y}(1)$
ystart $=-3.0491$
The measured output does not start at 0 .
Estimate a second-order transfer function sys and return the estimated initial condition ic.

```
[sys,ic] = tfest(zli,2,1);
ic
ic =
    initialCondition with properties:
    A: [2x2 double]
    X0: [2x1 double]
    C: [0.2957 5.2441]
    Ts: 0
```

ic is an initialCondition object that encapsulates the free response of sys, in state-space form, to the initial state vector in X0.

Simulate sys using the estimation data but without incorporating the initial conditions. Plot the simulated output with the measured output.
y_no_ic = sim(sys,zli);
plot(y_no_ic,zli)
legend('Módel Response','Output Data')


The measured and simulated outputs do not agree at the beginning of the simulation.
Incorporate the initial condition into the simOptions option set.

```
opt = simOptions('InitialCondition',ic);
y_ic = sim(sys,zli,opt);
plot(y_ic,zli)
legend('Model Response','Output Data')
```


## Input-Output Data



The simulation combines the model response to the input signal with the free response to the initial condition. The measured and simulated outputs now have better agreement at the beginning of the simulation. This initial condition is valid only for the estimation data z1i.

## Input Arguments

## data - Estimation data

iddata object | frd object | idfrd object
Estimation data, specified as an iddata object, an frd object, or an idfrd object.
For time-domain estimation, data must be an iddata object containing the input and output signal values.

Time-series models, which are models that contain no measured inputs, cannot be estimated using tfest. Use ar, arx, or armax for time-series models instead.

For frequency-domain estimation, data can be one of the following:

- Recorded frequency response data (frd or idfrd)
- iddata object with properties specified as follows:
- InputData - Fourier transform of the input signal
- OutputData - Fourier transform of the output signal
- Domain - 'Frequency'

Estimation data must be uniformly sampled.
For multiexperiment data, the sample times and intersample behavior of all the experiments must match.

You can estimate both continuous-time and discrete-time models (of sample time matching that of data) using time-domain data and discrete-time frequency-domain data. You can estimate only continuous-time models using continuous-time frequency-domain data.

## np - Number of poles

nonnegative integer | matrix
Number of poles in the estimated transfer function, specified as a nonnegative integer or a matrix.
For systems that have multiple inputs and/or multiple outputs, you can apply either a global value or individual values of np to the input-output pairs, as follows:

- Same number of poles for every pair - Specify np as a scalar.
- Individual number of poles for each pair - Specify $n p$ as an $n_{y}$-by- $n_{u}$ matrix. $n_{y}$ is the number of outputs and $n_{u}$ is the number of inputs.

For an example, see "Estimate Transfer Function Model by Specifying Number of Poles" on page 11548.

## nz - Number of zeros

nonnegative integer | matrix
Number of zeros in the estimated transfer function, specified as a nonnegative integer or a matrix.
For systems that have multiple inputs, multiple outputs, or both, you can apply either a global value or individual values of $n z$ to the input-output pairs, as follows:

- Same number of poles for every pair - Specify $n z$ as a scalar.
- Individual number of poles for each pair - Specify $n z$ as an $n_{y}$-by- $n_{u}$ matrix. $n_{y}$ is the number of outputs and $n_{u}$ is the number of inputs.

For a continuous-time model estimated using discrete-time data, set $\mathrm{nz}<=\mathrm{np}$.
For discrete-time model estimation, specify $n z$ as the number of zeros of the numerator polynomial of the transfer function. For example, tfest (data, 2, 1, 'Ts' , data.Ts) estimates a transfer function of the form $b_{1} z^{-1} /\left(1+a_{1} z^{-1}+b_{2} z^{-2}\right)$, while tfest (data, 2,2 , 'Ts' , data.Ts) estimates $\left(b_{1} z^{-1}+b_{2} z^{-2}\right) /\left(1+a_{1} z^{-1}+b_{2} z^{-2}\right)$. Here, $z^{-1}$ is the Z-transform lag variable. For more information about discrete-time transfer functions, see "Discrete-Time Representation". For an example, see "Estimate Discrete-Time Transfer Function" on page 1-1550.

## iodelay - Transport delay

[] (default) | nonnegative integer | matrix
Transport delay, specified as a nonnegative integer, an NaN scalar, or a matrix.

For continuous-time systems, specify transport delays in the time unit stored in the TimeUnit property of data. For discrete-time systems, specify transport delays as integers denoting delays of a multiple of the sample time Ts.

For a MIMO system with $N_{y}$ outputs and $N_{u}$ inputs, set iodelay to an $N_{y}$-by- $N_{u}$ array. Each entry of this array is a numerical value that represents the transport delay for the corresponding input-output pair. You can also set iodelay to a scalar value to apply the same delay to all input-output pairs.

The specified values are treated as fixed delays. To denote unknown transport delays, specify NaN in the iodelay matrix.

Use [ ] or 0 to indicate that there is no transport delay.
For an example, see "Estimate Transfer Function Containing Known Transport Delay" on page 11549.

## opt - Estimation options

n4sidOptions option set
Estimation options, specified as an tfestOptions option set. Options specified by opt include:

- Estimation objective
- Handling of initial conditions
- Numerical search method to be used in estimation

For an example, see "Estimate Transfer Function Using Estimation Option Set" on page 1-1555.

## init_sys - Linear system that configures initial parameterization of sys idtf model | linear model | structure

Linear system that configures the initial parameterization of sys, specified as an idtf model or as a structure. You obtain init_sys either by performing an estimation using measured data or by direct construction.

If init_sys is an idtf model, tfest uses the parameter values of init_sys as the initial guess for estimating sys.

Use the Structure property of init_sys to configure initial parameter values and constraints for the numerator, denominator, and transport lag. For instance:

- To specify an initial guess for the $A$ matrix of init_sys, set init_sys.Structure. Numerator. Value to the initial guess.
- To specify constraints for the $B$ matrix of init_sys:
- Set init_sys.Structure.Numerator.Minimum to the minimum numerator coefficient values.
- Set init_sys.Structure. Numerator. Maximum to the maximum numerator coefficient values.
- Set init_sys.Structure.Numerator.Free to indicate which numerator coefficients are free for estimation.

For an example, see "Estimate Transfer Function with Unknown, Constrained Transport Delays" on page 1-1557.

If init_sys is not an idtf model, the software first converts init_sys to a transfer function. tfest uses the parameters of the resulting model as the initial guess for estimation.

If you do not specify opt, and init_sys was obtained by estimation rather than construction, then the software uses estimation options from init_sys. Report. OptionsUsed.

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, . . . , NameN, ValueN.
Example: sys = tfest(data,np,nz,'Ts',0.1)

## Ts - Sample time of estimated model

0 (continuous time) (default) | positive scalar
Sample time of the estimated model, specified as the comma-separated pair consisting of ' Ts' and either 0 or a positive scalar.

- For continuous-time models, specify 'Ts' as 0.
- For discrete-time models, specify 'Ts' as the data sample time in the units stored in the TimeUnit property. In the discrete case, np and nz refer to the number of roots of $z^{-1}$ for the numerator and denominator polynomials.

For an example, see "Estimate Discrete-Time Transfer Function" on page 1-1550.

## InputDelay - Input delays

0 (default) | scalar | vector
Input delay for each input channel, specified as the comma-separated pair consisting of
'InputDelay' and a numeric vector.

- For continuous-time models, specify ' InputDelay ' in the time units stored in the TimeUnit property.
- For discrete-time models, specify ' InputDelay ' in integer multiples of the sample time Ts. For example, setting 'InputDelay' to 3 specifies a delay of three sampling periods.

For a system with $N_{u}$ inputs, set InputDelay to an $N_{u}$-by-1 vector. Each entry of this vector is a numerical value that represents the input delay for the corresponding input channel.

To apply the same delay to all channels, specify InputDelay as a scalar.
For an example, see "Specify Model Properties of Estimated Transfer Function" on page 1-1556.

## Feedthrough - Feedthrough for discrete-time transfer function

0 (default) | 1 | logical matrix
Feedthrough for discrete-time transfer functions, specified as the comma-separated pair consisting of ' Feedthrough ' a logical scalar or an $N_{y}$-by- $N_{u}$ logical matrix. $N_{y}$ is the number of outputs and $N_{u}$ is the number of inputs. To use the same feedthrough for all input-output channels, specify Feedthrough as a scalar.

Consider a discrete-time model with two poles and three zeros:
$H\left(z^{-1}\right)=\frac{b 0+b 1 z^{-1}+b 2 z^{-2}+b 3 z^{-3}}{1+a 1 z^{-1}+a 2 z^{-2}}$
When the model has direct feedthrough, $b 0$ is a free parameter whose value is estimated along with the rest of the model parameters $b 1, b 2, b 3, a 1$, and $a 2$. When the model has no feedthrough, $b 0$ is fixed to zero. For an example, see "Estimate Discrete-Time Transfer Function with Feedthrough" on page 1-1551.

## Output Arguments

## sys - Identified transfer function

idtf model
Identified transfer function, returned as an idtf object. This model is created using the specified model orders, delays, and estimation options.

Information about the estimation results and options used is stored in the Report property of the model. Report has the following fields.

| Report <br> Field | Description |
| :--- | :--- |
| Status | Summary of the model status, which indicates whether the model was created by <br> construction or obtained by estimation. |
| Method | Estimation command used. |
| Initializ <br> eMethod | Algorithm used to initialize the numerator and denominator for estimation of <br> continuous-time transfer functions using time-domain data, returned as one of the <br> following values: |
| - $\quad$ 'iv' - Instrument Variable approach |  |
| $-\quad$ 'svf' - State Variable Filters approach |  |
| - 'gpmf' - Generalized Poisson Moment Functions approach |  |
| $-\quad$ 'n4sid' - Subspace state-space estimation approach |  |


| Report Field | Description |  |
| :---: | :---: | :---: |
| InitialCo ndition | Handling of initial conditions during model estimation, returned as one of the following values: <br> - 'zero' - The initial conditions were set to zero. <br> - 'estimate' - The initial conditions were treated as independent estimation parameters. <br> - 'backcast' - The initial conditions were estimated using the best least squares fit. <br> This field is especially useful to view how the initial conditions were handled when the InitialCondition option in the estimation option set is 'auto'. |  |
| Fit | Quantitative assessment of the estimation, returned as a structure. See "Loss Function and Model Quality Metrics" for more information on these quality metrics. The structure has the following fields: |  |
|  | Field | Description |
|  | FitPerce nt | Normalized root mean squared error (NRMSE) measure of how well the response of the model fits the estimation data, expressed as the percentage fit $=100(1-$ NRMSE $)$. |
|  | LossFcn | Value of the loss function when the estimation completes. |
|  | MSE | Mean squared error (MSE) measure of how well the response of the model fits the estimation data. |
|  | FPE | Final prediction error for the model. |
|  | AIC | Raw Akaike Information Criteria (AIC) measure of model quality. |
|  | AICc | Small sample-size corrected AIC. |
|  | nAIC | Normalized AIC. |
|  | BIC | Bayesian Information Criteria (BIC). |
| Parameter <br> s | Estimated values of model parameters. |  |
| OptionsUs ed | Option set used for estimation. If no custom options were configured, this is a set of default options. See polyestOptions for more information. |  |
| RandState | State of the random number stream at the start of estimation. Empty, [ ], if randomization was not used during estimation. For more information, see rng. |  |


| Report Field | Description |  |
| :---: | :---: | :---: |
| DataUsed | Attributes of the data used for estimation, returned as a structure with the following fields: |  |
|  | Field | Description |
|  | Name | Name of the data set. |
|  | Type | Data type. |
|  | Length | Number of data samples. |
|  | Ts | Sample time. |
|  | InterSam ple | Input intersample behavior, returned as one of the following values: <br> - 'zoh ' - Zero-order hold maintains a piecewise-constant input signal between samples. <br> - ' foh ' - First-order hold maintains a piecewise-linear input signal between samples. <br> - 'bl' - Band-limited behavior specifies that the continuous-time input signal has zero power above the Nyquist frequency. |
|  | Input0ff set | Offset removed from time-domain input data during estimation. For nonlinear models, it is [ ]. |
|  | Output0f fset | Offset removed from time-domain output data during estimation. For nonlinear models, it is [ ]. |
| Terminati on | Termination conditions for the iterative search used for prediction error minimization, returned as a structure with the following fields: |  |
|  | Field | Description |
|  | WhyStop | Reason for terminating the numerical search. |
|  | Iteratio ns | Number of search iterations performed by the estimation algorithm. |
|  | First0rd erOptima lity | $\infty$-norm of the gradient search vector when the search algorithm terminates. |
|  | FcnCount | Number of times the objective function was called. |
|  | UpdateNo rm | Norm of the gradient search vector in the last iteration. Omitted when the search method is 'lsqnonlin' or 'fmincon'. |
|  | LastImpr ovement | Criterion improvement in the last iteration, expressed as a percentage. Omitted when the search method is 'lsqnonlin' or 'fmincon'. |
|  | Algorith | Algorithm used by ' lsqnonlin' or 'fmincon' search method. Omitted when other search methods are used. |
|  | For estimation methods that do not require numerical search optimization, the Termination field is omitted. |  |

For more information on using Report, see "Estimation Report".

## ic - Initial conditions

initialCondition object|object array of initialCondition values
Estimated initial conditions, returned as an initialCondition object or an object array of initialCondition values.

- For a single-experiment data set, ic represents, in state-space form, the free response of the transfer function model ( $A$ and $C$ matrices) to the estimated initial states ( $x_{0}$ ).
- For a multiple-experiment data set with $N_{e}$ experiments, ic is an object array of length $N_{e}$ that contains one set of initialCondition values for each experiment.

If tfest returns ic values of 0 and the you know that you have non-zero initial conditions, set the 'InitialCondition' option in tfestOptions to 'estimate' and pass the updated option set to tfest. For example:

```
opt = tfestOptions('InitialCondition,'estimate')
```

[sys,ic] = tfest(data,np,nz,opt)

The default 'auto' setting of 'InitialCondition' uses the 'zero' method when the initial conditions have a negligible effect on the overall estimation-error minimization process. Specifying 'estimate' ensures that the software estimates values for ic.

For more information, see initialCondition. For an example of using this argument, see "Obtain and Apply Estimated Initial Conditions" on page 1-1563.

## Algorithms

The details of the estimation algorithms used by tfest vary depending on various factors, including the sampling of the estimated model and the estimation data.

## Continuous-Time Transfer Function Estimation Using Time-Domain Data

## Parameter Initialization

The estimation algorithm initializes the estimable parameters using the method specified by the InitializeMethod estimation option. The default method is the Instrument Variable (IV) method.

The State-Variable Filters (SVF) approach and the Generalized Poisson Moment Functions (GPMF) approach to continuous-time parameter estimation use prefiltered data [1] [2]. The constant $\frac{1}{\lambda}$ in [1] and [2] corresponds to the initialization option (InitializeOptions) field FilterTimeConstant. IV is the simplified refined IV method and is called SRIVC in [3]. This method has a prefilter that is the denominator of the current model, initialized with SVF. This prefilter is iterated up to MaxIterations times, until the model change is less than Tolerance. MaxIterations and Tolerance are options that you can specify using the InitializeOptions structure. The 'n4sid' initialization option estimates a discrete-time model, using the N4SID estimation algorithm, that it transforms to continuous-time using d2c.

Use tfestOptions to create the option set used to estimate a transfer function.

## Parameter Update

The initialized parameters are updated using a nonlinear least-squares search method, specified by the SearchMethod estimation option. The objective of the search method is to minimize the weighted prediction error norm.

## Discrete-Time Transfer Function Estimation Using Time-Domain Data

For discrete-time data, tfest uses the same algorithm as oe to determine the numerator and denominator polynomial coefficients. In this algorithm, the initialization is performed using arx, followed by nonlinear least-squares search-based updates to minimize a weighted prediction error norm.

## Continuous-Time Transfer Function Estimation Using Continuous-Time Frequency-Domain Data

The estimation algorithm performs the following tasks:
1 Perform a bilinear mapping to transform the domain (frequency grid) of the transfer function. For continuous-time models, the imaginary axis is transformed to the unit disk. For discrete-time models, the original domain unit disk is transformed to another unit disk.
2 Perform S-K iterations [4] to solve a nonlinear least-squares problem - Consider a multi-input single-output system. The nonlinear least-squares problem is to minimize the following loss function:

$$
\underset{D, N_{i}}{\operatorname{minimize}} \sum_{k=1}^{n_{f}}\left|W\left(\omega_{k}\right)\left(y\left(\omega_{k}\right)-\sum_{i=1}^{n_{U}} \frac{N_{i}\left(\omega_{k}\right)}{D\left(\omega_{k}\right)} u_{i}\left(\omega_{k}\right)\right)\right|^{2}
$$

Here, $W$ is a frequency-dependent weight that you specify. $D$ is the denominator of the transfer function model that is to be estimated, and $N_{i}$ is the numerator corresponding to the ith input. $y$ and $u$ are the measured output and input data, respectively. $n_{f}$ and $n_{u}$ are the number of frequencies and inputs, and $w$ is the frequency. Rearranging the terms gives

$$
\underset{D, N_{i}}{\operatorname{minimize}} \sum_{k=1}^{n_{f}}\left|\frac{W\left(\omega_{k}\right)}{D\left(\omega_{k}\right)}\right| D\left(\omega_{k}\right) y\left(\omega_{k}\right)-\left.\sum_{i=1}^{n_{u}} N_{i}\left(\omega_{k}\right) u_{i}\left(\omega_{k}\right)\right|^{2}
$$

To perform the S-K iterations, the algorithm iteratively solves

$$
\underset{D_{m}, N_{i, m}}{\operatorname{minimize}} \sum_{k=1}^{n_{f}} \left\lvert\, \frac{W\left(\omega_{k}\right)}{D_{m-1}\left(\omega_{k}\right)}\left(D_{m}\left(\omega_{k}\right) y\left(\omega_{k}\right)-\left.\sum_{i=1}^{n_{u}} N_{i, m}\left(\omega_{k}\right) u_{i}\left(\omega_{k}\right)\right|^{2}\right.\right.
$$

Here, $m$ is the current iteration, and $D_{m-1}(\omega)$ is the denominator response identified at the previous iteration. Now each step of the iteration is a linear least-squares problem, where the identified parameters capture the responses $D_{m}(\omega)$ and $N_{i, m}(\omega)$ for $i=1,2, \ldots n_{u}$. The iteration is initialized by choosing $D_{0}(\omega)=1$.

- The first iteration of the algorithm identifies $D_{1}(\omega)$. The $D_{1}(\omega)$ and $N_{i, 1}(\omega)$ polynomials are expressed in monomial basis.
- The second and following iterations express the polynomials $D_{m}(\omega)$ and $N_{i, m}(\omega)$ in terms of orthogonal rational basis functions on the unit disk. These basis functions have the form

$$
B_{j, m}(\omega)=\left(\frac{\sqrt{1-\left|\lambda_{j, m-1}\right|^{2}}}{q-\lambda_{j, m-1}}\right)_{r=0}^{j-1} \frac{1-\left(\lambda_{j, m-1}\right)^{*} q(\omega)}{q(\omega)-\lambda_{r, m-1}}
$$

Here, $\lambda_{j, m-1}$ is the $j$ th pole that is identified at the previous step $m-1$ of the iteration. $\lambda_{j, m-1}{ }^{*}$ is the complex conjugate of $\lambda_{j, m-1}$, and $q$ is the frequency-domain variable on the unit disk.

- The algorithm runs for a maximum of 20 iterations. The iterations are terminated early if the relative change in the value of the loss function is less than 0.001 in the last three iterations.

If you specify bounds on transfer function coefficients, these bounds correspond to affine constraints on the identified parameters. If you have only equality constraints (fixed transfer function coefficients), the corresponding equality constrained least-squares problem is solved algebraically. To do so, the software computes an orthogonal basis for the null space of the equality constraint matrix, and then solves the least-squares problem within this null space. If you have upper or lower bounds on transfer function coefficients, the corresponding inequality constrained least-squares problem is solved using interior-point methods.
3 Perform linear refinements - The S-K iterations, even when they converge, do not always yield a locally optimal solution. To find a critical point of the optimization problem that can yield a locally optimal solution, a second set of iterations are performed. The critical points are solutions to a set of nonlinear equations. The algorithm searches for a critical point by successively constructing a linear approximation to the nonlinear equations and solving the resulting linear equations in the least-squares sense. The equations follow.

- Equation for the $j$ th denominator parameter:
$0=2 \sum_{k=1}^{n_{f}} \operatorname{Re}\left\{\frac{\left|W\left(\omega_{k}\right)\right|^{2} B_{j}^{*}\left(\omega_{k}\right) \sum_{i=1}^{n_{u}} N_{i, m-1}^{*}\left(\omega_{k}\right) u_{i}^{*}\left(\omega_{k}\right)}{D_{m-1}^{*}\left(\omega_{k}\right)\left|D_{m-1}\left(\omega_{k}\right)\right|^{2}}\left(D_{m}\left(\omega_{k}\right) y\left(\omega_{k}\right)-\sum_{i=1}^{n_{u}} N_{i, m}\left(\omega_{k}\right) u_{i}\left(\omega_{k}\right)\right)\right\}$
- Equation for the $j$ th numerator parameter that corresponds to input $l$ :
$0=-2 \sum_{k=1}^{n_{f}} \operatorname{Re}\left\{\frac{\left|W\left(\omega_{k}\right)\right|^{2} B_{j}^{*}\left(\omega_{k}\right) u_{l}^{*}\left(\omega_{k}\right)}{\left|D_{m-1}\left(\omega_{k}\right)\right|^{2}}\left(D_{m}\left(\omega_{k}\right) y\left(\omega_{k}\right)-\sum_{i=1}^{n_{u}} N_{i, m}\left(\omega_{k}\right) u_{i}\left(\omega_{k}\right)\right)\right\}$
The first iteration is started with the best solution found for the numerators $N_{i}$ and denominator $D$ parameters during S-K iterations. Unlike S-K iterations, the basis functions $B_{j}(\omega)$ are not changed at each iteration; the iterations are performed with the basis functions that yielded the best solution in the S-K iterations. As before, the algorithm runs for a maximum of 20 iterations. The iterations are terminated early if the relative change in the value of the loss function is less than 0.001 in the last three iterations.

If you specify bounds on transfer function coefficients, these bounds are incorporated into the necessary optimality conditions using generalized Lagrange multipliers. The resulting constrained linear least-squares problems are solved using the same methods explained in the SK iterations step.
4 Return the transfer function parameters corresponding to the optimal solution - Both the S-K and linear refinement iteration steps do not guarantee an improvement in the loss function value. The algorithm tracks the best parameter value observed during these steps, and returns these values.
5 Invert the bilinear mapping performed in step 1.
6 Perform an iterative refinement of the transfer function parameters using the nonlinear leastsquares search method specified in the SearchMethod estimation option. This step is implemented in the following situations:

- When you specify the EnforceStability estimation option as true (stability is requested), and the result of step 5 of this algorithm is an unstable model. The unstable poles are
reflected inside the stability boundary and the resulting parameters are iteratively refined. For information about estimation options, see tfestOptions.
- When you add a regularization penalty to the loss function using the Regularization estimation option. For an example about regularization, see "Regularized Identification of Dynamic Systems".
- You estimate a continuous-time model using discrete-time data (see "Discrete-Time Transfer Function Estimation Using Discrete-Time Frequency-Domain Data" on page 1-1576).
- You use frequency domain input-output data to identify a multi-input model.

If you are using the estimation algorithm from R2016a or earlier (see "tfest Estimation Algorithm Update" on page 1-1577) for estimating a continuous-time model using continuous-time frequencydomain data, then for continuous-time data and fixed delays, the Output-Error algorithm is used for model estimation. For continuous-time data and free delays, the state-space estimation algorithm is used. In this algorithm, the model coefficients are initialized using the N4SID estimation method. This initialization is followed by nonlinear least-squares search-based updates to minimize a weighted prediction error norm.

## Discrete-Time Transfer Function Estimation Using Discrete-Time Frequency-Domain Data

The estimation algorithm is the same as for continuous-time transfer function estimation using continuous-time frequency-domain data on page 1-1574, except discrete-time data is used.

If you are using the estimation algorithm from R2016a or earlier (see "tfest Estimation Algorithm Update" on page 1-1577), the algorithm is the same as the algorithm for discrete-time transfer function estimation using time-domain data on page 1-1574.

Note The software does not support estimation of a discrete-time transfer function using continuoustime frequency-domain data.

## Continuous-Time Transfer Function Estimation Using Discrete-Time Frequency-Domain Data

The tfest command first estimates a discrete-time model from the discrete-time data. The estimated model is then converted to a continuous-time model using the d2c command. The frequency response of the resulting continuous-time model is then computed over the frequency grid of the estimation data. A continuous-time model of the desired (user-specified) structure is then fit to this frequency response. The estimation algorithm for using the frequency-response data to obtain the continuoustime model is the same as the algorithm for continuous-time transfer function estimation using continuous-time data on page 1-1574.

If you are using the estimation algorithm from R2016a or earlier (see "tfest Estimation Algorithm Update" on page 1-1577), the state-space estimation algorithm is used for estimating continuous-time models from discrete-time data. In this algorithm, the model coefficients are initialized using the N4SID estimation method. This initialization is followed by nonlinear least-squares search-based updates to minimize a weighted prediction error norm.

## Delay Estimation

- When delay values are specified as NaN , they are estimated separate from the model numerator and denominator coefficients, using delayest. The delay values thus determined are treated as fixed values during the iterative update of the model using a nonlinear least-squares search method. Thus, the delay values are not iteratively updated.
- For an initial model, init_sys, with:
- init_sys.Structure.IODelay.Value specified as finite values
- init_sys.Structure.IODelay.Free specified as true
the initial delay values are left unchanged.
Estimation of delays is often a difficult problem. A best practice is to assess the presence and the value of a delay. To do so, use physical insight of the process being modeled and functions such as arxstruc, delayest, and impulseest. For an example of determining input delay, see "Model Structure Selection: Determining Model Order and Input Delay".


## Compatibility Considerations

## tfest Estimation Algorithm Update

Starting in R2016b, a new algorithm is used for performing transfer function estimation from frequency-domain data. You are likely to see faster and more accurate results with the new algorithm, particularly for data with dynamics over a large range of frequencies and amplitudes. However, the estimation results might not match results from previous releases. To perform estimation using the previous estimation algorithm, append '-R2016a' to the syntax.

For example, suppose that you are estimating a transfer function model with np poles using the frequency-domain data data.

```
sys = tfest(data,np)
```

To use the previous estimation algorithm, use the following syntax.

```
sys = tfest(data,np,'-R2016a')
```


## References

[1] Garnier, H., M. Mensler, and A. Richard. "Continuous-Time Model Identification from Sampled Data: Implementation Issues and Performance Evaluation." International Journal of Control 76, no. 13 (January 2003): 1337-57. https://doi.org/10.1080/0020717031000149636.
[2] Ljung, Lennart. "Experiments with Identification of Continuous Time Models." IFAC Proceedings Volumes 42, no. 10 (2009): 1175-80. https://doi.org/10.3182/20090706-3-FR-2004.00195.
[3] Young, Peter, and Anthony Jakeman. "Refined Instrumental Variable Methods of Recursive TimeSeries Analysis Part III. Extensions." International Journal of Control 31, no. 4 (April 1980): 741-64. https://doi.org/10.1080/00207178008961080.
[4] Drmač, Z., S. Gugercin, and C. Beattie. "Quadrature-Based Vector Fitting for Discretized $\mathrm{H}_{2}$ Approximation." SIAM Journal on Scientific Computing 37, no. 2 (January 2015): A625-52. https://doi.org/10.1137/140961511.
[5] Ozdemir, Ahmet Arda, and Suat Gumussoy. "Transfer Function Estimation in System Identification Toolbox via Vector Fitting." IFAC-PapersOnLine 50, no. 1 (July 2017): 6232-37. https://doi.org/ 10.1016/j.ifacol.2017.08.1026.

## Extended Capabilities

## Automatic Parallel Support

Accelerate code by automatically running computation in parallel using Parallel Computing Toolbox ${ }^{\mathrm{TM}}$.
Parallel computing support is available for estimation using the lsqnonlin search method (requires Optimization Toolbox). To enable parallel computing, use tfestOptions, set SearchMethod to 'lsqnonlin', and set SearchOptions.Advanced. UseParallel to true.

For example:

```
opt = tfestOptions;
opt.SearchMethod = 'lsqnonlin';
opt.SearchOptions.Advanced.UseParallel = true;
```


## See Also

ar|arx|bj|greyest|idtf|oe|polyest|procest|ssest|tfest0ptions

## Topics

"Estimate Transfer Function Models at the Command Line"
"Estimate Transfer Function Models with Transport Delay to Fit Given Frequency-Response Data"
"Estimate Transfer Function Models With Prior Knowledge of Model Structure and Constraints"
"Apply Initial Conditions when Simulating Identified Linear Models"
"Troubleshoot Frequency-Domain Identification of Transfer Function Models"
"What are Transfer Function Models?"
"Regularized Estimates of Model Parameters"
"Estimating Models Using Frequency-Domain Data"

## Introduced in R2012a

## tfestOptions

Option set for tfest

## Syntax

```
opt = tfestOptions
opt = tfestOptions(Name,Value)
```


## Description

opt $=$ tfestOptions creates the default option set for tfest.
opt $=$ tfestOptions(Name, Value) creates an option set with the options specified by one or more Name, Value pair arguments.

## Input Arguments

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Valuel, . . . , NameN, ValueN.

## InitializeMethod - Algorithm used to initialize the numerator and denominator 'iv' (default)|'svf'|'gpmf'|'n4sid'|'all'

Algorithm used to initialize the values of the numerator and denominator of the output of tfest, applicable only for estimation of continuous-time transfer functions using time-domain data, specified as one of the following values:

- 'iv' - Instrument Variable approach.
- 'svf' - State Variable Filters approach.
- 'gpmf ' - Generalized Poisson Moment Functions approach.
- 'n4sid' - Subspace state-space estimation approach.
- 'all' - Combination of all of the preceding approaches. The software tries all these methods and selects the method that yields the smallest value of prediction error norm.


## InitializeOptions - Option set for the initialization algorithm

structure
Option set for the initialization algorithm used to initialize the values of the numerator and denominator of the output of tfest, specified as a structure with the following fields:

- N4Weight - Calculates the weighting matrices used in the singular-value decomposition step of the 'n4sid' algorithm. Applicable when InitializeMethod is 'n4sid'.

N4Weight is specified as one of the following values:

- 'MOESP' - Uses the MOESP algorithm by Verhaegen.
- 'CVA' - Uses the canonical variate algorithm (CVA) by Larimore.
- 'SSARX' - A subspace identification method that uses an ARX estimation based algorithm to compute the weighting.

Specifying this option allows unbiased estimates when using data that is collected in closedloop operation. For more information about the algorithm, see [6].

- 'auto' - The software automatically determines if the MOESP algorithm or the CVA algorithm should be used in the singular-value decomposition step.

Default: 'auto'

- N4Horizon - Determines the forward and backward prediction horizons used by the 'n4sid' algorithm. Applicable when InitializeMethod is 'n4sid'.

N4Horizon is a row vector with three elements: [ $r$ sy su], where $r$ is the maximum forward prediction horizon. The algorithm uses up to $r$ step-ahead predictors. sy is the number of past outputs, and su is the number of past inputs that are used for the predictions. See pages 209 and 210 in [1] for more information. These numbers can have a substantial influence on the quality of the resulting model, and there are no simple rules for choosing them. Making 'N4Horizon' a k-by-3 matrix means that each row of 'N4Horizon' is tried, and the value that gives the best (prediction) fit to data is selected. k is the number of guesses of [ r sy su] combinations.

If N4Horizon = 'auto', the software uses an Akaike Information Criterion (AIC) for the selection of sy and su.

Default: 'auto'

- FilterTimeConstant - Time constant of the differentiating filter used by the iv, svf, and gpmf initialization methods (see [4] and [5]).

FilterTimeConstant specifies the cutoff frequency of the differentiating filter, $F_{\text {cutoff }}$ as:

$$
F_{\text {cutoff }}=\frac{\text { FilterTimeConstant }}{T_{S}}
$$

$T_{s}$ is the sample time of the estimation data.
Specify FilterTimeConstant as a positive number, typically less than 1. A good value of FilterTimeConstant is the ratio of $T_{s}$ to the dominating time constant of the system.

Default: 0.1

- MaxIterations - Maximum number of iterations. Applicable when InitializeMethod is 'iv'.

Default: 30

- Tolerance - Convergence tolerance. Applicable when InitializeMethod is 'iv'.

Default: 0.01

## InitialCondition - Handling of initial conditions

'auto' (default)|'zero'|'estimate'|'backcast'
Handling of initial conditions during estimation, specified as one of the following values:

- 'zero' - All initial conditions are taken as zero.
- 'estimate' - The necessary initial conditions are treated as estimation parameters.
- 'backcast' - The necessary initial conditions are estimated by a backcasting (backward filtering) process, described in [2].
- 'auto' - An automatic choice among the preceding options is made, guided by the data.


## WeightingFilter - Weighting prefilter

[] (default) | vector | matrix | cell array | linear system | 'inv' |'invsqrt'
Weighting prefilter applied to the loss function to be minimized during estimation. To understand the effect of WeightingFilter on the loss function, see "Loss Function and Model Quality Metrics".

Specify WeightingFilter as one of the following values:

- [] - No weighting prefilter is used.
- Passbands - Specify a row vector or matrix containing frequency values that define desired passbands. You select a frequency band where the fit between estimated model and estimation data is optimized. For example, [wl,wh] where wl and wh represent lower and upper limits of a passband. For a matrix with several rows defining frequency passbands,
[w1l,w1h;w2l,w2h;w3l,w3h;...], the estimation algorithm uses the union of the frequency ranges to define the estimation passband.

Passbands are expressed in rad/TimeUnit for time-domain data and in FrequencyUnit for frequency-domain data, where TimeUnit and FrequencyUnit are the time and frequency units of the estimation data.

- SISO filter - Specify a single-input-single-output (SISO) linear filter in one of the following ways:
- A SISO LTI model
- $\{A, B, C, D\}$ format, which specifies the state-space matrices of a filter with the same sample time as estimation data.
- \{numerator, denominator\} format, which specifies the numerator and denominator of the filter as a transfer function with the same sample time as estimation data.

This option calculates the weighting function as a product of the filter and the input spectrum to estimate the transfer function.

- Weighting vector - Applicable for frequency-domain data only. Specify a column vector of weights. This vector must have the same length as the frequency vector of the data set, Data. Frequency. Each input and output response in the data is multiplied by the corresponding weight at that frequency.
- 'inv' - Applicable for estimation using frequency-response data only. Use $1 /|G(\omega)|$ as the weighting filter, where $G(\omega)$ is the complex frequency-response data. Use this option for capturing relatively low amplitude dynamics in data, or for fitting data with high modal density. This option also makes it easier to specify channel-dependent weighting filters for MIMO frequency-response data.
- 'invsqrt ' - Applicable for estimation using frequency-response data only. Use $1 / \sqrt{|G(\omega)|}$ as the weighting filter. Use this option for capturing relatively low amplitude dynamics in data, or for fitting data with high modal density. This option also makes it easier to specify channel-dependent weighting filters for MIMO frequency-response data.


## EnforceStability - Control whether to enforce stability of model

false (default) | true

Control whether to enforce stability of estimated model, specified as the comma-separated pair consisting of 'EnforceStability' and either true or false.

Use this option when estimating models using frequency-domain data. Models estimated using timedomain data are always stable.

Data Types: logical

## EstimateCovariance - Control whether to generate parameter covariance data true (default) | false

Controls whether parameter covariance data is generated, specified as true or false.
If EstimateCovariance is true, then use getcov to fetch the covariance matrix from the estimated model.

## Display - Specify whether to display the estimation progress

'off' (default)|'on'
Specify whether to display the estimation progress, specified as one of the following values:

- 'on ' - Information on model structure and estimation results are displayed in a progress-viewer window.
- 'off ' - No progress or results information is displayed.


## InputOffset - Removal of offset from time-domain input data during estimation

[ ] (default) | vector of positive integers | matrix
Removal of offset from time-domain input data during estimation, specified as the comma-separated pair consisting of 'InputOffset' and one of the following:

- A column vector of positive integers of length $N u$, where $N u$ is the number of inputs.
- [ ] - Indicates no offset.
- $N u$-by-Ne matrix - For multi-experiment data, specify InputOffset as an Nu -by-Ne matrix. Nu is the number of inputs, and $N e$ is the number of experiments.

Each entry specified by InputOffset is subtracted from the corresponding input data.

## OutputOffset - Removal of offset from time-domain output data during estimation

[] (default) | vector | matrix
Removal of offset from time-domain output data during estimation, specified as the comma-separated pair consisting of 'OutputOffset' and one of the following:

- A column vector of length $N y$, where $N y$ is the number of outputs.
- [ ] - Indicates no offset.
- $N y$-by-Ne matrix - For multi-experiment data, specify OutputOffset as a $N y$-by-Ne matrix. $N y$ is the number of outputs, and $N e$ is the number of experiments.

Each entry specified by OutputOffset is subtracted from the corresponding output data.
OutputWeight - Weighting of prediction errors in multi-output estimations
[] (default) | 'noise' | positive semidefinite symmetric matrix
Weighting of prediction errors in multi-output estimations, specified as one of the following values:

- 'noise' - Minimize $\operatorname{det}\left(E^{*} E / N\right)$, where $E$ represents the prediction error and $N$ is the number of data samples. This choice is optimal in a statistical sense and leads to maximum likelihood estimates if nothing is known about the variance of the noise. It uses the inverse of the estimated noise variance as the weighting function.

Note OutputWeight must not be 'noise' if SearchMethod is 'lsqnonlin'.

- Positive semidefinite symmetric matrix ( W ) - Minimize the trace of the weighted prediction error matrix trace (E'*E*W/N) where:
- $E$ is the matrix of prediction errors, with one column for each output, and $W$ is the positive semidefinite symmetric matrix of size equal to the number of outputs. Use $W$ to specify the relative importance of outputs in multiple-output models, or the reliability of corresponding data.
- N is the number of data samples.
- [] - The software chooses between the 'noise' or using the identity matrix for W.

This option is relevant for only multi-output models.

## Regularization - Options for regularized estimation of model parameters

## structure

Options for regularized estimation of model parameters. For more information on regularization, see "Regularized Estimates of Model Parameters".

Regularization is a structure with the following fields:

- Lambda - Constant that determines the bias versus variance tradeoff.

Specify a positive scalar to add the regularization term to the estimation cost.
The default value of zero implies no regularization.
Default: 0

- R - Weighting matrix.

Specify a vector of nonnegative numbers or a square positive semi-definite matrix. The length must be equal to the number of free parameters of the model.

For black-box models, using the default value is recommended. For structured and grey-box models, you can also specify a vector of np positive numbers such that each entry denotes the confidence in the value of the associated parameter.

The default value of 1 implies a value of eye(npfree), where npfree is the number of free parameters.

## Default: 1

- Nominal - The nominal value towards which the free parameters are pulled during estimation.

The default value of zero implies that the parameter values are pulled towards zero. If you are refining a model, you can set the value to 'model' to pull the parameters towards the parameter values of the initial model. The initial parameter values must be finite for this setting to work.

Default: 0

## SearchMethod - Numerical search method used for iterative parameter estimation <br> 'auto' (default)|'gn'|'gna'|'lm'|'grad'|'lsqnonlin' |'fmincon'

Numerical search method used for iterative parameter estimation, specified as the comma-separated pair consisting of 'SearchMethod' and one of the following:

- 'auto' - A combination of the line search algorithms, 'gn', 'lm', 'gna', and 'grad' methods is tried in sequence at each iteration. The first descent direction leading to a reduction in estimation cost is used.
- 'gn' - Subspace Gauss-Newton least squares search. Singular values of the Jacobian matrix less than GnPinvConstant*eps*max(size(J))*norm(J) are discarded when computing the search direction. $J$ is the Jacobian matrix. The Hessian matrix is approximated as $J^{T} J$. If there is no improvement in this direction, the function tries the gradient direction.
- 'gna' - Adaptive subspace Gauss-Newton search. Eigenvalues less than gamma*max(sv) of the Hessian are ignored, where $s v$ contains the singular values of the Hessian. The Gauss-Newton direction is computed in the remaining subspace. gamma has the initial value
InitialGnaTolerance (see Advanced in 'Search0ptions' for more information). This value is increased by the factor LMStep each time the search fails to find a lower value of the criterion in fewer than five bisections. This value is decreased by the factor $2 *$ LMStep each time a search is successful without any bisections.
- 'lm' - Levenberg-Marquardt least squares search, where the next parameter value is - pinv (H $+\mathrm{d} * \mathrm{I}) * \mathrm{grad}$ from the previous one. $H$ is the Hessian, $I$ is the identity matrix, and grad is the gradient. $d$ is a number that is increased until a lower value of the criterion is found.
- 'grad ' - Steepest descent least squares search.
- 'lsqnonlin' - Trust-region-reflective algorithm of lsqnonlin. Requires Optimization Toolbox software.
- 'fmincon' - Constrained nonlinear solvers. You can use the sequential quadratic programming (SQP) and trust-region-reflective algorithms of the fmincon solver. If you have Optimization Toolbox software, you can also use the interior-point and active-set algorithms of the fmincon solver. Specify the algorithm in the SearchOptions.Algorithm option. The fmincon algorithms may result in improved estimation results in the following scenarios:
- Constrained minimization problems when there are bounds imposed on the model parameters.
- Model structures where the loss function is a nonlinear or non smooth function of the parameters.
- Multi-output model estimation. A determinant loss function is minimized by default for multioutput model estimation. fmincon algorithms are able to minimize such loss functions directly. The other search methods such as ' lm ' and 'gn' minimize the determinant loss function by alternately estimating the noise variance and reducing the loss value for a given noise variance value. Hence, the fmincon algorithms can offer better efficiency and accuracy for multi-output model estimations.


## SearchOptions - Option set for the search algorithm

search option set
Option set for the search algorithm, specified as the comma-separated pair consisting of
'SearchOptions ' and a search option set with fields that depend on the value of SearchMethod.

## SearchOptions Structure When SearchMethod is Specified as 'gn', 'gna', 'lm', 'grad', or 'auto'

| Field <br> Name | Description | Default |
| :--- | :--- | :--- |
| Toleran <br> ce | Minimum percentage difference between the current value of the loss <br> function and its expected improvement after the next iteration, specified as <br> a positive scalar. When the percentage of expected improvement is less <br> than Tolerance, the iterations stop. The estimate of the expected loss- <br> function improvement at the next iteration is based on the Gauss-Newton <br> vector computed for the current parameter value. | 0.01 |
| MaxIter <br> ations | Maximum number of iterations during loss-function minimization, specified <br> as a positive integer. The iterations stop when MaxIterations is reached <br> or another stopping criterion is satisfied, such as Tolerance. | 20 |
| Setting MaxIterations = 0 returns the result of the start-up procedure. <br> Use sys. Report. Termination. Iterations to get the actual number <br> of iterations during an estimation, where sys is an idtf model. |  |  |


| Field Name | Description |  | Default |
| :---: | :---: | :---: | :---: |
| Advance <br> d | Advanced search settings, specified as a structure with the following fields: |  |  |
|  | Field Name | Description | Default |
|  | GnPinvCons tant | Jacobian matrix singular value threshold, specified as a positive scalar. Singular values of the Jacobian matrix that are smaller than GnPinvConstant*max(size(J)*norm(J)*eps) are discarded when computing the search direction. Applicable when SearchMethod is 'gn'. | 10000 |
|  | InitialGna Tolerance | Initial value of gamma, specified as a positive scalar. Applicable when SearchMethod is 'gna'. | 0.0001 |
|  | LMStartVal ue | Starting value of search-direction length $d$ in the Levenberg-Marquardt method, specified as a positive scalar. Applicable when SearchMethod is ' 1 m '. | 0.001 |
|  | LMStep | Size of the Levenberg-Marquardt step, specified as a positive integer. The next value of the search-direction length $d$ in the Levenberg-Marquardt method is LMStep times the previous one. Applicable when SearchMethod is ' lm '. | 2 |
|  | MaxBisecti ons | Maximum number of bisections used for line search along the search direction, specified as a positive integer. | 25 |
|  | MaxFunctio nEvaluatio ns | Maximum number of calls to the model file, specified as a positive integer. Iterations stop if the number of calls to the model file exceeds this value. | Inf |
|  | MinParamet erChange | Smallest parameter update allowed per iteration, specified as a nonnegative scalar. | 0 |
|  | RelativeIm provement | Relative improvement threshold, specified as a nonnegative scalar. Iterations stop if the relative improvement of the criterion function is less than this value. | 0 |
|  | StepReduct ion | Step reduction factor, specified as a positive scalar that is greater than 1 . The suggested parameter update is reduced by the factor StepReduction after each try. This reduction continues until MaxBisections tries are completed or a lower value of the criterion function is obtained. <br> StepReduction is not applicable for SearchMethod ' lm' (Levenberg-Marquardt method). | 2 |

## SearchOptions Structure When SearchMethod is Specified as ' Isqnonlin'

| Field Name | Description | Default |
| :---: | :---: | :---: |
| Function Toleranc e | Termination tolerance on the loss function that the software minimizes to determine the estimated parameter values, specified as a positive scalar. <br> The value of FunctionTolerance is the same as that of opt.SearchOptions.Advanced.TolFun. | 1e-5 |
| StepTole rance | Termination tolerance on the estimated parameter values, specified as a positive scalar. <br> The value of StepTolerance is the same as that of opt. SearchOptions.Advanced.TolX. | 1e-6 |
| MaxItera tions | Maximum number of iterations during loss-function minimization, specified as a positive integer. The iterations stop when MaxIterations is reached or another stopping criterion is satisfied, such as FunctionTolerance. <br> The value of MaxIterations is the same as that of opt. SearchOptions.Advanced.MaxIter. | 20 |
| Advanced | Advanced search settings, specified as an option set for lsqnonlin. <br> For more information, see the Optimization Options table in "Optimization Options" (Optimization Toolbox). | Use optimset('lsqnonl in') to create a default option set. |

SearchOptions Structure When SearchMethod is Specified as 'fmincon'

| Field Name | Description | Default |
| :---: | :---: | :---: |
| Algorithm | fmincon optimization algorithm, specified as one of the following: <br> - 'sqp' - Sequential quadratic programming algorithm. The algorithm satisfies bounds at all iterations, and it can recover from NaN or Inf results. It is not a large-scale algorithm. For more information, see "Large-Scale vs. MediumScale Algorithms" (Optimization Toolbox). <br> - 'trust-regionreflective' - Subspace trust-region method based on the interior-reflective Newton method. It is a largescale algorithm. <br> - 'interior-point' -Large-scale algorithm that requires Optimization Toolbox software. The algorithm satisfies bounds at all iterations, and it can recover from NaN or Inf results. <br> - 'active-set' - Requires Optimization Toolbox software. The algorithm can take large steps, which adds speed. It is not a large-scale algorithm. <br> For more information about the algorithms, see "Constrained Nonlinear Optimization Algorithms" (Optimization Toolbox) and "Choosing the Algorithm" (Optimization Toolbox). | 'sqp' |


| Field Name | Description | Default |
| :--- | :--- | :--- |
| FunctionTolerance | Termination tolerance on the <br> loss function that the software <br> minimizes to determine the <br> estimated parameter values, <br> specified as a positive scalar. | $1 \mathrm{e}-6$ |
| StepTolerance | Termination tolerance on the <br> estimated parameter values, <br> specified as a positive scalar. | $1 \mathrm{e}-6$ |
| MaxIterations | Maximum number of iterations <br> during loss function <br> minimization, specified as a <br> positive integer. The iterations <br> stop when MaxIterations is <br> reached or another stopping <br> criterion is satisfied, such as <br> FunctionTolerance. | 100 |

## Advanced - Additional advanced options

structure
Additional advanced options, specified as a structure with the following fields:

- ErrorThreshold - Specifies when to adjust the weight of large errors from quadratic to linear.

Errors larger than ErrorThreshold times the estimated standard deviation have a linear weight in the loss function. The standard deviation is estimated robustly as the median of the absolute deviations from the median of the prediction errors, divided by 0.7. For more information on robust norm choices, see section 15.2 of [1].

ErrorThreshold $=0$ disables robustification and leads to a purely quadratic loss function. When estimating with frequency-domain data, the software sets ErrorThreshold to zero. For time-domain data that contains outliers, try setting ErrorThreshold to 1.6.

Default: 0

- MaxSize - Specifies the maximum number of elements in a segment when input-output data is split into segments.

MaxSize must be a positive, integer value.
Default: 250000

- StabilityThreshold - Specifies thresholds for stability tests.

StabilityThreshold is a structure with the following fields:

- s-Specifies the location of the right-most pole to test the stability of continuous-time models. A model is considered stable when its right-most pole is to the left of $s$.

Default: 0

- $z$ - Specifies the maximum distance of all poles from the origin to test stability of discretetime models. A model is considered stable if all poles are within the distance $z$ from the origin.

Default: $1+$ sqrt(eps)

- AutoInitThreshold - Specifies when to automatically estimate the initial conditions.

The initial condition is estimated when

$$
\frac{\left\|y_{p, z}-y_{\text {meas }}\right\|}{\left\|y_{p, e}-y_{\text {meas }}\right\|}>\text { AutoInitThreshold }
$$

- $y_{\text {meas }}$ is the measured output.
- $y_{p, z}$ is the predicted output of a model estimated using zero initial states.
- $y_{p, e}$ is the predicted output of a model estimated using estimated initial states.

Applicable when InitialCondition is 'auto'.
Default: 1.05

## Output Arguments

## opt - Option set for tfest

tfest0ptions option set
Option set for tfest, returned as an tfestOptions option set.

## Examples

## Create Default Options Set for Transfer Function Estimation

```
opt = tfestOptions;
```


## Specify Options for Transfer Function Estimation

Create an options set for tfest using the 'n4sid' initialization algorithm and set the Display to 'on'.
opt = tfestOptions('InitializeMethod','n4sid','Display','on');
Alternatively, use dot notation to set the values of opt.

```
opt = tfestOptions;
opt.InitializeMethod = 'n4sid';
opt.Display = 'on';
```


## Compatibility Considerations

## Renaming of Estimation and Analysis Options

The names of some estimation and analysis options were changed in R2018a. Prior names still work. For details, see the R2018a release note "Renaming of Estimation and Analysis Options".

## References

[1] Ljung, L. System Identification: Theory for the User. Upper Saddle River, NJ: Prentice-Hall PTR, 1999.
[2] Knudsen, T. "New method for estimating ARMAX models," In Proceedings of 10 th IFAC Symposium on System Identification, SYSID'94, Copenhagen, Denmark, July 1994, Vol. 2, pp. 611-617.
[3] Wills, Adrian, B. Ninness, and S. Gibson. "On Gradient-Based Search for Multivariable System Estimates." Proceedings of the 16th IFAC World Congress, Prague, Czech Republic, July 3-8, 2005. Oxford, UK: Elsevier Ltd., 2005.
[4] Garnier, H., M. Mensler, and A. Richard. "Continuous-time Model Identification From Sampled Data: Implementation Issues and Performance Evaluation" International Journal of Control, 2003, Vol. 76, Issue 13, pp 1337-1357.
[5] Ljung, L. "Experiments With Identification of Continuous-Time Models." Proceedings of the 15th IFAC Symposium on System Identification. 2009.
[6] Jansson, M. "Subspace identification and ARX modeling." 13th IFAC Symposium on System Identification, Rotterdam, The Netherlands, 2003.

## See Also

tfest

## Topics

"Loss Function and Model Quality Metrics"

## Introduced in R2012b

## timeoptions

Create list of time plot options

## Syntax

P = timeoptions
P = timeoptions('cstprefs')

## Description

$P=$ timeoptions returns a list of available options for time plots with default values set. You can use these options to customize the time value plot appearance from the command line.

P = timeoptions('cstprefs') initializes the plot options you selected in the Control System and System Identification Toolbox Preferences Editor. For more information about the editor, see "Toolbox Preferences Editor".

This table summarizes the available time plot options.

| Option | Description |
| :---: | :---: |
| Title, XLabel, YLabel | Label text and style |
| TickLabel | Tick label style |
| Grid | Show or hide the grid, specified as one of the following values: 'off' $\mid$ 'on' Default: ' off' |
| GridColor | Color of the grid lines, specified as one of the following: Vector of RGB values in the range [ 0,1$] \mid$ character vector of color name \|' none '. For example, for yellow color, specify as one of the following: [lllll 110 ], 'yellow', or 'y'. <br> Default: [0.15, 0.15, 0.15] |
| XlimMode, YlimMode | Limit modes |
| Xlim, Ylim | Axes limits |
| IOGrouping | Grouping of input-output pairs, specified as one of the following values: ' none' \|'inputs'|'outputs'|'all' Default: 'none' |
| InputLabels, OutputLabels | Input and output label styles |
| InputVisible, OutputVisible | Visibility of input and output channels |
| Normalize | Normalize responses, specified as one of the following values: 'on ' \|'off' Default: ' off' |
| SettleTimeThreshold | Settling time threshold |
| RiseTimeLimits | Rise time limits |


| Option | Description |
| :---: | :---: |
| TimeUnits | Time units, specified as one of the following values: <br> - 'nanoseconds' <br> - 'microseconds' <br> - 'milliseconds' <br> - 'seconds' <br> - 'minutes' <br> - 'hours' <br> - 'days' <br> - 'weeks' <br> - 'months' <br> - 'years' <br> Default: 'seconds ' <br> You can also specify 'auto ' which uses time units specified in the TimeUnit property of the input system. For multiple systems with different time units, the units of the first system is used. |

## Examples

## Plot Normalized Step Response

Create a default time options set.

```
opt = timeoptions;
```

Enable plotting of normalized responses.
opt.Normalize = 'on';
Plot the step response of two transfer function models using the specified options.

```
sys1 = tf(10,[1,1]);
sys2 = tf(5,[1,5]);
stepplot(sys1,sys2,opt);
```



The plot shows the normalized step response for the two transfer function models.

## See Also <br> getoptions|impulseplot|initialplot|lsimplot|setoptions|stepplot <br> Introduced in R2012a

## totaldelay

Total combined I/O delays for LTI model

## Syntax

td $=$ totaldelay(sys)

## Description

td = totaldelay(sys) returns the total combined I/O delays for an LTI model sys. The matrix td combines contributions from the InputDelay, OutputDelay, and ioDelayMatrix properties.

Delays are expressed in seconds for continuous-time models, and as integer multiples of the sample period for discrete-time models. To obtain the delay times in seconds, multiply td by the sample time sys.Ts.

## Examples

## Compute Combined Input-Output Delay for Transfer Function

Create the transfer function model, $1 / s$.
sys = tf(1,[10]);
Specify a 2 second input delay, and a 1.5 second output delay.
sys. InputDelay $=2$;
sys.Outputdelay = 1.5;
Compute the combined input-output delay for sys.
td = totaldelay(sys)
$\mathrm{td}=3.5000$
The resulting transfer function has the following form:

$$
e^{-2 s} x \frac{1}{s} e^{-1.5 s}=e^{-3.5 s} \frac{1}{s}
$$

This result is equivalent to specifying an input-output delay of 3.5 seconds for the original transfer function, $1 / \mathrm{s}$.

## See Also

absorbDelay | hasdelay

## Introduced in R2012a

## translatecov

Translate parameter covariance across model transformation operations

## Syntax

sys_new $=$ translatecov(fcn,sys)
sys_new $=$ translatecov(fcn,Input1,...,InputN)

## Description

sys_new = translatecov(fcn,sys) transforms sys into sys_new = fcn(sys), and translates the parameter covariance of sys to the parameter covariance of the transformed model. fcn is a transformation function that you specify. The command computes the parameter covariance of sys_new by applying the Gauss Approximation formula. To view the translated parameter covariance, use getcov.

Applying model transformations directly does not always translate the parameter covariance of the original model to that of the transformed model. For example, $\mathrm{d} 2 \mathrm{c}(\mathrm{sys})$ does not translate the parameter covariance of sys. In contrast, translatecov(@(x)d2c(x), sys) produces a transformed model that has the same coefficients as d2c (sys) and has the translated parameter covariance of sys.
sys_new = translatecov(fcn,Input1,...,InputN) returns the model sys_new = $\mathrm{fcn}($ Input1, ... InputN) and its parameter covariance. At least one of the $N$ inputs must be a linear model with parameter covariance information.

## Input Arguments

## fcn

Model transformation function, specified as a function handle.
For single-input functions, sys_new = fcn(sys). The input to fcn must be an identified model with parameter covariance information. Typical single-input transformation operations include:

- Model-type conversion, such as idpoly and idss. For example, fcn = @(x)idpoly (x) converts the model $x$ to a polynomial model.
- Sample time conversion, such as c2d and d2c. For example, $f c n=@(x) c 2 d(x, T s)$ converts the continuous-time model $x$ to discrete-time using the specified sample time Ts.

For multi-input functions, sys_new $=\mathrm{fcn}$ (Input1,..InputN). At least one of the input arguments Input1, ... , InputN must be an identified model with parameter covariance information. Typical multi-input operations include merging and concatenation of multiple models. For example, fcn $=$ $@(x, y)[x, y]$ performs horizontal concatenation of the models $x$ and $y$.
sys
Linear model with parameter covariance information, specified as one of the following model types:

- idtf
- idproc
- idss
- idpoly
- idgrey

The model must contain parameter covariance information, that is getcov(sys) must be nonempty.

## Input1, ..., InputN

Multiple input arguments to the translation function fcn . At least one of the $N$ inputs must be a linear identified model with parameter covariance information. The other inputs can be any MATLAB data type. For an example, see "Translate Parameter Covariance to Closed-Loop Model" on page 1-1599.

## Output Arguments

sys_new
Model resulting from transformation operation. The model includes parameter covariance information.

## Examples

## Translate Parameter Covariance During Model Conversion

Convert an estimated transfer function model into state-space model while also translating the estimated parameter covariance.

Estimate a transfer function model.

```
load iddata1
sys1 = tfest(z1,2);
```

Convert the estimated model to state-space form while also translating the estimated parameter covariance.

```
sys2 = translatecov(@(x)idss(x),sys1);
```

If you convert the transfer function model to state-space form directly, the estimated parameter covariance is lost (the output of getcov is empty).

```
sys3 = idss(sys1);
getcov(sys3)
ans =
    []
```

View the parameter covariance in the estimated and converted models.

```
covsys1 = getcov(sys1);
covsys2 = getcov(sys2);
```

Compare the confidence regions.
h = bodeplot(sys1,sys2);
showConfidence(h,2);


The confidence bounds for sys1 overlaps with sys2.

## Translate Parameter Covariance During Model Concatenation

Concatenate 3 single-output models such that the covariance data from the 3 models combine to produce the covariance data for the resulting model.

Construct a state-space model.

```
a = [-1.1008 0.3733;0.3733 -0.9561];
b = [0.7254 0.7147;-0.0631 -0.2050];
c = [-0.1241 0; 1.4897 0.6715; 1.4090 -1.2075];
d = [0 1.0347; 1.6302 0; 0.4889 0];
sys = idss(a,b,c,d,'Ts',0);
```

Generate multi-output estimation data.

```
t = (0:0.01:0.99)';
u = randn(100,2);
y = lsim(sys,u,t,'zoh');
```

```
y = y + rand(size(y))/10;
data = iddata(y,u,0.01);
```

Estimate a separate model for each output signal.

```
m1 = ssest(data(:,1,:),2,'feedthrough',true(1,2), 'DisturbanceModel', 'none');
m2 = ssest(data(:,2,:),2,'feedthrough',true(1,2), 'DisturbanceModel', 'none');
m3 = ssest(data(:,3,:),2,'feedthrough',true(1,2), 'DisturbanceModel', 'none');
```

Combine the estimated models while also translating the covariance information.

```
f = @(x,y,z)[x;y;z];
M2 = translatecov(f, m1, m2, m3);
```

The parameter covariance is not empty.
getcov(M2, 'factors')
If you combine the estimated models into one 3-output model directly, the covariance information is lost (the output of getcov is empty).

```
M1 = [m1;m2;m3];
getcov(M1)
```

Compare the confidence bounds.

```
h = bodeplot(M2, m1, m2, m3);
```

showConfidence(h);

The confidence bounds for M2 overlap with those of m1, m2 and m3 models on their respective plot axes.

## Translate Parameter Covariance to Closed-Loop Model

Consider a closed-loop feedback model consisting of a plant and controller. Translate the parameter covariance of the plant to the closed-loop feedback model.

Estimate a plant as a fourth-order state-space model using estimation data zl .
load iddatal z1
Plant = ssest(z1,4);
Plant contains parameter covariance information.
Create a controller as a continuous-time zero-pole-gain model with zeros, poles, and gain equal to -2 , -10, 5, respectively.

Controller $=\operatorname{zpk}(-2,-10,5)$;
Define a transformation function to generate the closed-loop feedback state-space model.
fcn = @(x,y)idss(feedback(x,y));
Translate the parameter covariance of the plant to the closed-loop feedback model.

```
sys_new = translatecov(fcn,Plant,Controller);
```

sys_new contains the translated parameter covariance information.
Plot the frequency-response of the transformed model sys_new, and view the confidence region of the response.

```
h = bodeplot(sys_new);
showConfidence(h)
```



The plot shows the effect of the uncertainty in Plant on the closed-loop response.

## Tips

- If you obtained sys through estimation and have access to the estimation data, you can use zeroiteration update to recompute the parameter covariance. For example:

```
load iddata1
m = ssest(z1,4);
opt = ssestOptions
opt.SearchOptions.MaxIterations = 0;
m_new = ssest(z1,m2,opt)
```

You cannot run a zero-iteration update in the following cases:

- If MaxIterations option, which depends on the SearchMethod option, is not available.
- For some model and data types. For example, a continuous-time idpoly model using timedomain data.


## Algorithms

translatecov uses numerical perturbations of individual parameters of sys to compute the Jacobian of fcn(sys) parameters with respect to parameters of sys. translatecov then applies Gauss Approximation formula cov_new $=J \times \operatorname{cov} \times J^{T}$ to translate the covariance, where $J$ is the Jacobian matrix. This operation can be slow for models containing a large number of free parameters.

## See Also

getcov|getpvec|rsample|setcov

## Topics

"What Is Model Covariance?"
"Types of Model Uncertainty Information"

## Introduced in R2012b

## treepartition

Tree-partitioned nonlinear function for nonlinear ARX models

## Description

A treepartition object implements a tree-partitioned nonlinear function, and is a nonlinear mapping function for estimating nonlinear ARX models. The mapping function, which is also referred to as a nonlinearity, uses a combination of linear weights, an offset and a nonlinear function to compute its output. The nonlinear function contains treepartition unit functions that operate on a radial combination of inputs.


Mathematically, treepartition is a nonlinear function $y=F(x)$ that maps $m$ inputs $X(t)=$ $\left[x\left(t_{1}\right), x_{2}(t), \ldots, x_{m}(t)\right]^{\mathrm{T}}$ to a scalar output $y(t) . F$ is a piecewise-linear (affine) function of $x$ :

$$
F(x)=x L+[1, x] C_{k}+d
$$

Here, $x$ belongs to the partition $P_{k}$. $L$ is a 1-by- $m$ vector, $C_{k}$ is a 1-by-m+1 vector, and $P_{k}$ is a partition of the $x$-space.

For more information about the mapping function $F(x)$ see "Algorithms" on page 1-1608.
Use treepartition as the value of the OutputFcn property of an idnlarx model. For example, specify treepartition when you estimate an idnlarx model with the following command.
sys $=$ nlarx(data, regressors,treepartition)
When nlarx estimates the model, it essentially estimates the parameters of the treepartition function.

You can configure the treepartition function to fix parameters. To omit the linear component, set LinearFcn. Use to false. Use evaluate to compute the output of the function for a given vector of inputs.

## Creation

## Syntax

$\mathrm{T}=$ treepartition
T = treepartition(numUnits)

## Description

$T=$ treepartition creates a treepartition object $t$ that is a binary tree nonlinear mapping object. The function computes the number of tree nodes $J$, represented by the property
NumberOfUnits, automatically during estimation. The tree has the number of leaves equal to $2^{\wedge}$ ( J $+1)-1$.
$T=$ treepartition(numUnits) specifies the number of treepartition nodes numUnits.

## Input Arguments

## numUnits - Number of units

'auto' (default)| positive integer
Number of units, specified as 'auto ' or a positive integer. numUnits determines the number of tree nodes.

This argument sets the T. NonlinearFcn. NumberOfUnits property.

## Properties

## Input - Input signal information

signal property values
Input signal information for signals used for estimation, specified as vectors of $m$ property-specific values, where $m$ is the number of input signals. The Input properties for each input signal are as follows:

- Name - Names of the input signals, specified as a 1-by-m string or character array, where $m$ is the number of inputs
- Mean - Mean of the input signals, specified as a numeric scalar
- Range - Ranges of the input signals, specified as a 2-by-m numeric array that contains the minimum and maximum values


## Output - Output signal information

signal property values
Output signal information, specified as property-specific values. The Output properties are as follows:

- Name - Name of the output signal, specified as a string or a character array
- Mean - Mean of the output signal, specified as a numeric scalar
- Range - Range of the output signal, specified as a 2-by-1 numeric array that contains the minimum and maximum values


## LinearFcn - Parameters of linear function

linear function property values (default)
Parameters of the linear function, specified as follows:

- Value - Value of $L$, specified as a 1-by- $m$ vector.
- Free - Option to update entries of Value during estimation. specified as a logical scalar. The software honors the Free specification only if the starting value of Value is finite. The default value is true.
- Minimum - Minimum bound on Value, specified as a 1-by-p vector. If Minimum is specified with a finite value and the starting value of Value is finite, then the software enforces that minimum bound during model estimation.
- Maximum - Maximum bound on Value, specified as a 1-by-p vector. If Maximum is specified with a finite value and the starting value of Value is finite, then the software enforces that maximum bound during model estimation.
- SelectedInputIndex - Indices of treepartition inputs (see Input.Name) that are used as inputs to the linear function, specified as an 1-by- $n_{r}$ integer vector, where $n_{r}$ is the number of inputs. The RegressorUsage property of the idnlarx model determines these indices.

The software computes the value of LinearFcn as (X-X0) ' $*$ L, where X 0 is the regressor mean.

## Offset - Parameters of offset term

offset property values
Parameters of the offset term, specified as follows:

- Value - Offset value, specified as a scalar.
- Free - Option to update Value during estimation, specified as a scalar logical. The software honors the Free specification of false only if the value of Value is finite. The default value is true.
- Minimum - Minimum bound on Value, specified as a numeric scalar or -Inf. If Minimum is specified with a finite value and the value of Value is finite, then the software enforces that minimum bound during model estimation. The default value is - Inf.
- Maximum - Maximum bound on Value, specified as a numeric scalar or Inf. If Maximum is specified with a finite value and the starting value of Value is finite, then the software enforces that maximum bound during model estimation. The default value is Inf.


## NonlinearFcn - Parameters of nonlinear function

nonlinear function property values
Parameters of the nonlinear function, specified as follows:

- NumberOfUnits - Number of units, specified as 'auto' or a positive integer. NumberOfUnits determines the number of nodes $N$ in the tree. When $N$ is set to:
- 'auto', the software selects $N$ by pruning.
- a positive integer before estimation, then the software sets $N$ to the largest value of the form $2^{\wedge}(\mathrm{J}+1)-1$ less than this integer.
- Parameters - estimated parameter values.treepartition, specified as in the following table:

| Field Name | Description |
| :---: | :---: |
| SampleLength | Length of the estimation data |
| NoiseVariance | Estimated variance of the noise in the estimation data |
| Tree | Structure that contains the tree parameters, as described in the following list: <br> - TreeLevelPntr: N -by-1 vector containing the levels j of each node. <br> - AncestorDescendantPntr: N -by-3 matrix, such that the entry ( $k, 1$ ) is the ancestor of node $k$, and entries $(k, 2)$ and $(k, 3)$ are the left and right descendants, respectively. <br> - LocalizingVectors: N -by- $(\mathrm{m}+1)$ matrix, such that the rth row is B_r. <br> - LocalParVector: N -by- $(\mathrm{m}+1)$ matrix, such that the kth row is C_k. <br> - LocalCovMatrix: N -by- $((\mathrm{m}+1) \mathrm{m} / 2)$ matrix such that the kth row is the covariance matrix of C_k. C_k is reshaped as a row vector. |

- Free - Option to estimate parameters, specified as a logical scalar. If all the parameters have finite values, such as when the treepartition object corresponds to a previously estimated model, then setting Free to false causes the parameters of the nonlinear component of the function $F(X)$ to remain unchanged during estimation. The default value is true.
- SelectedInputIndex - Indices of treepartition inputs (see Input.Name) that are used as inputs to the nonlinear function, specified as an 1-by- $n_{r}$ integer vector, where $n_{r}$ is the number of inputs. The RegressorUsage property determines these indices.
- Structure - Advanced options that affect the initial model.

| Property | Description | Default |
| :--- | :--- | :--- |
| FinestCell | Integer or character vector <br> specifying the minimum <br> number of data points in the <br> smallest partition. | 'auto' |


| Property | Description | Default |
| :--- | :--- | :--- |
| Threshold | Threshold parameter used by <br> the adaptive pruning <br> algorithm. Smaller threshold <br> value corresponds to a shorter <br> branch that is terminated by <br> the active partition D_a. <br> Higher threshold value results <br> in a longer branch | 1.0 |
| Stabilizer | Penalty parameter of the <br> penalized least-squares <br> algorithm used to compute <br> local parameter vectors C_k. | $1 \mathrm{e}-6$ |
| Higher stabilizer value <br> improves stability, but may <br> deteriorate the accuracy of <br> the least-square estimate. |  |  |

## Examples

## Estimate Nonlinear ARX Model with treepartition as Output Function

Load the data
load iddatal z1
Create a treepartition object with default settings.

```
T = treepartition
T =
Tree Partition
Nonlinear Function: Tree Partition with number of units chosen automatically.
Linear Function: uninitialized
Output Offset: uninitialized
            Input: [1\times1 idpack.Channel]
            Output: [1×1 idpack.Channel]
            LinearFcn: [1\times1 nlident.internal.StructuredFcn]
    NonlinearFcn: [1×1 nlident.internal.TreeFcn]
            Offset: [1×1 nlident.internal.Offset]
```

Estimate a nonlinear ARX model using T.
sys = nlarx(z1,[2 2 1],T);
View the output function of sys.
disp(sys.OutputFcn)
Tree Partition
Inputs: $\mathrm{y} 1(\mathrm{t}-1), \mathrm{y} 1(\mathrm{t}-2), \mathrm{u}(\mathrm{t}-1), \mathrm{u}(\mathrm{t}-2)$

```
Output: y1
Nonlinear Function: Tree Partition with 31 units.
Linear Function: initialized to [1.19 -0.419 0.873 0.844]
Output Offset: initialized to 0.249
    Input: [1×1 idpack.Channel]
        Output: [1×1 idpack.Channel]
        LinearFcn: [1×1 nlident.internal.StructuredFcn]
    NonlinearFcn: [1×1 nlident.internal.TreeFcn]
        Offset: [1×1 nlident.internal.Offset]
```

T now has 31 nodes.

## Specify treepartition Parameters

Load the data
load iddata7 z7
ze = z7(1:300);
Create a treepartition object and use dot notation to set parameters.
$\mathrm{T}=$ treepartition;
T.Offset.Value = 0.2;
T.Offset. Free = false;
T.NonlinearFcn.NumberOfUnits = 30;

Specify model regressors.
Reg1 = linearRegressor(\{'y1','u1'\},\{1:4, 0:4\});
Reg2 = polynomialRegressor(\{'y1','u1'\},\{1:2, 0:2\},2);
Estimate a nonlinear ARX model.
sys = nlarx(ze, [Reg1;Reg2], T);
View postestimation OutputFcn properties.

```
disp(sys.OutputFcn)
Tree Partition
Inputs: y1(t-1), y1(t-2), y1(t-3), y1(t-4), u1(t), u1(t-1), u1(t-2), u1(t-3), u1(t-4), y1(t-1)^2
Output: y1
Nonlinear Function: Tree Partition with 15 units.
Linear Function: initialized to [0.0725 0.895 -0.0727 -0.475 0.0725 -0.106 0.0304 1.02 1.43 0.0(
Output Offset: fixed to 0.2
                    Input: [1×1 idpack.Channel]
            Output: [1×1 idpack.Channel]
            LinearFcn: [1\times1 nlident.internal.StructuredFcn]
    NonlinearFcn: [1×1 nlident.internal.TreeFcn]
            Offset: [1\times1 nlident.internal.Offset]
disp(sys.OutputFcn.Input)
```

```
    Channel with properties:
    Name: [1×14 string]
    Mean: [0.2033 0.2035 0.2149 0.2159 0.0405 0.0338 0.0338 0.0338 0.0405 10.0493 10.0491 1 1 1
    Range: [2×14 double]
disp(sys.OutputFcn.Offset)
Output Offset (fixed to 0.2)
    Value: 0.2000
    Minimum: -Inf
    Maximum: Inf
        Free: 0
disp(sys.OutputFcn.NonlinearFcn)
    TreeFcn with properties:
        NumberOfUnits: 15
            Structure: [1\times1 nlident.internal.TreeStructure]
            Parameters: [1×1 nlident.internal.TreeFcnParameters]
                    Free: 1
    SelectedInputIndex: [11 2 3 4 5 6 7 8 9 10 11 12 13 14]
```


## Algorithms

The mapping $F$ is defined by a dyadic partition $P$ of the $x$-space, such that on each partition element $P_{k}, F$ is a linear mapping. When $x$ belongs to $P_{k}, F(x)$ is given by:

$$
F(x)=x L+[1, x] C_{k}+d
$$

where $L$ is 1 -by- $m$ vector and $d$ is a scalar common for all elements of partition. $C_{\mathrm{k}}$ is a 1-by- $(m+1)$ vector.

The mapping $F$ and associated partition $P$ of the $x$-space are computed as follows:
1 Given the value of $J$, a dyadic tree with $J$ levels and $N=2^{\mathrm{J}-1}$ nodes is initialized.
2 Each node at level $1<j<J$ has two descendants at level $j+1$ and one parent at level $j-1$.

- The root node at level 1 has two descendants.
- Nodes at level $J$ are terminating leaves of the tree and have one parent.

3 One partition element is associated to each node $k$ of the tree.

- The vector of coefficients $C_{k}$ is computed using the observations on the corresponding partition element $P_{k}$ by the penalized least-squares algorithm.
- When the node $k$ is not a terminating leaf, the partition element $P_{\mathrm{k}}$ is cut into two to obtain the partition elements of descendant nodes. The cut is defined by the half-spaces $(1, x) B_{k}>0$ or $<=0$ (move to left or right descendant), where $B_{k}$ is chosen to improve the stability of leastsquare computation on the partitions at the descendant nodes.
4 When the value of the mapping $F$, defined by the treepartition object, is computed at $x$, an adaptive algorithm selects the active node $k$ of the tree on the branch of partitions that contain $x$.

When the Focus option in nlarx0ptions is 'prediction', treepartition uses a noniterative technique for estimating parameters. Iterative refinements are not possible for models containing this nonlinearity estimator.

You cannot use treepartition when Focus is 'simulation' because this nonlinear mapping object is not differentiable. Minimization of simulation error requires differentiable nonlinear functions.

## Compatibility Considerations

## Use of previous nonlinearity estimator properties is not recommended

Not recommended starting in R2021a
Starting in R2021a, the properties of the mapping objects, previously known as nonlinearity estimators, have been reorganized. These objects are wavenet (W), sigmoidnet (S), treepartition (T), customnet (C), and linear (L). The property changes do not apply to neuralnet. The use of the pre-R2021a properties in the following table is discouraged. However, the software still accepts commands that set these properties. There are no plans to exclude such commands at this time.

| Pre-R2021a Property | R2021a Property | Applicable Mapping Objects |
| :--- | :--- | :--- |
| NumberOfUnits | NonlinearFcn.NumberOfUni <br> ts | W,S,T,C |
| LinearTerm | LinearFcn.Use, Offset.Use | W,S,C |
| Parameters | Split into three pieces: <br> - LinearFcn.Value <br> - Offset.Value <br> - NonlinearFcn.Parameter <br> s | W,S,T,C,L <br> linear (L) excludes <br> NonlinearFcn.Parameters. |
| Options | NonlinearFcn.Structure | W,T |

## See Also

customnet |evaluate |idnlarx|linear|linearRegressor|neuralnet|nlarx| polynomialRegressor|sigmoidnet|wavenet

Introduced in R2007a

## TrendInfo

Offset and linear trend slope values for detrending data

## Description

TrendInfo class represents offset and linear trend information of input and output data.
Constructing the corresponding object lets you:

- Compute and store mean values or best-fit linear trends of input and output data signals.
- Define specific offsets and trends to be removed from input-output data.

By storing offset and trend information, you can apply it to multiple data sets.
After estimating a linear model from detrended data, you can simulate the model at original operation conditions by adding the saved trend to the simulated output using retrend.

## Construction

For transient data, if you want to define a specific offset or trend to be removed from this data, create the TrendInfo object using getTrend. For example:
$\mathrm{T}=$ getTrend(data)
where data is the iddata object from which you will be removing the offset or linear trend, and T is the TrendInfo object. You must then assign specific offset and slope values as properties of this object before passing the object as an argument to detrend.

For steady-state data, if you want to detrend the data and store the trend information, use the det rend command with the output argument for storing trend information.

## Properties

After creating the object, you can use get or dot notation to access the object property values.

| Property Name | Default | Description |
| :--- | :--- | :--- |
| DataName | ' ' | Name of the iddata object from which trend information is <br> derived (if any) |
| InputOffset | zeros (1, nu), where nu <br> is the number of inputs | - For transient data, the physical equilibrium offset you <br> specify for each input signal. <br> Gor steady-state data, the mean of input values. Computed <br> automatically when detrending the data. <br> If removing a linear trend from the input-output data, the <br> value of the line at t0, where t0 is the start time. <br> For multiple experiment data, this is a cell array of size equal to <br> the number of experiments in the data set. |

$\left.\begin{array}{|l|l|l|}\hline \text { Property Name } & \text { Default } & \text { Description } \\ \hline \text { InputSlope } & \begin{array}{l}\text { zeros (1, nu), where nu } \\ \text { is the number of inputs }\end{array} & \begin{array}{l}\text { Slope of linear trend in input data, computed automatically } \\ \text { when using the detrend command to remove the linear trend } \\ \text { in the data. }\end{array} \\ \hline \text { Output0ffset } & \begin{array}{l}\text { zeros (1, ny), where ny } \\ \text { is the number of outputs }\end{array} & \begin{array}{l}\text { For multiple experiment data, this is a cell array of size equal to } \\ \text { the number of experiments in the data set. }\end{array} \\ \hline \text { For transient data, the physical equilibrium offset you } \\ \text { specify for each output signal } \\ \text { For steady-state data, the mean of output values. Computed } \\ \text { automatically when detrending the data. } \\ \text { If removing a linear trend from the input-output data, the } \\ \text { value of the line at t0, where t0 is the start time. }\end{array}\right\}$

## Examples

## Remove Offsets from Data

Remove a specified offset from input and output signals.
Load SISO data containing vectors u 2 and y 2 .
load dryer2
Create a data object with a sample time of 0.08 seconds and plot it.

```
data = iddata(y2,u2,0.08);
```

plot(data)

## Input-Output Data



The data has a nonzero mean value.
Store the data offset and trend information in a TrendInfo object.
T = getTrend(data);
Assign offset values to the TrendInfo object.
T. InputOffset = 5;
T.OutputOffset = 5;

Subtract the offsets from the data.
data_d = detrend(data, T );
Plot the detrended data on the same plot.
hold on
plot(data_d)

## Input-Output Data



View the mean value removed from the data.

```
get(T)
ans = struct with fields:
                                    DataName: 'data'
    InputOffset: 5
    OutputOffset: 5
        InputSlope: 0
        OutputSlope: 0
```


## Store Trend Information

Construct the TrendInfo object that stores trend information as part of data detrending.
Load SISO data containing vectors u2 and y2.
load dryer2
Create data object with sample time of 0.08 seconds.
data $=$ iddata( $\mathrm{y} 2, \mathrm{u} 2,0.08$ );
Detrend the mean from the data and store the mean as a TrendInfo object T.
[data_d,T] = detrend(data,0);
View the mean value removed from the data.

```
get(T)
```

ans = struct with fields:
DataName: 'data'
InputOffset: 5.0000
OutputOffset: 4.8901
InputSlope: 0
OutputSlope: 0

## See Also

detrend | getTrend | retrend
Topics
"Handling Offsets and Trends in Data"
Introduced in R2009a

## unitgain

Specify absence of nonlinearities for specific input or output channels in Hammerstein-Wiener models

## Syntax

unit=unitgain

## Description

unit=unitgain instantiates an object that specifies an identity mapping $F(x)=x$ to exclude specific input and output channels from being affected by a nonlinearity in Hammerstein-Wiener models.

Use the unitgain object as an argument in the $n l h w$ estimator to set the corresponding channel nonlinearity to unit gain.

For example, for a two-input and one-output model, to exclude the second input from being affected by a nonlinearity, use the following syntax:

```
m = nlhw(data,orders,['saturation''unitgain'],'deadzone')
```

In this case, the first input saturates and the output has an associated deadzone nonlinearity.

## unitgain Properties

unitgain does not have properties.

## Examples

For example, for a one-input and one-output model, to exclude the output from being affected by a nonlinearity, use the following syntax:
m = nlhw(Data,Orders,'saturation','unitgain')
In this case, the input has a saturation nonlinearity.
If nonlinearities are absent in input or output channels, you can replace unitgain with an empty matrix. For example, to specify a Wiener model with a sigmoid nonlinearity at the output and a unit gain at the input, use the following command:
m = nlhw(Data,Orders,[],'sigmoid');

## Tips

Use the unitgain object to exclude specific input and output channels from being affected by a nonlinearity in Hammerstein-Wiener models. unitgain is a linear function $y=F(x)$, where $F(x)=x$.

## See Also

deadzone|nlhw| saturation | sigmoidnet

Introduced in R2007a

## unscentedKalmanFilter

Create unscented Kalman filter object for online state estimation

## Syntax

```
obj = unscentedKalmanFilter(StateTransitionFcn,MeasurementFcn,InitialState)
obj = unscentedKalmanFilter(StateTransitionFcn,MeasurementFcn,InitialState,
Name,Value)
obj = unscentedKalmanFilter(StateTransitionFcn,MeasurementFcn)
obj = unscentedKalmanFilter(StateTransitionFcn,MeasurementFcn,Name,Value)
obj = unscentedKalmanFilter(Name,Value)
```


## Description

obj = unscentedKalmanFilter(StateTransitionFcn,MeasurementFcn,InitialState) creates an unscented Kalman filter object for online state estimation of a discrete-time nonlinear system. StateTransitionFcn is a function that calculates the state of the system at time $k$, given the state vector at time $k-1$. MeasurementFcn is a function that calculates the output measurement of the system at time $k$, given the state at time $k$. InitialState specifies the initial value of the state estimates.

After creating the object, use the correct and predict commands to update state estimates and state estimation error covariance values using a discrete-time unscented Kalman filter algorithm and real-time data.
obj = unscentedKalmanFilter(StateTransitionFcn,MeasurementFcn,InitialState, Name, Value) specifies additional attributes of the unscented Kalman filter object using one or more Name, Value pair arguments.
obj = unscentedKalmanFilter(StateTransitionFcn,MeasurementFcn) creates an unscented Kalman filter object using the specified state transition and measurement functions. Before using the predict and correct commands, specify the initial state values using dot notation. For example, for a two-state system with initial state values $[1 ; 0]$, specify obj. State $=[1 ; 0]$.
obj = unscentedKalmanFilter(StateTransitionFcn,MeasurementFcn, Name,Value) specifies additional attributes of the unscented Kalman filter object using one or more Name, Value pair arguments. Before using the predict and correct commands, specify the initial state values using Name, Value pair arguments or dot notation.
obj = unscentedKalmanFilter(Name,Value) creates an unscented Kalman filter object with properties specified using one or more Name, Value pair arguments. Before using the predict and correct commands, specify the state transition function, measurement function, and initial state values using Name, Value pair arguments or dot notation.

## Object Description

unscentedKalmanFilter creates an object for online state estimation of a discrete-time nonlinear system using the discrete-time unscented Kalman filter algorithm.

Consider a plant with states $x$, input $u$, output $y$, process noise $w$, and measurement noise $v$. Assume that you can represent the plant as a nonlinear system.


The algorithm computes the state estimates $\widehat{x}$ of the nonlinear system using state transition and measurement functions specified by you. The software lets you specify the noise in these functions as additive or nonadditive:

- Additive Noise Terms - The state transition and measurements equations have the following form:

$$
\begin{aligned}
& x[k]=f\left(x[k-1], u_{s}[k-1]\right)+w[k-1] \\
& y[k]=h\left(x[k], u_{m}[k]\right)+v[k]
\end{aligned}
$$

Here $f$ is a nonlinear state transition function that describes the evolution of states x from one time step to the next. The nonlinear measurement function $h$ relates $x$ to the measurements $y$ at time step k. w and v are the zero-mean, uncorrelated process and measurement noises, respectively. These functions can also have additional input arguments that are denoted by $\mathrm{u}_{\mathrm{s}}$ and $u_{m}$ in the equations. For example, the additional arguments could be time step $k$ or the inputs $u$ to the nonlinear system. There can be multiple such arguments.

Note that the noise terms in both equations are additive. That is, $x(k)$ is linearly related to the process noise $w(k-1)$, and $y(k)$ is linearly related to the measurement noise $v(k)$.

- Nonadditive Noise Terms - The software also supports more complex state transition and measurement functions where the state $x[k]$ and measurement $y[k]$ are nonlinear functions of the process noise and measurement noise, respectively. When the noise terms are nonadditive, the state transition and measurements equation have the following form:

$$
\begin{aligned}
& x[k]=f\left(x[k-1], w[k-1], u_{S}[k-1]\right) \\
& y[k]=h\left(x[k], v[k], u_{m}[k]\right)
\end{aligned}
$$

When you perform online state estimation, you first create the nonlinear state transition function $f$ and measurement function $h$. You then construct the unscentedKalmanFilter object using these nonlinear functions, and specify whether the noise terms are additive or nonadditive. After you create the object, you use the predict command to predict state estimates at the next time step, and correct to correct state estimates using the unscented Kalman filter algorithm and real-time data. For information about the algorithm, see "Extended and Unscented Kalman Filter Algorithms for Online State Estimation".

You can use the following commands with unscentedKalmanFilter objects:

| Command | Description |
| :--- | :--- |
| correct | Correct the state and state estimation error <br> covariance at time step $k$ using measured data at <br> time step $k$. |
| residual | Predict the state and state estimation error <br> covariance at time the next time step. |
| clone | Return the difference between the actual and <br> predicted measurements. |
| Create another object with the same object <br> property values. <br> Do not create additional objects using syntax <br> obj2 = obj. Any changes made to the <br> properties of the new object created in this way <br> (obj2) also change the properties of the original <br> object (obj). |  |

For unscentedKalmanFilter object properties, see "Properties" on page 1-1623.

## Examples

## Create Unscented Kalman Filter Object for Online State Estimation

To define an unscented Kalman filter object for estimating the states of your system, you write and save the state transition function and measurement function for the system.

In this example, use the previously written and saved state transition and measurement functions, vdpStateFcn.m and vdpMeasurementFcn.m. These functions describe a discrete-approximation to van der Pol oscillator with nonlinearity parameter, mu, equal to 1 . The oscillator has two states.

Specify an initial guess for the two states. You specify the initial state guess as an M-element row or column vector, where M is the number of states.
initialStateGuess = [1;0];
Create the unscented Kalman filter object. Use function handles to provide the state transition and measurement functions to the object.
obj = unscentedKalmanFilter(@vdpStateFcn,@vdpMeasurementFcn,initialStateGuess);
The object has a default structure where the process and measurement noise are additive.
To estimate the states and state estimation error covariance from the constructed object, use the correct and predict commands and real-time data.

## Specify Process and Measurement Noise Covariances in Unscented Kalman Filter Object

Create an unscented Kalman filter object for a van der Pol oscillator with two states and one output. Use the previously written and saved state transition and measurement functions, vdpStateFcn.m
and vdpMeasurementFcn.m. These functions are written for additive process and measurement noise terms. Specify the initial state values for the two states as [2;0].

Since the system has two states and the process noise is additive, the process noise is a 2 -element vector and the process noise covariance is a 2-by-2 matrix. Assume there is no cross-correlation between process noise terms, and both the terms have the same variance 0.01 . You can specify the process noise covariance as a scalar. The software uses the scalar value to create a 2 -by- 2 diagonal matrix with 0.01 on the diagonals.

Specify the process noise covariance during object construction.

```
obj = unscentedKalmanFilter(@vdpStateFcn,@vdpMeasurementFcn,[2;0],...
    'ProcessNoise',0.01);
```

Alternatively, you can specify noise covariances after object construction using dot notation. For example, specify the measurement noise covariance as 0.2 .
obj. MeasurementNoise $=0.2$;
Since the system has only one output, the measurement noise is a 1 -element vector and the MeasurementNoise property denotes the variance of the measurement noise.

## Specify Nonadditive Measurement Noise in Unscented Kalman Filter Object

Create an unscented Kalman filter object for a van der Pol oscillator with two states and one output. Assume that the process noise terms in the state transition function are additive. That is, there is a linear relation between the state and process noise. Also assume that the measurement noise terms are nonadditive. That is, there is a nonlinear relation between the measurement and measurement noise.

```
obj = unscentedKalmanFilter('HasAdditiveMeasurementNoise',false);
```

Specify the state transition function and measurement functions. Use the previously written and saved functions, vdpStateFcn.m and vdpMeasurementNonAdditiveNoiseFcn.m.

The state transition function is written assuming the process noise is additive. The measurement function is written assuming the measurement noise is nonadditive.

```
obj.StateTransitionFcn = @vdpStateFcn;
obj.MeasurementFcn = @vdpMeasurementNonAdditiveNoiseFcn;
```

Specify the initial state values for the two states as [2;0].

```
obj.State = [2;0];
```

You can now use the correct and predict commands to estimate the state and state estimation error covariance values from the constructed object.

## Specify Additional Inputs in State Transition and Measurement Functions

Consider a nonlinear system with input $u$ whose state $x$ and measurement $y$ evolve according to the following state transition and measurement equations:

$$
\begin{aligned}
& x[k]=\sqrt{x[k-1]+u[k-1]}+w[k-1] \\
& y[k]=x[k]+2 * u[k]+v[k]^{2}
\end{aligned}
$$

The process noise $w$ of the system is additive while the measurement noise $v$ is nonadditive.
Create the state transition function and measurement function for the system. Specify the functions with an additional input $u$.

```
f = @(x,u)(sqrt(x+u));
h = @(x,v,u)(x+2*u+v^2);
```

$f$ and $h$ are function handles to the anonymous functions that store the state transition and measurement functions, respectively. In the measurement function, because the measurement noise is nonadditive, v is also specified as an input. Note that v is specified as an input before the additional input $u$.

Create an unscented Kalman filter object for estimating the state of the nonlinear system using the specified functions. Specify the initial value of the state as 1, and the measurement noise as nonadditive.
obj = unscentedKalmanFilter(f,h,1,'HasAdditiveMeasurementNoise',false);
Specify the measurement noise covariance.
obj. MeasurementNoise $=0.01$;
You can now estimate the state of the system using the predict and correct commands. You pass the values of $u$ to predict and correct, which in turn pass them to the state transition and measurement functions, respectively.

Correct the state estimate with measurement $\mathrm{y}[\mathrm{k}]=0.8$ and input $\mathrm{u}[\mathrm{k}]=0.2$ at time step k .
correct (obj, 0.8, 0.2)
Predict the state at next time step, given $u[k]=0.2$.
predict(obj,0.2)

## Input Arguments

## StateTransitionFcn - State transition function

## function handle

State transition function $f$, specified as a function handle. The function calculates the $N s$-element state vector of the system at time step $k$, given the state vector at time step $k-1$. Ns is the number of states of the nonlinear system.

You write and save the state transition function for your nonlinear system, and use it to construct the object. For example, if vdpStateFcn.m is the state transition function, specify
StateTransitionFcn as @vdpStateFcn. You can also specify StateTransitionFcn as a function handle to an anonymous function.

The inputs to the function you write depend on whether you specify the process noise as additive or nonadditive in the HasAdditiveProcessNoise property of the object:

- HasAdditiveProcessNoise is true - The process noise w is additive, and the state transition function specifies how the states evolve as a function of state values at the previous time step:
$x(k)=f(x(k-1), U s 1, \ldots, U s n)$
Where $x(k)$ is the estimated state at time $k$, and Us1, ... Usn are any additional input arguments required by your state transition function, such as system inputs or the sample time. During estimation, you pass these additional arguments to the predict command, which in turn passes them to the state transition function.
- HasAdditiveProcessNoise is false - The process noise is nonadditive, and the state transition function also specifies how the states evolve as a function of the process noise:

```
x(k) = f(x(k-1),w(k-1),Us1,...,Usn)
```

To see an example of a state transition function with additive process noise, type edit vdpStateFcn at the command line.

## MeasurementFcn - Measurement function

function handle
Measurement function $h$, specified as a function handle. The function calculates the $N$-element output measurement vector of the nonlinear system at time step $k$, given the state vector at time step $k . N$ is the number of measurements of the system. You write and save the measurement function, and use it to construct the object. For example, if vdpMeasurementFcn.m is the measurement function, specify MeasurementFcn as @vdpMeasurementFcn. You can also specify MeasurementFcn as a function handle to an anonymous function.

The inputs to the function depend on whether you specify the measurement noise as additive or nonadditive in the HasAdditiveMeasurementNoise property of the object:

- HasAdditiveMeasurementNoise is true - The measurement noise v is additive, and the measurement function specifies how the measurements evolve as a function of state values:

$$
y(k)=h(x(k), U m 1, \ldots, U m n)
$$

Where $y(k)$ and $x(k)$ are the estimated output and estimated state at time $k$, and Um1, ..., Umn are any optional input arguments required by your measurement function. For example, if you are using multiple sensors for tracking an object, an additional input could be the sensor position. During estimation, you pass these additional arguments to the correct command, which in turn passes them to the measurement function.

- HasAdditiveMeasurementNoise is false - The measurement noise is nonadditive, and the measurement function also specifies how the output measurement evolves as a function of the measurement noise:

```
y(k) = h(x(k),v(k),Um1,\ldots,..Umn)
```

To see an example of a measurement function with additive process noise, type edit vdpMeasurementFcn at the command line. To see an example of a measurement function with nonadditive process noise, type edit vdpMeasurementNonAdditiveNoiseFcn.

## InitialState - Initial state estimates

vector

Initial state estimates, specified as an $N s$-element vector, where $N s$ is the number of states in the system. Specify the initial state values based on your knowledge of the system.

The specified value is stored in the State property of the object. If you specify InitialState as a column vector then State is also a column vector, and predict and correct commands return state estimates as a column vector. Otherwise, a row vector is returned.

If you want a filter with single-precision floating-point variables, specify InitialState as a singleprecision vector variable. For example, for a two-state system with state transition and measurement functions vdpStateFcn.m and vdpMeasurementFcn.m, create the unscented Kalman filter object with initial states [1;2] as follows:
obj = unscentedKalmanFilter(@vdpStateFcn,@vdpMeasurementFcn,single([1;2]))
Data Types: double | single

## Name-Value Pair Arguments

Specify optional comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside quotes. You can specify several name and value pair arguments in any order as Name1, Value1, ... , NameN, ValueN.

Use Name, Value arguments to specify properties on page 1-1623 of unscentedKalmanFilter object during object creation. For example, to create an unscented Kalman filter object and specify the process noise covariance as 0.01 :
obj = unscentedKalmanFilter(StateTransitionFcn,MeasurementFcn,InitialState,'ProcessNoise',0.01);

## Properties

unscentedKalmanFilter object properties are of three types:

- Tunable properties that you can specify multiple times, either during object construction using Name, Value arguments, or any time afterwards during state estimation. After object creation, use dot notation to modify the tunable properties.
obj = unscentedKalmanFilter(StateTransitionFcn,MeasurementFcn,InitialState);
obj. ProcessNoise = 0.01;
The tunable properties are State, StateCovariance, ProcessNoise, MeasurementNoise, Alpha, Beta, and Kappa.
- Nontunable properties that you can specify once, either during object construction or afterward using dot notion. Specify these properties before state estimation using correct and predict. The StateTransitionFcn and MeasurementFcn properties belong to this category.
- Nontunable properties that you must specify during object construction. The HasAdditiveProcessNoise and HasAdditiveMeasurementNoise properties belong to this category.


## Alpha - Spread of sigma points

1e-3 (default) | scalar value between 0 and 1
Spread of sigma points around mean state value, specified as a scalar value between 0 and 1 ( $0<$ Alpha <= 1).

The unscented Kalman filter algorithm treats the state of the system as a random variable with mean value State and variance StateCovariance. To compute the state and its statistical properties at
the next time step, the algorithm first generates a set of state values distributed around the mean State value by using the unscented transformation. These generated state values are called sigma points. The algorithm uses each of the sigma points as an input to the state transition and measurement functions to get a new set of transformed state points and measurements. The transformed points are used to compute the state and state estimation error covariance value at the next time step.

The spread of the sigma points around the mean state value is controlled by two parameters Alpha and Kappa. A third parameter, Beta, impacts the weights of the transformed points during state and measurement covariance calculations:

- Alpha - Determines the spread of the sigma points around the mean state value. It is usually a small positive value. The spread of sigma points is proportional to Alpha. Smaller values correspond to sigma points closer to the mean state.
- Kappa - A second scaling parameter that is usually set to 0 . Smaller values correspond to sigma points closer to the mean state. The spread is proportional to the square-root of Kappa.
- Beta - Incorporates prior knowledge of the distribution of the state. For Gaussian distributions, Beta $=2$ is optimal.

If you know the distribution of state and state covariance, you can adjust these parameters to capture the transformation of higher-order moments of the distribution. The algorithm can track only a single peak in the probability distribution of the state. If there are multiple peaks in the state distribution of your system, you can adjust these parameters so that the sigma points stay around a single peak. For example, choose a small Alpha to generate sigma points close to the mean state value.

For more information, see "Unscented Kalman Filter Algorithm".
Alpha is a tunable property. You can change it using dot notation.

## Beta - Characterization of state distribution

2 (default) | scalar value greater than or equal to 0
Characterization of the state distribution that is used to adjust weights of transformed sigma points, specified as a scalar value greater than or equal to 0 . For Gaussian distributions, Beta $=2$ is an optimal choice.

For more information, see the Alpha property description.
Beta is a tunable property. You can change it using dot notation.

## HasAdditiveMeasurementNoise - Measurement noise characteristics

true (default) | false
Measurement noise characteristics, specified as one of the following values:

- true - Measurement noise $v$ is additive. The measurement function $h$ that is specified in MeasurementFcn has the following form:
$y(k)=h(x(k), U m 1, \ldots, U m n)$
Where $y(k)$ and $x(k)$ are the estimated output and estimated state at time $k$, and Um1, . . , Umn are any optional input arguments required by your measurement function.
- false - Measurement noise is nonadditive. The measurement function specifies how the output measurement evolves as a function of the state and measurement noise:

$$
y(k)=h(x(k), v(k), U m 1, \ldots, U m n)
$$

HasAdditiveMeasurementNoise is a nontunable property, and you can specify it only during object construction. You cannot change it using dot notation.

HasAdditiveProcessNoise - Process noise characteristics
true (default) | false
Process noise characteristics, specified as one of the following values:

- true - Process noise w is additive. The state transition function $f$ specified in StateTransitionFcn has the following form:
$x(k)=f(x(k-1), U s 1, \ldots, U s n)$
Where $x(k)$ is the estimated state at time $k$, and Us $1, \ldots$ Usn are any additional input arguments required by your state transition function.
- false - Process noise is nonadditive. The state transition function specifies how the states evolve as a function of the state and process noise at the previous time step:

$$
x(k)=f(x(k-1), w(k-1), U s 1, \ldots, U s n)
$$

HasAdditiveProcessNoise is a nontunable property, and you can specify it only during object construction. You cannot change it using dot notation.

## Kappa - Spread of sigma points

0 (default) | scalar value between 0 and 3
Spread of sigma points around mean state value, specified as a scalar value between 0 and 3 ( $0<=$ Kappa <= 3). Kappa is typically specified as 0 . Smaller values correspond to sigma points closer to the mean state. The spread is proportional to the square-root of Kappa. For more information, see the Alpha property description.

Kappa is a tunable property. You can change it using dot notation.

## MeasurementFcn - Measurement function

function handle
Measurement function $h$, specified as a function handle. The function calculates the $N$-element output measurement vector of the nonlinear system at time step $k$, given the state vector at time step $k . N$ is the number of measurements of the system. You write and save the measurement function and use it to construct the object. For example, if vdpMeasurementFcn.m is the measurement function, specify MeasurementFcn as @vdpMeasurementFcn. You can also specify MeasurementFcn as a function handle to an anonymous function.

The inputs to the function depend on whether you specify the measurement noise as additive or nonadditive in the HasAdditiveMeasurementNoise property of the object:

- HasAdditiveMeasurementNoise is true - The measurement noise $v$ is additive, and the measurement function specifies how the measurements evolve as a function of state values:
$y(k)=h(x(k), U m 1, \ldots, U m n)$
Where $y(k)$ and $x(k)$ are the estimated output and estimated state at time $k$, and Um1, ..., Umn are any optional input arguments required by your measurement function. For example, if you are
using multiple sensors for tracking an object, an additional input could be the sensor position. During estimation, you pass these additional arguments to the correct command which in turn passes them to the measurement function.
- HasAdditiveMeasurementNoise is false - The measurement noise is nonadditive, and the measurement function also specifies how the output measurement evolves as a function of the measurement noise:
$y(k)=h(x(k), v(k), U m 1, \ldots, U m n)$
To see an example of a measurement function with additive process noise, type edit vdpMeasurementFcn at the command line. To see an example of a measurement function with nonadditive process noise, type edit vdpMeasurementNonAdditiveNoiseFcn.

MeasurementFcn is a nontunable property. You can specify it once before using the correct command either during object construction or using dot notation after object construction. You cannot change it after using the correct command.

## MeasurementNoise - Measurement noise covariance

1 (default) | scalar | matrix
Measurement noise covariance, specified as a scalar or matrix depending on the value of the HasAdditiveMeasurementNoise property:

- HasAdditiveMeasurementNoise is true - Specify the covariance as a scalar or an $N$-by- $N$ matrix, where $N$ is the number of measurements of the system. Specify a scalar if there is no cross-correlation between measurement noise terms and all the terms have the same variance. The software uses the scalar value to create an $N-b y-N$ diagonal matrix.
- HasAdditiveMeasurementNoise is false - Specify the covariance as a $V$-by- $V$ matrix, where $V$ is the number of measurement noise terms. MeasurementNoise must be specified before using correct. After you specify MeasurementNoise as a matrix for the first time, to then change MeasurementNoise you can also specify it as a scalar. Specify as a scalar if there is no crosscorrelation between the measurement noise terms and all the terms have the same variance. The software extends the scalar to a $V$-by- $V$ diagonal matrix with the scalar on the diagonals.

MeasurementNoise is a tunable property. You can change it using dot notation.

## ProcessNoise - Process noise covariance

1 (default) | scalar | matrix
Process noise covariance, specified as a scalar or matrix depending on the value of the HasAdditiveProcessNoise property:

- HasAdditiveProcessNoise is true - Specify the covariance as a scalar or an Ns-by-Ns matrix, where Ns is the number of states of the system. Specify a scalar if there is no cross-correlation between process noise terms, and all the terms have the same variance. The software uses the scalar value to create an $N s$-by- $N s$ diagonal matrix.
- HasAdditiveProcessNoise is false - Specify the covariance as a $W$-by- $W$ matrix, where $W$ is the number of process noise terms. ProcessNoise must be specified before using predict. After you specify ProcessNoise as a matrix for the first time, to then change ProcessNoise you can also specify it as a scalar. Specify as a scalar if there is no cross-correlation between the process noise terms and all the terms have the same variance. The software extends the scalar to a $W$-by- $W$ diagonal matrix.

ProcessNoise is a tunable property. You can change it using dot notation.

## State - State of nonlinear system

[] (default) | vector
State of the nonlinear system, specified as a vector of size $N s$, where $N s$ is the number of states of the system.

When you use the predict command, State is updated with the predicted value at time step $k$ using the state value at time step $k-1$. When you use the correct command, State is updated with the estimated value at time step $k$ using measured data at time step $k$.

The initial value of State is the value you specify in the InitialState input argument during object creation. If you specify InitialState as a column vector, then State is also a column vector, and the predict and correct commands return state estimates as a column vector. Otherwise, a row vector is returned. If you want a filter with single-precision floating-point variables, you must specify State as a single-precision variable during object construction using the InitialState input argument.

State is a tunable property. You can change it using dot notation.

## StateCovariance - State estimation error covariance <br> 1 (default) | scalar | matrix

State estimation error covariance, specified as a scalar or an Ns-by-Ns matrix, where Ns is the number of states of the system. If you specify a scalar, the software uses the scalar value to create an $N s$-by-Ns diagonal matrix.

Specify a high value for the covariance when you do not have confidence in the initial state values that you specify in the InitialState input argument.

When you use the predict command, StateCovariance is updated with the predicted value at time step $k$ using the state value at time step $k-1$. When you use the correct command, StateCovariance is updated with the estimated value at time step $k$ using measured data at time step $k$.

StateCovariance is a tunable property. You can change it using dot notation after using the correct or predict commands.

## StateTransitionFcn - State transition function

function handle
State transition function $f$, specified as a function handle. The function calculates the Ns-element state vector of the system at time step $k$, given the state vector at time step $k-1$. Ns is the number of states of the nonlinear system.

You write and save the state transition function for your nonlinear system and use it to construct the object. For example, if vdpStateFcn.m is the state transition function, specify StateTransitionFcn as @vdpStateFcn. You can also specify StateTransitionFcn as a function handle to an anonymous function.

The inputs to the function you write depend on whether you specify the process noise as additive or nonadditive in the HasAdditiveProcessNoise property of the object:

- HasAdditiveProcessNoise is true - The process noise w is additive, and the state transition function specifies how the states evolve as a function of state values at previous time step:
$x(k)=f(x(k-1), U s 1, \ldots, U s n)$
Where $x(k)$ is the estimated state at time $k$, and Us1, . . , Usn are any additional input arguments required by your state transition function, such as system inputs or the sample time. During estimation, you pass these additional arguments to the predict command, which in turn passes them to the state transition function.
- HasAdditiveProcessNoise is false - The process noise is nonadditive, and the state transition function also specifies how the states evolve as a function of the process noise:

$$
x(k)=f(x(k-1), w(k-1), U s 1, \ldots, U s n)
$$

To see an example of a state transition function with additive process noise, type edit vdpStateFcn at the command line.

StateTransitionFcn is a nontunable property. You can specify it once before using the predict command either during object construction or using dot notation after object construction. You cannot change it after using the predict command.

## Output Arguments

## obj - unscented Kalman filter object for online state estimation

unscentedKalmanFilter object
Unscented Kalman filter object for online state estimation, returned as an unscentedKalmanFilter object. This object is created using the specified properties on page 1-1623. Use the correct and predict commands to estimate the state and state estimation error covariance using the unscented Kalman filter algorithm.

When you use predict, obj. State and obj. StateCovariance are updated with the predicted value at time step $k$ using the state value at time step $k-1$. When you use correct, obj. State and obj. StateCovariance are updated with the estimated values at time step $k$ using measured data at time step $k$.

## Compatibility Considerations

## Numerical Changes

Behavior changed in R2020b
Starting in R2020b, numerical improvements in the unscentedKalmanFilter algorithm might produce results that are different from the results you obtained in previous versions.

## Extended Capabilities

## $\mathbf{C} / \mathbf{C + +}$ Code Generation

Generate C and $\mathrm{C}++$ code using MATLAB® $\mathrm{Coder}^{\mathrm{TM}}$.
Usage notes and limitations:
For more information, see "Generate Code for Online State Estimation in MATLAB".
Generated code uses an algorithm that is different from the algorithm that the unscentedKalmanFilter function uses. You might see some numerical differences in the results obtained using the two methods.

Supports MATLAB Function block: No

## See Also

## Functions

clone | correct | extendedKalmanFilter | predict | residual
Blocks
Extended Kalman Filter | Kalman Filter | Unscented Kalman Filter

## Topics

"Nonlinear State Estimation Using Unscented Kalman Filter and Particle Filter"
"Generate Code for Online State Estimation in MATLAB"
"What Is Online Estimation?"
"Extended and Unscented Kalman Filter Algorithms for Online State Estimation"
"Validate Online State Estimation at the Command Line"
"Troubleshoot Online State Estimation"

## External Websites

Understanding Kalman Filters: Nonlinear State Estimators - MATLAB Video Series
Introduced in R2016b

## wavenet

Wavelet network function for nonlinear ARX and Hammerstein-Wiener models

## Description

A wavenet object implements a wavelet network function, and is a nonlinear mapping function for estimating nonlinear ARX and Nonlinear Hammerstein-Weiner models. The mapping function, which is also referred to as a nonlinearity, uses a combination of linear weights, an offset and a nonlinear function to compute its output. The nonlinear function contains wavelet unit functions that operate on a radial combination of inputs.


Mathematically, a wavenet network is a function that maps $m$ inputs $X(t)=\left[x\left(t_{1}\right), \chi_{2}(t), \ldots, \chi_{m}(t)\right]^{T}$ to a scalar output $y(t)$ using the following relationship:

$$
y(t)=y_{0}+(X(t)-\bar{X})^{T} P L+W(X(t))+S(X(t))
$$

Here:

- $X(t)$ is an $m$-by- 1 vector of inputs, or regressors, with mean $\bar{X}$.
- $y_{0}$ is the output offset, a scalar.
- $\quad P$ is an $m$-by- $p$ projection matrix, where $m$ is the number of regressors and is $p$ is the number of linear weights. $m$ must be greater than or equal to $p$.
- $L$ is a $p$-by- 1 vector of weights.
- $W(X)$ and $S(X)$ together constitute the nonlinear function of the wavelet network. $W(X)$ is a sum of dilated and translated wavelets while $S(X)$ is a sum of dilated and translated scaling functions (also known as scalelets). The total number of wavelet $d_{w}$ and scaling functions $d_{s}$ is referred to as the number of units of the network.

For definitions of the wavelet function term $W(X)$ and the scaling function term $S(X)$, see "More About" on page 1-1640.

Use wavenet as the value of the OutputFcn property of an idnlarx model or the InputNonlinearity and OutputLinearity properties of an idnlhw object. For example, specify wavenet when you estimate an idnlarx model with the following command.
sys = nlarx(data, regressors, wavenet)
When nlarx estimates the model, it essentially estimates the parameters of the wavenet function.
You can configure the wavenet function to disable components and fix parameters. To omit the linear component, set LinearFcn. Use to false. To omit the offset, set Offset. Use to false. To specify known values for the linear function and the offset, set their Value attributes directly and set the corresponding Free attributes to False. Use evaluate to compute the output of the function for a given vector of inputs.

## Creation

## Syntax

W = wavenet
$W=$ wavenet(numUnits)
W = wavenet (numUnits, UseLinearFcn)
W = wavenet (numUnits, UseLinearFcn, UseOffset)

## Description

$W$ = wavenet creates a wavenet object $W$, for which the function computes the number of units automatically during model estimation.
$W=$ wavenet (numUnits) specifies the number of units numUnits. This syntax includes an option that allows you to interactively assess the relationship between the number of units and unexplained variance.
$W=$ wavenet (numUnits, UseLinearFcn) specifies whether the function uses a linear function as a subcomponent.
$W$ = wavenet (numUnits,UseLinearFcn,UseOffset) specifies whether the function uses an offset term.

## Input Arguments

numUnits - Number of units
'auto' (default)|'interactive' | positive integer
Number of units, specified as the string or character vector that represents 'auto ' or 'interactive', or as a positive integer. numUnits determines the number of wavelets or scaling functions, or, if both elements are present, the combined number of wavelets and scaling functions. Typically, the wavelet network contains either wavelets or scaling functions, but not both. Specify numUnits as one of the following values:

- 'auto' - The software determines the number of units automatically during model estimation.
- 'interactive' - During model estimation, the software displays an interactive bar plot that relates unexplained variance to the number of units. Click on a bar to view the achievable fit to the estimation data for the selected number of units. A blue bar indicates the optimal choice, based on the generalized cross-validation (GCV) criterion. A general rule for the selection of the number of units is to use the smallest number of units that capture most of the variance.
- Positive integer - The software uses the specified value directly.

This argument sets the W.NonlinearFcn.NumberOfUnits property.

## UseLinearFcn - Option to use linear function

true (default) | false
Option to use the linear function subcomponent, specified as true or false. This argument sets the value of the $W$. LinearFcn. Use property.

## UseOffset - Option to use offset term <br> true (default) | false

Option to use an offset term, specified as true or false. This argument sets the value of the W.Offset.Use property.

## Properties

## Input - Input signal information

signal property values
Input signal information for signals used for estimation, specified as vectors of $m$ property-specific values, where $m$ is the number of input signals. The Input properties for each input signal are as follows:

- Name - Names of the input signals, specified as a 1-by-m string or character array, where $m$ is the number of inputs
- Mean - Mean of the input signals, specified as a numeric scalar
- Range - Ranges of the input signals, specified as a 2 -by-m numeric array that contains the minimum and maximum values


## Output - Output signal information

signal property values
Output signal information, specified as property-specific values. The Output properties are as follows:

- Name - Name of the output signal, specified as a string or a character array
- Mean - Mean of the output signal, specified as a numeric scalar
- Range - Range of the output signal, specified as a 2-by-1 numeric array that contains the minimum and maximum values


## LinearFcn - Parameters of linear function

## linear function property values (default)

Parameters of the linear function, specified as follows:

- Use - Option to use the linear function in the wavelet network, specified as a scalar logical. The default value is true. For an example of setting this option, see "Exclude Linear Term from Wavelet Network Mapping Object" on page 1-1636.
- Value - Linear weights that compose $L^{\prime}$, specified as a 1-by- $p$ vector.
- InputProjection - Input projection matrix $P$, specified as an $m$-by- $p$ matrix, that transforms the input vector of length $m$ into a vector of length $p$. For Hammerstein-Wiener models, InputProjection is equal to 1 .
- Free - Option to update entries of Value during estimation, specified as a 1-by-p logical vector. The software honors the Free specification only if the starting value of Value is finite. The default value is true.
- Minimum - Minimum bound on Value, specified as a 1-p vector. If Minimum is specified with a finite value and the starting value of Value is finite, then the software enforces that minimum bound during model estimation.
- Maximum - Maximum bound on Value, specified as a 1- $p$ vector. If Maximum is specified with a finite value and the starting value of Value is finite, then the software enforces that maximum bound during model estimation.
- SelectedInputIndex - Indices of wavenet inputs (see Input . Name) that are used as inputs to the linear function, specified as an 1-by- $n_{r}$ integer vector, where $n_{r}$ is the number of inputs. For nonlinear ARX models, the RegressorUsage property determines these indices. For Hammerstein-Wiener models, SelectedInputIndex is always 1.


## Offset - Parameters of offset term

offset property values
Parameters of the offset term, specified as follows:

- Use - Option to use the offset in the wavelet network, specified as a scalar logical. The default value is true.
- Value - Offset value, specified as a scalar.
- Free - Option to update Value during estimation, specified as a scalar logical. The software honors the Free specification of false only if the value of Value is finite. The default value is true.
- Minimum - Minimum bound on Value, specified as a numeric scalar or -Inf. If Minimum is specified with a finite value and the value of Value is finite, then the software enforces that minimum bound during model estimation. The default value is - Inf.
- Maximum - Maximum bound on Value, specified as a numeric scalar or Inf. If Maximum is specified with a finite value and the starting value of Value is finite, then the software enforces that maximum bound during model estimation. The default value is Inf.


## NonlinearFcn - Parameters of nonlinear function

nonlinear function property values
Parameters of the nonlinear function, specified as follows:

- NumberOfUnits - Number of units, specified as 'auto', 'interactive', or a positive integer. NumberOfUnits determines the number of wavelets or scaling functions, or, if both elements are present, the combined number of wavelets and scaling functions. Typically, the wavelet network contains either wavelets or scaling functions, but not both. The options for NumberOfUnits are as follows:
- 'auto' - The software determines the number of units automatically during model estimation.
- 'interactive' - During model estimation, the software displays an interactive bar plot that relates unexplained variance to the number of units. Click on a bar to view the achievable fit to the estimation data for the selected number of units. A blue bar indicates the optimal choice, based on the generalized cross-validation (GCV) criterion. A general rule for the selection of the number of units is to use the smallest number of units that capture most of the variance.
- Positive integer - The software uses the specified value directly.
- Structure - Advanced options that control the structure of the wavelet and scaling functions, specified as in the following table.

| Property | Description | Default |
| :--- | :--- | :--- |
| FinestCell | Minimum number of data <br> points in the smallest cell, <br> specified as 'auto ' or a <br> positive integer. A cell is the <br> area covered by the part of a <br> wavelet that is significantly <br> nonzero. The default setting <br> of 'auto ' specifies that the <br> software determines this <br> value during estimation. | 'auto' |
| MinimumCells | Minimum number of cells in <br> the partition, specified as a <br> positive integer. | 16 |
| MaximumCells | Maximum number of cells in <br> the partition, specified as a <br> positive integer. | 16 |
| MaximumLevels | Maximum number of wavelet <br> levels, specified as a positive <br> integer. | 10 |
| DilationStep | Dilation step size, specified as <br> a positive integer. | 2 |
| TranslationStep | Translation step size, <br> specified as a positive integer. | 1 |

- Parameters - Parameters of wavenet, specified as in the following table.

| Field Name | Description | Default |
| :---: | :---: | :---: |
| InputProject ion | Projection matrix $Q$, specified as an $m$-by- $q$ matrix. $Q$ transforms the detrended input vector ( $X-\bar{X}$ ) of length $m$ into a vector of length $q$. Typically, $Q$ has the same dimensions as the linear projection matrix $P$. In this case, $q$ is equal to $p$, which is the number of linear weights. <br> For Hammerstein-Wiener models, InputProjection is equal to 1. | [] |
| ScalingCoeff icient | Scaling function coefficients $s_{i}$, specified as a ds-by-1 vector. | [] |
| ScalingTrans lation | Scaling translation matrix, specified as a $d s$-by- $q$ matrix of scaling translation row vectors $e_{i}$. | [] |
| $\begin{aligned} & \hline \begin{array}{l} \text { ScalingDilat } \\ \text { ion } \end{array} \\ & \hline \end{aligned}$ | Scaling function dilation coefficients $d_{i}$, specified as an $d s$ -by-1 vector. | [] |
| WaveletCoeff icient | Wavelet function coefficients $w_{i}$, specified as a dw-by-1 vector. | [] |
| WaveletTrans lation | Wavelet translation matrix, $e_{i}$, specified as a $d w$-by- $q$ matrix of wavelet translation row vectors $c_{i}$. | [] |
| WaveletDilat ion | Wavelet dilation coefficients $b_{i}$, specified as an $d w$-by-1 vector. | [] |

- Free - Option to estimate parameters, specified as a logical scalar. If all the parameters have finite values, such as when the wavenet object corresponds to a previously estimated model, then setting Free to false causes the parameters of the nonlinear functions $W(X)$ and $S(X)$ to remain unchanged during estimation. The default value is true.
- SelectedInputIndex - Indices of wavenet inputs (see Input. Name) that are used as inputs to the nonlinear function, specified as an 1-by- $n_{r}$ integer vector, where $n_{r}$ is the number of inputs. For nonlinear ARX models, the RegressorUsage property determines these indices. For Hammerstein-Wiener models, SelectedInputIndex is always 1.


## Examples

## Create Wavelet Network Mapping Object

MO = wavenet;
View the wavenet object.
disp(MO)
Wavelet Network
Nonlinear Function: Wavelet network with number of units chosen automatically.
Linear Function: uninitialized
Output Offset: uninitialized
Input: [1×1 idpack.Channel]
Output: [1×1 idpack.Channel]
LinearFcn: [1×1 nlident.internal.UseProjectedLinearFcn]

```
NonlinearFcn: [1\times1 nlident.internal.WavenetFcn]
    Offset: [1×1 nlident.internal.ChooseableOffset]
```


## Exclude Linear Term from Wavelet Network Mapping Object

Create the wavenet mapping object MO.
MO = wavenet;
Exclude the linear term from MO.
MO.LinearFcn.Use = false;
View the wavenet object.
disp(MO)
Wavelet Network

```
Nonlinear Function: Wavelet network with number of units chosen automatically.
Linear Function: not in use
Output Offset: uninitialized
            Input: [1×1 idpack.Channel]
            Output: [1×1 idpack.Channel]
        LinearFcn: [1\times1 nlident.internal.UseProjectedLinearFcn]
    NonlinearFcn: [1×1 nlident.internal.WavenetFcn]
            Offset: [1\times1 nlident.internal.ChooseableOffset]
```


## Estimate Nonlinear ARX Model with Specific Mapping Function

Load the estimation data.
load twotankdata y u;
Create an iddata object from the estimation data.
z = iddata(y,u,0.2);
Create a wavelet network mapping object with five units.
MO = wavenet(5);
Estimate the nonlinear ARX model.

```
sys = nlarx(z,[4 4 1],MO)
sys =
Nonlinear ARX model with 1 output and 1 input
    Inputs: u1
    Outputs: y1
Regressors:
    Linear regressors in variables y1, ul
```

```
    List of all regressors
Output function: Wavelet Network with 5 units
Sample time: 0.2 seconds
Status:
Estimated using NLARX on time domain data "z".
Fit to estimation data: 96.8% (prediction focus)
FPE: 3.553e-05, MSE: 3.515e-05
```


## Estimate MIMO Hammerstein-Wiener Model

Load the estimation data.
load motorizedcamera;
Create an iddata object.
z = iddata(y,u,0.02,'Name','Motorized Camera','TimeUnit','s');
$z$ is an iddata object with six inputs and two outputs.
Specify the model orders and delays.
Orders $=[$ ones $(2,6)$, ones $(2,6)$, ones $(2,6)]$;
Specify the same nonlinearity estimator for each input channel.
InputNL = wavenet;
Specify different nonlinearity estimators for each output channel.

```
OutputNL = [deadzone,wavenet];
```

Estimate the Hammerstein-Wiener model.
sys $=$ nlhw(z,Orders,InputNL,OutputNL);
To see the shape of the estimated input and output nonlinearities, plot the nonlinearities.
plot(sys)


Click on the input and output nonlinearity blocks on the top of the plot to see the nonlinearities.

## Apply wavenet Constraints when Estimating Nonlinear ARX Model

Load the estimation data.

```
load iddata7 z7;
```

Specify wavenet for the model output function. Configure the wavenet object to have a fixed offset value of 1 and to use five units in the nonlinear component.

```
w = wavenet;
w.Offset.Value = 1;
w.Offset.Free = false;
w.NonlinearFcn.NumberOfUnits = 5;
```

Specify a linear model regressor set using a lag array that produces four consecutive output regressors and five consecutive input regressors.

```
reg = linearRegressor({'y1','ul'},{1:4,0:4});
```

Estimate the nonlinear ARX model.

```
sys = nlarx(z7,reg,w);
```

Examine the postestimation properties of the output function.

```
disp(sys.OutputFcn)
Wavelet Network
Inputs: y1(t-1), y1(t-2), y1(t-3), y1(t-4), ul(t), u1(t-1), ul(t-2), u1(t-3), u1(t-4)
Output: y1
    Nonlinear Function: Wavelet network with 5 units.
    Linear Function: initialized to [-1.12 0.469 1.25 0.556 -0.81 -0.261 -0.074 0.711 1.15]
    Output Offset: fixed to 1
            Input: [1×1 idpack.Channel]
            Output: [1×1 idpack.Channel]
            LinearFcn: [1\times1 nlident.internal.UseProjectedLinearFcn]
            NonlinearFcn: [1>1 nlident.internal.WavenetFcn]
            Offset: [1×1 nlident.internal.ChooseableOffset]
disp(sys.OutputFcn.Input)
    Channel with properties:
        Name: [1\times9 string]
        Mean: [0.0795 0.0848 0.1082 0.1134 0.0253 0.0253 0.0202 0.0202 0.0202]
        Range: [2\times9 double]
disp(sys.OutputFcn.NonlinearFcn)
    WavenetFcn with properties:
            NumberOfUnits: 5
            Structure: [1\times1 nlident.internal.WavenetStructure]
            Parameters: [1\times1 nlident.internal.WavenetFcnParameters]
                            Free: 1
        SelectedInputIndex: [1 2 3 4 5 6 7 8 9]
```


## Apply wavenet Constraints when Estimating Hammerstein-Wiener Model

Load the estimation data.
load throttledata

Specify wavenet for the model output nonlinearity and set the arguments for NumUnits to 5 and UseLinearFcn and UseOffset to false.
w = wavenet(5,false,false);
An alternative method for removing the linear function and offset is to use dot notation after creating the nonlinearity.
w.LinearFcn.Use = false;
w.Offset.Use = false;

Estimate the model, using an order specification of $\left[\begin{array}{lll}4 & 4 & 1\end{array}\right]$.
sys $=$ nlhw(ThrottleData,[4 4 1],[],w);
Compare the simulated model response with the estimation data.
compare(ThrottleData,sys);

Simulated Response Comparison


## More About

## Wavelet Nonlinear Function $\mathbf{W}(X)$

The wavelet nonlinear function is a sum of the dilated and translated wavelets, and is described by the following equation:

$$
W(X)=\sum_{i=1}^{d_{W}} w_{i} f_{w}\left(b_{i}(X-\bar{X})^{T} Q-c_{i}\right)
$$

Here:

- $\quad Q$ is an $m$-by- $q$ projection matrix, where $m \geq q$
- $w_{1}, w_{2}, \ldots, w_{d w}$ are scalar coefficients called wavelet coefficients.
- $b_{1}, b_{2}, \ldots, b_{d w}$ are scalars called wavelet dilations that multiply the detrended input matrix $(X-\bar{X})$.
- $c_{1}, c_{2}, \ldots, c_{d w}$ are 1-by- $q$ row vectors called wavelet translations.
- $f_{w}(x)=e^{-x x^{T} / 2}$ is a radial function that depends only upon the squared magnitude of the vector $x$. $x$ is a row vector that is composed of a linear combination of inputs with an offset $c_{i}$.


## Scaling Nonlinear Function $\mathbf{S}(X)$

The scaling nonlinear function is a sum of the dilated and translated scaling functions, and is described by the following equation:

$$
S(X)=\sum_{i=1}^{d_{S}} s_{i} f_{S}\left(b_{i}(X-\bar{X})^{T} Q-e_{i}\right)
$$

Here:

- $\quad Q$ is an $m$-by- $q$ projection matrix, where $m \geq q$.
- $s_{1}, s_{2}, \ldots, s_{d s}$ are scalar coefficients called scaling coefficients.
- $d_{1}, d_{2}, \ldots, d_{d s}$ are scalars called scaling dilations that multiply the detrended input matrix $(X-\bar{X})$.
- $e_{1}, e_{2}, \ldots, e_{d s}$ are 1-by- $q$ row vectors called scaling translations.
- $f_{S}(x)=\left(\operatorname{dim}(x)-x x^{T}\right) e^{-x x^{T} / 2}$ is a radial function that depends only upon the squared magnitude of the vector $x . x$ is a row vector that is composed of a linear combination of inputs with an offset $e_{i}$.


## Algorithms

You can use wavenet in both nonlinear ARX and Hammerstein-Wiener models. The algorithms for estimating wavenet parameters depend on which model you are estimating.

- In a nonlinear ARX model, wavenet uses either a noniterative or an iterative technique for predicting the parameters, depending on option settings in nlarx0ptions.
- If the Focus option is set to 'prediction', then wavenet uses a fast noniterative technique to estimate parameters [1]. Successive refinements after the first estimation use an iterative algorithm.
- If the Focus option is set to 'simulation', then wavenet uses an iterative technique to estimate parameters.
- To always use either an iterative or a noniterative algorithm, specify the IterativeWavenet property of nlarx0ptions as 'on' or 'off', respectively.
- In a Hammerstein-Wiener model, wavenet uses iterative minimization to determine the parameters.


## Compatibility Considerations

Use of previous nonlinearity estimator properties is not recommended
Not recommended starting in R2021a
Starting in R2021a, the properties of the mapping objects, previously known as nonlinearity estimators, have been reorganized. These objects are wavenet (W), sigmoidnet (S), treepartition (T), customnet (C), and linear (L). The property changes do not apply to neuralnet. The use of the pre-R2021a properties in the following table is discouraged. However, the software still accepts commands that set these properties. There are no plans to exclude such commands at this time.

| Pre-R2021a Property | R2021a Property | Applicable Mapping Objects |
| :--- | :--- | :--- |
| NumberOfUnits | NonlinearFcn. NumberOfUni <br> ts | W,S,T,C |
| LinearTerm | LinearFcn.Use, Offset.Use | W,S,C |
| Parameters | Split into three pieces: <br> - LinearFcn.Value <br> - <br> Offset.Value <br> NonlinearFcn.Parameter <br> s | W,S,T,C,L <br> linear (L) excludes <br> NonlinearFcn.Parameters. |
| Options | NonlinearFcn.Structure | W,T |

## References

[1] Qinghua Zhang. "Using Wavelet Network in Nonparametric Estimation." IEEE Transactions on Neural Networks 8, no. 2 (March 1997): 227-36. https://doi.org/10.1109/72.557660.

## See Also

customnet | deadzone |evaluate |idnlarx|idnlhw|linear|neuralnet|nlarx|nlhw| polyld|pwlinear|saturation|sigmoidnet|treepartition|unitgain

## Introduced in R2007a

## xperm

Reorder states in state-space models

## Syntax

sys $=$ xperm(sys, $P$ )

## Description

sys $=$ xperm(sys,P) reorders the states of the state-space model sys according to the permutation P . The vector P is a permutation of 1:NX, where $N X$ is the number of states in sys. For information about creating state-space models, see ss and dss.

## Examples

## Alphabetically Order States of State-Space Model

Load a previously saved state space model ssF8 with four states.

```
load ltiexamples
ssF8
ssF8 =
    A =
\begin{tabular}{lrrrr} 
& PitchRate & Velocity & A0A & PitchAngle \\
PitchRate & -0.7 & -0.0458 & -12.2 & 0 \\
Velocity & 0 & -0.014 & -0.2904 & -0.562 \\
A0A & 1 & -0.0057 & -1.4 & 0 \\
PitchAngle & 1 & 0 & 0 & 0
\end{tabular}
    B =
\begin{tabular}{lrr} 
& Elevator & Flaperon \\
PitchRate & -19.1 & -3.1 \\
Velocity & -0.0119 & -0.0096 \\
A0A & -0.14 & -0.72 \\
PitchAngle & 0 & 0
\end{tabular}
    C =
\begin{tabular}{lrrrr} 
& PitchRate & Velocity & AOA & PitchAngle \\
FlightPath & 0 & 0 & -1 & 1 \\
Acceleration & 0 & 0 & 0.733 & 0
\end{tabular}
    D =
\begin{tabular}{lrr} 
& Elevator & Flaperon \\
FlightPath & 0 & 0 \\
Acceleration & 0.0768 & 0.1134
\end{tabular}
Continuous-time state-space model.
```

Order the states in alphabetical order.

```
[y,P] = sort(ssF8.StateName);
sys = xperm(ssF8,P)
sys =
    A =
\begin{tabular}{lrrrr} 
& AOA & PitchAngle & PitchRate & Velocity \\
AOA & -1.4 & 0 & 1 & -0.0057 \\
PitchAngle & 0 & 0 & 1 & 0 \\
PitchRate & -12.2 & 0 & -0.7 & -0.0458 \\
Velocity & -0.2904 & -0.562 & 0 & -0.014
\end{tabular}
    B =
\begin{tabular}{lrr} 
& Elevator & Flaperon \\
AOA & -0.14 & -0.72 \\
PitchAngle & 0 & 0 \\
PitchRate & -19.1 & -3.1 \\
Velocity & -0.0119 & -0.0096
\end{tabular}
    C =
    FlightPath
    Acceleration
    -0
    D =
\begin{tabular}{lrr} 
& Elevator & Flaperon \\
FlightPath & 0 & 0 \\
Acceleration & 0.0768 & 0.1134
\end{tabular}
Continuous-time state-space model.
```

The states in ssF8 now appear in alphabetical order.

## See Also

dss | ss

Introduced in R2008b

## zero

Zeros and gain of SISO dynamic system

## Syntax

Z = zero(sys)
[Z,gain] = zero(sys)
[Z, gain] = zero(sys,J1,...,JN)

## Description

Z = zero(sys) returns the zeros of the single-input, single-output (SISO) dynamic system model, sys. The output is expressed as the reciprocal of the time units specified in sys.TimeUnit.
[Z,gain] = zero(sys) also returns the zero-pole-gain of sys.
[Z,gain] = zero(sys,J1, .., JN) returns the zeros and gain of the entries in the model array sys with subscripts J1, ..., JN.

## Examples

## Zeros of Transfer Function

Compute the zeros of the following transfer function:
$\operatorname{sys}(s)=\frac{4.2 s^{2}+0.25 s-0.004}{s^{2}+9.6 s+17}$
sys $=t f([4.2,0.25,-0.004],[1,9.6,17])$;
Z = zero(sys)
Z $=2 \times 1$
$-0.0726$
0.0131

## Zeros and Gain of Transfer Function

Calculate the zero locations and zero-pole gain of the following transfer function:

$$
\operatorname{sys}(s)=\frac{4.2 s^{2}+0.25 s-0.004}{s^{2}+9.6 s+17}
$$

sys $=\operatorname{tf}([4.2,0.25,-0.004],[1,9.6,17])$;
[z,gain] = zero(sys)

```
z = 2×1
    -0.0726
    0.0131
gain = 4.2000
```

The zero locations are expressed in second ${ }^{-1}$, because the time unit of the transfer function (H.TimeUnit) is seconds.

## Zeros and Gain of Models in an Array

For this example, load a 3-by-1 array of transfer function models.

```
load('tfArray.mat','sys');
size(sys)
3x1 array of transfer functions.
Each model has 1 outputs and l inputs.
```

Find the zeros and gain values of the models in the array.

```
[Z, gain] = zero(sys);
Z(:,:,1,1)
ans =
    0x1 empty double column vector
gain(:,:,1,1)
ans = 1
```

zero returns an array each for the zeros and the gain values respectively. Here, $Z(:,:, 1,1)$ and gain (:,:,1,1) corresponds to the zero and the gain value of the first model in the array, that is, sys(:,:,1,1).

## Input Arguments

## sys - Dynamic system

dynamic system model | model array
Dynamic system, specified as a SISO dynamic system model, or an array of SISO dynamic system models. Dynamic systems that you can use include continuous-time or discrete-time numeric LTI models such as tf , zpk , or ss models.

If sys is a generalized state-space model genss or an uncertain state-space model uss, zero returns the zeros of the current or nominal value of sys. If sys is an array of models, zero returns the zeros of the model corresponding to its subscript J1, . . , JN in sys. For more information on model arrays, see "Model Arrays" (Control System Toolbox).

## J1, . . . , JN - Indices of models in array whose zeros you want to extract positive integer

Indices of models in array whose zeros you want to extract, specified as a positive integer. You can provide as many indices as there are array dimensions in sys. For example, if sys is a 4 -by- 5 array of dynamic system models, the following command extracts the zeros for entry $(2,3)$ in the array.

Z = zero(sys,2,3);

## Output Arguments

## Z - Zeros of the dynamic system

column vector | array
Zeros of the dynamic system, returned as a column vector or an array. If sys is:

- A single model, then $Z$ is a column vector of zeros of the dynamic system model sys
- A model array, then $Z$ is an array containing the zeros of each model in sys
$Z$ is expressed as the reciprocal of the time units specified in sys.TimeUnit. For example, zero is expressed in 1 /minute if sys.TimeUnit = 'minutes'.


## gain - Zero-pole-gain of the dynamic system

scalar
Zero-pole-gain of the dynamic system, returned as a scalar. In other words, gain is the value of K when the model is written in zpk form.

## Tips

- If sys has internal delays, zero sets all internal delays to zero, creating a zero-order Padé approximation. This approximation ensures that the system has a finite number of zeros. zero returns an error if setting internal delays to zero creates singular algebraic loops. To assess the stability of models with internal delays, use step or impulse.
- To calculate the transmission zeros of a multi-input, multi-output (MIMO) system, use tzero.


## See Also

impulse|pole | pzmap|pzplot|step|tzero
Topics
"Pole and Zero Locations" (Control System Toolbox)

## Introduced in R2012a

## zgrid

Generate z-plane grid of constant damping factors and natural frequencies

## Syntax

```
zgrid
zgrid(T)
zgrid(zeta,wn)
zgrid(zeta,wn,T)
zgrid(__,'new')
zgrid(AX,
```

$\qquad$

## Description

zgrid generates a grid of constant damping factors from 0 to 1 in steps of 0.1 and natural frequencies from 0 to $\pi / T$ in steps of $0.1 * \pi / T$ for root locus and pole-zero maps. The default steps of $0.1 * \pi / T$ represent fractions of the Nyquist frequencies. zgrid then plots the grid over the current axis. zgrid creates the grid over the plot without altering the current axis limits if the current axis contains a discrete z-plane root locus diagram or pole-zero map. Use this syntax to plot multiple systems with different sample times.

Alternatively, you can select Grid from the context menu in the plot window to generate the same zplane grid.
zgrid( $T$ ) generates the z-plane grid by using default values for damping factor and natural frequency relative to the sample time T .
zgrid(zeta,wn) plots a grid of constant damping factor and natural frequency lines for the damping factors and normalized natural frequencies in the vectors zeta and wn, respectively. When the sample time is not specified, the frequency values in wn are interpreted as normalized values, that is, wn/T.
zgrid(zeta, wn, T ) plots a grid of constant damping factor and natural frequency lines for the damping factors and natural frequencies in the vectors zeta and wn, relative to sample time T . zeta lines are independent for $T$ but the wn lines depend on the sample time value. Use this syntax to create the z -plane grid with specific values of wn .
zgrid (__, ' new') clears the current axes first and sets hold on.
zgrid(AX, $\qquad$ ) plots the z-plane grid on the Axes or UIAxes object in the current figure with the handle AX. Use this syntax when creating apps with zgrid in the App Designer.

## Examples

## Plot z-plane Grid Lines on the Root Locus

To see the z-plane grid on the root locus plot, type

```
H=tf([[2 -3.4 1.5],[1 -1.6 0.8],-1)
H =
    2 z^2 - 3.4 z + 1.5
    ----------------
Sample time: unspecified
Discrete-time transfer function.
rlocus(H)
zgrid
axis equal
```



## Normalized and True z-plane Grid Lines on the Pole-Zero Map

For this example, consider a discrete-time transfer function sys with a sample time of 0.1 s . Now plot the pole-zero map of sys and visualize the default z-plane grid without specifying the sample time.

```
sys = tf([2 -3.4 1.5],[1 -1.6 0.8],0.1);
Ts = 0.1;
figure()
pzmap(sys)
zgrid()
axis equal
```



Observe that the frequencies on the z-plane grid are normalized in terms of $f \frac{\Pi}{T}$. To obtain the true frequency values on the grid, specify the sample time with the zgrid command.

```
figure()
```

pzmap(sys)
zgrid(Ts)
axis equal


Now, observe that the frequency values on the plot are true values, that is, they are non-normalized.

## Input Arguments

## zeta - Damping ratio

vector
Damping ratio, specified as a vector in the same order as wn.

## wn - Natural frequency values

vector
Natural frequency values, specified as a vector. Natural frequencies are plotted as true values when $T$ is specified. When the sample time is not specified, zgrid normalizes the values as wn/T.

## T - Sample time

positive scalar |-1
Sample time, specified as:

- A positive scalar representing the sampling period of a discrete-time system. The actual frequency values are displayed on the frequency grid.
-     - 1 for a discrete-time system with an unspecified sample time. The frequency values are displayed as normalized values $f * \pi / T$ for the default grid.
zeta lines are independent of T while wn lines are dependent on the sample time. You must specify T to plot specific values of wn. When the sample time T is not specified, the required wn values are interpreted as normalized values, that is, wn/T.


## AX - Object handle

Axes object | UIAxes object
Object handle, specified as an Axes or UIAxes object. Use AX to create apps with zgrid in the App Designer.

## See Also

pzmap|rlocus | sgrid
Introduced before R2006a

## zpkdata

Access zero-pole-gain data

## Syntax

[z,p,k] = zpkdata(sys)
[z,p,k,Ts] = zpkdata(sys)
[z, p,k,Ts,covz,covp,covk] = zpkdata(sys)

## Description

$[z, p, k]=z p k d a t a(s y s)$ returns the zeros $z$, poles $p$, and gain(s) $k$ of the zero-pole-gain model sys. The outputs $z$ and $p$ are cell arrays with the following characteristics:

- $z$ and $p$ have as many rows as outputs and as many columns as inputs.
- The ( $i, j$ ) entries $z\{i, j\}$ and $p\{i, j\}$ are the (column) vectors of zeros and poles of the transfer function from input $j$ to output $i$.

The output k is a matrix with as many rows as outputs and as many columns as inputs such that $k(i, j)$ is the gain of the transfer function from input $j$ to output $i$. If sys is a transfer function or state-space model, it is first converted to zero-pole-gain form using zpk.

For SISO zero-pole-gain models, the syntax
[z,p,k] = zpkdata(sys,'v')
forces zpkdata to return the zeros and poles directly as column vectors rather than as cell arrays (see example below).
[z,p,k,Ts] = zpkdata(sys) also returns the sample time Ts.
[z,p,k,Ts,covz,covp,covk] = zpkdata(sys) also returns the covariances of the zeros, poles and gain of the identified model sys. covz is a cell array such that covz\{ky, ku\} contains the covariance information about the zeros in the vector $z\{k y, k u\} . \operatorname{covz}\{k y, k u\}$ is a 3-D array of dimension 2-by-2-by-Nz, where $N z$ is the length of $z\{k y, k u\}$, so that the $(1,1)$ element is the variance of the real part, the $(2,2)$ element is the variance of the imaginary part, and the $(1,2)$ and $(2,1)$ elements contain the covariance between the real and imaginary parts. covp has a similar relationship to $p$. covk is a matrix containing the variances of the elements of $k$.

You can access the remaining LTI properties of sys with get or by direct referencing, for example,
sys.Ts
sys.inputname

## Examples

## Example 1

Given a zero-pole-gain model with two outputs and one input

```
H = zpk({[0];[-0.5]},{[0.3];[0.1+i 0.1-i]},[1;2],-1)
Zero/pole/gain from input to output...
            z
#1: -------
        (z-0.3)
            2(z+0.5)
    #2: ------------------
        (z^2 - 0.2z + 1.01)
Sample time: unspecified
you can extract the zero/pole/gain data embedded in }\textrm{H}\mathrm{ with
```

```
[z,p,k] = zpkdata(H)
```

[z,p,k] = zpkdata(H)
z =
z =
[ 0]
[ 0]
[-0.5000]
[-0.5000]
p =
p =
[ 0.3000]
[ 0.3000]
[2x1 double]
[2x1 double]
k =
k =
1
1
2

```
    2
```

To access the zeros and poles of the second output channel of H , get the content of the second cell in $z$ and $p$ by typing

```
z{2,1}
```

ans =
-0.5000
$\mathrm{p}\{2,1\}$
ans $=$
$0.1000+1.0000 i$
0.1000-1.0000i

## Example 2

Extract the ZPK matrices and their standard deviations for a 2-input, 1 output identified transfer function.

```
load iddata7
```

transfer function model
sys1 = tfest(z7, 2, 1, 'InputDelay',[1 0]);
an equivalent process model

```
sys2 = procest(z7, {'P2UZ', 'P2UZ'}, 'InputDelay',[1 0]);
```

1, p1, k1, ~, dz1, dp1, dk1] = zpkdata(sys1);
[z2, p2, k2, ~, dz2, dp2, dk2] = zpkdata(sys2);

Use iopzplot to visualize the pole-zero locations and their covariances

```
h = iopzplot(sys1, sys2);
showConfidence(h)
```


## See Also

get | ssdata|tfdata|zpk

Introduced before R2006a

## Blocks

## Extended Kalman Filter

Estimate states of discrete-time nonlinear system using extended Kalman filter
Library:
Control System Toolbox / State Estimation
System Identification Toolbox / Estimators


## Description

The Extended Kalman Filter block estimates the states of a discrete-time nonlinear system using the first-order discrete-time extended Kalman filter algorithm.

Consider a plant with states $x$, input $u$, output $y$, process noise $w$, and measurement noise $v$. Assume that you can represent the plant as a nonlinear system.


Using the state transition and measurement functions of the system and the extended Kalman filter algorithm, the block produces state estimates $\widehat{x}$ for the current time step. For information about the algorithm, see "Extended and Unscented Kalman Filter Algorithms for Online State Estimation".

You create the nonlinear state transition function and measurement functions for the system and specify these functions in the block. The block supports state estimation of a system with multiple sensors that are operating at different sampling rates. You can specify up to five measurement functions, each corresponding to a sensor in the system. You can also specify the Jacobians of the state transition and measurement functions. If you do not specify them, the software numerically computes the Jacobians. For more information, see "State Transition and Measurement Functions" on page 2-14.

## Ports

## Input

## $\mathrm{y} 1, \mathrm{y} 2, \mathrm{y} 3, \mathrm{y} 4, \mathrm{y} 5$ - Measured system outputs <br> vector

Measured system outputs corresponding to each measurement function that you specify in the block. The number of ports equals the number of measurement functions in your system. You can specify up to five measurement functions. For example, if your system has two sensors, you specify two
measurement functions in the block. The first port $\mathbf{y 1}$ is available by default. When you click Apply, the software generates port $\mathbf{y 2}$ corresponding to the second measurement function.

Specify the ports as $N$-dimensional vectors, where $N$ is the number of quantities measured by the corresponding sensor. For example, if your system has one sensor that measures the position and velocity of an object, then there is only one port $\mathbf{y 1}$. The port is specified as a 2-dimensional vector with values corresponding to position and velocity.

## Dependencies

The first port $\mathbf{y 1}$ is available by default. Ports $\mathbf{y} \mathbf{2}$ to $\mathbf{y 5}$ are generated when you click Add Measurement, and click Apply.
Data Types: single | double
StateTransitionFcnInputs - Additional optional input argument to state transition function
scalar | vector | matrix
Additional optional input argument to the state transition function $f$ other than the state x and process noise w. For information about state transition functions see, "State Transition and Measurement Functions" on page 2-14.

Suppose that your system has nonadditive process noise, and the state transition function $f$ has the following form:
$x(k+1)=f(x(k), w(k)$,StateTransitionFcnInputs)
Here k is the time step, and StateTransitionFcnInputs is an additional input argument other than $x$ and $w$.

If you create f using a MATLAB function (.m file), the software generates the port
StateTransitionFcnInputs when you click Apply. You can specify the inputs to this port as a scalar, vector, or matrix.

If your state transition function has more than one additional input, use a Simulink Function block to specify the function. When you use a Simulink Function block, you provide the additional inputs directly to the Simulink Function block using Inport blocks. No input ports are generated for the additional inputs in the Extended Kalman Filter block.

## Dependencies

This port is generated only if both of the following conditions are satisfied:

- You specify $f$ in Function using a MATLAB function, and $f$ is on the MATLAB path.
- frequires only one additional input argument apart from $x$ and $w$.

Data Types: single | double
MeasurementFcn1Inputs, MeasurementFcn2Inputs, MeasurementFcn3Inputs, Measurement Fcn4Inputs, MeasurementFcn5Inputs - Additional optional input argument to each measurement function
scalar | vector | matrix

Additional optional inputs to the measurement functions other than the state $x$ and measurement noise v. For information about measurement functions see, "State Transition and Measurement Functions" on page 2-14.

MeasurementFcn1Inputs corresponds to the first measurement function that you specify, and so on. For example, suppose that your system has three sensors and nonadditive measurement noise, and the three measurement functions h1, h2, and h3 have the following form:

```
\(y 1[k]=h 1(x[k], v 1[k], M e a s u r e m e n t F c n 1 I n p u t s)\)
\(\mathrm{y} 2[\mathrm{k}]=\mathrm{h} 2(\mathrm{x}[\mathrm{k}], \mathrm{v} 2[\mathrm{k}]\), MeasurementFcn2Inputs)
\(y 3[k]=h 3(x[k], v 3[k])\)
```

Here $k$ is the time step, and MeasurementFcn1Inputs and MeasurementFcn2Inputs are the additional input arguments to h1 and h2.

If you specify h1, h2, and h3 using MATLAB functions (. m files) in Function, the software generates ports MeasurementFcn1Inputs and MeasurementFcn2Inputs when you click Apply. You can specify the inputs to these ports as scalars, vectors, or matrices.

If your measurement functions have more than one additional input, use Simulink Function blocks to specify the functions. When you use a Simulink Function block, you provide the additional inputs directly to the Simulink Function block using Inport blocks. No input ports are generated for the additional inputs in the Extended Kalman Filter block.

## Dependencies

A port corresponding to a measurement function $h$ is generated only if both of the following conditions are satisfied:

- You specify $h$ in Function using a MATLAB function, and $h$ is on the MATLAB path.
- $h$ requires only one additional input argument apart from $x$ and $v$.


## Data Types: single | double

## Q - Time-varying process noise covariance

scalar | vector | matrix
Time-varying process noise covariance, specified as a scalar, vector, or matrix depending on the value of the Process noise parameter:

- Process noise is Additive - Specify the covariance as a scalar, an Ns-element vector, or an Ns-by-Ns matrix, where $N s$ is the number of states of the system. Specify a scalar if there is no crosscorrelation between process noise terms, and all the terms have the same variance. Specify a vector of length $N s$, if there is no cross-correlation between process noise terms, but all the terms have different variances.
- Process noise is Nonadditive - Specify the covariance as a $W$-by- $W$ matrix, where $W$ is the number of process noise terms in the state transition function.


## Dependencies

This port is generated if you specify the process noise covariance as Time-Varying. The port appears when you click Apply.

Data Types: single|double

## $R 1, R 2, R 3, R 4, R 5$ - Time-varying measurement noise covariance matrix

Time-varying measurement noise covariances for up to five measurement functions of the system, specified as matrices. The sizes of the matrices depend on the value of the Measurement noise parameter for the corresponding measurement function:

- Measurement noise is Additive - Specify the covariance as an $N$-by- $N$ matrix, where $N$ is the number of measurements of the system.
- Measurement noise is Nonadditive - Specify the covariance as a $V$-by- $V$ matrix, where $V$ is the number of measurement noise terms in the corresponding measurement function.


## Dependencies

A port is generated if you specify the measurement noise covariance as Time-Varying for the corresponding measurement function. The port appears when you click Apply.
Data Types: single | double

## Enable1,Enable2,Enable3,Enable4,Enable5 - Enable correction of estimated states when measured data is available

 scalarSuppose that measured output data is not available at all time points at the port $\mathbf{y 1}$ that corresponds to the first measurement function. Use a signal value other than 0 at the Enable1 port to enable the correction of estimated states when measured data is available. Specify the port value as 0 when measured data is not available. Similarly, if measured output data is not available at all time points at the port yi for the $i^{\text {th }}$ measurement function, specify the corresponding port Enablei as a value other than 0.

## Dependencies

A port corresponding to a measurement function is generated if you select Add Enable port for that measurement function. The port appears when you click Apply.

Data Types: single | double | Boolean

## Output

## xhat - Estimated states

vector
Estimated states, returned as a vector of size $N s$, where $N s$ is the number of states of the system. To access the individual states, use the Selector block.

When the Use the current measurements to improve state estimates parameter is selected, the block outputs the corrected state estimate $\widehat{x}[k \mid k]$ at time step k , estimated using measured outputs until time $k$. If you clear this parameter, the block returns the predicted state estimate $\widehat{x}[k \mid k-1]$ for time $k$, estimated using measured output until a previous time $k-1$. Clear this parameter if your filter is in a feedback loop and there is an algebraic loop in your Simulink model.

## Data Types: single \| double

## P - State estimation error covariance matrix

State estimation error covariance, returned as an Ns-by-Ns matrix, where Ns is the number of states of the system. To access the individual covariances, use the Selector block.

## Dependencies

This port is generated if you select Output state estimation error covariance in the System Model tab, and click Apply.
Data Types: single | double

## Parameters

## System Model Tab

State Transition

## Function - State transition function name

myStateTransitionFcn (default) | function name
The state transition function calculates the $N s$-element state vector of the system at time step $k+1$, given the state vector at time step $k$. $N s$ is the number of states of the nonlinear system. You create the state transition function and specify the function name in Function. For example, if vdpStateFcn.m is the state transition function that you created and saved, specify Function as vdpStateFcn.

The inputs to the function you create depend on whether you specify the process noise as additive or nonadditive in Process noise.

- Process noise is Additive - The state transition function $f$ specifies how the states evolve as a function of state values at previous time step:

$$
x(k+1)=f(x(k), U s 1(k), \ldots, U s n(k)),
$$

where $x(k)$ is the estimated state at time $k$, and Us1, ... Usn are any additional input arguments required by your state transition function, such as system inputs or the sample time. To see an example of a state transition function with additive process noise, type edit vdpStateFcn at the command line.

- Process noise is Nonadditive - The state transition function also specifies how the states evolve as a function of the process noise $w$ :

$$
x(k+1)=f(x(k), w(k), U s 1(k), \ldots, U s n(k)) .
$$

For more information, see "State Transition and Measurement Functions" on page 2-14.
You can create $f$ using a Simulink Function block or as a MATLAB function (.m file).

- You can use a MATLAB function only if $f$ has one additional input argument Us1 other than x and w.
$x(k+1)=f(x(k), w(k), U s 1(k))$
The software generates an additional input port StateTransitionFcnInputs to specify this argument.
- If you are using a Simulink Function block, specify x and w using Argument Inport blocks and the additional inputs Us1, . . . Usn using Inport blocks in the Simulink Function block. You do not provide Us1, . . . , Usn to the Extended Kalman Filter block.

Programmatic Use
Block Parameter: StateTransitionFcn
Type: character vector, string
Default: 'myStateTransitionFcn'

## Jacobian - Jacobian of state transition function <br> off (default) | on

Jacobian of state transition function $f$, specified as one of the following:

- off - The software computes the Jacobian numerically. This computation may increase processing time and numerical inaccuracy of the state estimation.
- on - You create a function to compute the Jacobian, and specify the name of the function in Jacobian. For example, if vdpStateJacobianFcn.m is the Jacobian function, specify Jacobian as vdpStateJacobianFcn. If you create the state transition function $f$ using a Simulink Function block, then create the Jacobian using a Simulink Function block. If you create $f$ using a MATLAB function (. m file), then create the Jacobian using a MATLAB function.

The function calculates the partial derivatives of the state transition function with respect to the states and process noise. The number of inputs to the Jacobian function must equal the number of inputs of the state transition function and must be specified in the same order in both functions. The number of outputs of the Jacobian function depends on the Process noise parameter:

- Process noise is Additive - The function calculates the partial derivative of the state transition function $f$ with respect to the states ( $\partial f / \partial x$ ). The output is an $N s$-by-Ns Jacobian matrix, where $N s$ is the number of states.

To see an example of a Jacobian function for additive process noise, type edit vdpStateJacobianFcn at the command line.

- Process noise is Nonadditive - The function must also return a second output that is the partial derivative of the state transition function $f$ with respect to the process noise terms $(\partial f / \partial w)$. The second output is returned as an $N s$-by- $W$ matrix, where $W$ is the number of process noise terms in the state transition function.


## Programmatic Use

Block Parameter: HasStateTransitionJacobianFcn
Type: character vector
Values: 'off', 'on'
Default: 'off'
Block Parameter: StateTransitionJacobianFcn
Type: character vector, string
Default: ' '

## Process noise - Process noise characteristics

Additive (default) | Nonadditive
Process noise characteristics, specified as one of the following values:

- Additive - Process noise $w$ is additive, and the state transition function $f$ that you specify in Function has the following form:
$x(k+1)=f(x(k), \operatorname{Us} 1(k), \ldots, U s n(k))$,
where $\mathrm{x}(\mathrm{k})$ is the estimated state at time k , and Us1, . . , Usn are any additional input arguments required by your state transition function.
- Nonadditive - Process noise is nonadditive, and the state transition function specifies how the states evolve as a function of the state and process noise at the previous time step:

$$
x(k+1)=f(x(k), w(k), U s 1(k), \ldots, U s n(k)) .
$$

Programmatic Use
Block Parameter: HasAdditiveProcessNoise
Type: character vector
Values: 'Additive', 'Nonadditive'
Default: 'Additive'

## Covariance - Time-invariant process noise covariance

1 (default) | scalar | vector | matrix
Time-invariant process noise covariance, specified as a scalar, vector, or matrix depending on the value of the Process noise parameter:

- Process noise is Additive - Specify the covariance as a scalar, an Ns-element vector, or an Ns-by-Ns matrix, where Ns is the number of states of the system. Specify a scalar if there is no crosscorrelation between process noise terms and all the terms have the same variance. Specify a vector of length $N s$, if there is no cross-correlation between process noise terms but all the terms have different variances.
- Process noise is Nonadditive - Specify the covariance as a $W$-by- $W$ matrix, where $W$ is the number of process noise terms.

If the process noise covariance is time-varying, select Time-varying. The block generates input port $\mathbf{Q}$ to specify the time-varying covariance.

## Dependencies

This parameter is enabled if you do not specify the process noise as Time-Varying.

## Programmatic Use

Block Parameter: ProcessNoise
Type: character vector, string
Default: ' 1 '

## Time-varying - Time-varying process noise covariance

'off' (default)|'on'
If you select this parameter, the block includes an additional input port $\mathbf{Q}$ to specify the time-varying process noise covariance.

## Programmatic Use

Block Parameter: HasTimeVaryingProcessNoise
Type: character vector
Values: 'off', 'on'
Default: 'off'

## Initialization

## Initial state - Initial state estimate <br> 0 (default) | vector

Initial state estimate value, specified as an $N s$-element vector, where $N s$ is the number of states in the system. Specify the initial state values based on your knowledge of the system.

```
Programmatic Use
Block Parameter: InitialState
Type: character vector, string
Default: '0'
Initial covariance - State estimation error covariance
1 (default) | scalar | vector | matrix
```

State estimation error covariance, specified as a scalar, an Ns-element vector, or an $N s$-by-Ns matrix, where $N s$ is the number of states of the system. If you specify a scalar or vector, the software creates an $N s$-by- $N s$ diagonal matrix with the scalar or vector elements on the diagonal.

Specify a high value for the covariance when you do not have confidence in the initial state values that you specify in Initial state.

Programmatic Use
Block Parameter: InitialStateCovariance
Type: character vector, string
Default: ' 1 '
Measurement
Function - Measurement function name
myMeasurementFcn (default) | function name
The measurement function calculates the $N$-element output measurement vector of the nonlinear system at time step $k$, given the state vector at time step $k$. You create the measurement function and specify the function name in Function. For example, if vdpMeasurementFcn.m is the measurement function that you created and saved, specify Function as vdpMeasurementFcn.

The inputs to the function you create depend on whether you specify the measurement noise as additive or nonadditive in Measurement noise.

- Measurement noise is Additive - The measurement function $h$ specifies how the measurements evolve as a function of state Values:
$y(k)=h(x(k), \operatorname{Um1}(k), \ldots, \operatorname{Umn}(k))$,
where $y(k)$ and $x(k)$ are the estimated output and estimated state at time $k$, and Um1, ..., Umn are any optional input arguments required by your measurement function. For example, if you are using a sensor for tracking an object, an additional input could be the sensor position.

To see an example of a measurement function with additive process noise, type edit vdpMeasurementFcn at the command line.

- Measurement noise is Nonadditive- The measurement function also specifies how the output measurement evolves as a function of the measurement noise $v$ :

```
y(k) = h(x(k),v(k),Um1(k),\ldots,Umn(k)).
```

To see an example of a measurement function with nonadditive process noise, type edit vdpMeasurementNonAdditiveNoiseFcn.

For more information, see "State Transition and Measurement Functions" on page 2-14.
You can create $h$ using a Simulink Function block or as a MATLAB function (.m file).

- You can use a MATLAB function only if $h$ has one additional input argument Um1 other than x and v.
$y[k]=h(x[k], v[k], \operatorname{Um1}(k))$
The software generates an additional input port MeasurementFcniInputs to specify this argument for the $i$ th measurement function.
- If you are using a Simulink Function block, specify $x$ and $v$ using Argument Inport blocks and the additional inputs Um1, . . . Umn using Inport blocks in the Simulink Function block. You do not provide Um1, . . . , Umn to the Extended Kalman Filter block.

If you have multiple sensors in your system, you can specify multiple measurement functions. You can specify up to five measurement functions using the Add Measurement button. To remove measurement functions, use Remove Measurement.

```
Programmatic Use
Block Parameter: MeasurementFcn1, MeasurementFcn2, MeasurementFcn3,
MeasurementFcn4, MeasurementFcn5
Type: character vector, string
Default: 'myMeasurementFcn'
Jacobian - Jacobian of measurement function
off (default) | on
```

Jacobian of measurement function $h$, specified as one of the following:

- off - The software computes the Jacobian numerically. This computation may increase processing time and numerical inaccuracy of the state estimation.
- on - You create a function to compute the Jacobian of the measurement function $h$, and specify the name of the function in Jacobian. For example, if vdpMeasurementJacobianFcn.m is the Jacobian function, specify MeasurementJacobianFcn as vdpMeasurementJacobianFcn. If you create $h$ using a Simulink Function block, then create the Jacobian using a Simulink Function block. If you create $h$ using a MATLAB function (. m file), then create the Jacobian using a MATLAB function.

The function calculates the partial derivatives of the measurement function $h$ with respect to the states and measurement noise. The number of inputs to the Jacobian function must equal the number of inputs to the measurement function and must be specified in the same order in both functions. The number of outputs of the Jacobian function depends on the Measurement noise parameter:

- Measurement noise is Additive - The function calculates the partial derivatives of the measurement function with respect to the states ( $\partial h / \partial x$ ). The output is as an $N$-by-Ns Jacobian matrix, where $N$ is the number of measurements of the system and $N s$ is the number of states.

To see an example of a Jacobian function for additive measurement noise, type edit vdpMeasurementJacobianFcn at the command line.

- Measurement noise is Nonadditive - The function also returns a second output that is the partial derivative of the measurement function with respect to the measurement noise terms ( $\partial h / \partial v$ ). The second output is returned as an $N$-by- $V$ Jacobian matrix, where $V$ is the number of measurement noise terms.

```
Programmatic Use
Block Parameter: HasMeasurementJacobianFcn1, HasMeasurementJacobianFcn2,
HasMeasurementJacobianFcn3,
HasMeasurementJacobianFcn4,HasMeasurementJacobianFcn5
Type: character vector
Values:'off','on'
Default: 'off'
Block Parameter: MeasurementJacobianFcn1, MeasurementJacobianFcn2,
MeasurementJacobianFcn3, MeasurementJacobianFcn4, MeasurementJacobianFcn5
Type: character vector
Default: ''
Measurement noise - Measurement noise characteristics
Additive (default) | Nonadditive
```

Measurement noise characteristics, specified as one of the following values:

- Additive - Measurement noise $v$ is additive, and the measurement function $h$ that you specify in Function has the following form:

$$
y(k)=h(x(k), \operatorname{Um} 1(k), \ldots, \operatorname{Umn}(k)),
$$

where $y(k)$ and $x(k)$ are the estimated output and estimated state at time $k$, and Um1, ..., Umn are any optional input arguments required by your measurement function.

- Nonadditive - Measurement noise is nonadditive, and the measurement function specifies how the output measurement evolves as a function of the state and measurement noise:

$$
y(k)=h(x(k), v(k), \operatorname{Um1}(k), \ldots, \operatorname{Umn}(k)) .
$$

## Programmatic Use

Block Parameter: HasAdditiveMeasurementNoise1, HasAdditiveMeasurementNoise2, HasAdditiveMeasurementNoise3, HasAdditiveMeasurementNoise4,
HasAdditiveMeasurementNoise5
Type: character vector
Values: 'Additive','Nonadditive'
Default: 'Additive'
Covariance - Time-invariant process noise covariance
1 (default) | scalar | vector | matrix
Time-invariant process noise covariance, specified as a scalar, vector, or matrix depending on the value of the Process noise parameter:

- Process noise is Additive - Specify the covariance as a scalar, an Ns-element vector, or an Ns-by-Ns matrix, where Ns is the number of states of the system. Specify a scalar if there is no crosscorrelation between process noise terms and all the terms have the same variance. Specify a vector of length $N s$, if there is no cross-correlation between process noise terms but all the terms have different variances.
- Process noise is Nonadditive - Specify the covariance as a $W$-by- $W$ matrix, where $W$ is the number of process noise terms.

If the process noise covariance is time-varying, select Time-varying. The block generates input port $\mathbf{Q}$ to specify the time-varying covariance.

## Dependencies

This parameter is enabled if you do not specify the process noise as Time-Varying.
Programmatic Use
Block Parameter: ProcessNoise
Type: character vector, string
Default: '1'
Time-varying - Time-varying measurement noise covariance off (default) |on

If you select this parameter for the measurement noise covariance of the first measurement function, the block includes an additional input port R1. You specify the time-varying measurement noise covariance in R1. Similarly, if you select Time-varying for the $i^{\text {th }}$ measurement function, the block includes an additional input port Ri to specify the time-varying measurement noise covariance for that function.

Programmatic Use
Block Parameter: HasTimeVaryingMeasurementNoise1,
HasTimeVaryingMeasurementNoise2, HasTimeVaryingMeasurementNoise3,
HasTimeVaryingMeasurementNoise4, HasTimeVaryingMeasurementNoise5
Type: character vector
Values: 'off', 'on'
Default: 'off'
Add Enable Port - Enable correction of estimated states only when measured data is available
off (default) | on
Suppose that measured output data is not available at all time points at the port $\mathbf{y} \mathbf{1}$ that corresponds to the first measurement function. Select Add Enable port to generate an input port Enable1. Use a signal at this port to enable the correction of estimated states only when measured data is available. Similarly, if measured output data is not available at all time points at the port yi for the $i^{\text {th }}$ measurement function, select the corresponding Add Enable port.
Programmatic Use
Block Parameter: HasMeasurementEnablePort1, HasMeasurementEnablePort2, HasMeasurementEnablePort3, HasMeasurementEnablePort4, HasMeasurementEnablePort5
Type: character vector
Values: 'off', 'on'
Default: 'off'

## Settings

Use the current measurements to improve state estimates - Choose between corrected or predicted state estimate
on (default) | off
When this parameter is selected, the block outputs the corrected state estimate $\widehat{x}[k \mid k]$ at time step k , estimated using measured outputs until time $k$. If you clear this parameter, the block returns the
predicted state estimate $\widehat{x}[k \mid k-1]$ for time $k$, estimated using measured output until a previous time $\mathrm{k}-1$. Clear this parameter if your filter is in a feedback loop and there is an algebraic loop in your Simulink model.

```
Programmatic Use
Block Parameter: UseCurrentEstimator
Type: character vector
Values: 'off', 'on'
Default: 'on'
Output state estimation error covariance - Output state estimation error covariance
off (default)| on
```

If you select this parameter, a state estimation error covariance output port $\mathbf{P}$ is generated in the block.

```
Programmatic Use
Block Parameter: OutputStateCovariance
Type: character vector
Values: 'off','on'
Default: 'off'
Data type - Data type for block parameters
double (default)| single
```

Use this parameter to specify the data type for all block parameters.

## Programmatic Use

Block Parameter: DataType
Type: character vector
Values: 'single', 'double'
Default: 'double'

## Sample time - Block sample time

1 (default) | positive scalar
Block sample time, specified as a positive scalar. If the sample times of your state transition and measurement functions are different, select Enable multirate operation in the Multirate tab, and specify the sample times in the Multirate tab instead.

## Dependencies

This parameter is available if in the Multirate tab, the Enable multirate operation parameter is off.

## Programmatic Use

Block Parameter: SampleTime
Type: character vector, string
Default: ' 1 '
Multirate Tab
Enable multirate operation - Enable specification of different sample times for state transition and measurement functions
off (default) | on

Select this parameter if the sample times of the state transition and measurement functions are different. You specify the sample times in the Multirate tab, in Sample time.

Programmatic Use
Block Parameter: EnableMultirate
Type: character vector
Values: 'off', 'on'
Default: 'off'

## Sample times - State transition and measurement function sample times positive scalar

If the sample times for state transition and measurement functions are different, specify Sample time. Specify the sample times for the measurement functions as positive integer multiples of the state transition sample time. The sample times you specify correspond to the following input ports:

- Ports corresponding to state transition function - Additional input to state transition function StateTransitionFcnInputs and time-varying process noise covariance $\mathbf{Q}$. The sample times of these ports must always equal the state transition function sample time, but can differ from the sample time of the measurement functions.
- Ports corresponding to $i^{\text {th }}$ measurement function - Measured output yi, additional input to measurement function MeasurementFcniInputs, enable signal at port Enablei, and timevarying measurement noise covariance Ri. The sample times of these ports for the same measurement function must always be the same, but can differ from the sample time for the state transition function and other measurement functions.


## Dependencies

This parameter is available if in the Multirate tab, the Enable multirate operation parameter is on.

Programmatic Use
Block Parameter: StateTransitionFcnSampleTime, MeasurementFcn1SampleTime1, MeasurementFcn1SampleTime2, MeasurementFcn1SampleTime3, MeasurementFcn1SampleTime4, MeasurementFcn1SampleTime5
Type: character vector, string
Default: '1'

## More About

## State Transition and Measurement Functions

The algorithm computes the state estimates $\widehat{x}$ of the nonlinear system using state transition and measurement functions specified by you. You can specify up to five measurement functions, each corresponding to a sensor in the system. The software lets you specify the noise in these functions as additive or nonadditive.

- Additive Noise Terms - The state transition and measurements equations have the following form:

$$
\begin{aligned}
& x[k+1]=f\left(x[k], u_{s}[k]\right)+w[k] \\
& y[k]=h\left(x[k], u_{m}[k]\right)+v[k]
\end{aligned}
$$

Here $f$ is a nonlinear state transition function that describes the evolution of states x from one time step to the next. The nonlinear measurement function $h$ relates $x$ to the measurements $y$ at
time step k. w and v are the zero-mean, uncorrelated process and measurement noises, respectively. These functions can also have additional optional input arguments that are denoted by $u_{s}$ and $u_{m}$ in the equations. For example, the additional arguments could be time step $k$ or the inputs $u$ to the nonlinear system. There can be multiple such arguments.

Note that the noise terms in both equations are additive. That is, $x(k+1)$ is linearly related to the process noise $w(k)$, and $y(k)$ is linearly related to the measurement noise $v(k)$. For additive noise terms, you do not need to specify the noise terms in the state transition and measurement functions. The software adds the terms to the output of the functions.

- Nonadditive Noise Terms - The software also supports more complex state transition and measurement functions where the state $\chi[k]$ and measurement $y[k]$ are nonlinear functions of the process noise and measurement noise, respectively. When the noise terms are nonadditive, the state transition and measurements equation have the following form:

$$
\begin{aligned}
& x[k+1]=f\left(x[k], w[k], u_{S}[k]\right) \\
& y[k]=h\left(x[k], v[k], u_{m}[k]\right)
\end{aligned}
$$

## Compatibility Considerations

## Numerical Changes

Behavior changed in R2020b
Starting in R2020b, numerical improvements in the Extended Kalman Filter algorithm might produce results that are different from the results you obtained in previous versions.

## Extended Capabilities

## C/C++ Code Generation

Generate C and C++ code using Simulink $\circledR^{\circledR}$ Coder ${ }^{\mathrm{TM}}$.
The state transition, measurement, and Jacobian functions that you specify must use only the MATLAB commands and Simulink blocks that support code generation. For a list of blocks that support code generation, see "Simulink Built-In Blocks That Support Code Generation" (Simulink Coder). For a list of commands that support code generation, see "Functions and Objects Supported for C/C++ Code Generation" (MATLAB Coder).

Generated code uses an algorithm that is different from the algorithm that the Extended Kalman Filter block itself uses. You might see some numerical differences in the results obtained using the two methods.

## See Also

## Blocks

Kalman Filter | Particle Filter | Unscented Kalman Filter

## Functions

extendedKalmanFilter|particleFilter|unscentedKalmanFilter

## Topics

"What Is Online Estimation?"
"Extended and Unscented Kalman Filter Algorithms for Online State Estimation"
"Validate Online State Estimation in Simulink"
"Troubleshoot Online State Estimation"

## External Websites

Understanding Kalman Filters: Nonlinear State Estimators - MATLAB Video Series

Introduced in R2017a

## Iddata Sink

Export simulation data as iddata object to MATLAB workspace
Library:
System Identification Toolbox

```
\(\lambda\) Input
    IDDATA SINK
\(>\) Output
    Iddata Sink
```


## Description

The Iddata Sink block exports simulation data as an iddata object to the MATLAB workspace. The object stores the input and simulated output signals, sampled at the sample time that you specify. If you simulate your model from the model window, the block exports the object to the MATLAB base workspace. If you simulate the model programmatically, the object is exported to the MATLAB caller workspace. The caller workspace is the workspace of the function that called the currently running function.

## Ports

## Input

## Input - Input of iddata object

scalar | vector
Input of iddata object, specified as a scalar for single-input data. For multichannel data with Nu inputs, specify Input as a vector of length $N u$.
Data Types: double

## Output - Output of iddata object

scalar | vector
Output of iddata object, specified as a scalar for single-output data. For multichannel data with Ny outputs, specify Output as a vector of length Ny .

## Data Types: double

## Parameters

## IDDATA Name - Name of iddata object

data (default) | variable name
Name of iddata object, specified as a MATLAB variable name. The object is exported with this name to the MATLAB workspace.

## Sample Time - Sample time in seconds

0.1 (default) | finite positive number

Sample time in seconds, specified as a finite positive number. The iddata object stores the input and output signals, sampled at the sample time that you specify.

## See Also

Iddata Source

## Topics

"Simulate Identified Model in Simulink"
Introduced in R2008a

## Iddata Source

Import time-domain data stored in iddata object in MATLAB workspace Library: System Identification Toolbox


## Description

The Iddata Source block imports the input-output time-domain data stored in an iddata object in the MATLAB workspace. You can use this block to import data for simulating a model in Simulink.

## Ports

## Output

## Input - Input data stored in iddata object <br> scalar | vector

Input data stored in iddata object, returned as a scalar for single-input data and a vector of length $N u$ for multichannel data with $N u$ inputs. If $z$ is the iddata object, the output at this port at simulation time $t$ is z. InputData at time $t$. If $t$ is greater than $z$. SamplingInstants (end), the maximum time in $z$, the input data is returned as 0 for each input channel.

Data Types: double

## Output - Output data stored in iddata object

scalar | vector
Output data stored in iddata object, returned as a scalar for single-output data and as a vector of length $N y$ for multichannel data with $N y$ outputs. If $z$ is the iddata object, the output at this port at simulation time $t$ is $z$. OutputData at time $t$. If $t$ is greater than $z$. SamplingInstants (end), the maximum time in $z$, the output data is returned as 0 for each output channel.

Data Types: double

## Parameters

IDDATA object - Time-domain data to be imported
iddata(1,1) (default) |iddata object
Time-domain data that is to be imported, specified as an iddata object that you have created in the MATLAB workspace. The iddata object must contain only one experiment. For a multiple-experiment iddata object $z$, to specify an iddata object for experiment number kexp, specify IDDATA object as getexp (z,kexp).

## Extended Capabilities

C/C++ Code Generation
Generate C and C++ code using Simulink ${ }^{\circledR}$ Coder $^{\mathrm{TM}}$.

## See Also

Iddata Sink

## Topics

"Simulate Identified Model in Simulink"

Introduced in R2008a

## Idmodel

Simulate identified linear model in Simulink software
Library: System Identification Toolbox / Models


## Description

The Idmodel block simulates the output of an identified linear model using time-domain input data. The model is a state-space (idss), linear grey-box (idgrey), polynomial (idpoly), transfer function (idtf), or process (idproc) model that you previously estimated or created. For the simulation of state-space and linear grey-box models, you can specify the initial state values. For other linear models, initial conditions are set to zero. You can also add noise to the simulated output.

## Ports

## Input

## Port_1(In1) - Simulation input data

scalar | vector
Simulation input data, specified as a scalar for a single-input model. The data must be time-domain data. For multi-input models, specify the input as an $N u$-element vector, where $N u$ is the number of inputs. For example, you can use a Vector Concatenate block to concatenate scalar signals into a vector signal.

Note Do not use a Bus Creator or Mux block to produce the vector signal.

Data Types: double

## Output

Port_1(Out1) - Simulated output
scalar | vector
Simulated output from linear model, returned as a scalar for a single-output model and an Ny element vector for a model with Ny outputs.
Data Types: double

## Parameters

Identified model - Linear model to be simulated
idss(-1,1,1,0,'Ts',1) (default)|idss object|idgrey object|idpoly object|idtf object | idproc object

Linear model to be simulated, specified as an idss, idgrey, idpoly, idtf, or idproc object. The model can be continuous-time or discrete-time, with or without input-output delays. You previously estimate or construct the linear model in the MATLAB workspace.

## Initial states (state space only: idss,idgrey) - Initial state values <br> 0 (default) | vector

Initial state values of state-space (idss) and linear grey-box (idgrey) models, specified as an $N x-$ element vector, where $N x$ is the number of states of the model. To estimate the initial states that provide a best fit between measured data and the simulated response of the model for the same input, use the findstates command.

For example, to compute initial states such that the response of the model M matches the simulated output data in the data set z , specify $\mathrm{X0} 0$, such that:

```
X0 = findstates(M,z)
```

For linear models other than idss or idgrey, the block assumes that initial conditions are zero.
If you want to reproduce the simulation results that you get in the Model Output plot window in the System Identification app, or from the compare command:

1 If the identified model $m$ is not a state-space or grey-box model, convert the model into statespace form (idss model), and specify the state-space model mss in the block.

```
mss = idss(m);
```

2 Compute the initial state values that produce the best fit between the model output and the measured output signal using findstates. Specify the prediction horizon as Inf, that is, minimize the simulation error.

X0 = findstates(mss,z,Inf);
3 Use the model mss and initial states X0 in the Idmodel block to perform the simulation. Specify the same input signal $z$ for simulation that you used as validation data in the app or compare.

## Add noise - Add noise to simulated output

```
on (default) | off
```

When you select this parameter, the block derives the noise amplitude from the linear model property model. NoiseVariance. The software filters random Gaussian white noise with the noise transfer function of the model and adds the resulting noise to the simulated model response. If you want to add the same noise every time you run the Simulink model, specify the Noise seed(s) property.

For continuous-time models, the ideal variance of the noise term is infinite. In reality, you see a bandlimited noise that accounts for the time constants of the system. You can interpret the resulting simulated output as filtered using a lowpass filter with a passband that does not distort the dynamics from the input.

## Noise seed(s) - Add same noise to output for multiple simulations

[ ] (default) | nonnegative integer | vector
The Noise seed(s) property seeds the random number generator such that the block adds the same noise to the simulated output every time you run the Simulink model. For information about using seeds, see rng.

For multi-output models, you can use independent noise realizations that generate the outputs with additive noise. Enter a vector of $N y$ nonnegative integer entries, where $N y$ is the number of output channels.

For random restarts that vary from one simulation to another, specify Noise seed(s) as [ ].

## Dependency

To enable this parameter, select Add noise.

## Extended Capabilities

C/C++ Code Generation
Generate C and C++ code using Simulink® Coder ${ }^{\mathrm{TM}}$.

## See Also

## Functions

findstates |idgrey|idpoly|idproc|idss|idtf|sim
Blocks
Iddata Sink | Iddata Source
Topics
"Simulate Identified Model in Simulink"

Introduced in R2008a

## Nonlinear ARX Model

Simulate nonlinear ARX model in Simulink software

## Library: <br> System Identification Toolbox / Models



## Description

The Nonlinear ARX Model block simulates the output of a nonlinear ARX model using time-domain input data. The model is an idnlarx model that you previously estimated or constructed in the MATLAB workspace. You specify initial conditions for the simulation as either steady-state input and output signal levels or as an initial state vector.

## Ports

## Input

Port_1(In1) - Simulation input data
scalar | vector
Simulation input data, specified as a scalar for a single-input model. The data must be time-domain data. For multi-input models, specify the input as an $N u$-element vector, where $N u$ is the number of inputs. For example, you can use a Vector Concatenate block to concatenate scalar signals into a vector signal.

Note Do not use a Bus Creator or Mux block to produce the vector signal.

## Data Types: double

## Output

Port_1(Out1) - Simulated output
scalar | vector
Simulated output from nonlinear ARX model, returned as a scalar for a single-output model and an $N y$-element vector for a model with $N y$ outputs.
Data Types: double

## Parameters

Model - Nonlinear ARX model to be simulated
idnlarx object
Nonlinear ARX model to be simulated, specified as an idnlarx object. You previously estimate or construct the idnlarx model in the MATLAB workspace.

## Initial conditions - Initial condition specification for simulation

Input and output values (default)|State values
The states of a nonlinear ARX model correspond to the dynamic elements of the nonlinear ARX model structure. The dynamic elements are the model regressors. Regressors can be the delayed input or output variables (standard regressors) or user-defined transformations of delayed input-output variables (custom regressors). For more information about the states of a nonlinear ARX model, see the idnlarx reference page.

For simulating nonlinear ARX models, you can specify the initial conditions one of the following:

- Input and output values - Specify steady-state input and output signal levels in Input level and Output level, respectively.
- State values - Specify a vector of length equal to the number of states in the model in Specify initial states as a vector.


## Input level - Steady-state input signal level

0 (default) | scalar
Steady-state input signal level before simulation, specified as a scalar.

## Dependency

To enable this parameter, specify Initial conditions as Input and output values.

## Output level - Steady-state output signal level

0 (default) | scalar
Steady-state output signal level before simulation, specified as a scalar.

## Dependency

To enable this parameter, specify Initial conditions as Input and output values.

## Specify initial states as a vector - Initial state values

0 (default) | vector
Initial state values of the model, specified as an $N x$-element vector, where $N x$ is the number of states of the model. This parameter is named Vector of state values until you specify Model.

If you do not know the initial states, you can estimate these states as follows:

- To simulate the model around a given input level when you do not know the corresponding output level, estimate the equilibrium state values using the findop command. For example, to simulate a model M about a steady-state point where the input is 1 and the output is unknown, specify the initial state values as $\mathrm{X0} 0$, where

X0 = findop ( $\mathrm{M},{ }^{\prime}$ steady ${ }^{\prime}, 1, \mathrm{NaN}$ )

- To estimate the initial states that provide a best fit between measured data and the simulated response of the model for the same input, use the findstates command. For example, to compute initial states such that the response of the model M matches the output data in the data set $z$, specify $X 0$, such that:

X0 = findstates(M,z,Inf)

- To continue a simulation from a previous simulation run, use the simulated input-output values from the previous simulation to compute the initial states X 0 for the current simulation. Use the
data2state command to compute X0. For example, suppose that firstSimData is a variable that stores the input and output values from a previous simulation. For a model M, you can specify X0, such that:

```
X0 = data2state(M,firstSimData)
```

Dependency
To enable this parameter, specify Initial conditions as State values.

## Extended Capabilities

## C/C++ Code Generation

Generate C and C++ code using Simulink ${ }^{\circledR}$ Coder ${ }^{\mathrm{TM}}$.

## See Also

## Functions

findstates|idnlarx|idnlarx/findop|sim

## Blocks

Iddata Sink | Iddata Source

## Topics

"Identifying Nonlinear ARX Models"
"Simulate Identified Model in Simulink"

Introduced in R2008a

## Nonlinear Grey-Box Model

Simulate nonlinear grey-box model in Simulink software


## Library

System Identification Toolbox

## Description

Simulates systems of nonlinear grey-box (idnlgrey) models.

## Input

Input signal to the model.

## Output

Output signal from the model.

## Parameters

## IDNLGREY model

Name of idnlgrey variable in the MATLAB workspace.

## Initial state

Specify the initial states as one of the following:

- 'z': Specifies zero, which corresponds to a system starting from rest.
- ' $m$ ': Specifies the internal initial states of the model.
- Vector of size equal to the number of states in the idnlgrey object.
- An initial state structure array. For information about creating this structure, type help idnlgrey/sim in the MATLAB Command Window.


## See Also

## Functions

idnlgrey

## Blocks

Iddata Sink | Iddata Source

## Topics

"Estimate Nonlinear Grey-Box Models"

Introduced in R2008a

## Hammerstein-Wiener Model

Library: System Identification Toolbox / Models


## Description

The Hammerstein-Wiener Model block simulates the output of a Hammerstein-Wiener model using time-domain input data. The model is an idnlhw model that you previously estimated or constructed in the MATLAB workspace. You specify initial conditions for the simulation as one of the following:

- Zero for all states
- Initial state vector representing the initial states of the linear block

For information about the structure of a Hammerstein-Wiener model, see "What are HammersteinWiener Models?".

## Ports

## Input

## Port_1(In1) - Simulation input data

scalar | vector
Simulation input data, specified as a scalar for a single-input model. The data must be time-domain data. For multi-input models, specify the input as an $N u$-element vector, where $N u$ is the number of inputs. For example, you can use a Vector Concatenate block to concatenate scalar signals into a vector signal.

Note Do not use a Bus Creator or Mux block to produce the vector signal.

Data Types: double

## Output

Port_1(Out1) - Simulated output
scalar | vector
Simulated output from Hammerstein-Wiener model, returned as a scalar for a single-output model and as an $N y$-element vector for a model with $N y$ outputs.
Data Types: double

## Parameters

Model - Hammerstein-Wiener model to be simulated
idnlhw object
Hammerstein-Wiener model to be simulated, specified as an idnlhw object. You previously estimate or construct the idnlhw model in the MATLAB workspace.

## Initial conditions - Initial condition specification for simulation <br> Zero (default)|State values

The states of a Hammerstein-Wiener model correspond to the states of the embedded linear idpoly or idss model. For more information about the states, see the idnlhw reference page. You specify Initial conditions as one of the following:

- Zero - Specifies zero initial state values, which correspond to a simulation starting from a state of rest.
- State values - You specify the state values in Specify a vector of state values. Specify the states as a vector of length equal to the number of states in the model.

If you do not know the initial states, you can estimate these states as follows:

- To simulate the model around a given input level when you do not know the corresponding output level, you can estimate the equilibrium state values using the findop command.

For example, to simulate a model $M$ about a steady-state point where the input is 1 and the output is unknown, you can specify the initial state values as X0, where:

```
X0 = findop(M,'steady',1,NaN)
```

- To estimate the initial states that provide a best fit between measured data and the simulated response of the model for the same input, use the findstates command.

For example, to compute initial states such that the response of the model $M$ matches the simulated output data in the data set z , specify $\mathrm{X0}$, such that:

```
X0 = findstates(M,z)
```


## Extended Capabilities

## C/C++ Code Generation

Generate C and C++ code using Simulink ${ }^{\circledR}$ Coder ${ }^{\mathrm{TM}}$.

## See Also

Functions
findstates|idnlhw|idnlhw/findop|sim
Blocks
Iddata Sink | Iddata Source

## Topics

"Identifying Hammerstein-Wiener Models"
"Simulate Identified Model in Simulink"

Introduced in R2008a

## Kalman Filter

Estimate states of discrete-time or continuous-time linear system


## Library

Estimators

## Description

Use the Kalman Filter block to estimate states of a state-space plant model given process and measurement noise covariance data. The state-space model can be time-varying. A steady-state Kalman filter implementation is used if the state-space model and the noise covariance matrices are all time-invariant. A time-varying Kalman filter is used otherwise.

Kalman filter provides the optimal solution to the following continuous or discrete estimation problems:

## Continuous-Time Estimation

Given the continuous plant

$$
\begin{aligned}
& \dot{x}(t)=A(t) x(t)+B(t) u(t)+G(t) w(t) \quad \text { (state equation) } \\
& y(t)=C(t) x(t)+D(t) u(t)+H(t) w(t)+v(t) \quad \text { (measurement equation) }
\end{aligned}
$$

with known inputs $u$, white process noise $w$, and white measurement noise $v$ satisfying:

$$
\begin{aligned}
& E[w(t)]=E[v(t)]=0 \\
& E\left[w(t) w^{T}(t)\right]=Q(t) \\
& E\left[w(t) v^{T}(t)\right]=N(t) \\
& E\left[v(t) v\left(^{(t}\right]=R(t)\right.
\end{aligned}
$$

construct a state estimate $\widehat{x}$ that minimizes the state estimation error covariance $P(t)=E\left[(x-\widehat{x})(x-\widehat{x})^{T}\right]$.

The optimal solution is the Kalman filter with equations

$$
\begin{aligned}
& L(t)=\left(P(t) C^{T}(t)+\bar{N}(t) \bar{R}\left({ }^{t},\right.\right. \\
& \dot{P}(t)=A(t) P(t)+P(t) A^{T}(t)+\bar{Q}(t)-L(t) \bar{R}(t) L^{T}(t), \\
& \dot{\hat{x}}(t)=A(t) \widehat{X}(t)+B(t) u(t)+L(t)(y(t)-C(t) \widehat{x}(t)-D(t) u(t)),
\end{aligned}
$$

where

$$
\begin{aligned}
& \bar{Q}(t)=G(t) Q(t) G^{T}(t), \\
& \bar{R}(t)=R(t)+H(t) N(t)+N^{T}(t) H^{T}(t)+H(t) Q(t) H^{T}(t), \\
& \bar{N}(t)=G(t)\left(Q(t) H^{T}(t)+N(t)\right) .
\end{aligned}
$$

The Kalman filter uses the known inputs $u$ and the measurements $y$ to generate the state estimates $\widehat{x}$. If you want, the block can also output the estimates of the true plant output $\widehat{y}$.


The block implements the steady-state Kalman filter when the system matrices $(A(t), B(t), C(t)$, $D(t), G(t), H(t))$ and noise covariance matrices $(Q(t), R(t), N(t))$ are constant (specified in the Block Parameters dialog box). The steady-state Kalman filter uses a constant matrix P that minimizes the steady-state estimation error covariance and solves the associated continuous-time algebraic Riccati equation:

$$
P=\lim _{t \rightarrow \infty} E\left[(x-\widehat{x})(x-\widehat{x})^{T}\right] .
$$

## Discrete-Time Estimation

Given the discrete plant

$$
\begin{aligned}
& x[n+1]=A[n] x[n]+B[n] u[n]+G[n] w[n], \\
& y[n]=C[n] x[n]+D[n] u[n]+H[n] w[n]+v[n],
\end{aligned}
$$

with known inputs $u$, white process noise $w$ and white measurement noise $v$ satisfying

$$
\begin{aligned}
& E[w[n]]=E[v[n]]=0, \\
& E\left[w[n] w^{T}[n]\right]=Q[n], \\
& E\left[v[n] v^{T}[n]\right]=R[n], \\
& E\left[w[n] v^{T}[n]\right]=N[n] .
\end{aligned}
$$

The estimator has the following state equation

$$
\widehat{x}[n+1 \mid n]=A[n] \widehat{x}[n \mid n-1]+B[n] u[n]+L[n](y[n]-C[n] \widehat{x}[n \mid n-1]-D[n] u[n]),
$$

where the gain $\mathrm{L}[\mathrm{n}]$ is calculated through the discrete Riccati equation:

$$
\begin{aligned}
& L[n]=\left(A[n] P[n] C^{T}[n]+\bar{N}[n]\right) C^{C}, \\
& M[n]=P[n] C^{T}[n] C^{C}, \\
& Z[n]=(I-M[n] C[n]) P[n]\left(^{I}+M[n] \bar{R}[n] M^{T}[n],\right. \\
& P[n+1]=\left(A[n]-\bar{N}[n] \bar{R}^{-1}[n] C[n]\right) Z\left(^{A}+\bar{Q}[n]-N[n] \bar{R}^{-1}[n] \bar{N}^{T}[n],\right.
\end{aligned}
$$

where I is the identity matrix of appropriate size and

$$
\begin{aligned}
& \bar{Q}[\mathrm{n}]=G[\mathrm{n}] Q[\mathrm{n}] G^{T}[\mathrm{n}], \\
& \bar{R}[\mathrm{n}]=R[\mathrm{n}]+H[\mathrm{n}] N[\mathrm{n}]+N^{T}[\mathrm{n}] H^{T}[\mathrm{n}]+H[\mathrm{n}] Q[\mathrm{n}] H^{T}[\mathrm{n}], \\
& \bar{N}[\mathrm{n}]=G[\mathrm{n}]\left(Q[\mathrm{n}] H^{T}[\mathrm{n}]+N[\mathrm{n}]\right), \\
& \text { and } \\
& P[\mathrm{n}]=E\left[(x-\widehat{x}[\mathrm{n} \mid n-1])\left({ }^{x}\right],\right. \\
& Z[\mathrm{n}]=E\left[(x-\widehat{x}[n \mid n])\left({ }^{\chi}\right],\right.
\end{aligned}
$$

The steady-state Kalman filter uses a constant matrix P that minimizes the steady-state estimation error covariance and solves the associated discrete-time algebraic Riccati equation.

There are two variants of discrete-time Kalman filters:

- The current estimator generates the state estimates $\widehat{x}[n \mid n]$ using all measurement available, including $y[n]$. The filter updates $\widehat{x}[n \mid n-1]$ with $y[n]$ and outputs:

$$
\begin{aligned}
& \widehat{x}[n \mid n]=\widehat{x}[n \mid n-1]+M[n](y[n]-C[n] \widehat{x}[n \mid n-1]-D[n] u[n]), \\
& \widehat{y}[n \mid n]=C[n] \widehat{x}[n \mid n]+D[n] u[n] .
\end{aligned}
$$

- The delayed estimator generates the state estimates $\widehat{x}[n \mid n-1]$ using measurements up to $y[n-1]$. The filter outputs $\widehat{x}[n \mid n-1]$ as defined previously, along with the optional output $\widehat{y}[n \mid n-1]$

$$
\widehat{y}[n \mid n-1]=C[n] \widehat{x}[n \mid n-1]+D[n] u[n]
$$

The current estimator has better estimation accuracy compared to the delayed estimator, which is important for slow sample times. However, it has higher computational cost, making it harder to implement inside control loops. More specifically, it has direct feedthrough. This leads to an algebraic loop if the Kalman filter is used in a feedback loop that does not contain any delays (the feedback loop itself also has direct feedthrough). The algebraic loop can impact the speed of simulation. You cannot generate code if your model contains algebraic loops.

The Kalman Filter block differs from the kalman command in the following ways:

- When calling kalman(sys,...), sys includes the $G$ and $H$ matrices. Specifically, sys.B has [B G] and sys.D has [D H]. When you provide a LTI variable to the Kalman Filter block, it does not assume that the LTI variable provided contains G and H . They are optional and separate.
- The kalman command outputs [yhat;xhat] by default. The block only outputs xhat by default.


## Parameters

The following table summarizes the Kalman Filter block parameters, accessible via the Block Parameter dialog box.

| Task | Parameters |
| :--- | :--- |
| Specify filter settings | - Time domain on page 2-35 <br> -Use the current measurement y[n] to <br> improve xhat[n] on page 2-35 $\mathbf{l}$ |


| Task | Parameters |
| :---: | :---: |
| Specify the system model | Model source on page 2-35 in Model Parameters tab |
| Specify initial state estimates | Source on page 2-36 in Model Parameters tab |
| Specify noise characteristics | In Model Parameters tab: <br> - Use $\mathbf{G}$ and $\mathbf{H}$ matrices (default $\mathbf{G}=\mathrm{I}$ and $\mathbf{H = 0 )}$ on page 2-37 <br> - $\mathbf{Q}$ on page 2-37, Time-invariant $\mathbf{Q}$ on page 2-37 <br> - $\mathbf{R}$ on page 2-38, Time-invariant $\mathbf{R}$ on page 2-38 <br> - $\mathbf{N}$ on page 2-38, Time-invariant $\mathbf{N}$ on page 2-38 |
| Specify additional inports | In Options tab: <br> - Add input port u on page 2-38 <br> - Add input port Enable to control measurement updates on page 2-38 <br> - External reset on page 2-38 |
| Specify additional outports | In Options tab: <br> - Output estimated model output $y$ on page 2-39 <br> - Output state estimation error covariance $\mathbf{Z}$ on page 2-39 |

## Time domain

Specify whether to estimate continuous-time or discrete-time states:

- Discrete-Time (Default) - Block estimates discrete-time states
- Continuous-Time - Block estimates continuous-time states

When the Kalman Filter block is in a model with synchronous state control (see the State Control block), you cannot select Continuous-time.

## Use the current measurement $y[n]$ to improve xhat[n]

Use the current estimator variant of the discrete-time Kalman filter. When not selected, the delayed estimator (variant) is used.

This option is available only when Time Domain is Discrete-Time.

## Model source

Specify how the A, B, C, D matrices are provided to the block. Must be one of the following:

- Dialog: LTI State-Space Variable - Use the values specified in the LTI state-space variable. You must also specify the variable name in Variable. The sample time of the model must
match the setting in the Time domain option, i.e. the model must be discrete-time if the Time domain is discrete-time.
- Dialog: Individual A, B, C, D matrices - Specify values in the following block parameters:
- A - Specify the A matrix. It must be real and square.
- B - Specify the B matrix. It must be real and have as many rows as the A matrix. This option is available only when Add input port $\mathbf{u}$ is selected in the Options tab.
- C - Specify the C matrix. It must be real and have as many columns as the A matrix.
- D - Specify the D matrix. It must be real. It must have as many rows as the C matrix and as many columns as the B matrix. This option is available only when Add input port $\mathbf{u}$ is selected in the Options tab.
- External - Specify the A, B, C, D matrices as input signals to the Kalman Filter block. If you select this option, the block includes additional input ports A, B, C and D. You must also specify the following in the block parameters:
- Number of states - Number of states to be estimated, specified as a positive integer. The default value is 2 .
- Number of inputs - Number of known inputs in the model, specified as a positive integer. The default value is 2 . This option is only available when Add input port $\mathbf{u}$ is selected.
- Number of outputs - Number of measured outputs in the model, specified as a positive integer. The default value is 2 .


## Sample Time

Block sample time, specified as -1 or a positive scalar.
This option is available only when Time Domain is Discrete Time and Model Source is Dialog: Individual A, B, C, D matrices or External. The sample time is obtained from the LTI statespace variable if the Model Source is Dialog: LTI State-Space Variable.

The default value is -1 , which implies that the block inherits its sample time based on the context of the block within the model. All block input ports must have the same sample time.

## Source

Specify how to enter the initial state estimates and initial state estimation error covariance:

- Dialog - Specify the values directly in the dialog box. You must also specify the following parameters:
- Initial states $\mathbf{x [ 0 ]}$ - Specify the initial state estimate as a real scalar or vector. If you specify a scalar, all initial state estimates are set to this scalar. If you specify a vector, the length of the vector must match with the number of states in the model.
- State estimation error covariance P[0] (only when time-varying Kalman filter is used) Specify the initial state estimation error covariance $\mathrm{P}[0]$ for discrete-time Kalman filter or $\mathrm{P}(0)$ for continuous-time Kalman filter. Must be specified as one of the following:
- Real nonnegative scalar. P is an Ns-by-Ns diagonal matrix with the scalar on the diagonals. Ns is the number of states in the model.
- Vector of real nonnegative scalars. $P$ is an Ns-by-Ns diagonal matrix with the elements of the vector on the diagonals of $P$.
- Ns-by-Ns positive semi-definite matrix.
- External - Inherit the values from input ports. The block includes an additional input port X0. A second additional input port P0 is added when time-varying Kalman filter is used. X0 and P0 must satisfy the same conditions described previously when you specify them in the dialog box.


## Use the Kalman Gain K from the model variable

Specify whether to use the pre-identified Kalman Gain contained in the state-space plant model. This option is available only when:

- Model Source is Dialog: LTI State-Space Variable and Variable is an identified statespace model (idss) with a nonzero K matrix.
- Time Invariant $\mathbf{Q}$, Time Invariant $\mathbf{R}$ and Time Invariant $\mathbf{N}$ options are selected.

If the Use G and $\mathbf{H}$ matrices (default $\mathbf{G}=\mathbf{I}$ and $\mathbf{H}=\mathbf{0}$ ) option is selected, Time Invariant $\mathbf{G}$ and Time Invariant $\mathbf{H}$ options must also be selected.

## Use G and $H$ matrices (default $G=I$ and $H=0$ )

Specify whether to use non-default values for the G and H matrices. If you select this option, you must specify:

- G - Specify the G matrix. It must be a real matrix with as many rows as the A matrix. The default value is 1.
- Time-invariant G - Specify if the G matrix is time invariant. If you unselect this option, the block includes an additional input port G.
- $\mathbf{H}$ - Specify the H matrix. It must be a real matrix with as many rows as the C matrix and as many columns as the G matrix. The default value is 0 .
- Time-invariant $\mathbf{H}$ - Specify if the H matrix is time invariant. If you unselect this option, the block includes an additional input port G.
- Number of process noise inputs - Specify the number of process noise inputs in the model. The default value is 1 .

This option is available only when Time-invariant $\mathbf{G}$ and Time-invariant $\mathbf{H}$ are cleared. Otherwise, this information is inferred from the G or H matrix.

## Q

Process noise covariance matrix, specified as one of the following:

- Real nonnegative scalar. Q is an Nw-by-Nw diagonal matrix with the scalar on the diagonals. Nw is the number of process noise inputs in the model.
- Vector of real nonnegative scalars. Q is an Nw -by-Nw diagonal matrix with the elements of the vector on the diagonals of Q .
- Nw-by-Nw positive semi-definite matrix.


## Time Invariant Q

Specify if the Q matrix is time invariant. If you unselect this option, the block includes an additional input port Q.

## R

Measurement noise covariance matrix, specified as one of the following:

- Real positive scalar. R is an Ny-by-Ny diagonal matrix with the scalar on the diagonals. Ny is the number of measured outputs in the model.
- Vector of real positive scalars. R is an Ny-by-Ny diagonal matrix with the elements of the vector on the diagonals of R .
- Ny-by-Ny positive-definite matrix.


## Time Invariant R

Specify if the R matrix is time invariant. If you unselect this option, the block includes an additional input port R.

## N

Process and measurement noise cross-covariance matrix. Specify it as a Nw-by-Ny matrix. The matrix [Q $\mathrm{N} ; \mathrm{N}^{\mathrm{T}} \mathrm{R}$ ] must be positive definite.

## Time Invariant $\mathbf{N}$

Specify if the N matrix is time invariant. If you unselect this option, the block includes an additional input port N .

## Add input port u

Select this option if your model contains known inputs $u(t)$ or $u[k]$. The option is selected by default. Unselecting this option removes the input port u from the block and removes the B, D and Number of inputs parameters from the block dialog box.

## Add input port Enable to control measurement updates

Select this option if you want to control the measurement updates. The block includes an additional inport Enable. The Enable input port takes a scalar signal. This option is cleared by default.

By default the block does measurement updates at each time step to improve the state and output estimates $\widehat{x}$ and $\widehat{y}$ based on measured outputs. The measurement update is skipped for the current sample time when the signal in the Enable port is 0 . Concretely, the equation for state estimates become $\dot{\widehat{x}}(t)=A(t) \widehat{X}(t)+B(t) u(t)$ for continuous-time Kalman filter and $\widehat{x}[n+1 \mid n]=A[n] \widehat{x}[n \mid n-1]+B[n] u[n]$ for discrete-time.

## External Reset

Option to reset estimated states and parameter covariance matrix using specified initial values.
Suppose you reset the block at a time step, $t$. If the block is enabled at $t$, the software uses the initial parameter values specified either in the block dialog or the input ports P0 and X0 to estimate the states. In other words, at $t$, the block performs a time update and if it is enabled, a measurement update after the reset. The block outputs these updated estimates.

Specify one of the following:

- None (Default) - Estimated states $\widehat{x}$ and state estimation error covariance matrix $P$ values are not reset.
- Rising - Triggers a reset when the control signal rises from a negative or zero value to a positive value. If the initial value is negative, rising to zero triggers a reset.
- Falling - Triggers a reset when the control signal falls from a positive or a zero value to a negative value. If the initial value is positive, falling to zero triggers a reset.
- Either - Triggers a reset when the control signal is either rising or falling.
- Level - Triggers a reset in either of these cases:
- The control signal is nonzero at the current time step.
- The control signal changes from nonzero at the previous time step to zero at the current time step.
- Level hold - Triggers reset when the control signal is nonzero at the current time step.

When you choose an option other than None, a Reset input port is added to the block to provide the reset control input signal.

## Output estimated model output y

Add $\widehat{y}$ output port to the block to output the estimated model outputs. The option is cleared by default.

## Output state estimation error covariance $\mathbf{P}$ or $\mathbf{Z}$

Add P output port or Z output port to the block. The Z matrix is provided only when Time Domain is Discrete Time and the Use the current measurement $\mathbf{y}[\mathbf{n}]$ to improve xhat[n] is selected. Otherwise, the P matrix, as described in the "Description" on page 2-32 section previously, is provided.

The option is cleared by default.

## Ports

| Port Name | Port <br> Type <br> (In/ <br> Out) | Description |
| :--- | :--- | :--- |
| u (Optional) | In | Known inputs, specified as a real scalar or vector. |
| y | In | Measured outputs, specified as a real scalar or vector. |
| xhat | Out | Estimated states, returned as a real scalar or vector. |
| yhat (Optional) | Out | Estimated outputs, returned as a real scalar or vector. |
| P or Z (Optional) | Out | State estimation error covariance, returned as a matrix. |
| A (Optional) | In | A matrix, specified as a real matrix. |
| B (Optional) | In | B matrix, specified as a real matrix. |
| C (Optional) | In | C matrix, specified as a real matrix. |
| D (Optional) | In | D matrix, specified as a real matrix. |
| G (Optional) | In | G matrix, specified as a real matrix. |
| H (Optional) | In | H matrix, specified as a real matrix. |


| Port Name | Port <br> Type <br> (In/ <br> Out) | Description |
| :--- | :--- | :--- |
| Q (Optional) | In | Q matrix, specified as a real scalar, vector or matrix. |
| R (Optional) | In | R matrix, specified as a real scalar, vector or matrix. |
| N (Optional) | In | N matrix, specified as a real matrix. |
| P0 (Optional) | In | P matrix at initial time, specified as a real scalar, vector, or matrix. |
| X0 (Optional) | In | Initial state estimates, specified as a real scalar or vector. |
| Enable <br> (Optional) | In | Control signal to enable measurement updates, specified as a real scalar. |
| Reset (Optional) | In | Control signal to reset state estimates, specified as a real scalar. |

## Supported Data Types

- Double-precision floating point
- Single-precision floating point (for discrete-time Kalman filter only)


## Note

- All input ports except Enable and Reset must have the same data type (single or double).
- Enable and Reset ports support single, double, int8, uint8, int16, uint16, int32, uint32, and boolean data types.


## Limitations

- The plant and noise data must satisfy:
- $(C, A)$ detectable
- $\bar{R}>0$ and $\bar{Q}-\bar{N} \bar{R}^{-1} \bar{N}^{T} \geq 0$
- (A- $\bar{N} \bar{R}^{-1} C, \bar{Q}-\bar{N} \bar{R}^{-1} \bar{N}^{T}$ ) has no uncontrollable mode on the imaginary axis (or unit circle in discrete time) with the notation

$$
\begin{aligned}
& \bar{Q}=G Q G^{T} \\
& \bar{R}=R+H N+N^{T} H^{T}+H Q H^{T} \\
& \bar{N}=G\left(Q H^{T}+N\right)
\end{aligned}
$$

- The continuous-time Kalman filter cannot be used in Function-Call Subsystems or Triggered Subsystems.


## Compatibility Considerations

Kalman Filter block: Numerical changes
Behavior changed in R2021a

Starting in 2021a, numerical improvements in the algorithms used by the Kalman FIlter block might produce results that are different from the results you obtained using previous versions.

## References

[1] Franklin, G.F., J.D. Powell, and M.L. Workman, Digital Control of Dynamic Systems, Second Edition, Addison-Wesley, 1990.
[2] Lewis, F., Optimal Estimation, John Wiley \& Sons, Inc, 1986.

## Extended Capabilities

C/C++ Code Generation
Generate C and C++ code using Simulink $\circledR^{\circledR}$ Coder ${ }^{\mathrm{TM}}$.
PLC Code Generation
Generate Structured Text code using Simulink ${ }_{\circledR}{ }^{\circledR}$ PLC Coder ${ }^{\text {TM }}$.

## See Also

## Functions

extendedKalmanFilter|kalman | particleFilter|unscentedKalmanFilter

## Blocks

Extended Kalman Filter | Particle Filter | Unscented Kalman Filter

## Topics

"State Estimation Using Time-Varying Kalman Filter"
"What Is Online Estimation?"
"Validate Online State Estimation in Simulink"
"Troubleshoot Online State Estimation"

Introduced in R2014b

## Model Type Converter

Convert polynomial model coefficients to state-space model matrices


Model Type Converter

## Library

Estimators

## Description

Use the Model Type Converter block to convert the ARX, ARMAX, OE, or BJ model coefficients into state-space model matrices.

The block inport, u , requires a bus. The number of elements depends on the input polynomial model type:

- ARX - A, B
- ARMAX - A, B, C
- OE - B, F
- BJ - B,C, D, F

These bus elements must contain row vectors of the estimated coefficient values as outputted by the Recursive Polynomial Model Estimator block. For MISO data, specify $B$ polynomial coefficients as a matrix where the $i$-th row parameters correspond to the $i$-th input. The coefficient values can vary with time. The Model Type Converter block converts these coefficients into the A, B, C, and D matrices of a discrete-time state-space model. The Model Type Converter block outport, y, returns a bus with elements that correspond to the A, B, C, and D matrices of the state-space model. If the signals in $u$ are time-varying, then the state-space matrices are time-varying too.

You can also estimate a state-space model online by using the Recursive Polynomial Model Estimator and Model Type Converter blocks together. Connect the outport of the Recursive Polynomial Model Estimator block to the inport of the Model Type Converter block to obtain online values of the statespace matrices. The conversion ignores the noise component of the models. In other words, the statespace matrices only capture the $\mathrm{y}(\mathrm{t}) / \mathrm{u}(\mathrm{t})$ relationship.

## Parameters

## Input model type

Specify the model type coefficients to convert to state-space model matrices. Specify one of the following model types:

- ARX
- ARMAX
- OE
- BJ


## Ports

| Port | Port <br> Type <br> (ln/ <br> Out) | Description |
| :--- | :--- | :--- |
| u | In | Estimated A, B, C, D and F polynomial coefficients, specified as a bus <br> with elements: A, B, C, D and F. |
| y | Out | State-space model, returned as a bus with elements that correspond to <br> the A, B, C, and D matrices. |

## Supported Data Types

- Double-precision floating point
- Single-precision floating point


## Extended Capabilities

## C/C++ Code Generation

Generate C and C++ code using Simulink ${ }^{\circledR}$ Coder ${ }^{\mathrm{TM}}$.

## PLC Code Generation

Generate Structured Text code using Simulink® PLC Coder ${ }^{\mathrm{TM}}$.

## See Also

Recursive Polynomial Model Estimator
Introduced in R2014a

## Particle Filter

Estimate states of discrete-time nonlinear system using particle filter
Library:
Control System Toolbox / State Estimation
System Identification Toolbox / Estimators


## Description

The Particle Filter block estimates the states of a discrete-time nonlinear system using the discretetime particle filter algorithm.

Consider a plant with states $x$, input $u$, output $m$, process noise $w$, and measurement $y$. Assume that you can represent the plant as a nonlinear system.


The algorithm computes the state estimates $\widehat{x}$ of the nonlinear system using the state transition and measurement likelihood functions you specify.

You create the nonlinear state transition function and measurement likelihood functions for the system and specify these functions in the block. The block supports state estimation of a system with multiple sensors that are operating at different sampling rates. You can specify up to five measurement likelihood functions, each corresponding to a sensor in the system.

## Ports

## Input

## $\mathrm{y} 1, \mathrm{y} 2, \mathrm{y} 3, \mathrm{y} 4, \mathrm{y} 5$ - Measured system outputs

vector
Measured system outputs corresponding to each measurement likelihood function that you specify in the block. The number of ports equals the number of measurement likelihood functions in your system. You can specify up to five measurement likelihood functions. For example, if your system has two sensors, you specify two measurement likelihood functions in the block. The first port $\mathbf{y 1}$ is available by default. Click Add Measurement, to generate port y2 corresponding to the second measurement likelihood function.

Specify the ports as $N$-dimensional vectors, where $N$ is the number of quantities measured by the corresponding sensor. For example, if your system has one sensor that measures the position and
velocity of an object, then there is only one port $\mathbf{y 1}$. The port is specified as a two-dimensional vector with values corresponding to position and velocity.

## Dependencies

The first port $\mathbf{y 1}$ is available by default. Ports y2 to $\mathbf{y 5}$ are generated when you click Add Measurement.

## StateTransitionFcnInputs - Optional input argument to state transition function scalar | vector | matrix

Optional input argument to the state transition function f other than the state x .
If you create f using a MATLAB function (.m file), the software generates the port StateTransitionFcnInputs when you enter the name of your function, and click Apply.

If your state transition function has more than one additional input, use a Simulink Function block to specify the function. When you use a Simulink Function block, you provide the additional inputs directly to the Simulink Function block using Inport blocks. No input ports are generated for the additional inputs in the Particle Filter block.

## Dependencies

This port is generated only if both of the following conditions are satisfied:

- You specify $f$ in Function using a MATLAB function, and $f$ is on the MATLAB path.
- f requires only one additional input argument apart from particles.

MeasurementLikelihoodFcn1Inputs, ... , MeasurementLikelihoodFcn5Inputs - Optional input argument to each measurement likelihood function scalar | vector | matrix

Optional inputs to the measurement likelihood functions other than the state x and measurement y .
MeasurementLikelihoodFcn1Inputs corresponds to the first measurement likelihood function that you specify, and so on.

If you specify two measurement inputs using MATLAB functions (.m files) in Function, the software generates ports MeasurementLikelihoodFcn1Inputs and MeasurementLikelihoodFcn2Inputs when you click Apply. You can specify the inputs to these ports as scalars, vectors, or matrices.

If your measurement likelihood functions have more than one additional input, use Simulink Function blocks to specify the functions. When you use a Simulink Function block, you provide the additional inputs directly to the Simulink Function block using Inport blocks. No input ports are generated for the additional inputs in the Particle Filter block.

## Dependencies

A port corresponding to a measurement likelihood function $h$ is generated only if both of the following conditions are satisfied:

- You specify measurement input h in Function using a MATLAB function, and h is on the MATLAB path.
- $h$ requires only one additional input argument apart from particles and measurement.


## Enable1, Enable2,Enable3, Enable4, Enable5 - Enable correction of estimated states when measured data is available

scalar
Enable correction of estimated states when measured data is available.
For example, consider that measured output data is not available at all time points at the port $\mathbf{y} \mathbf{1}$ that corresponds to the first measurement likelihood function. Then, use a signal value other than 0 at the Enable1 port to enable the correction of estimated states when measured data is available. Specify the port value as 0 when measured data is not available. Similarly, if measured output data is not available at all time points at the port yi for the $i^{\text {th }}$ measurement likelihood function, specify the corresponding port Enablei as a value other than 0.

## Dependencies

If you select Add Enable port for a measurement likelihood function, a port corresponding to that measurement likelihood function is generated. The port appears when you click Apply.

## Output

## xhat - Estimated states

vector
Estimated states, returned as a vector of size $N s$, where $N s$ is the number of states of the system. To access the individual states, use the Selector block.

When the Use the current measurements to improve state estimates parameter is selected, the block outputs the corrected state estimate $\widehat{x}[k \mid k]$ at time step k , estimated using measured outputs until time $k$. If you clear this parameter, the block returns the predicted state estimate $\widehat{x}[k \mid k-1]$ for time $k$, estimated using measured output until a previous time $k-1$. Clear this parameter if your filter is in a feedback loop and there is an algebraic loop in your Simulink model.

## P - State estimation error covariance matrix

State estimation error covariance, returned as an $N s$-by- $N s$ matrix, where $N s$ is the number of states of the system. To access the individual covariances, use the Selector block.

You can output the error covariance only if you select Output state estimation error covariance in the Block outputs, Multirate tab, and click Apply.

## Dependencies

This parameter is available if in the Block outputs, Multirate tab, the State estimation method parameter is set to 'Mean'.

## Particles - Particle values used for state estimation array

Particle values used for state estimation, returned as an $N s$-by- $N p$ or $N p$-by- $N s$ array. $N s$ is the number of states of the system, and $N p$ is the number of particles.

- If the State0rientation parameter is specified as 'column' , then Particles is returned as an $N s$-by-Np array.
- If the State0rientation parameter is specified as ' row', then Particles is returned as an $N p$ -by-Ns array.


## Dependencies

This port is generated if you select Output all particles in the Block outputs, Multirate tab, and click Apply.

## Weights - Particle weights used for state estimation

vector

Particle weights used for state estimation, returned as a 1-by- $N p$ or $N p$-by- 1 vector, where $N p$ is the number of particles used for state estimation.

- If the State0rientation parameter is specified as ' column', then Weights is returned as a 1-by- $N p$ vector, where each weight is associated with the particle in the same column in the Particles array.
- If the StateOrientation parameter is specified as 'row', then Weights is returned as a $N p$ -by-1 vector, where each weight is associated with the particle in the same row in the Particles array.


## Dependencies

This port is generated if you select Output weights in the Block outputs, Multirate tab, and click Apply.

## Parameters

## System Model Tab

State Transition

## Function - State transition function name

'vdpParticleFilterStateFcn' (default)|function name
The particle filter state transition function calculates the particles at time step $k+1$, given particles at time step $k$ per the dynamics of your system and process noise. This function has the syntax:
particlesNext $=$ f(particles, param1, param2, ...)
where, particles and particlesNext have dimensions $N s$-by-Np if State Orientation is specified as ' column ' , or $N p$-by-Ns if State Orientation is specified as 'row'. Also, param_i represents optional input arguments you may specify. For more information on optional input arguments, see "StateTransitionFcnInputs" on page 2-0.

You create the state transition function and specify the function name in Function. For example, if vdpParticleFilterStateFcn.m is the state transition function that you created and saved, specify Function as 'vdpParticleFilterStateFcn'.

You can create Function using a Simulink Function block or as a MATLAB function (.m file).

## Programmatic Use

Block Parameter: StateTransitionFcn
Type: character vector, string
Default: 'vdpParticleFilterStateFcn'

## Initialization

## Number of Particles - Number of particles used in the filter

1000 (default) | positive scalar integer
Number of particles used in the filter, specified as a positive scalar integer. Each particle represents a state hypothesis in the system. A higher number of particles increases the state estimation accuracy, but also increases the computational effort required to run the filter.

Programmatic Use
Block Parameter: NumberOfParticles
Type: positive scalar integer
Default: 1000

## Distribution - Initial distribution of particles

'Gaussian' (default)|'Uniform'| 'Custom'
Initial distribution of particles, specified as 'Gaussian', 'Uniform', or 'Custom'.
If you choose 'Gaussian ', the initial set of particles or state hypotheses are distributed per the multivariate Gaussian distribution, where you specify the Mean and Covariance. The initial weight of all particles is assumed to be equal.

If you choose 'Uniform' , the initial set of particles are distributed per the uniform distribution, where you specify the upper and lower State bounds. The initial weight of all particles is assumed to be equal.
'Custom ' allows you to specify your own set of initial particles and their weights. You can use arbitrary probability distributions for Particles and Weights to initialize the filter.

```
Programmatic Use
Block Parameter: InitialDistribution
Type: character vector
Values: 'Gaussian', 'Uniform', 'Custom'
Default: 'Gaussian'
Mean - Initial mean value of particles
[0;0] (default) | vector
```

Initial mean value of particles, specified as a vector. The number of states to be estimated defines the length of the vector.

## Dependencies

This parameter is available if in the System model tab, the Distribution parameter is set to Gaussian.

Programmatic Use
Block Parameter: InitialMean
Type: array
Default: [0, 0]

## Covariance - Initial covariance of particles

1 (default) | scalar | vector | matrix
Initial covariance of particles, specified as a scalar, vector, or matrix.

If Covariance is specified as:

- A scalar, then it must be positive. The covariance is assumed to be a [Ns Ns] matrix with this scalar on the diagonals. Here, $N s$ is the number of states.
- A vector, then each element must be positive. The covariance is assumed to be a [Ns Ns] matrix with the elements of the vector on the diagonals.
- A matrix, then it must be positive semidefinite.


## Dependencies

This parameter is available if in the System model tab, the Distribution parameter is set to Gaussian.

Programmatic Use
Block Parameter: InitialCovariance
Type: scalar, vector, or matrix
Default: 1
Circular Variables - Circular variables used for state estimation
0 (default) | scalar | vector
Circular variables used for state estimation, specified as a scalar, or Ns-element vector, where $N s$ is the number of states.

If Circular Variables is specified as a scalar, the software extends it to a vector where each element is equal to this scalar. Circular (or angular) distributions use a probability density function with a range of $[-\pi \pi]$. Use circular variables if some of the states in your system represent angular quantities like the orientation of an object.

## Programmatic Use

Block Parameter: CircularVariables
Type: scalar, vector
Default: 0
State Orientation - Orientation of input system states
'column' (default)|'row'
Orientation of system states, specified as 'column' or 'row'.
If State Orientation is specified as:

- ' column' , then the first input argument to the state transition and measurement likelihood function is [ $N s N p$ ]. In this case, $i^{\text {th }}$ column of this matrix is the $i^{\text {th }}$ particle (state hypothesis). Also, the states estimates xhat is output as a [ $N s$ 1] vector. Here, $N s$ is the number of states, and $N p$ is the number of particles.
- ' row' , then the first input argument to the state transition and measurement likelihood function is [ $N p N s$ ], and each row of this matrix contains a particle. Also, the states estimates xhat is output as a [1 Ns] vector.

Programmatic Use
Block Parameter: State0rientation
Type: character vector
Values: 'column', 'row'
Default: 'column'

## State bounds - Initial bounds on system states

[-3 3;-3 3] (default) | array
Initial bounds on system states, specified as an $N s$-by-2 array, where $N s$ is the number of states.
The $i^{\text {th }}$ row lists the lower and upper bound of the uniform distribution for the initial distribution of particles of the $i^{\text {th }}$ state.

## Dependencies

This parameter is available if in the System model tab, the Distribution parameter is set to Uniform.

Programmatic Use
Block Parameter: InitialStateBounds
Type: array
Default: [-3 3;-3 3]

## Particles - Custom particle distribution for state estimation

[] (default) | array
Custom particle distribution for state estimation, specified as an $N s$-by- $N p$ or $N p$-by- $N s$ array. $N s$ is the number of states of the system, and $N p$ is the number of particles.

- If the StateOrientation parameter is specified as ' column ', then Particles is an $N s$-by- $N p$ array.
- If the StateOrientation parameter is specified as 'row', then Particles is an $N p$-by-Ns array.


## Dependencies

This parameter is available if in the System model tab, the Distribution parameter is set to Custom.

## Programmatic Use

Block Parameter: InitialParticles
Type: array
Default: []

## Weights - Custom particle weight values for state estimation

[] (default) | positive vector
Custom particle weight values for state estimation, specified as a 1-by-Np or $N p-$ by-1 positive vector, where $N p$ is the number of particles used for state estimation.

- If the StateOrientation parameter is specified as 'column', then Weights is a 1-by-Np vector. Each weight in the vector is associated with the particle in the same column in the Particles array.
- If the StateOrientation parameter is specified as 'row', then Weights is a $N p$-by-1 vector. Each weight in the vector is associated with the particle in the same row in the Particles array.


## Dependencies

This parameter is available if in the System model tab, the Distribution parameter is set to Custom.

Programmatic Use
Block Parameter: InitialWeights
Type: positive vector
Default: []

## Measurement

Function - Measurement likelihood function name
'vdpMeasurementLikelihoodFcn' (default)|function name
The measurement likelihood function calculates the likelihood of particles (state hypotheses) using the sensor measurements. For each state hypothesis (particle), the function first calculates an Nmelement measurement hypothesis vector. Then the likelihood of each measurement hypothesis is calculated based on the sensor measurement and the measurement noise probability distribution. this function has the syntax:
likelihood = h(particles, measurement, param1, param2, ...)
where, likelihood is an $N p$-element vector, where $N p$ is the number of particles. particles have dimensions $N s$-by- $N p$ if State Orientation is specified as ' column ', or $N p$-by-Ns if State Orientation is specified as 'row'. measurement is an $N m$-element vector where, $N m$ is the number of measurements your sensor provides. param_i represents optional input arguments you may specify. For more information on optional input arguments, see
"MeasurementLikelihoodFcn1Inputs,...,MeasurementLikelihoodFcn5Inputs" on page 2-0 .
You create the measurement likelihood function and specify the function name in Function. For example, if vdpMeasurementLikelihoodFcn.m is the measurement likelihood function that you created and saved, specify Function as 'vdpMeasurementLikelihoodFcn'.

You can create Function using a Simulink Function block or as a MATLAB function (.m file).

- You can use a MATLAB function only if $h$ has zero or one additional input argument param_i other than Particles and Measurement.

The software generates an additional input port MeasurementLikelihoodFcniInputs to specify this argument for the $i^{\text {th }}$ measurement likelihood function, and click Apply.

- If you are using a Simulink Function block, specify $x$ and $y$ using Argument Inport blocks and the additional inputs param_i using Inport blocks in the Simulink Function block. You do not provide param_i to the Particle Filter block.

If you have multiple sensors in your system, you can specify multiple measurement likelihood functions. You can specify up to five measurement likelihood functions using the Add Measurement button. To remove measurement likelihood functions, use Remove Measurement.

Programmatic Use
Block Parameter: MeasurementLikelihoodFcn1, MeasurementLikelihoodFcn2, MeasurementLikelihoodFcn3, MeasurementLikelihoodFcn4, MeasurementLikelihoodFcn5 Type: character vector, string
Default: 'vdpMeasurementLikelihoodFcn'
Add Enable Port - Enable correction of estimated states only when measured data is available
off (default) | on
Suppose that measured output data is not available at all time points at the port y1 that corresponds to the first measurement likelihood function. To generate an input port Enable1, select Add Enable
port. Use a signal at this port to enable the correction of estimated states only when measured data is available. Similarly, if measured output data is not available at all time points at the port yi for the $i^{\text {th }}$ measurement likelihood function, select the corresponding Add Enable port.

Programmatic Use
Block Parameter: HasMeasurementEnablePort1, HasMeasurementEnablePort2,
HasMeasurementEnablePort3, HasMeasurementEnablePort4, HasMeasurementEnablePort5
Type: character vector
Values: 'off', 'on'
Default: 'off'
Resampling
Resampling method - Method used for particle resampling
'Multinomial' (default)|'Systemic'|'Stratified'
Method used for particle resampling, specified as one of the following:

- 'Multinomial'
- 'Systematic'
- 'Stratified'

Programmatic Use
Block Parameter: ResamplingMethod
Type: character vector
Values: 'Multinomial', 'Systemic', 'Stratified'
Default: 'Multinomial'

## Trigger method - Method to determine when resampling occurs <br> 'Ratio' (default)|'Interval'

Method to determine when resampling occurs, specified as either 'Ratio' or 'Interval '. The 'Ratio' value triggers resampling based on the ratio of effective total particles. The 'Interval' value triggers resampling at regular time steps of the particle filter operation.

## Programmatic Use

Block Parameter: TriggerMethod
Type: character vector
Values: 'Ratio', 'Interval'
Default: 'Ratio'

## Minimum effective particle ratio - Minimum desired ratio of the effective number of particles to the total number of particles

0.5 (default) | positive scalar

Minimum desired ratio of the effective number of particles to the total number of particles, specified as a positive scalar. The effective number of particles is a measure of how well the current set of particles approximates the posterior distribution. A lower effective particle ratio implies that a lower number of particles are contributing to the estimation and resampling is required.

If the ratio of the effective number of particles to the total number of particles falls below the minimum effective particle ratio, a resampling step is triggered.

Specify minimum effective particle ratio as any value from 0 through 1.

## Dependencies

This parameter is available if in the System model tab, the Trigger method parameter is set to Ratio.

Programmatic Use
Block Parameter: MinEffectiveParticleRatio
Type: scalar
Values: Range [0,1]
Default: 0.5
Sampling Interval - Fixed interval between resampling
1 (default) | positive scalar integer
Fixed interval between resampling, specified as a positive scalar integer. The sampling interval determines during which correction steps the resampling is executed. For example, a value of two means the resampling is executed every second correction step. A value of inf means that resampling is never executed.

## Dependencies

This parameter is available if in the System model tab, the Trigger method parameter is set to Interval.

Programmatic Use
Block Parameter: SamplingInterval
Type: positive scalar integer
Default: 1
Random Number Generator Options
Randomness - Whether the random numbers are repeatable
'Repeatable' (default)|'Not repeatable'
Whether the random numbers are repeatable, specified as either 'Repeatable' or 'Not repeatable'. If you want to be able to produce the same result more than once, set Randomness to 'Repeatable', and specify the same random number generator seed value in Seed.

## Programmatic Use

Block Parameter: Randomness
Type: character vector
Values: 'Repeatable', 'Not repeatable'
Default: 'Repeatable'

## Seed - Seed value for repeatable random numbers

0 (default) | scalar
Seed value for repeatable random numbers, specified as a scalar.

## Dependencies

This parameter is available if in the System model tab, the Randomness parameter is set to 'Repeatable'.

Programmatic Use
Block Parameter: Seed
Type: scalar

Default: 0

## Settings

Data type - Data type for block parameters
double (default)| single
Use this parameter to specify the data type for all block parameters.
Programmatic Use
Block Parameter: DataType
Type: character vector
Values: 'single', 'double'
Default: 'double'

## Sample time - Block sample time

1 (default) | positive scalar
Block sample time, specified as a positive scalar.
Use the Sample time parameter if your state transition and all measurement likelihood functions have the same sample time. Otherwise, select the Enable multirate operation option in the Multirate tab, and specify sample times in the same tab.

## Dependencies

This parameter is available if in the Block output, Multirate tab, the Enable multirate operation parameter is off.

Programmatic Use
Block Parameter: SampleTime
Type: character vector, string
Default: ' 1'

## Block Outputs, Multirate Tab

## Outputs

## State Estimation Method - Method used for extracting a state estimate from particles 'Mean' (default)|'MaxWeight'| 'None'

Method used for extracting a state estimate from particles, specified as one of the following:

- 'Mean ' - The Particle Filter block outputs the weighted mean of the particles, depending on the parameters Weights and Particles, as the state estimate.
- 'Maxweight ' - The Particle Filter block outputs the particle with the highest weight as the state estimate.
- 'None ' - Use this option to implement a custom state estimation method by accessing all particles using the Output all particles parameter from the Block outputs, Multirate tab.

Programmatic Use
Block Parameter: StateEstimationMethod
Type: character vector, string
Values: 'Mean', 'MaxWeight', 'None'
Default: 'Mean'

## Output all particles - Output all particles

'off' (default)|'on'
If you select this parameter, an output port for particles used in the estimation, Particles is generated in the block.

- If the StateOrientation parameter is specified as 'column', then the particles are output as an $N s$-by- $N p$ array. $N s$ is the number of states of the system, and $N p$ is the number of particles.
- If the StateOrientation parameter is specified as 'row', then the particles are output as an $N p$-by-Ns array.

Programmatic Use
Block Parameter: OutputParticles
Type: character vector
Values: 'off', 'on'
Default: 'off'

## Output weights - Output particle weights

'off' (default)|'on'
If you select this parameter, an output port for particle weights used in the estimation, Weights is generated in the block.

- If the State0rientation parameter is specified as 'column', then the particle weights are output as a $1-$ by- $N p$ vector. Here, where each weight is associated with the particle in the same column in the Particles array. $N p$ is the number of particles used for state estimation.
- If the State0rientation parameter is specified as 'row', then the particle weights are output as a $N p$-by-1 vector.

Programmatic Use
Block Parameter: OutputWeights
Type: character vector
Values: 'off', 'on'
Default: 'off'
Output state estimation error covariance - Output state estimation error covariance 'off' (default)|'on'

If you select this parameter, a state estimation error covariance output port, $\mathbf{P}$ is generated in the block.

## Dependencies

This parameter is available if in the Block outputs, Multirate tab, the State estimation method parameter is set to 'Mean'.
Programmatic Use
Block Parameter: OutputStateCovariance
Type: character vector
Values: 'off', 'on'
Default: 'off'
Use the current measurements to improve state estimates - Option to use current measurements for state estimation
'on' (default)|'off'

When this parameter is selected, the block outputs the corrected state estimate $\widehat{x}[k \mid k]$ at time step $k$, estimated using measured outputs until time $k$. If you clear this parameter, the block returns the predicted state estimate $\widehat{x}[k \mid k-1]$ for time $k$, estimated using measured output until a previous time $\mathrm{k}-1$. Clear this parameter if your filter is in a feedback loop and there is an algebraic loop in your Simulink model.

Programmatic Use
Block Parameter: UseCurrentEstimator
Type: character vector
Values: 'on', 'off'
Default: 'on'

## Multirate

## Enable multirate operation - Enable specification of different sample times for state transition and measurement likelihood functions

'off' (default)|'on'
Select this parameter if the sample times of the state transition or any of the measurement likelihood functions differ from the rest. You specify the sample times in the Multirate tab, in Sample time.

```
Programmatic Use
Block Parameter: EnableMultirate
Type: character vector
Values: 'off','on'
Default: 'off'
```

Sample times - State transition and measurement likelihood function sample times
positive scalar

If the sample times for state transition and measurement likelihood functions are different, specify Sample time. Specify the sample times for the measurement functions as positive integer multiples of the state transition sample time. The sample times you specify correspond to the following input ports:

- Ports corresponding to state transition function - Additional input to state transition function StateTransitionFcnInputs. The sample times of these ports must always equal the state transition function sample time, but can differ from the sample time of the measurement likelihood functions.
- Ports corresponding to $i^{\text {th }}$ measurement likelihood function - Measured output yi, additional input to measurement likelihood function MeasurementLikelihoodFcniInputs, enable signal at port Enablei. The sample times of these ports for the same measurement likelihood function must always be the same, but can differ from the sample time for the state transition function and other measurement likelihood functions.


## Dependencies

This parameter is available if in the Block outputs, Multirate tab, the Enable multirate operation parameter is on.

```
Programmatic Use
Block Parameter: StateTransitionFcnSampleTime,
MeasurementLikelihoodFcn1SampleTime1,MeasurementLikelihoodFcn1SampleTime2,
```

MeasurementLikelihoodFcn1SampleTime3, MeasurementLikelihoodFcn1SampleTime4, MeasurementLikelihoodFcn1SampleTime5
Type: character vector, string
Default: ' 1'

## Extended Capabilities

## C/C++ Code Generation

Generate C and $\mathrm{C}++$ code using Simulink ${ }^{\circledR}$ Coder $^{\mathrm{TM}}$.
The state transition and measurement likelihood functions that you specify must use only the MATLAB commands and Simulink blocks that support code generation. For a list of blocks that support code generation, see "Simulink Built-In Blocks That Support Code Generation" (Simulink Coder). For a list of commands that support code generation, see "Functions and Objects Supported for C/C++ Code Generation" (MATLAB Coder).

## See Also

## Blocks

Extended Kalman Filter | Kalman Filter | Unscented Kalman Filter

## Functions

extendedKalmanFilter| particleFilter|unscentedKalmanFilter

## Topics

"Parameter and State Estimation in Simulink Using Particle Filter Block"
"Validate Online State Estimation in Simulink"
"Troubleshoot Online State Estimation"
"Estimate States of Nonlinear System with Multiple, Multirate Sensors"

## External Websites

Understanding Kalman Filters: Nonlinear State Estimators - MATLAB Video Series

## Introduced in R2018a

## Recursive Least Squares Estimator

Estimate model coefficients using recursive least squares (RLS) algorithm
Library:
System Identification Toolbox / Estimators


## Description

The Recursive Least Squares Estimator estimates the parameters of a system using a model that is linear in those parameters. Such a system has the following form:

$$
y(t)=H(t) \theta(t) .
$$

$y$ and $H$ are known quantities that you provide to the block to estimate $\theta$. The block can provide both infinite-history [1] and finite-history [2] (also known as sliding-window), estimates for $\theta$. For more information on these methods, see "Recursive Algorithms for Online Parameter Estimation".

The block supports several estimation methods and data input formats. Configurable options in the block include:

- Sample-based or frame-based data format - See the Input Processing parameter.
- Infinite-history or finite- history estimation - See the History parameter.
- Multiple infinite-history estimation methods - See the Estimation Method parameter.
- Initial conditions, enable flag, and reset trigger - See the Initial Estimate, Add enable port, and External Reset parameters.

For a given time step $t, y(t)$ and $H(t)$ correspond to the Output and Regressors inports of the Recursive Least Squares Estimator block, respectively. $\theta(t)$ corresponds to the Parameters outport.

For example, suppose that you want to estimate a scalar gain, $\theta$, in the system $y=h^{2} \theta$. Here, $y$ is linear with respect to $\theta$. You can use the Recursive Least Squares Estimator block to estimate $\theta$. Specify $y$ and $h^{2}$ as inputs to the Output and Regressor inports.

## Ports

Input
Regressors - Regressors signal
vector | matrix
Regressors input signal $H(t)$. The Input Processing and Number of Parameters parameters define the dimensions of the signal:

- Sample-based input processing and $N$ estimated parameters - 1-by- $N$ vector
- Frame-based input processing with $M$ samples per frame and $N$ estimated parameters $-M$-by- $N$ matrix

Data Types: single |double

## Output - Measured output

scalar | vector
Measured output signal $y(t)$. The Input Processing parameter defines the dimensions of the signal:

- Sample-based input processing - Scalar
- Frame-based input processing with $M$ samples per frame $-M$-by-1 vector

Data Types: single | double

## Enable - Enable estimation updates <br> true (default) | false

External signal that allows you to enable and disable estimation updates. If the signal value is:

- true - Estimate and output the parameter values for the time step.
- false - Do not estimate the parameter values, and output the most recent previously estimated value.


## Dependencies

To enable this port, select the Add enable port parameter.
Data Types: single|double|Boolean|int8|int16|int32|int64|uint8|uint16|uint32

## Reset - Reset trigger

scalar
Reset parameter estimation to its initial conditions. The value of the External reset parameter determines the trigger type. The trigger type dictates whether the reset occurs on a signal that is rising, falling, either rising or falling, level, or on level hold.

## Dependencies

To enable this port, select any option other than None in the External reset dropdown.
Data Types: single|double | Boolean | int8|int16|int32|uint8|uint16|uint32

## InitialParameters - Initial parameter estimates

vector
Initial parameter estimates, supplied from a source external to the block. The block uses this inport at the beginning of the simulation or when you trigger an algorithm reset using the Reset signal.

The Number of Parameters parameter defines the dimensions of the signal. If there are $N$ parameters, the signal is $N$-by- 1 .

## Dependencies

To enable this port, set History to Infinite and Initial Estimate to External.
Data Types: single|double

## InitialCovariance - Initial covariance of parameters <br> positive scalar (default) | vector of positive scalars | symmetric positive-definite matrix

Initial parameter covariances, supplied from a source external to the block. For details, see the Parameter Covariance Matrix parameter. The block uses this inport at the beginning of the simulation or when you trigger an algorithm reset using the Reset signal.

## Dependencies

To enable this port, set the following parameters:

- History to Infinite
- Estimation Method to Forgetting Factor or Kalman Filter
- Initial Estimate to External

Data Types: single | double

## InitialRegressors - Initial values of the regressors matrix

Initial values of the regressors in the initial data window when using finite-history (sliding-window) estimation, supplied from an external source. The Window length parameter $W$ and the Number of Parameters parameter $N$ define the dimensions of this signal, which is $W$-by- $N$.

The InitialRegressors signal controls the initial behavior of the algorithm. The block uses this inport at the beginning of the simulation or whenever the Reset signal triggers.

If the initial buffer is set to 0 or does not contain enough information, you see a warning message during the initial phase of your estimation. The warning should clear after a few cycles. The number of cycles it takes for sufficient information to be buffered depends upon the order of your polynomials and your input delays. If the warning persists, you should evaluate the content of your signals.

## Dependencies

To enable this port, set History to Finite and Initial Estimate to External.
Data Types: single | double
Initial0utputs - Initial value of the measured output buffer
vector
Initial set of output measurements when using finite-history (sliding-window) estimation, supplied from an external source. The signal to this port must be a $W$-by- 1 vector, where $W$ is the window length.

The InitialOutputs signal controls the initial behavior of the algorithm. The block uses this inport at the beginning of the simulation or whenever the Reset signal triggers.

If the initial buffer is set to 0 or does not contain enough information, you see a warning message during the initial phase of your estimation. The warning should clear after a few cycles. The number of cycles it takes for sufficient information to be buffered depends upon the order of your polynomials and your input delays. If the warning persists, you should evaluate the content of your signals.

Dependencies
To enable this port, set History to Finite, and Initial Estimate to External.

Data Types: single | double

## Output

## Parameters - Estimated parameters

vector
Estimated parameters $\theta(t)$, returned as an $N$-by-1 vector where $N$ is the number of parameters.
Data Types: single|double

## Error - Estimation error

scalar | vector
Estimation error, returned as:

- Scalar - Sample-based input processing
- $M$-by- 1 vector - Frame-based input processing with $M$ samples per frame


## Dependencies

To enable this port, select the Output estimation error parameter.
Data Types: single|double

## Covariance - Parameter estimation error covariance P matrix

Parameter estimation error covariance $P$, returned as an $N$-by- $N$ matrix, where $N$ is the number of parameters. For details, see the Output Parameter Covariance Matrix parameter.

## Dependencies

To enable this port:

- If History is Infinite, set Estimation Method to Forgetting Factor or Kalman Filter.
- Whether History is Infinite or Finite, select the Output parameter covariance matrix parameter.

Data Types: single | double

## Parameters

## Model Parameters

## Initial Estimate - Source of initial parameter estimates <br> None (default) | Internal | External

Specify how to provide initial parameter estimates to the block:

- None - Do not specify initial estimates.
- If History is Infinite, the block uses 1 as the initial parameter estimate.
- If History is Finite, the block calculates the initial parameter estimates from the initial Regressors and Outputs signals.

Specify Number of Parameters, and also, if History is Infinite, Parameter Covariance Matrix.

- Internal - Specify initial parameter estimates internally to the block
- If History is Infinite, specify the Initial Parameter Values and Parameter Covariance Matrix parameters.
- If History is Finite, specify the Number of Parameters, the Initial Regressors, and the Initial Outputs parameters.
- External - Specify initial parameter estimates as an input signal to the block.

Specify the Number of Parameters parameter. Your setting for the History parameter determines which additional signals to connect to the relevant ports:

- If History is Infinite - InitialParameters and InitialCovariance
- If History is Finite - InitialRegressors and InitialOutputs

Programmatic Use
Block Parameter: InitialEstimateSource
Type: character vector, string
Values: 'None', 'Internal', 'External'
Default: 'None'

## Number of Parameters - Number of parameters to estimate

2 (default) | positive integer
Specify the number of parameters to estimate in the model, equal to the number of elements in the parameter $\theta(t)$ vector.

## Dependencies

To enable this parameter, set either:

- History to Infinite and Initial Estimate to either None or External
- History to Finite

An alternative way to specify the number of parameters $N$ to estimate is by using the Initial Parameter Values parameter, for which you define an initial estimate vector with $N$ elements. This approach covers the one remaining combination, where History is Infinite and Initial Estimate is Internal. For more information, see Initial Parameter Values.

## Programmatic Use

Block Parameter: InitialParameterData
Type: positive integer
Default: 2

## Parameter Covariance Matrix - Initial parameter covariance

le4 (default) | scalar | vector | matrix
Specify Parameter Covariance Matrix as a:

- Real positive scalar, $\alpha$ - Covariance matrix is an $N$-by- $N$ diagonal matrix, with $\alpha$ as the diagonal elements.
- Vector of real positive scalars, $\left[\alpha_{1}, \ldots, \alpha_{N}\right]$ Covariance matrix is an $N$-by- $N$ diagonal matrix, with [ $\alpha_{1}, \ldots, \alpha_{N}$ ] as the diagonal elements.
- $N$-by- $N$ symmetric positive-definite matrix.

Here, $N$ is the number of parameters to be estimated.

## Dependencies

To enable this parameter, set the following parameters:

## - History to Infinite

- Initial Estimate to None or Internal
- Estimation Method to Forgetting Factor or Kalman Filter

Programmatic Use
Block Parameter: P0
Type: scalar, vector, or matrix
Default: 1e4
Initial Parameter Values - Initial values of the parameters to estimate
[1 1] (default)|vector
Specify initial parameter values as a vector of length $N$, where $N$ is the number of parameters to estimate.

## Dependencies

To enable this parameter, set History to Infinite and Initial Estimate to Internal.
Programmatic Use
Block Parameter: InitialParameterData
Type: real vector
Default: [1 1]
Initial Regressors - Initial values of the regressors buffer
0 (default) | matrix
Specify the initial values of the regressors buffer when using finite-history (sliding window) estimation. The Window length parameter $W$ and the Number of Parameters parameter $N$ define the dimensions of the regressors buffer, which is $W$-by- $N$.

The Initial Regressors parameter controls the initial behavior of the algorithm. The block uses this parameter at the beginning of the simulation or whenever the Reset signal triggers.

When the initial value is set to 0 , the block populates the buffer with zeros.
If the initial buffer is set to 0 or does not contain enough information, you see a warning message during the initial phase of your estimation. The warning should clear after a few cycles. The number of cycles it takes for sufficient information to be buffered depends upon the order of your polynomials and your input delays. If the warning persists, you should evaluate the content of your signals.

## Dependencies

To enable this parameter, set History to Finite and Initial Estimate to Internal.

## Programmatic Use

Block Parameter: InitialRegressors
Type: real matrix

## Default: 0

Initial Outputs - Initial values of the measured outputs buffer
0 (default) | vector
Specify initial values of the measured outputs buffer when using finite-history (sliding-window) estimation. This parameter is a $W$-by- 1 vector, where $W$ is the window length.

When the initial value is set to 0 , the block populates the buffer with zeros.
If the initial buffer is set to 0 or does not contain enough information, you see a warning message during the initial phase of your estimation. The warning should clear after a few cycles. The number of cycles it takes for sufficient information to be buffered depends upon the order of your polynomials and your input delays. If the warning persists, you should evaluate the content of your signals.

The Initial Outputs parameter controls the initial behavior of the algorithm. The block uses this parameter at the beginning of the simulation or whenever the Reset signal triggers.

## Dependencies

To enable this parameter, set History to Finite and Initial Estimate to Internal.

## Programmatic Use

Block Parameter: InitialOutputs
Type: real vector
Default: 0

## Input Processing and Sample Time

## Input Processing - Choose sample-based or frame-based processing <br> Sample-based (default) | Frame-based

- Sample-based processing operates on signals streamed one sample at a time.
- Frame-based processing operates on signals containing samples from multiple time steps. Many machine sensor interfaces package multiple samples and transmit these samples together in frames. Frame-based processing allows you to input this data directly without having to first unpack it.

Specifying frame-based data adds an extra dimension of $M$ to some of your data inports and outports, where $M$ is the number of time steps in a frame. These ports are:

## - Regressors

- Output
- Error

For more information, see the port descriptions in "Ports" on page 2-58.

## Programmatic Use

Block Parameter: InputProcessing
Type: character vector, string
Values: 'Sample-based', 'Frame-based'
Default: 'Sample-based'

## Sample Time - Block sample time

- 1 (default) | positive scalar

Specify the data sample time, whether by individual samples for sample-based processing $\left(t_{s}\right)$, or by frames for frame-based processing $\left(t_{f}=M t_{s}\right)$, where $M$ is the frame length. When you set Sample Time to its default value of -1 , the block inherits its $t_{s}$ or $t_{f}$ based on the signal.

Specify Sample Time as a positive scalar to override the inheritance.
Programmatic Use
Block Parameter: Ts
Type: real scalar
Default: - 1

## Algorithm and Block Options

## Algorithm Options

## History - Choose infinite or finite data history

Infinite (default) | Finite
The History parameter determines what type of recursive algorithm you use:

- Infinite - Algorithms in this category aim to produce parameter estimates that explain all data since the start of the simulation. These algorithms retain the history in a data summary. The block maintains this summary within a fixed amount of memory that does not grow over time.

The block provides multiple algorithms of the Infinite type. Selecting this option enables the Estimation Method parameter with which you specify the algorithm.

- Finite - Algorithms in this category aim to produce parameter estimates that explain only a finite number of past data samples. The block uses all of the data within a finite window, and discards data once that data is no longer within the window bounds. This method is also called sliding-window estimation.

Selecting this option enables the Window Length parameter that sizes the sliding window.
For more information on recursive estimation methods, see "Recursive Algorithms for Online Parameter Estimation"

Programmatic Use
Block Parameter: History
Type: character vector, string
Values: 'Infinite', 'Finite'
Default: 'Infinite'

## Window Length - Window size for finite sliding-window estimation

200 (default) | positive integer
The Window Length parameter determines the number of time samples to use for the slidingwindow estimation method. Choose a window size that balances estimation performance with computational and memory burden. Sizing factors include the number and time variance of the parameters in your model. Always specify Window Length in samples, even if you are using framebased input processing.

Window Length must be greater than or equal to the number of estimated parameters.
Suitable window length is independent of whether you are using sample-based or frame-based input processing. However, when using frame-based processing, Window Length must be greater than or equal to the number of samples (time steps) contained in the frame.

## Dependencies

To enable this parameter, set History to Finite.
Programmatic Use
Block Parameter: WindowLength
Type: positive integer
Default: 200
Estimation Method - Recursive estimation algorithm
Forgetting Factor (default)|Kalman Filter|Normalized Gradient|Gradient
Specify the estimation algorithm when performing infinite-history estimation. When you select any of these methods, the block enables additional related parameters.

Forgetting factor and Kalman filter algorithms are more computationally intensive than gradient and normalized gradient methods. However, these more intensive methods have better convergence properties than the gradient methods. For more information about these algorithms, see "Recursive Algorithms for Online Parameter Estimation".

## Programmatic Use

Block Parameter: EstimationMethod
Type: character vector, string
Values: 'Forgetting Factor','Kalman Filter','Normalized Gradient','Gradient'
Default: 'Forgetting Factor'

## Forgetting Factor - Discount old data using forgetting factor

1 (default) | positive scalar in (0 1] range
The forgetting factor $\lambda$ specifies if and how much old data is discounted in the estimation. Suppose that the system remains approximately constant over $T_{0}$ samples. You can choose $\lambda$ such that:

$$
T_{0}=\frac{1}{1-\lambda}
$$

- Setting $\lambda=1$ corresponds to "no forgetting" and estimating constant coefficients.
- Setting $\lambda<1$ implies that past measurements are less significant for parameter estimation and can be "forgotten." Set $\lambda<1$ to estimate time-varying coefficients.

Typical choices of $\lambda$ are in the [0.98 0.995] range.

## Dependencies

To enable this parameter, set History to Infinite and Estimation Method to Forgetting Factor.

## Programmatic Use

Block Parameter: AdaptationParameter
Type: scalar
Values: (0 1] range
Default: 1

## Process Noise Covariance - Process noise covariance for Kalman filter estimation method

1 (default) | nonnegative scalar | vector of nonnegative scalars | symmetric positive semidefinite matrix

Process Noise Covariance prescribes the elements and structure of the noise covariance matrix for the Kalman filter estimation. Using $N$ as the number of parameters to estimate, specify the Process Noise Covariance as one of the following:

- Real nonnegative scalar, $\alpha$ - Covariance matrix is an $N$-by- $N$ diagonal matrix, with $\alpha$ as the diagonal elements.
- Vector of real nonnegative scalars, $\left[\alpha_{1}, \ldots, \alpha_{N}\right]$ - Covariance matrix is an $N$-by- $N$ diagonal matrix, with $\left[\alpha_{1}, \ldots, \alpha_{N}\right]$ as the diagonal elements.
- $\quad N$-by- $N$ symmetric positive semidefinite matrix.

The Kalman filter algorithm treats the parameters as states of a dynamic system and estimates these parameters using a Kalman filter. Process Noise Covariance is the covariance of the process noise acting on these parameters. Zero values in the noise covariance matrix correspond to constant coefficients, or parameters. Values larger than 0 correspond to time-varying parameters. Use large values for rapidly changing parameters. However, expect the larger values to result in noisier parameter estimates. The default value is 1 .

## Dependencies

To enable this parameter, set History to Infinite and Estimation Method to Kalman Filter.
Programmatic Use
Block Parameter: AdaptationParameter
Type: scalar, vector, matrix
Default: 1

## Adaptation Gain - Adaptation gain specification for gradient estimation methods

1 (default) | positive scalar
The adaptation gain $\gamma$ scales the influence of new measurement data on the estimation results for the gradient and normalized gradient methods. When your measurements are trustworthy, or in other words have a high signal-to-noise ratio, specify a larger value for $\gamma$. However, setting $\gamma$ too high can cause the parameter estimates to diverge. This divergence is possible even if the measurements are noise free.

When Estimation Method is NormalizedGradient, Adaptation Gain should be less than 2. With either gradient method, if errors are growing in time (in other words, estimation is diverging), or parameter estimates are jumping around frequently, consider reducing Adaptation Gain.

## Dependencies

To enable this parameter, set History to Infinite and Estimation Method to Normalized Gradient or to Gradient.

Programmatic Use
Block Parameter: AdaptationParameter
Type: scalar
Default: 1
Normalization Bias - Bias for adaptation gain scaling for normalized gradient estimation method
eps (default) | nonnegative scalar
The normalized gradient algorithm scales the adaptation gain at each step by the square of the twonorm of the gradient vector. If the gradient is close to zero, the near-zero denominator can cause
jumps in the estimated parameters. Normalization Bias is the term introduced to the denominator to prevent these jumps. Increase Normalization Bias if you observe jumps in estimated parameters.

## Dependencies

To enable this parameter, set History to Infinite and Estimation Method to Normalized Gradient.

Programmatic Use
Block Parameter: NormalizationBias
Type: scalar
Default: eps

## Block Options

## Output estimation error - Add Error outport to block <br> off (default) | on

Use the Error outport signal to validate the estimation. For a given time step $t$, the estimation error $e(t)$ is calculated as:

$$
e(t)=y(t)-y_{\text {est }}(t),
$$

where $y(t)$ is the measured output that you provide, and $y_{\text {est }}(t)$ is the estimated output using the regressors $H(t)$ and parameter estimates $\theta(t-1)$.

```
Programmatic Use
Block Parameter: OutputError
Type: character vector, string
Values: 'off','on',
Default: 'off'
Output parameter covariance matrix - Add covariance outport to block
off (default)| on
```

Use the Covariance outport signal to examine parameter estimation uncertainty. The software computes parameter covariance P assuming that the residuals, $e(t)$, are white noise, and the variance of these residuals is 1 .

The interpretation of P depends on the estimation approach you specify in History and Estimation Method as follows:

- If History is Infinite, then your Estimation Method selection results in:
- Forgetting Factor $-\left(R_{2} / 2\right) \mathrm{P}$ is approximately equal to the covariance matrix of the estimated parameters, where $R_{2}$ is the true variance of the residuals. The block outputs the residuals in the Error port.
- Kalman Filter $-R_{2} \mathrm{P}$ is the covariance matrix of the estimated parameters, and $R_{1} / R_{2}$ is the covariance matrix of the parameter changes. Here, $R_{1}$ is the covariance matrix that you specify in Parameter Covariance Matrix.
- Normalized Gradient or Gradient - Covariance $P$ is not available.
- If History is Finite (sliding-window estimation) $-R_{2} P$ is the covariance of the estimated parameters. The sliding-window algorithm does not use this covariance in the parameterestimation process. However, the algorithm does compute the covariance for output so that you can use it for statistical evaluation.

Programmatic Use
Block Parameter: OutputP
Type: character vector, string
Values: 'off', 'on'
Default: 'off'

## Add enable port - Add Enable inport to block

off (default) | on
Use the Enable signal to provide a control signal that enables or disables parameter estimation. The block estimates the parameter values for each time step that parameter estimation is enabled. If you disable parameter estimation at a given step, $t$, then the software does not update the parameters for that time step. Instead, the block outputs the last estimated parameter values.

- You can use this option, for example, when or if:
- Your regressors or output signal become too noisy, or do not contain information at some time steps
- Your system enters a mode where the parameter values do not change in time


## Programmatic Use

Block Parameter: AddEnablePort
Type: character vector, string
Values: 'off', 'on'
Default: 'off'

## External reset - Specify trigger for external reset

None (default) | Rising | Falling|Either| Level | Level hold
Set the External reset parameter to both add a Reset inport and specify the inport signal condition that triggers a reset of algorithm states to their specified initial values. Reset the estimation, for example, if parameter covariance is becoming too large because of lack of either sufficient excitation or information in the measured signals.

Suppose that you reset the block at a time step, $t$. If the block is enabled at $t$, the software uses the initial parameter values specified in Initial Estimate to estimate the parameter values. In other words, at $t$, the block performs a parameter update using the initial estimate and the current values of the inports.

If the block is disabled at $t$ and you reset the block, the block outputs the values specified in Initial Estimate.

Specify this option as one of the following:

- None - Algorithm states and estimated parameters are not reset.
- Rising - Trigger reset when the control signal rises from a negative or zero value to a positive value. If the initial value is negative, rising to zero triggers reset.
- Falling - Trigger reset when the control signal falls from a positive or a zero value to a negative value. If the initial value is positive, falling to zero triggers reset.
- Either - Trigger reset when the control signal is either rising or falling.
- Level - Trigger reset in either of these cases:
- Control signal is nonzero at the current time step.
- Control signal changes from nonzero at the previous time step to zero at the current time step.
- Level hold - Trigger reset when the control signal is nonzero at the current time step.

When you choose any option other than None, the software adds a Reset inport to the block. You provide the reset control input signal to this inport.

Programmatic Use
Block Parameter: ExternalReset
Type: character vector, string
Values: 'None','Rising','Falling', 'Either', 'Level', 'Level hold'
Default: 'None'

## References

[1] Ljung, L. System Identification: Theory for the User. Upper Saddle River, NJ: Prentice-Hall PTR, 1999, pp. 363-369.
[2] Zhang, Q. "Some Implementation Aspects of Sliding Window Least Squares Algorithms." IFAC Proceedings. Vol. 33, Issue 15, 2000, pp. 763-768.

## Extended Capabilities

C/C++ Code Generation
Generate C and $\mathrm{C}++$ code using Simulink ${ }^{\circledR}$ Coder $^{\mathrm{TM}}$.

## PLC Code Generation

Generate Structured Text code using Simulink ${ }^{\circledR}$ PLC Coder ${ }^{\mathrm{TM}}$.

## See Also

Kalman Filter | Recursive Polynomial Model Estimator

## Topics

"Estimate Parameters of System Using Simulink Recursive Estimator Block"
"Online Recursive Least Squares Estimation"
"Preprocess Online Parameter Estimation Data in Simulink"
"Validate Online Parameter Estimation Results in Simulink"
"Generate Online Parameter Estimation Code in Simulink"
"Recursive Algorithms for Online Parameter Estimation"

Introduced in R2014a

## Recursive Polynomial Model Estimator

Estimate input-output and time-series polynomial model coefficients
Library:
System Identification Toolbox / Estimators


## Description

## Model Structures

Use the Recursive Polynomial Model Estimator block to estimate discrete-time input-output polynomial and time-series models.

These model structures are:

- $\quad \mathrm{AR}-A(q) y(t)=e(t)$
- ARMA $-A(q) y(t)=C(q) e(t)$
- $\operatorname{ARX}-A(q) y(t)=B(q) u\left(t-n_{k}\right)+e(t)$
- $\operatorname{ARMAX}-A(q) y(t)=B(q) u\left(t-n_{k}\right)+C(q) e(t)$
- $\mathrm{OE}-y(t)=\frac{B(q)}{F(q)} u\left(t-n_{k}\right)+e(t)$
- BJ $-y(t)=\frac{B(q)}{F(q)} u\left(t-n_{k}\right)+\frac{C(q)}{D(q)} e(t)$
$q$ is the time-shift operator and $n k$ is the input delay. $u(t)$ is the input, $y(t)$ is the output, and $e(t)$ is the error. For MISO models, there are as many $B(q)$ polynomials as the number of inputs.

The orders of these models correspond to the maximum number of time shifts, as represented by the exponent of $q$. For instance, the order na is represented in the $A(q)$ polynomial by:
$1+a_{1} q^{-1}+a_{2} q^{-2}+\ldots+a_{n a} q^{-n a}$.
An equivalent representation applies to the $C(q), D(q)$, and $F(q)$ polynomials and their corresponding orders $n c, n d$, and $n f$.

The $B(q)$ polynomial is unique with respect to the others, because this polynomial operates on the input and contains the system zeros. For $B(q)$, the order $n b$ is the order of the polynomial $B(q)+1$ :
$b_{1}+b_{2} q^{-1}+b_{3} q^{-2}+\ldots+b_{n b} q^{(n b-1)}$.
The orders $n a, n b, n c, n d, n f$, and input delay $n k$ are known ahead of time. Specify these values as block parameters. Provide $u(t)$ and $y(t)$ through the Inputs and Outputs inports, respectively. The block estimates the set of $A(q), B(q), C(q), D(q)$, and $F(q)$ coefficients that the model structure uses and outputs them in the Parameters outport. During the estimation, the block constrains the estimated $C, D$, and $F$ polynomials to a stable region with roots in the unit disk, while allowing the
estimated $A$ and $B$ polynomials to be unstable. The Parameters outport provides a bus signal with the following elements:

- A - Vector containing $\left[1 a_{1}(t) \ldots a_{n a}(t)\right]$.
- B - Vector containing $\left[0_{1} \ldots 0_{n k}, b_{1}(t) \ldots b_{n b}(t)\right]$. For MISO data, $B$ is a matrix where the $i$-th row parameters correspond to the $i$-th input.
- C - Vector containing $\left[1 c_{1}(t) \ldots c_{n c}(t)\right]$.
- $\mathrm{D}-$ Vector containing $\left[1 d_{1}(t) \ldots d_{n d}(t)\right]$.
- $\mathrm{F}-$ Vector containing $\left[1 f_{1}(t) \ldots f_{n f}(t)\right]$.

For example, suppose that you want to estimate the coefficients for the following SISO ARMAX model:
$y(t)+a_{1} y(t-1)+\ldots+a_{n a} y(t-n a)=b_{1} u(t-n k)+\ldots+b_{n b} u(t-n b-n k+1)+e(t)+c_{1} e(t-1)+\ldots+$ $c_{n c} e(t-n c)$
$y, u, n a, n b, n c$, and $n k$ are known quantities that you provide to the block. For each time step, $t$, the block estimates the $A, B$, and $C$ parameter values, constraining only the $C$ polynomial to a stable region. The block then outputs these estimated values using the Parameters outport.

## Block Capabilities

The block supports several estimation methods and data input formats. The block can provide both infinite-history [1] and finite-history [2] (also known as sliding-window) estimates for $\theta$. Configurable options in the block include:

- Multiple inputs (ARX model structure only) - See the Inputs port.
- Sample-based or frame-based data format - See the Input Processing parameter.
- Multiple infinite-history estimation methods [1] - See the Estimation Method parameter.
- Infinite-history (all model structures) or finite-history (AR, ARX, or OE model structures only) See the History parameter.
- Initial conditions, enable flag, and reset trigger - See the Initial Estimate, Add enable port, and External Reset parameters.

For more information on the estimation methods, see "Recursive Algorithms for Online Parameter Estimation".

## Ports

## Input

## Inputs - Input signal

vector | matrix
Input signal $u(t)$. The Input Processing parameter and the number of inputs $n u$ define the dimensions of the signal. Only the ARX model structure can have multiple inputs, with nu greater than 1.

- Sample-based input processing and $n u$ inputs - nu-by-1 vector
- Frame-based input processing with $M$ samples per frame and $n u$ inputs $-M$-by-nu matrix


## Dependencies

To enable this port, set the Model Structure parameter to ARX, ARMAX, BJ, or 0 E .
Data Types: single|double

## Output - Measured output signal

scalar | vector
Measured output signal $y(t)$. The Input Processing parameter defines the dimensions of the signal:

- Sample-based input processing - Scalar
- Frame-based input processing with $M$ samples per frame $-M$-by-1 vector

Data Types: single | double

## Enable - Enable estimation updates

true (default) | false
External signal that allows you to enable and disable estimation updates. If the signal value is:

- true - Estimate and output the parameter values for the time step.
- false - Do not estimate the parameter values, and output the most recent previously estimated value.


## Dependencies

To enable this port, select the Add enable port parameter.
Data Types: single | double | Boolean | int8| int16|int32|int64|uint8|uint16|uint32

## Reset - Reset trigger

scalar
Reset parameter estimation to its initial conditions. The value of the External reset parameter determines the trigger type. The trigger type dictates whether the reset occurs on a signal that is rising, falling, either rising or falling, level, or on level hold.

## Dependencies

To enable this port, select any option other than None in the External reset dropdown.

## Data Types: single | double | Boolean | int8 | int16|int32|uint8|uint16|uint32

## InitialParameters - Initial parameter estimates <br> bus object

Initial parameter estimates, supplied from a source external to the block. The block uses this inport at the beginning of the simulation or when you trigger an algorithm reset using the Reset signal.

For information on the contents of the InitialParameters bus object, see the Parameters port description.

## Dependencies

To enable this port, set History to Infinite and Initial Estimate to External.
Data Types: single | double

## InitialCovariance - Initial covariance of parameters

positive scalar (default) | vector of positive scalars | symmetric positive-definite matrix
Initial parameter covariances, supplied from a source external to the block. For details, see the Parameter Covariance Matrix parameter. The block uses this inport at the beginning of the simulation or when you trigger an algorithm reset using the Reset signal.

## Dependencies

To enable this port, set the following parameters:

- History to Infinite
- Estimation Method to Forgetting Factor or Kalman Filter
- Initial Estimate to External

Data Types: single | double

## InitialInputs - Initial values of the inputs <br> matrix

Initial set of inputs when using finite-history (sliding-window) estimation, supplied from an external source.

- If Model Structure is ARX, then the signal to this port must be a $(W-1+\max (n b)+\max (n k)$ )-by-nu matrix, where $W$ is the window length and $n u$ is the number of inputs. $n b$ is the vector of $B(q)$ polynomial orders and $n k$ is the vector of input delays.
- If Model Structure is 0 E , then the signal to this port must be a $(W-1+n b+n k)$-by- 1 vector, where $W$ is the window length. $n b$ is the vector of $B(q)$ polynomial orders and $n k$ is the vector of input delays.

The block uses this inport at the beginning of the simulation or whenever the Reset signal triggers.
If the initial buffer is set to 0 or does not contain enough information, you see a warning message during the initial phase of your estimation. The warning should clear after a few cycles. The number of cycles it takes for sufficient information to be buffered depends upon the order of your polynomials and your input delays. If the warning persists, you should evaluate the content of your signals.

## Dependencies

To enable this port, set:

- History to Finite
- Model Structure to ARX or OE
- Initial Estimate to External

Data Types: single | double

## Initial0utputs - Initial values of the measured outputs buffer vector

Initial set of output measurements when using finite-history (sliding-window) estimation, supplied from an external source.

- If Model Structure is AR or ARX, then the signal to this port must be a ( $W+n a$ )-by- 1 vector, where $W$ is the window length and $n a$ is the polynomial order of $A(q)$.
- If Model Structure is 0 E , then the signal to this port must be a ( $W+n f$ )-by-1 vector, where $W$ is the window length and $n f$ is the polynomial order of $F(q)$.

The block uses this inport at the beginning of the simulation or whenever the Reset signal triggers.
If the initial buffer is set to 0 or does not contain enough information, you see a warning message during the initial phase of your estimation. The warning should clear after a few cycles. The number of cycles it takes for sufficient information to be buffered depends upon the order of your polynomials and your input delays. If the warning persists, you should evaluate the content of your signals.

## Dependencies

To enable this port, set:

- History to Finite
- Model Structure to AR, ARX, or OE
- Initial Estimate to External

Data Types: single | double
Output

## Parameters - Estimated parameters

bus object
Estimated polynomial coefficients, returned as a bus. The bus contains an element for each of the $A$, $B, C, D$, and $F$ polynomials that correspond to the structure that you specify in Model Structure (see "Model Structures" on page 2-71 ).

Each bus element is a vector signal containing the associated polynomial coefficients. For example, the $A$ element contains $\left[1 a_{1}(t) \ldots a_{n a}(t)\right]$.

Estimated $C, D$, and $F$ values are constrained to be stable discrete-time polynomials. That is, these polynomials all have roots within the unit circle. Estimated $A$ and $B$ polynomials are allowed to be unstable.
Data Types: single | double

## Error - Estimation error

scalar | vector
Estimation error, returned as:

- Scalar - Sample-based input processing
- $M$-by-1 vector - Frame-based input processing with $M$ samples per frame


## Dependencies

To enable this port, select the Output estimation error parameter.

## Data Types: single|double

Covariance - Parameter estimation error covariance $\mathbf{P}$
matrix

Parameter estimation error covariance $P$, returned as an $N$-by- $N$ matrix, where $N$ is the number of parameters. For details, see the Output Parameter Covariance Matrix parameter.

## Dependencies

To enable this port:

- Select the Output parameter covariance matrix parameter.
- If History is Infinite, set Estimation Method to Forgetting Factor or Kalman Filter.

Data Types: single | double

## Parameters

## Model Structure

Estimated model structure, specified as one of the following:

- ARX - SISO or MISO ARX model
- ARMAX - SISO ARMAX model
- OE - SISO OE model
- BJ - SISO BJ model
- AR - Time-series AR model
- ARMA - Time-series ARMA model


## Model Parameters

## Initial Estimate - Source of initial parameter estimates

None (default) | Internal | External
Specify how to provide initial parameter estimates to the block:

- None - Do not specify initial estimates.

The block uses 0 as the initial parameter estimate.
Specify the parameters that the block enables based on your choice of model structure and estimation method.

- Specify the set of Number of Parameters () parameters that the block enables based on your Model Structure. For instance, if your setting for Model Structure is AR, specify the Number of Parameters in A(q) (na) parameter.
- Specify the Input Delay (nk) parameter that the block enables when your model structure uses a $B(q)$ element.
- Specify the Parameter Covariance Matrix if Estimation Method is Forgetting Factor or Kalman Filter.
- Internal - Specify initial parameter estimates internally to the block.
- Specify the initial parameter values Initial () parameters that the block enables based on your Model Structure and History. For instance, if your setting for Model Structure is AR and History is Infinite, specify the Initial A(q) parameter.
- Specify the Input Delay (nk) parameter that the block enables when your model structure uses a $B(q)$ element.
- Specify the Parameter Covariance Matrix parameter if Estimation Method is Forgetting Factor or Kalman Filter.
- Specify the Initial Inputs parameter (ARX and OE only) and the Initial Outputs parameter (ARX, AR, and OE) if History is Finite.
- External - Specify initial parameter estimates as an input signal to the block.

Specify the Number of Parameters () parameters that the block enables based on your Model Structure. Your setting for Model Structure and for the History parameter determines which signals to connect to the relevant ports:

- If History is Infinite - InitialParameters and InitialCovariance
- If History is Finite - InitialOutputs for the AR, ARX, and OE model structures, and InitialInputs for the ARX and OE model structures

Programmatic Use
Block Parameter: InitialEstimateSource
Type: character vector, string
Values: 'None', 'Internal', 'External'
Default: 'None'
Number of Parameters in $A(q)$ (na) - Number of estimated parameters in the $A(q)$ polynomial
1 (default) | non-negative integer
Specify the number of estimated parameters na in the $A(q)$ polynomial.

## Dependencies

To enable this parameter, either:

- Set History to Infinite, Model Structure to AR, ARX, ARMA, or ARMAX, and Initial Estimate to None or External.
- Set History to Finite, Model Structure to AR or ARX, and Initial Estimate to None or External.

Programmatic Use
Block Parameter: A0
Type: non-negative integer
Default: 1

## Number of Parameters in $B(q)(n b)$ - Number of estimated parameters in the $B(q)$ polynomial <br> 1 (default) | vector of positive integers

Specify the number of estimated parameters $n b$ in the $B(q)$ polynomial.
For MISO systems using an ARX model structure, specify $n b$ as a vector with as many elements as there are inputs. Each element of this vector represents the order of the $B(q)$ polynomial associated with the corresponding input. For example, suppose that you have a two-input MISO system whose $B(q)$ elements are: $\left[\begin{array}{l}B_{1} \\ B_{2}\end{array}\right]=\left[\begin{array}{ccc}0 & b_{11} & 0 \\ 0 & b_{21} & b_{22}\end{array}\right]$. The zero at the beginning of each polynomial represents a
single input delay for each input (see the Initial B(q) parameter description). The trailing zero in $B_{1}$ is for equalizing the length of the polynomials and has no impact on estimation. nb for each polynomial is equal to the number of estimated parameters following the initial zero, or 1 for input 1 and 2 for input 2. Specify Number of Parameters in $\mathbf{B ( q )}$ (nb) as [1 2], and Input Delay (nk) as [11].

## Dependencies

To enable this parameter, either:

- Set History to Infinite, Model Structure to ARX, ARMAX, BJ, or OE, and Initial Estimate to None or External.
- Set History to Finite with Model Structure of ARX or OE and Initial Estimate to None or External.

Programmatic Use
Block Parameter: B0
Type: positive integer
Default: 1

```
Number of Parameters in C(q) (nc) - Number of estimated parameters in the C(q)
polynomial
1 (default) | positive integer
```

Specify the number of estimated parameters $n c$ in the $C(q)$ polynomial.

## Dependencies

To enable this parameter, set History to Infinite, Model Structure to ARMA, ARMAX, or BJ, and Initial Estimate to None or External

## Programmatic Use

Block Parameter: C0
Type: positive integer
Default: 1
Number of Parameters in $D(q)(n d)$ - Number of estimated parameters in the $D(q)$ polynomial
1 (default) | positive integer
Specify the number of estimated parameters nd in the $D(q)$ polynomial.

## Dependencies

To enable this parameter, set History to Infinite, Model Structure to BJ, and Initial Estimate to None or External.

Programmatic Use
Block Parameter: D0
Type: positive integer
Default: 1
Number of Parameters in $F(q)(n f)$ - Number of estimated parameters in the $F(q)$ polynomial
1 (default) | positive integer
Specify the number of estimated parameters $n f$ in the $F(q)$ polynomial.

## Dependencies

To enable this parameter, set Initial Estimate to None or External and either:

- History to Infinite, Model Structure to OE or BJ, and Initial Estimate to None or External
- History to Finite, Model Structure to OE, and Initial Estimate to None or External

Programmatic Use
Block Parameter: F0
Type: positive integer
Default: 1
Input Delay (nk) - Input delay
1 (default) | vector of non-negative integers
Specify the input delay as an integer representing the number of time steps that occur before the input affects the output. This delay is also called the dead time in the system. The block encodes the input delay as fixed leading zeros of the $B(q)$ polynomial. For more information, see the $B(q)$ parameter description.

For MISO systems with ARX model structure, specify $n k$ as a vector with elements specifying the delay for each input. This vector is of length $n u$, where $n u$ is the number of inputs.

For example, suppose that you have a two-input MISO system whose $B(q)$ elements are:
$\left[\begin{array}{l}B_{1} \\ B_{2}\end{array}\right]=\left[\begin{array}{ccc}0 & b_{11} & b_{12} \\ 0 & 0 & b_{21}\end{array}\right]$. The zeros at the beginning of the polynomials represent a single-sample delay for the first input, and a two-sample delay for the second input (see the Initial B(q) parameter description). $n b$ for each polynomial is equal to the number of estimated parameters following the initial zeros, or 2 for input 1 and 1 for input 2. Specify Input Delay (nk) as [1 2], and Number of Parameters in B(q) (nb) as [2 1].

## Dependencies

To enable this parameter, either:

- Set History to Infinite, Model Structure to ARX, ARMAX, OE, or BJ, and Initial Estimate to None or External.
- Set History to Finite, Model Structure to ARX or OE, and Initial Estimate to None or External.


## Programmatic Use

Block Parameter: nk
Type: non-negative integer vector
Default: 1

## Parameter Covariance Matrix - Initial parameter covariance

## le4 (default) | scalar | vector | matrix

- Real positive scalar, $\alpha$ - Covariance matrix is an $N$-by- $N$ diagonal matrix, with $\alpha$ as the diagonal elements.
- Vector of real positive scalars, $[\alpha(a), \alpha(b), \alpha(c), \alpha(d), \alpha(f)]$ - Covariance matrix is an $N$-by- $N$ diagonal matrix, with $[\alpha(a), \alpha(b), \alpha(c), \alpha(d), \alpha(f)]$ as the diagonal elements. $\alpha(a)$ is a vector of the
covariance for each coefficient of the $A$ polynomial. Similarly, $\alpha(b), \alpha(c), \alpha(d)$ and $\alpha(f)$ are vectors containing the covariance of the coefficients of the $B, C, D$ and $F$ polynomials, respectively.
- $N$-by- $N$ symmetric positive-definite matrix.
$N$ can be one of the following:
- $\mathrm{AR}-N=n a$
- ARX $-N=n a+\sum_{i=1}^{N_{u}} n b_{i}$
- ARMA $-N=n a+n c$
- ARMAX $-N=n a+n b+n c$
- $\mathrm{OE}-N=n b+n f$
- BJ $-N=n b+n c+n d+n f$


## Dependencies

To enable this parameter, set

- History to Infinite
- Initial Estimate to None or Internal
- Estimation Method to Forgetting Factor or Kalman Filter

The block uses this parameter at the beginning of the simulation or whenever the Reset signal triggers.

## Programmatic Use

Block Parameter: P0
Type: scalar, vector, or matrix
Default: 1e4

## Initial $A(q)$ - Initial values of the $A(q)$ polynomial coefficients

[1 eps] (default)|vector
Specify the initial estimate of the $A(q)$ polynomial coefficients as a row vector of length na+1.
The leading coefficient of $A$ must be 1 .

## Dependencies

To enable this parameter, set:

- Model Structure to AR, ARX, ARMA, or ARMAX
- Initial Estimate to Internal

Programmatic Use
Block Parameter: A0
Type: real vector
Default: [1 eps]

## Initial $B(q)$ - Initial values of the $B(q)$ polynomial coefficients

[0 eps] (default) | vector | matrix

Specify the initial estimate of the $B(q)$ polynomial coefficients as a row vector of length $n b+n k$. For multiple-input models, specify a matrix where the $i$ th row corresponds to the ith input.

The block counts the leading zeros in $B(q)$ and interprets them as input delay $n k$. Those zeros are fixed throughout the estimation. $n b$ is the number of elements after the first nonzero element in $B(q)$. The block estimates the value of these $n b$ elements.

For example:

- [0 eps] corresponds to $n k=1$ and $n b=1$.
- [0 00 eps] corresponds to $n k=2$ and $n b=1$.
- [0 0 eps 0 eps] corresponds to $n k=2$ and $n b=3$.

The default value is [0 eps].

## Dependencies

To enable this parameter, set:

- Model Structure to ARX, ARMAX, OE, or BJ
- Initial Estimate to Internal

Programmatic Use
Block Parameter: B0
Type: real vector or matrix
Default: [0 eps]

## Initial $C(q)$ - Initial values of the $C(q)$ polynomial coefficients <br> [1 eps] (default)|vector

Specify the initial estimate of the $C(q)$ polynomial coefficients as a row vector of length $n c+1$.
The leading coefficient of $C(q)$ must be 1 .
The coefficients must define a stable discrete-time polynomial, that is, have all polynomial roots within the unit circle.

## Dependencies

To enable this parameter, set:

- History to Infinite
- Model Structure to ARMA, ARMAX, or BJ
- Initial Estimate to Internal

Programmatic Use
Block Parameter: C0
Type: real vector
Default: [1 eps]

## Initial $D(q)$ - Initial values of the $D(q)$ polynomial coefficients

[1 eps] (default)|vector
Specify the initial estimate of the $D(q)$ polynomial coefficients as a row vector of length $n d+1$.

The leading coefficient of $D(q)$ must be 1.
The coefficients must define a stable discrete-time polynomial, that is, have all polynomial roots within the unit circle.

## Dependencies

To enable this parameter, set:

- History to Infinite
- Model Structure to BJ
- Initial Estimate to Internal

Programmatic Use
Block Parameter: D0
Type: real vector
Default: [1 eps]
Initial $F(q)$ - Initial values of the $F(q)$ polynomial coefficients
[1 eps] (default)|vector
Specify the initial estimate of the $F(q)$ polynomial coefficients as a row vector of length $n f+1$.
The leading coefficient of $F(q)$ must be 1 .
The coefficients must define a stable discrete-time polynomial, that is, have all polynomial roots within the unit circle.

## Dependencies

To enable this parameter, set:

- Model Structure to OE or to BJ
- Initial Estimate to Internal

Programmatic Use
Block Parameter: F0
Type: real vector
Default: [1 eps]

## Initial Inputs - Initial values of the inputs buffer <br> 0 (default) | matrix

Specify initial values of the inputs buffer when using finite-history (sliding window) estimation. The buffer dimensions accommodate the specified window length, the regressors associated with polynomials within that window, the input delays, and the number of inputs. These elements drive a matrix size of:

- ARX model structure $-(W-1+\max (n b)+\max (n k))$-by-nu
- OE model structure $-(W-1+n b+n k)$-by- 1
where $W$ is the window length and $n u$ is the number of inputs. $n b$ is the vector of $B(q)$ polynomial orders and $n k$ is the vector of input delays.

When the initial value is set to 0 , the block populates the buffer with zeros.

If the initial buffer is set to 0 or does not contain enough information, you see a warning message during the initial phase of your estimation. The warning should clear after a few cycles. The number of cycles it takes for sufficient information to be buffered depends upon the order of your polynomials and your input delays. If the warning persists, you should evaluate the content of your signals.

The block uses this parameter at the beginning of the simulation or whenever the Reset signal triggers.

## Dependencies

To enable this parameter, set

- History to Finite
- Model Structure to ARX or OE
- Initial Estimate to Internal

```
Programmatic Use
Block Parameter: InitialInputs
Type: real matrix
Default: 0
```


## Initial Outputs - Initial values of the measured outputs buffer <br> 0 (default) | vector

Specify initial values of the measured outputs buffer when using finite-history (sliding-window) estimation. The buffer dimensions accommodate the specified window length and the regressors associated with the polynomials within that window.

- AR or ARX model structure - $(W+n a)$-by- 1 vector, where $W$ is the window length and $n a$ is the polynomial order of $A(q)$.
- OE model structure - $(W+n f)$-by-1 vector, where $W$ is the window length and $n f$ is the polynomial order of $F(q)$.

When the initial value is set to 0 , the block populates the buffer with zeros.
If the initial buffer is set to 0 or does not contain enough information, you see a warning message during the initial phase of your estimation. The warning should clear after a few cycles. The number of cycles it takes for sufficient information to be buffered depends upon the order of your polynomials and your input delays. If the warning persists, you should evaluate the content of your signals.

The block uses this parameter at the beginning of the simulation or whenever the Reset signal triggers.

## Dependencies

To enable this parameter, set:

- History to Finite
- Model Structure to AR, ARX, or OE
- Initial Estimate to Internal

```
Programmatic Use
Block Parameter: Initial0utputs
Type: real vector
Default: 0
```


## Input Processing and Sample Time

## Input Processing - Choose sample-based or frame-based processing <br> Sample-based (default) | Frame-based

- Sample-based processing operates on signals streamed one sample at a time.
- Frame-based processing operates on signals containing samples from multiple time steps. Many machine sensor interfaces package multiple samples and transmit these samples together in frames. Frame-based processing allows you to input this data directly without having to first unpack it.

Specifying frame-based data adds an extra dimension of $M$ to some of your data inports and outports, where $M$ is the number of time steps in a frame. These ports are:

## - Inputs

- Output
- Error

For more information, see the port descriptions in "Ports" on page 2-72.

## Programmatic Use

Block Parameter: InputProcessing
Type: character vector, string
Values: 'Sample-based', 'Frame-based '
Default: 'Sample-based'

## Sample Time - Block sample time

-1 (default) | positive scalar
Specify the data sample time, whether by individual samples for sample-based processing $\left(t_{s}\right)$, or by frames for frame-based processing $\left(t_{f}=M t_{s}\right)$, where $M$ is the frame length. When you set Sample Time to its default value of -1 , the block inherits its $t_{s}$ or $t_{f}$ based on the signal.

Specify Sample Time as a positive scalar to override the inheritance.

## Programmatic Use <br> Block Parameter: Ts

Type: real scalar
Values: -1 , positive scalar
Default: -1

## Algorithm and Block Options

## Algorithm Options

## History - Choose infinite or finite data history <br> Infinite (default)|Finite

The History parameter determines what type of recursive algorithm you use:

- Infinite - Algorithms in this category aim to produce parameter estimates that explain all data since the start of the simulation. These algorithms retain the history in a data summary. The block maintains this summary within a fixed amount of memory that does not grow over time.

The block provides multiple algorithms of the Infinite type. Selecting this option enables the Estimation Method parameter with which you specify the algorithm.

- Finite - Algorithms in this category aim to produce parameter estimates that explain only a finite number of past data samples. The block uses all of the data within a finite window, and discards data once that data is no longer within the window bounds. This method is also called sliding-window estimation.

The block provides one algorithm of the Finite type. You can use this option only with the AR, ARX, and OE model structures.

Selecting this option enables the Window Length parameter.
For more information on recursive estimation methods, see "Recursive Algorithms for Online Parameter Estimation"

Programmatic Use
Block Parameter: History
Type: character vector, string
Values: 'Infinite', 'Finite'
Default: 'Infinite'

## Window Length - Window size for finite sliding-window estimation

200 (default) | positive integer
The Window Length parameter determines the number of time samples to use for the finite-history (sliding-window) estimation method. Choose a window size that balances estimation performance with computational and memory burden. Sizing factors include the number and time variance of the parameters in your model. Always specify Window Length in samples, even if you are using framebased input processing.

Window Length must be greater than or equal to the number of estimated parameters.
Suitable window length is independent of whether you are using sample-based or frame-based input processing. However, when using frame-based processing, Window Length must be greater than or equal to the number of samples (time steps) contained in the frame.

## Dependencies

To enable this parameter, set History to Finite.

## Programmatic Use

Block Parameter: WindowLength
Type: positive integer
Default: 200

## Estimation Method - Recursive estimation algorithm

Forgetting Factor (default)|Kalman Filter|Normalized Gradient|Gradient
Specify the estimation algorithm when performing infinite-history estimation. When you select any of these methods, the block enables additional related parameters.

Forgetting factor and Kalman filter algorithms are more computationally intensive than gradient and normalized gradient methods. However, these more intensive methods have better convergence properties than the gradient methods. For more information about these algorithms, see "Recursive Algorithms for Online Parameter Estimation".

Programmatic Use
Block Parameter: EstimationMethod
Type: character vector, string
Values: 'Forgetting Factor','Kalman Filter','Normalized Gradient','Gradient'
Default: 'Forgetting Factor'

## Forgetting Factor - Discount old data using forgetting factor

1 (default) | positive scalar in (0 1] range
The forgetting factor $\lambda$ specifies if and how much old data is discounted in the estimation. Suppose that the system remains approximately constant over $T_{0}$ samples. You can choose $\lambda$ such that:

$$
T_{0}=\frac{1}{1-\lambda}
$$

- Setting $\lambda=1$ corresponds to "no forgetting" and estimating constant coefficients.
- Setting $\lambda<1$ implies that past measurements are less significant for parameter estimation and can be "forgotten." Set $\lambda<1$ to estimate time-varying coefficients.

Typical choices of $\lambda$ are in the [0.98 0.995] range.

## Dependencies

To enable this parameter, set History to Infinite and Estimation Method to Forgetting Factor.

## Programmatic Use

Block Parameter: AdaptationParameter
Type: scalar
Values: (0 1] range
Default: 1

## Process Noise Covariance - Process noise covariance for Kalman filter estimation method

1 (default) | nonnegative scalar | vector of nonnegative scalars | symmetric positive semidefinite matrix

Process Noise Covariance prescribes the elements and structure of the noise covariance matrix for the Kalman filter estimation. Using $N$ as the number of parameters to estimate, specify the Process Noise Covariance as one of the following:

- Real nonnegative scalar, $\alpha$ - Covariance matrix is an $N$-by- $N$ diagonal matrix, with $\alpha$ as the diagonal elements.
- Vector of real nonnegative scalars, $\left[\alpha_{1}, \ldots, \alpha_{N}\right]$ - Covariance matrix is an $N$-by- $N$ diagonal matrix, with $\left[\alpha_{1}, \ldots, \alpha_{N}\right]$ as the diagonal elements.
- $N$-by- $N$ symmetric positive semidefinite matrix.

The Kalman filter algorithm treats the parameters as states of a dynamic system and estimates these parameters using a Kalman filter. Process Noise Covariance is the covariance of the process noise acting on these parameters. Zero values in the noise covariance matrix correspond to constant
coefficients, or parameters. Values larger than 0 correspond to time-varying parameters. Use large values for rapidly changing parameters. However, expect the larger values to result in noisier parameter estimates. The default value is 1 .

## Dependencies

To enable this parameter, set History to Infinite and Estimation Method to Kalman Filter.

## Programmatic Use

Block Parameter: AdaptationParameter
Type: scalar, vector, matrix
Default: 1

## Adaptation Gain - Adaptation gain specification for gradient estimation methods

1 (default) | positive scalar
The adaptation gain $\gamma$ scales the influence of new measurement data on the estimation results for the gradient and normalized gradient methods. When your measurements are trustworthy, or in other words have a high signal-to-noise ratio, specify a larger value for $\gamma$. However, setting $\gamma$ too high can cause the parameter estimates to diverge. This divergence is possible even if the measurements are noise free.

When Estimation Method is NormalizedGradient, Adaptation Gain should be less than 2. With either gradient method, if errors are growing in time (in other words, estimation is diverging), or parameter estimates are jumping around frequently, consider reducing Adaptation Gain.

## Dependencies

To enable this parameter, set History to Infinite and Estimation Method to Normalized Gradient or to Gradient.

Programmatic Use
Block Parameter: AdaptationParameter
Type: scalar
Default: 1

## Normalization Bias - Bias for adaptation gain scaling for normalized gradient estimation method <br> eps (default) | nonnegative scalar

The normalized gradient algorithm scales the adaptation gain at each step by the square of the twonorm of the gradient vector. If the gradient is close to zero, the near-zero denominator can cause jumps in the estimated parameters. Normalization Bias is the term introduced to the denominator to prevent these jumps. Increase Normalization Bias if you observe jumps in estimated parameters.

## Dependencies

To enable this parameter, set History to Infinite and Estimation Method to Normalized Gradient.

Programmatic Use
Block Parameter: NormalizationBias
Type: scalar
Default: eps

## Block Options

## Output estimation error - Add Error outport to block <br> off (default) | on

Use the Error outport signal to validate the estimation. For a given time step $t$, the estimation error $e(t)$ is calculated as:

$$
e(t)=y(t)-y_{\text {est }}(t),
$$

where $y(t)$ is the measured output that you provide, and $y_{\text {est }}(t)$ is the estimated output using the regressors $H(t)$ and parameter estimates $\theta(t-1)$.

Programmatic Use
Block Parameter: OutputError
Type: character vector, string
Values: 'off','on',
Default: 'off'
Output parameter covariance matrix - Add covariance outport to block
off (default) | on
Use the Covariance outport signal to examine parameter estimation uncertainty. The software computes parameter covariance P assuming that the residuals, $e(t)$, are white noise, and the variance of these residuals is 1 .

The interpretation of P depends on the estimation approach you specify in History and Estimation Method as follows:

- If History is Infinite, then your Estimation Method selection results in:
- Forgetting Factor $-\left(R_{2} / 2\right) \mathrm{P}$ is approximately equal to the covariance matrix of the estimated parameters, where $R_{2}$ is the true variance of the residuals. The block returns these residuals through the Error port.
- Kalman Filter $-R_{2} \mathrm{P}$ is the covariance matrix of the estimated parameters, and $R_{1} / R_{2}$ is the covariance matrix of the parameter changes. Here, $R_{1}$ is the covariance matrix that you specify in Parameter Covariance Matrix.
- Normalized Gradient or Gradient - Covariance P is not available.
- If History is Finite (sliding-window estimation) $-R_{2} \mathrm{P}$ is the covariance of the estimated parameters. The sliding-window algorithm does not use this covariance in the parameterestimation process. However, the algorithm does compute the covariance for output so that you can use it for statistical evaluation.

Programmatic Use
Block Parameter: OutputP
Type: character vector, string
Values: 'off', 'on'
Default: ' off'
Add enable port - Add Enable inport to block
off (default) | on
Use the Enable signal to provide a control signal that enables or disables parameter estimation. The block estimates the parameter values for each time step that parameter estimation is enabled. If you
disable parameter estimation at a given step, $t$, then the software does not update the parameters for that time step. Instead, the block output contains the last estimated parameter values.

You can use this option, for example, when or if:

- Your regressors or output signal become too noisy, or do not contain information at some time steps
- Your system enters a mode where the parameter values do not change in time

Programmatic Use
Block Parameter: AddEnablePort
Type: character vector, string
Values: 'off', 'on'
Default: 'off'

## External reset - Specify trigger for external reset

None (default) | Rising | Falling | Either | Level | Level hold
Set the External reset parameter to both add a Reset inport and specify the inport signal condition that triggers a reset of algorithm states to their specified initial values. Reset the estimation, for example, if parameter covariance is becoming too large because of lack of either sufficient excitation or information in the measured signals. The External reset parameter determines the timing for the reset.

Suppose that you reset the block at a time step, $t$. If the block is enabled at $t$, the software uses the initial parameter values specified in Initial Estimate to estimate the parameter values. In other words, at $t$, the block performs a parameter update using the initial estimate and the current values of the inports.

If the block is disabled at $t$ and you reset the block, the block output contains the values specified in Initial Estimate.

Specify this option as one of the following:

- None - Algorithm states and estimated parameters are not reset.
- Rising - Trigger reset when the control signal rises from a negative or zero value to a positive value. If the initial value is negative, rising to zero triggers reset.
- Falling - Trigger reset when the control signal falls from a positive or a zero value to a negative value. If the initial value is positive, falling to zero triggers reset.
- Either - Trigger reset when the control signal is either rising or falling.
- Level - Trigger reset in either of these cases:
- Control signal is nonzero at the current time step.
- Control signal changes from nonzero at the previous time step to zero at the current time step.
- Level hold - Trigger reset when the control signal is nonzero at the current time step.

When you choose any option other than None, the software adds a Reset inport to the block. You provide the reset control input signal to this inport.
Programmatic Use
Block Parameter: ExternalReset
Type: character vector, string

Values: 'None','Rising','Falling', 'Either', 'Level', 'Level hold'
Default: 'None'

## References

[1] Ljung, L. System Identification: Theory for the User. Upper Saddle River, NJ: Prentice-Hall PTR, 1999, pp. 363-369.
[2] Zhang, Q. "Some Implementation Aspects of Sliding Window Least Squares Algorithms." IFAC Proceedings. Vol. 33, Issue 15, 2000, pp. 763-768.

## Extended Capabilities

C/C++ Code Generation
Generate C and C++ code using Simulink ${ }^{\circledR}$ Coder ${ }^{\mathrm{TM}}$.

## PLC Code Generation

Generate Structured Text code using Simulink ${ }^{\circledR}$ PLC Coder ${ }^{\text {TM }}$.

## See Also

Kalman Filter | Recursive Least Squares Estimator

## Topics

"Estimate Parameters of System Using Simulink Recursive Estimator Block"
"Online Recursive Least Squares Estimation"
"Preprocess Online Parameter Estimation Data in Simulink"
"Validate Online Parameter Estimation Results in Simulink"
"Generate Online Parameter Estimation Code in Simulink"
"Recursive Algorithms for Online Parameter Estimation"
Introduced in R2014a

## Unscented Kalman Filter

Estimate states of discrete-time nonlinear system using unscented Kalman filter
Library:
Control System Toolbox / State Estimation
System Identification Toolbox / Estimators


## Description

The Unscented Kalman Filter block estimates the states of a discrete-time nonlinear system using the discrete-time unscented Kalman filter algorithm.

Consider a plant with states $x$, input $u$, output $y$, process noise $w$, and measurement noise $v$. Assume that you can represent the plant as a nonlinear system.


Using the state transition and measurement functions of the system and the unscented Kalman filter algorithm, the block produces state estimates $\widehat{x}$ for the current time step. For information about the algorithm, see "Extended and Unscented Kalman Filter Algorithms for Online State Estimation".

You create the nonlinear state transition function and measurement functions for the system and specify these functions in the block. The block supports state estimation of a system with multiple sensors that are operating at different sampling rates. You can specify up to five measurement functions, each corresponding to a sensor in the system. For more information, see "State Transition and Measurement Functions" on page 2-103.

## Ports

## Input

## $\mathrm{y} 1, \mathrm{y} 2, \mathrm{y} 3, \mathrm{y} 4, \mathrm{y} 5$ - Measured system outputs

vector
Measured system outputs corresponding to each measurement function that you specify in the block. The number of ports equals the number of measurement functions in your system. You can specify up to five measurement functions. For example, if your system has two sensors, you specify two measurement functions in the block. The first port y1 is available by default. When you click Apply, the software generates port $\mathbf{y} \mathbf{2}$ corresponding to the second measurement function.

Specify the ports as $N$-dimensional vectors, where $N$ is the number of quantities measured by the corresponding sensor. For example, if your system has one sensor that measures the position and velocity of an object, then there is only one port $\mathbf{y 1}$. The port is specified as a 2-dimensional vector with values corresponding to position and velocity.

## Dependencies

The first port $\mathbf{y 1}$ is available by default. Ports $\mathbf{y} \mathbf{2}$ to $\mathbf{y} 5$ are generated when you click Add Measurement, and click Apply.
Data Types: single|double

## StateTransitionFcnInputs - Additional optional input argument to state transition function <br> scalar | vector | matrix

Additional optional input argument to the state transition function $f$ other than the state $x$ and process noise w. For information about state transition functions see, "State Transition and Measurement Functions" on page 2-103.

Suppose that your system has nonadditive process noise, and the state transition function $f$ has the following form:
$x(k+1)=f(x(k), w(k)$, StateTransitionFcnInputs).
Here k is the time step, and StateTransitionFcnInputs is an additional input argument other than $x$ and $w$.

If you create f using a MATLAB function (.m file), the software generates the port
StateTransitionFcnInputs when you click Apply. You can specify the inputs to this port as a scalar, vector, or matrix.

If your state transition function has more than one additional input, use a Simulink Function block to specify the function. When you use a Simulink Function block, you provide the additional inputs directly to the Simulink Function block using Inport blocks. No input ports are generated for the additional inputs in the Unscented Kalman Filter block.

## Dependencies

This port is generated only if both of the following conditions are satisfied:

- You specify $f$ in Function using a MATLAB function, and $f$ is on the MATLAB path.
- f requires only one additional input argument apart from $x$ and $w$.

Data Types: single | double
MeasurementFcn1Inputs, MeasurementFcn2Inputs, MeasurementFcn3Inputs, Measurement Fcn4Inputs, MeasurementFcn5Inputs - Additional optional input argument to each measurement function
scalar | vector | matrix
Additional optional inputs to the measurement functions other than the state $x$ and measurement noise v. For information about measurement functions see, "State Transition and Measurement Functions" on page 2-103.

MeasurementFcn1Inputs corresponds to the first measurement function that you specify, and so on. For example, suppose that your system has three sensors and nonadditive measurement noise, and the three measurement functions h1, h2, and h3 have the following form:

```
y1[k] = h1(x[k],v[k],MeasurementFcn1Inputs)
y2[k] = h2(x[k],v[k],MeasurementFcn2Inputs)
y3[k] = h3(x[k],v[k])
```

Here k is the time step, and MeasurementFcn1Inputs and MeasurementFcn2Inputs are the additional input arguments to h 1 and h 2 .

If you specify h1, h2, and h3 using MATLAB functions (.m files) in Function, the software generates ports MeasurementFcn1Inputs and MeasurementFcn2Inputs when you click Apply. You can specify the inputs to these ports as scalars, vectors, or matrices.

If your measurement functions have more than one additional input, use Simulink Function blocks to specify the functions. When you use a Simulink Function block, you provide the additional inputs directly to the Simulink Function block using Inport blocks. No input ports are generated for the additional inputs in the Unscented Kalman Filter block.

## Dependencies

A port corresponding to a measurement function $h$ is generated only if both of the following conditions are satisfied:

- You specify $h$ in Function using a MATLAB function, and $h$ is on the MATLAB path.
- $h$ requires only one additional input argument apart from $x$ and $v$.


## Data Types: single | double

## Q - Time-varying process noise covariance

scalar | vector | matrix
Time-varying process noise covariance, specified as a scalar, vector, or matrix depending on the value of the Process noise parameter:

- Process noise is Additive - Specify the covariance as a scalar, an Ns-element vector, or an Ns-by-Ns matrix, where Ns is the number of states of the system. Specify a scalar if there is no crosscorrelation between process noise terms, and all the terms have the same variance. Specify a vector of length $N s$, if there is no cross-correlation between process noise terms, but all the terms have different variances.
- Process noise is Nonadditive - Specify the covariance as a $W$-by- $W$ matrix, where $W$ is the number of process noise terms in the state transition function.


## Dependencies

This port is generated if you specify the process noise covariance as Time-Varying. The port appears when you click Apply.
Data Types: single | double

## R1, R2, R3, R4, R5 - Time-varying measurement noise covariance matrix

Time-varying measurement noise covariances for up to five measurement functions of the system, specified as matrices. The sizes of the matrices depend on the value of the Measurement noise parameter for the corresponding measurement function:

- Measurement noise is Additive - Specify the covariance as an $N$-by- $N$ matrix, where $N$ is the number of measurements of the system.
- Measurement noise is Nonadditive - Specify the covariance as a $V$-by- $V$ matrix, where $V$ is the number of measurement noise terms in the corresponding measurement function.


## Dependencies

A port is generated if you specify the measurement noise covariance as Time-Varying for the corresponding measurement function. The port appears when you click Apply.
Data Types: single|double

## Enable1, Enable2, Enable3, Enable4, Enable5 - Enable correction of estimated states when measured data is available

scalar
Suppose that measured output data is not available at all time points at the port $\mathbf{y} \mathbf{1}$ that corresponds to the first measurement function. Use a signal value other than 0 at the Enable 1 port to enable the correction of estimated states when measured data is available. Specify the port value as 0 when measured data is not available. Similarly, if measured output data is not available at all time points at the port yi for the $i^{\text {th }}$ measurement function, specify the corresponding port Enablei as a value other than 0.

## Dependencies

A port corresponding to a measurement function is generated if you select Add Enable port for that measurement function. The port appears when you click Apply.

## Data Types: single|double | Boolean

## Output

## xhat - Estimated states

vector
Estimated states, returned as a vector of size $N s$, where $N s$ is the number of states of the system. To access the individual states, use the Selector block.

When the Use the current measurements to improve state estimates parameter is selected, the block outputs the corrected state estimate $\widehat{x}[k \mid k]$ at time step k , estimated using measured outputs until time k . If you clear this parameter, the block returns the predicted state estimate $\widehat{x}[k \mid k-1]$ for time $k$, estimated using measured output until a previous time $k-1$. Clear this parameter if your filter is in a feedback loop and there is an algebraic loop in your Simulink model.

Data Types: single | double

## P - State estimation error covariance <br> matrix

State estimation error covariance, returned as an Ns-by-Ns matrix, where Ns is the number of states of the system. To access the individual covariances, use the Selector block.

## Dependencies

This port is generated if you select Output state estimation error covariance in the System Model tab, and click Apply.

Data Types: single | double

## Parameters

## System Model Tab

## State Transition

Function - State transition function name
myStateTransitionFcn (default) | function name
The state transition function calculates the $N s$-element state vector of the system at time step $k+1$, given the state vector at time step $k$. $N s$ is the number of states of the nonlinear system. You create the state transition function and specify the function name in Function. For example, if vdpStateFcn.m is the state transition function that you created and saved, specify Function as vdpStateFcn.

The inputs to the function you create depend on whether you specify the process noise as additive or nonadditive in Process noise.

- Process noise is Additive - The state transition function $f$ specifies how the states evolve as a function of state values at previous time step:
$x(k+1)=f(x(k), U s 1(k), \ldots, U s n(k))$,
where $x(k)$ is the estimated state at time $k$, and Us1, ... Usn are any additional input arguments required by your state transition function, such as system inputs or the sample time. To see an example of a state transition function with additive process noise, type edit vdpStateFcn at the command line.
- Process noise is Nonadditive - The state transition function also specifies how the states evolve as a function of the process noise w :
$x(k+1)=f(x(k), w(k), U s 1(k), \ldots, U s n(k))$.
For more information, see "State Transition and Measurement Functions" on page 2-103.
You can create $f$ using a Simulink Function block or as a MATLAB function (.m file).
- You can use a MATLAB function only if $f$ has one additional input argument Us1 other than $x$ and w.
$x(k+1)=f(x(k), w(k), U s 1(k))$
The software generates an additional input port StateTransitionFenInputs to specify this argument.
- If you are using a Simulink Function block, specify x and w using Argument Inport blocks and the additional inputs Us1, . . . Usn using Inport blocks in the Simulink Function block. You do not provide Us1, . . . Usn to the Unscented Kalman Filter block.


## Programmatic Use <br> Block Parameter: StateTransitionFcn <br> Type: character vector, string <br> Default: 'myStateTransitionFcn' <br> Process noise - Process noise characteristics <br> Additive (default) | Nonadditive

Process noise characteristics, specified as one of the following values:

- Additive - Process noise $w$ is additive, and the state transition function $f$ that you specify in Function has the following form:
$x(k+1)=f(x(k), U s 1(k), \ldots, U s n(k))$,
where $x(k)$ is the estimated state at time $k$, and Us1, ... Usn are any additional input arguments required by your state transition function.
- Nonadditive - Process noise is nonadditive, and the state transition function specifies how the states evolve as a function of the state and process noise at the previous time step:
$x(k+1)=f(x(k), w(k), U s 1(k), \ldots, U s n(k))$.
Programmatic Use
Block Parameter: HasAdditiveProcessNoise
Type: character vector
Values: 'Additive','Nonadditive'
Default: 'Additive'


## Covariance - Time-invariant process noise covariance

1 (default) | scalar | vector | matrix
Time-invariant process noise covariance, specified as a scalar, vector, or matrix depending on the value of the Process noise parameter:

- Process noise is Additive - Specify the covariance as a scalar, an Ns-element vector, or an Ns-by-Ns matrix, where Ns is the number of states of the system. Specify a scalar if there is no crosscorrelation between process noise terms and all the terms have the same variance. Specify a vector of length $N s$, if there is no cross-correlation between process noise terms but all the terms have different variances.
- Process noise is Nonadditive - Specify the covariance as a $W$-by- $W$ matrix, where $W$ is the number of process noise terms.

If the process noise covariance is time-varying, select Time-varying. The block generates input port $\mathbf{Q}$ to specify the time-varying covariance.

## Dependencies

This parameter is enabled if you do not specify the process noise as Time-Varying.

```
Programmatic Use
Block Parameter: ProcessNoise
Type: character vector, string
Default: '1'
Time-varying - Time-varying process noise covariance
'off'(default)|'on'
```

If you select this parameter, the block includes an additional input port $\mathbf{Q}$ to specify the time-varying process noise covariance.

Programmatic Use
Block Parameter: HasTimeVaryingProcessNoise
Type: character vector
Values: 'off', 'on'
Default: 'off'
Initialization
Initial state - Initial state estimate
0 (default) | vector
Initial state estimate value, specified as an $N s$-element vector, where $N s$ is the number of states in the system. Specify the initial state values based on your knowledge of the system.

```
Programmatic Use
Block Parameter: InitialState
Type: character vector, string
Default: ' 0 '
Initial covariance - State estimation error covariance
1 (default) | scalar | vector | matrix
```

State estimation error covariance, specified as a scalar, an Ns-element vector, or an Ns-by-Ns matrix, where $N s$ is the number of states of the system. If you specify a scalar or vector, the software creates an $N s$-by- $N s$ diagonal matrix with the scalar or vector elements on the diagonal.

Specify a high value for the covariance when you do not have confidence in the initial state values that you specify in Initial state.

```
Programmatic Use
Block Parameter: InitialStateCovariance
Type: character vector, string
Default: '1'
Unscented Transformation Parameters
Alpha - Spread of sigma points
1e-3 (default)| scalar value between 0 and 1
```

The unscented Kalman filter algorithm treats the state of the system as a random variable with a mean state value and variance. To compute the state and its statistical properties at the next time step, the algorithm first generates a set of state values distributed around the mean value by using the unscented transformation. These generated state values are called sigma points. The algorithm uses each of the sigma points as an input to the state transition and measurement functions to get a new set of transformed state points and measurements. The transformed points are used to compute the state and state estimation error covariance value at the next time step.

The spread of the sigma points around the mean state value is controlled by two parameters Alpha and Kappa. A third parameter, Beta, impacts the weights of the transformed points during state and measurement covariance calculations:

- Alpha - Determines the spread of the sigma points around the mean state value. Specify as a scalar value between 0 and $1(0<$ Alpha $<=1)$. It is usually a small positive value. The spread of
sigma points is proportional to Alpha. Smaller values correspond to sigma points closer to the mean state.
- Kappa - A second scaling parameter that is typically set to 0 . Smaller values correspond to sigma points closer to the mean state. The spread is proportional to the square-root of Kappa.
- Beta - Incorporates prior knowledge of the distribution of the state. For Gaussian distributions, Beta $=2$ is optimal.

If you know the distribution of state and state covariance, you can adjust these parameters to capture the transformation of higher-order moments of the distribution. The algorithm can track only a single peak in the probability distribution of the state. If there are multiple peaks in the state distribution of your system, you can adjust these parameters so that the sigma points stay around a single peak. For example, choose a small Alpha to generate sigma points close to the mean state value.

For more information, see "Unscented Kalman Filter Algorithm".
Programmatic Use
Block Parameter: Alpha
Type: character vector, string
Default: '1e-3'

## Beta - Characterization of state distribution

2 (default) | scalar value greater than or equal to 0
Characterization of the state distribution that is used to adjust weights of transformed sigma points, specified as a scalar value greater than or equal to 0 . For Gaussian distributions, Beta $=2$ is the optimal choice.

For more information, see the description for Alpha.
Programmatic Use
Block Parameter: Beta
Type: character vector, string
Default: '2'

## Kappa - Spread of sigma points

0 (default) | scalar value between 0 and 3
Spread of sigma points around mean state value, specified as a scalar value between 0 and 3 ( $0<=$ Kappa <= 3). Kappa is typically specified as 0 . Smaller values correspond to sigma points closer to the mean state. The spread is proportional to the square root of Kappa. For more information, see the description for Alpha.

```
Programmatic Use
Block Parameter: Kappa
Type: character vector, string
Default: '0'
Measurement
Function - Measurement function name
myMeasurementFcn (default)| function name
```

The measurement function calculates the $N$-element output measurement vector of the nonlinear system at time step $k$, given the state vector at time step $k$. You create the measurement function and
specify the function name in Function. For example, if vdpMeasurementFcn.m is the measurement function that you created and saved, specify Function as vdpMeasurementFcn.

The inputs to the function you create depend on whether you specify the measurement noise as additive or nonadditive in Measurement noise.

- Measurement noise is Additive - The measurement function $h$ specifies how the measurements evolve as a function of state Values:
$y(k)=h(x(k), \operatorname{Um1}(k), \ldots, \operatorname{Umn}(k))$,
where $\mathrm{y}(\mathrm{k})$ and $\mathrm{x}(\mathrm{k})$ are the estimated output and estimated state at time k , and Um1, ..., Umn are any optional input arguments required by your measurement function. For example, if you are using a sensor for tracking an object, an additional input could be the sensor position.

To see an example of a measurement function with additive process noise, type edit vdpMeasurementFcn at the command line.

- Measurement noise is Nonadditive- The measurement function also specifies how the output measurement evolves as a function of the measurement noise v :
$y(k)=h(x(k), v(k), U m 1(k), \ldots, \operatorname{Umn}(k))$.
To see an example of a measurement function with nonadditive process noise, type edit vdpMeasurementNonAdditiveNoiseFcn.

For more information, see "State Transition and Measurement Functions" on page 2-103.
You can create $h$ using a Simulink Function block or as a MATLAB function (.m file).

- You can use a MATLAB function only if $h$ has one additional input argument Um1 other than $x$ and v.

$$
y[k]=h(x[k], v[k], \operatorname{Um} 1(k))
$$

The software generates an additional input port MeasurementFcnInput to specify this argument.

- If you are using a Simulink Function block, specify $x$ and $v$ using Argument Inport blocks and the additional inputs Um1, . . . Umn using Inport blocks in the Simulink Function block. You do not provide Um1, . . . , Umn to the Unscented Kalman Filter block.

If you have multiple sensors in your system, you can specify multiple measurement functions. You can specify up to five measurement functions using the Add Measurement button. To remove measurement functions, use Remove Measurement.

```
Programmatic Use
Block Parameter: MeasurementFcn1, MeasurementFcn2, MeasurementFcn3,
MeasurementFcn4, MeasurementFcn5
Type: character vector, string
Default: 'myMeasurementFcn'
Measurement noise - Measurement noise characteristics
Additive (default)| Nonadditive
```

Measurement noise characteristics, specified as one of the following values:

- Additive - Measurement noise $v$ is additive, and the measurement function $h$ that you specify in Function has the following form:

```
y(k) = h(x(k),Um1(k),...,Umn(k)),
```

where $y(k)$ and $x(k)$ are the estimated output and estimated state at time $k$, and Um1, ... Umn are any optional input arguments required by your measurement function.

- Nonadditive - Measurement noise is nonadditive, and the measurement function specifies how the output measurement evolves as a function of the state and measurement noise:

$$
y(k)=h(x(k), v(k), U m 1(k), \ldots, \operatorname{Umn}(k)) .
$$

## Programmatic Use

Block Parameter: HasAdditiveMeasurementNoise1, HasAdditiveMeasurementNoise2, HasAdditiveMeasurementNoise3, HasAdditiveMeasurementNoise4,
HasAdditiveMeasurementNoise5
Type: character vector
Values: 'Additive','Nonadditive'
Default: 'Additive'

## Covariance - Time-invariant measurement noise covariance

1 (default) | matrix
Time-invariant measurement noise covariance, specified as a matrix. The size of the matrix depends on the value of the Measurement noise parameter:

- Measurement noise is Additive - Specify the covariance as an $N$-by- $N$ matrix, where $N$ is the number of measurements of the system.
- Measurement noise is Nonadditive - Specify the covariance as a $V$-by- $V$ matrix, where $V$ is the number of measurement noise terms.

If the measurement noise covariance is time-varying, select Time-varying. The block generates input port $\mathbf{R i}$ to specify the time-varying covariance for the $i^{\text {th }}$ measurement function.

## Dependencies

This parameter is enabled if you do not specify the process noise as Time-Varying.

```
Programmatic Use
Block Parameter: MeasurementNoise1, MeasurementNoise2, MeasurementNoise3,
MeasurementNoise4, MeasurementNoise5
Type: character vector, string
Default: '1'
```


## Time-varying - Time-varying measurement noise covariance

``` off (default) | on
```

If you select this parameter for the measurement noise covariance of the first measurement function, the block includes an additional input port R1. You specify the time-varying measurement noise covariance in R1. Similarly, if you select Time-varying for the $i^{\text {th }}$ measurement function, the block includes an additional input port Ri to specify the time-varying measurement noise covariance for that function.

Programmatic Use
Block Parameter: HasTimeVaryingMeasurementNoisel, HasTimeVaryingMeasurementNoise2, HasTimeVaryingMeasurementNoise3, HasTimeVaryingMeasurementNoise4, HasTimeVaryingMeasurementNoise5
Type: character vector
Values: 'off', 'on'
Default: 'off'

Add Enable Port - Enable correction of estimated states only when measured data is available
off (default) | on
Suppose that measured output data is not available at all time points at the port y1 that corresponds to the first measurement function. Select Add Enable port to generate an input port Enable1. Use a signal at this port to enable the correction of estimated states only when measured data is available. Similarly, if measured output data is not available at all time points at the port yi for the $i^{\text {th }}$ measurement function, select the corresponding Add Enable port.

```
Programmatic Use
Block Parameter: HasMeasurementEnablePort1, HasMeasurementEnablePort2,
HasMeasurementEnablePort3, HasMeasurementEnablePort4, HasMeasurementEnablePort5
Type: character vector
Values: 'off', 'on'
Default: 'off'
Settings
Use the current measurements to improve state estimates - Choose between
corrected or predicted state estimate
on (default)| off
```

When this parameter is selected, the block outputs the corrected state estimate $\widehat{x}[k \mid k]$ at time step k , estimated using measured outputs until time $k$. If you clear this parameter, the block returns the predicted state estimate $\widehat{x}[k \mid k-1]$ for time $k$, estimated using measured output until a previous time $\mathrm{k}-1$. Clear this parameter if your filter is in a feedback loop and there is an algebraic loop in your Simulink model.

Programmatic Use
Block Parameter: UseCurrentEstimator
Type: character vector
Values: 'off', 'on'
Default: 'on'
Output state estimation error covariance - Output state estimation error covariance off (default) | on

If you select this parameter, a state estimation error covariance output port $\mathbf{P}$ is generated in the block.

Programmatic Use
Block Parameter: OutputStateCovariance
Type: character vector
Values: 'off','on'
Default: 'off'

## Data type - Data type for block parameters <br> double (default) | single

Use this parameter to specify the data type for all block parameters.
Programmatic Use
Block Parameter: DataType
Type: character vector
Values: 'single', 'double'
Default: 'double'
Sample time - Block sample time
1 (default) | positive scalar
Block sample time, specified as a positive scalar. If the sample times of your state transition and measurement functions are different, select Enable multirate operation in the Multirate tab, and specify the sample times in the Multirate tab instead.

## Dependencies

This parameter is available if in the Multirate tab, the Enable multirate operation parameter is off.

Programmatic Use
Block Parameter: SampleTime
Type: character vector, string
Default: '1'
Multirate Tab
Enable multirate operation - Enable specification of different sample times for state transition and measurement functions
off (default) | on
Select this parameter if the sample times of the state transition and measurement functions are different. You specify the sample times in the Multirate tab, in Sample time.

```
Programmatic Use
Block Parameter: EnableMultirate
Type: character vector
Values: 'off', 'on'
Default: 'off'
Sample times - State transition and measurement function sample times
positive scalar
```

If the sample times for state transition and measurement functions are different, specify Sample time. Specify the sample times for the measurement functions as positive integer multiples of the state transition sample time. The sample times you specify correspond to the following input ports:

- Ports corresponding to state transition function - Additional input to state transition function StateTransitionFcnInputs and time-varying process noise covariance $\mathbf{Q}$. The sample times of these ports must always equal the state transition function sample time, but can differ from the sample time of the measurement functions.
- Ports corresponding to $i^{\text {th }}$ measurement function - Measured output $\mathbf{y i}$, additional input to measurement function MeasurementFcniInputs, enable signal at port Enablei, and time-
varying measurement noise covariance Ri. The sample times of these ports for the same measurement function must always be the same, but can differ from the sample time for the state transition function and other measurement functions.


## Dependencies

This parameter is available if in the Multirate tab, the Enable multirate operation parameter is on.

## Programmatic Use

Block Parameter: StateTransitionFcnSampleTime, MeasurementFcn1SampleTime1, MeasurementFcn1SampleTime2, MeasurementFcn1SampleTime3,
MeasurementFcn1SampleTime4, MeasurementFcn1SampleTime5
Type: character vector, string
Default: ' 1 '

## More About

## State Transition and Measurement Functions

The algorithm computes the state estimates $\widehat{x}$ of the nonlinear system using state transition and measurement functions specified by you. You can specify up to five measurement functions, each corresponding to a sensor in the system. The software lets you specify the noise in these functions as additive or nonadditive.

- Additive Noise Terms - The state transition and measurements equations have the following form:

$$
\begin{aligned}
& x[k+1]=f\left(x[k], u_{s}[k]\right)+w[k] \\
& y[k]=h\left(x[k], u_{m}[k]\right)+v[k]
\end{aligned}
$$

Here $f$ is a nonlinear state transition function that describes the evolution of states $x$ from one time step to the next. The nonlinear measurement function $h$ relates $x$ to the measurements $y$ at time step k. w and v are the zero-mean, uncorrelated process and measurement noises, respectively. These functions can also have additional optional input arguments that are denoted by $u_{s}$ and $u_{m}$ in the equations. For example, the additional arguments could be time step $k$ or the inputs $u$ to the nonlinear system. There can be multiple such arguments.

Note that the noise terms in both equations are additive. That is, $x(k+1)$ is linearly related to the process noise $w(k)$, and $y(k)$ is linearly related to the measurement noise $v(k)$. For additive noise terms, you do not need to specify the noise terms in the state transition and measurement functions. The software adds the terms to the output of the functions.

- Nonadditive Noise Terms - The software also supports more complex state transition and measurement functions where the state $x[k]$ and measurement $y[k]$ are nonlinear functions of the process noise and measurement noise, respectively. When the noise terms are nonadditive, the state transition and measurements equation have the following form:

$$
\begin{aligned}
& x[k+1]=f\left(x[k], w[k], u_{s}[k]\right) \\
& y[k]=h\left(x[k], v[k], u_{m}[k]\right)
\end{aligned}
$$

## Compatibility Considerations

## Numerical Changes

Behavior changed in R2020b
Starting in R2020b, numerical improvements in the Unscented Kalman Filter algorithm might produce results that are different from the results you obtained in previous versions.

## Extended Capabilities

C/C++ Code Generation
Generate C and C++ code using Simulink ${ }^{\circledR}$ Coder $^{\text {TM }}$.
The state transition and measurement functions that you specify must use only the MATLAB commands and Simulink blocks that support code generation. For a list of blocks that support code generation, see "Simulink Built-In Blocks That Support Code Generation" (Simulink Coder). For a list of commands that support code generation, see "Functions and Objects Supported for C/C++ Code Generation" (MATLAB Coder).

Generated code uses an algorithm that is different from the algorithm that the Unscented Kalman Filter block itself uses. You might see some numerical differences in the results obtained using the two methods.

## See Also

## Blocks

Extended Kalman Filter | Kalman Filter | Particle Filter
Functions
extendedKalmanFilter|particleFilter|unscentedKalmanFilter

## Topics

"What Is Online Estimation?"
"Extended and Unscented Kalman Filter Algorithms for Online State Estimation"
"Validate Online State Estimation in Simulink"
"Troubleshoot Online State Estimation"
External Websites
Understanding Kalman Filters: Nonlinear State Estimators - MATLAB Video Series

## Introduced in R2017a


[^0]:    - Specify estimation initialization

